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Statistical description of the proton spin with a large gluon helicity distribution

Claude Bourrely [a], Jacques Soffer [b]

[a] Aix-Marseille Université, Département de Physique, Faculté des Sciences site de Luminy, 13288 Marseille cedex 09, France
[b] Physics Department, Temple University, 1835 N, 12th Street, Philadelphia, PA 19122-6082, USA

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ABSTRACT

The quantum statistical parton distributions approach proposed more than one decade ago is revisited by considering a larger set of recent and accurate Deep Inelastic Scattering (DIS) experimental results. It enables us to improve the description of the data by means of a new determination of the parton distributions. We will see that a large gluon polarization emerges, giving a significant contribution to the proton spin.

1. Introduction

Several years ago a new set of parton distribution functions (PDF) was constructed in the framework of a statistical approach of the nucleon [1]. For quarks (antiquarks), the building blocks are the helicity dependent distributions $q^{\pm}(x)$ ($\bar{q}^{\pm}(x)$). This allows to describe simultaneously the unpolarized distributions $q(x) = q^{+}(x) + q^{-}(x)$ and the helicity distributions $\Delta q(x) = q^{+}(x) - q^{-}(x)$ (similarly for antiquarks). At the initial energy scale, these distributions are given by the sum of two terms, a quasi Fermi–Dirac function and a helicity independent diffractive contribution. The flavor asymmetry for the light sea, *i.e.* $\bar{d}(x) > \bar{u}(x)$, observed in the data is built in. This is simply understood in terms of the Pauli exclusion principle, based on the fact that the proton contains two up-quarks and only one down-quark. The chiral properties of QCD lead to strong relations between $q(x)$ and $\bar{q}(x)$. For example, it is found that the well established result $\Delta u(x) > 0$ implies $\Delta\bar{u}(x) > 0$ and similarly $\Delta d(x) < 0$ leads to $\Delta\bar{d}(x) < 0$. This earlier prediction was confirmed by recent data. In addition we found the approximate equality of the flavor asymmetries, namely $\bar{d}(x) - \bar{u}(x) \sim \Delta\bar{u}(x) - \Delta\bar{d}(x)$. Concerning the gluon, the unpolarized distribution $G(x, Q_0^2)$ is given in terms of a quasi Bose–Einstein function, with only *one free parameter*, and for simplicity, we were assuming zero gluon polarization, *i.e.* $\Delta G(x, Q_0^2) = 0$, at the initial energy scale Q_0^2. As we will see below, the new analysis of a larger set of recent accurate DIS data, has enforced us to give up this assumption. It leads to an unexpected large gluon helicity distribution and this is the major point, which is emphasized in this letter. In our previous analysis all unpolarized and helicity light quark distributions were depending upon *eight free parameters*, which were determined in 2002 (see Ref. [1]), from a next-to-leading (NLO) fit of a small set of accurate DIS data. Concerning the strange quarks and antiquarks distributions, the statistical approach was applied using slightly different expressions, with four additional parameters [2]. Since the first determination of the free parameters, new tests against experimental (unpolarized and polarized) data turned out to be very satisfactory, in particular in hadronic reactions, as reported in Refs. [3–5].

In this letter, after a brief review of the statistical approach, we will only give some elements of the new determination of the parton distributions, but we will focus on the gluon helicity distribution, a fundamental contribution to the proton spin.

2. Basic review on the statistical approach

Let us now recall the main features of the statistical approach for building up the PDF, as oppose to the standard polynomial type parameterizations of the PDF, based on Regge theory at low x and on counting rules at large x. The fermion distributions are given by the sum of two terms, a quasi Fermi–Dirac function and a helicity independent diffractive contribution:

E-mail address: jacques.soffer@gmail.com (J. Soffer).

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$$xq^h(x, Q_0^2) = \frac{A_q X_{0q}^h x^{b_q}}{\exp[(x - X_{0q}^h)/\bar{x}] + 1} + \frac{\tilde{A}_q x^{\tilde{b}_q}}{\exp(x/\bar{x}) + 1}, \quad (1)$$

$$x\bar{q}^h(x, Q_0^2) = \frac{\bar{A}_q (X_{0q}^{-h})^{-1} x^{b_{\bar{q}}}}{\exp[(x + X_{0q}^{-h})/\bar{x}] + 1} + \frac{\tilde{A}_q x^{\tilde{b}_q}}{\exp(x/\bar{x}) + 1}, \quad (2)$$

at the input energy scale $Q_0^2 = 1\ \mathrm{GeV}^2$. We note that the diffractive term is absent in the quark helicity distribution Δq and in the quark valence contribution $q - \bar{q}$.

In Eqs. (1), (2) the multiplicative factors X_{0q}^h and $(X_{0q}^{-h})^{-1}$ in the numerators of the non-diffractive parts of the q's and $\bar{q}$'s distributions, imply a modification of the quantum statistical form, we were led to propose in order to agree with experimental data. The presence of these multiplicative factors was justified in our earlier attempt to generate the transverse momentum dependence (TMD) [6], which was revisited recently [7]. The parameter $\bar{x}$ plays the role of a *universal temperature* and $X_{0q}^{\pm}$ are the two *thermodynamical potentials* of the quark q, with helicity $h = \pm$. Notice the change of sign of the potentials and helicity for the antiquarks.[1]

For a given flavor q the corresponding quark and antiquark distributions involve *eight* free parameters: $X_{0q}^{\pm}$, A_q, $\bar{A}_q$, $\tilde{A}_q$, b_q, $\bar{b}_q$ and $\tilde{b}_q$. It reduces to *seven* since one of them is fixed by the valence sum rule, $\int (q(x) - \bar{q}(x))dx = N_q$, where $N_q = 2, 1, 0$ for u, d, s, respectively. For the light quarks $q = u, d$, the total number of free parameters is reduced to *eight* by taking, as in Ref. [1], $A_u = A_d$, $\bar{A}_u = \bar{A}_d$, $\tilde{A}_u = \tilde{A}_d$, $b_u = b_d$, $\bar{b}_u = \bar{b}_d$ and $\tilde{b}_u = \tilde{b}_d$. For the strange quark and antiquark distributions, the simple choice made in Ref. [1] was improved in Ref. [2], but here they are expressed in terms of *seven* free parameters.

For the gluons we consider the black-body inspired expression

$$xG(x, Q_0^2) = \frac{A_G x^{b_G}}{\exp(x/\bar{x}) - 1}, \quad (3)$$

a quasi Bose–Einstein function, with b_G being the only free parameter, since A_G is determined by the momentum sum rule. In our earlier works [1,4], we were assuming that, at the input energy scale, the polarized gluon, distribution vanishes, so

$$x\Delta G(x, Q_0^2) = 0. \quad (4)$$

However in our recent analysis of a much larger set of very accurate unpolarized and polarized DIS data, we must give up this simplifying assumption. We are now taking

$$\Delta G(x, Q_0^2) = P(x) G(x, Q_0^2), \quad (5)$$

where $P(x)$ is expressed in terms of *four* free parameters, as follows

$$P(x) = \bar{A}_G x^{\bar{b}_G} / (c_G + x^{d_G}). \quad (6)$$

We must have $|P(x)| \leq 1$, to insure that positivity is satisfied.

It is clear that we don't have a serious justification of the functional form of $\Delta G(x)$. However the above expression shows that it is strongly related to $G(x)$, since it is constructed by means of a Bose–Einstein distribution with zero potential. A simpler expression would be $P(x) = Ax^b$, but the additional term x^{d_G} in the denominator is needed in order to get a reasonable fit of the data as we will discuss below.

To summarize the new determination of the PDF involves a total of *twenty one* free parameters: in addition to the temperature $\bar{x}$ and the exponent b_G of the gluon distribution, we have *eight* free parameters for the light quarks (u, d), *seven* free parameters for the strange quarks and *four* free parameters for the gluon helicity distribution. These parameters were determined from a next-to leading order (NLO) fit of a large set of accurate DIS data, (the unpolarized structure functions $F_2^{p,n,d}(x, Q^2)$, the polarized structure functions $g_1^{p,n,d}(x, Q^2)$, the structure function $xF_3^{\nu N}(x, Q^2)$ in νN DIS, etc...) a total of 2140 experimental points with an average χ^2/pt of 1.33. For the polarized structure functions, it is slightly better since for a total of 271 experimental points, we have an average χ^2/pt of 1.21. Although the full details of these new results will be presented in a forthcoming paper [8], we just want to make a general remark. By comparing with the results of 2002 [1], we have observed a remarkable stability of some important parameters, the light quarks potentials $X_{0u}^{\pm}$ and $X_{0d}^{\pm}$, whose numerical values are almost unchanged. The new temperature is slightly lower. As a result the main features of the new light quark and antiquark distributions are only hardly modified, which is not surprising, since our 2002 PDF set has proven to have a rather good predictive power.

We now turn to the gluon helicity distribution which is the main purpose of this letter.

3. The gluon helicity distribution and the proton spin

The gluon helicity distribution $\Delta G(x, Q^2)$ of the proton is a fundamental physical quantity for our understanding of the proton spin. Its integral over x, $\Delta G(Q^2)$, may be interpreted, in the light-cone gauge $A^+ = 0$, as the gluon spin contribution to the proton spin [9]. If $\Delta\Sigma(Q^2)$ denotes the total sum of quark and antiquark helicity contributions and $L_{q,\bar{q},G}(Q^2)$ are the quark, antiquark and gluon orbital angular momentum contributions, the proton helicity sum rule reads

$$1/2 = 1/2\Delta\Sigma(Q^2) + \Delta G(Q^2) + L_q(Q^2) + L_{\bar{q}}(Q^2) + L_G(Q^2). \quad (7)$$

In the above sum rule $\Delta\Sigma(Q^2)$ is certainly the contribution which is best known, with a typical value ~ 0.3 according to Refs. [10, 11] and also from our own result. This contribution is therefore too small to satisfy the sum rule and it is crucial to find out how much the gluon contributes to the proton spin, a long standing problem.

The new determination of the PDF leads, in the gluon sector, to the following parameters:

$$A_G = 36.778 \pm 0.085, \qquad b_G = 1.020 \pm 0.0014,$$
$$\bar{A}_G = 0.731 \pm 0.001, \qquad \bar{b}_G = 5.214 \pm 0.250,$$
$$c_G = 0.006 \pm 0.0005, \qquad d_G = 6.072 \pm 0.35, \quad (8)$$

and the new temperature is $\bar{x} = 0.090 \pm 0.002$. We display in Fig. 1 the gluon helicity distribution versus x at the initial scale $Q_0^2 = 1\ \mathrm{GeV}^2$ and $Q^2 = 10\ \mathrm{GeV}^2$. At the initial scale it is sharply peaked around $x = 0.4$, but this feature lessens after some evolution. We find that $P(x) = 0.731x^{5.210}/(x^{6.072} + 0.006)$, which is such that $0 < P(x) < 1$ for $0 < x < 1$, so positivity is satisfied and in addition the gluon helicity distribution remains positive.

As already mentioned the term x^{d_G} plays an important role. We found that it has a strong effect on the quality of the fit of the polarized structure functions since the chi2 increases substantially when d_G decreases. The chi2 which is 328 for $d_G = 6.072$, becomes 337 for $d_G = 5$, 368 for $d_G = 4.5$ and 465 for $d_G = 4$. Its value also affects the shape of the gluon helicity distribution, which becomes larger towards the smaller x-values, for smaller d_G.

[1] At variance with statistical mechanics where the distributions are expressed in terms of the energy, here one uses x which is clearly the natural variable entering in all the sum rules of the parton model.

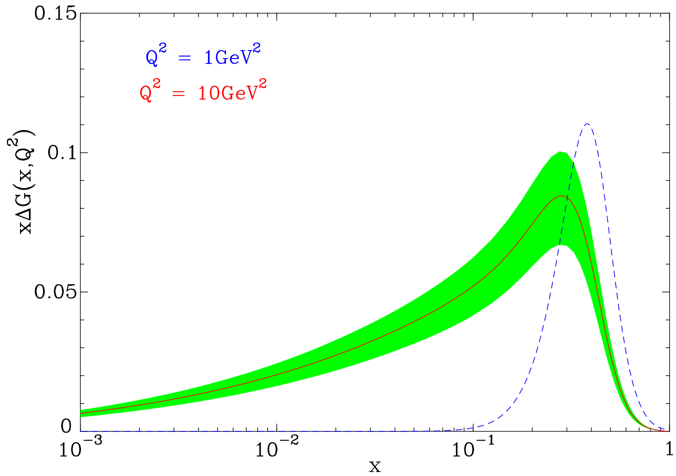

Fig. 1. The gluon helicity distribution $x\Delta G(x, Q^2)$ versus x, for $Q^2 = 1$ GeV^2 (dashed curve) and $Q^2 = 10$ GeV^2 (solid curve), with the corresponding error band.

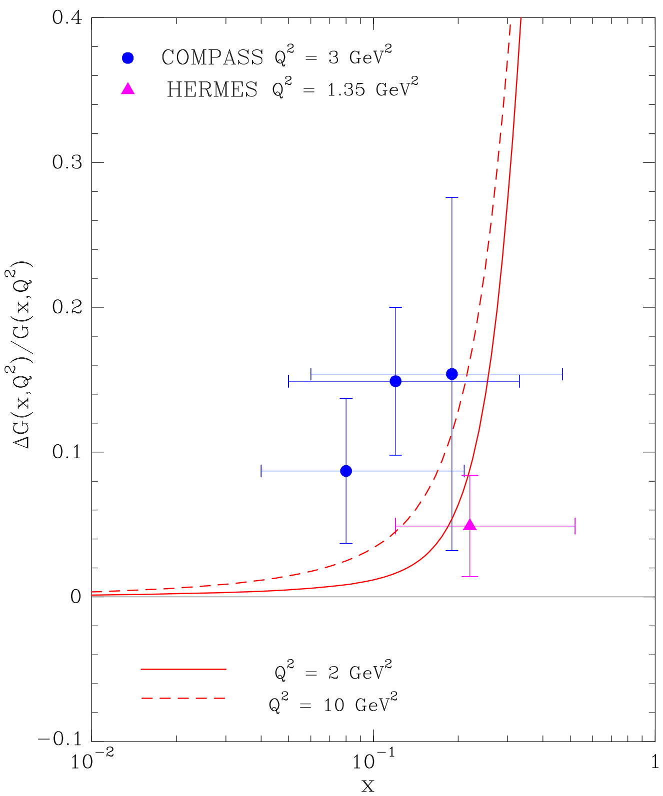

Fig. 2. $\Delta G(x, Q^2)/G(x, Q^2)$ versus x, for $Q^2 = 2$ GeV^2 (solid curve) and $Q^2 = 10$ GeV^2 (dashed curve). The data are from HERMES [12] and COMPASS [13].

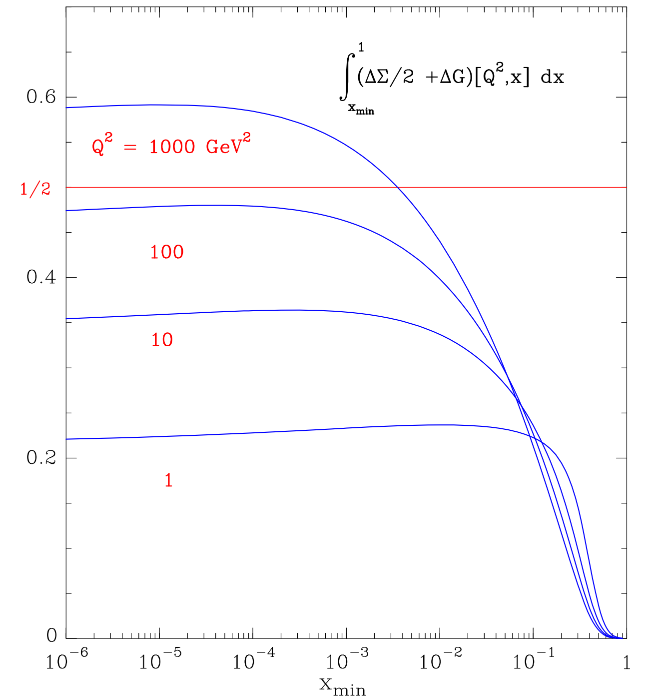

Fig. 3. Total contribution of the quark, antiquark $\Delta\Sigma(x, Q^2)$ and gluon $\Delta G(x, Q^2)$ helicity densities, to the proton helicity sum rule versus the lower limit of the integral $x_{\min}$, for different Q^2 values.

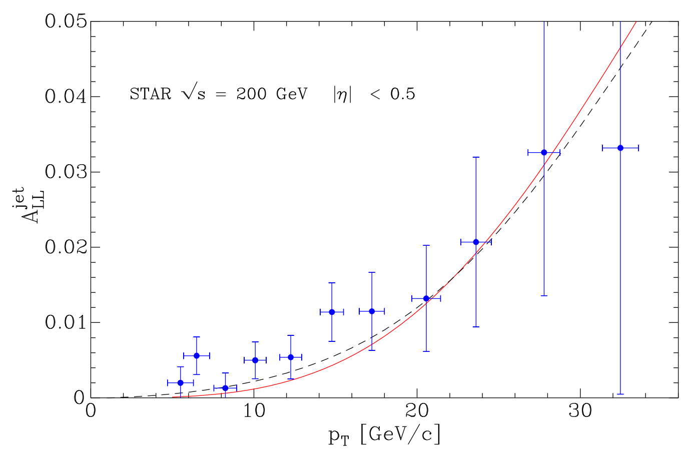

Fig. 4. *Solid curve*: Our predicted double-helicity asymmetry A_{LL}^{jet} for jet production at BNL–RHIC in the near-forward rapidity region, versus p_T and the data points from Ref. [14]. *Dashed curve*: The approximate expression Eq. (9) with $k = 1/7$.

We display $\Delta G(x, Q^2)/G(x, Q^2)$ in Fig. 2 for two Q^2 values and some data points [12,13] suggesting that the gluon helicity distribution is positive indeed. According to the constraints of the counting rules this ratio should go to 1 when $x = 1$, but we observe that this is not the case here, since for example at the initial scale $P(x = 1) = 0.726$. In some other parameterizations in the current literature, this ratio goes to zero, since the large x behavior of $x\Delta G(x)$ is $(1 - x)^\beta$ with $\beta \gg 3$ [10,11]. Clearly one needs a better knowledge of $\Delta G(x, Q^2)/G(x, Q^2)$ for $x > 0.2$.

Let us now examine the consequences of this new gluon helicity distribution, with a rather strong Q^2 dependence, on the proton helicity sum rule Eq. (7). By considering only the quark, antiquark and gluon helicity densities, we display in Fig. 3 their contributions versus the lower limit of the integral $x_{\min}$, for different Q^2 values. For increasing Q^2 they slowly saturate the sum rule, allowing a decreasing positive contribution to the orbital angular momentum, which definitely must change sign for $Q^2 = 1000$ GeV^2.

Finally, we would like to test our new gluon helicity distribution in a pure hadronic reaction. In a very recent paper, the STAR Collaboration at BNL–RHIC has reported the observation, in one-jet inclusive production, of a non-vanishing positive double-helicity asymmetry A_{LL}^{jet} for $5 \le p_T \le 30$ GeV, in the near-forward rapidity region [14]. We show in Fig. 4 our prediction[2] compared with these high-statistics data points

[2] We are grateful to Prof. W. Vogelsang for providing us with the code to make this calculation.

and the agreement is very reasonable. There is a simple way to understand the trend of this double-helicity asymmetry. In this kinematic region, where the jet has a pseudo-rapidity η close to zero and a moderate p_T, the dominant subprocess is $uG \to uG$, so we can write the following approximate expression

$$A_{LL}^{jet} = k\frac{\Delta G(x_T)}{G(x_T)} \cdot \frac{\Delta u(x_T)}{u(x_T)}, \quad (9)$$

where $x_T = 2p_T/\sqrt{s}$ and k is a normalization factor such that $0 \leq k \leq 1$. It exhibits the strong correlation of A_{LL}^{jet} on the sign and magnitude of ΔG and the validity of this approximation is clearly shown in Fig. 4.

This new STAR data has been used recently by the DSSV Collaboration [15] to perform a new global polarized fit which leads them to extract also a rather large positive gluon helicity distribution (see also the recent NNPDF Collaboration paper [16]).

Although we cannot yet firmly claim the discovery of a large positive gluon helicity distribution, giving a significant contribution to the proton spin, these new results are strongly suggesting that we may have reached a benchmark in our knowledge of the nucleon structure. Other independent processes sensitive to $\Delta G(x, Q^2)$ must be investigated and in particular in pp collisions, the di-jet production at forward rapidity is now strongly considered at BNL–RHIC.

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