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## Fine tuned vortices in lattice $SU(2)$ gluodynamics

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### Abstract

We report measurements of the action associated with center vortices in  $SU(2)$  pure lattice gauge theory. In the lattice units the excess of the action on the plaquettes belonging to the vortex is approximately a constant, independent on the lattice spacing  $a$ . Therefore, the action of the center vortex is of order  $A/a^2$ , where  $A$  is its area. Since the area  $A$  is known to scale in the physical units, the measurements imply that the suppression due to the surface action is balanced, or fine tuned to the entropy factor which is to be an exponential of  $A/a^2$ .

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### 1. Monopoles and vortices

Lattice measurements allow for a direct study of field fluctuations in the vacuum state of Yang–Mills theories. Generically, the probability to find a field configuration is a product of the entropy and action factors:

$$P = \exp(\mathfrak{S}) \cdot \exp(-S), \quad (1)$$

where  $S$  is the action and the entropy  $\exp(\mathfrak{S})$  is the number of ways in which the field configuration can be realized. Let us mention two simple examples to illustrate (1). In case of instantons, the (classical) action is  $S_{\text{inst}} = 8\pi^2/g^2$  while the entropy is provided by counting small (quantum) fluctuations in the instanton background. Another example is the zero-point vacuum fluctuations. They are dominated by the phase space, or by the entropy.

These two examples demonstrate relevance to various fluctuations of two distinct scales, that is of the lattice spacing  $a$  and  $\Lambda_{\text{QCD}}^{-1}$ . The latter is in physical units and does not depend on the lattice. The lattice spacing serves as an ultraviolet cutoff. In particular the average action density  $\langle s \rangle$  is ultraviolet divergent,

$$\langle s \rangle \sim (N_c^2 - 1)a^{-4}, \quad (2)$$

where  $N_c$  is the number of colors.<sup>1</sup> The ultraviolet divergence,  $a^{-4}$ , is due to the zero-point fluctuations and is well known in field theory. On the other hand, the quasi-classical fluctuations, like instantons, are driven to the infrared scale of order  $\Lambda_{\text{QCD}}^{-1}$ .

On the lattice, there were also observed fluctuations which are defined as geometrical objects, that is closed worldlines and closed surfaces. We mean now

<sup>1</sup> From now on we will consider only the  $SU(2)$  case,  $N_c = 2$ .

monopoles and center vortices, for review see, e.g., Ref. [1]. What is common for the monopoles and vortices is that they are defined not within the original theory but within a projected theory. In case of monopoles one projects the original  $SU(2)$  to  $U(1)$  theory and in case of the P-vortices—to  $Z(2)$  gauge theory. In more detail, the monopoles and P-vortices are defined as follows.<sup>2</sup> First, one fixes the maximal Abelian gauge by maximizing the functional:

$$R_{\text{Abel}} = \sum_{x,\mu} \text{Tr}[U_{x,\mu} \sigma^3 U_{x,\mu}^+ \sigma^3], \quad (3)$$

where  $U_{x,\mu}$  are link matrices. Next, Abelian projection is made by replacing

$$U_{x,\mu} \rightarrow U_{x,\mu}^{\text{Abel}} = \frac{\zeta_{x,\mu}}{|\zeta_{x,\mu}|},$$

$$\zeta_{x,\mu} = \text{Tr}[(1 + \sigma^3)U_{x,\mu}]. \quad (4)$$

Then the monopoles are defined as the topological defects in  $U_{x,\mu}^{\text{Abel}}$  fields. By construction they form a closed worldlines on the dual lattice.

Since the functional (3) leaves the  $U(1)$  gauge freedom unfixd one can fix the gauge further by maximizing

$$R_{\text{center}} = \sum_{x,\mu} (\text{Re } U_{x,\mu}^{\text{Abel}})^2 \quad (5)$$

with respect to  $U(1)$  gauge rotations. Then the maximal center projection amounts to replacing

$$U_{x,\mu}^{\text{Abel}} \rightarrow Z_{x,\mu} = \text{sign Re } U_{x,\mu}^{\text{Abel}}. \quad (6)$$

Plaquettes constructed in a standard way from  $Z_{x,\mu}$  have values  $\pm 1$ . Finally, P-vortices are defined as union of all the negative plaquettes and are closed surfaces on the dual lattice.

Knowing only definitions of these geometrical objects, lines and surfaces, it is not easy to figure out what kind of physics can be revealed by their studies. However, there emerged phenomenology indicating that there are some physical objects detected through the projections. “Physical” in the present context means first of all that the area, (respectively, length) of the percolating vortices (monopoles) is in physical

units, see, e.g., [2,3]:

$$A_{\text{vort}} = 6\rho_{\text{vort}} \cdot V_4, \quad \rho_{\text{vort}} \approx 4 \text{ (fm)}^{-2}, \quad (7)$$

$$L_{\text{mon}} = 4\rho_{\text{mon}} \cdot V_4, \quad \rho_{\text{mon}} \approx 6 \text{ (fm)}^{-3}, \quad (8)$$

where  $V_4 = a^4 L^4$  is the volume of the lattice.

Note that at this stage we do not have yet any information on the action and entropy factors entering Eq. (1) and one is free to speculate theoretically about them. The common viewpoint is that there are objects of the size of order  $\Lambda_{\text{QCD}}^{-1}$  behind mathematically thin lines [4] or vortices [5] defined in the projected theories. The thin geometrical objects then mark these bulky structures and their position within the “thick” fluctuations is more or less accidental. The only indirect evidence in favor of the physical objects being thin is that both monopoles and vortices generate a linear piece in the heavy quark potential even at short distances [6].

On the other hand, one can try to measure the action and even the entropy associated with the geometrical objects directly. Such measurements have been performed mostly for the monopoles, see [7,8], and the results can be interpreted only as fine tuning [9]. Namely, both the action and the entropy are ultraviolet divergent<sup>3</sup> but cancel each other to order  $\Lambda_{\text{QCD}}$ :

$$|\mathcal{S}_{\text{mon}} - S_{\text{mon}}| = |(\mathfrak{s}_{\text{mon}} - s_{\text{mon}})| \cdot L/a \sim \Lambda_{\text{QCD}} \cdot L, \quad (9)$$

where  $L$  is the length of the monopole trajectory. Moreover the action associated with the monopoles is measured on the lattice (see Ref. [7] for details) and turns out to be ultraviolet divergent. The entropy factor is then easy to calculate,  $\mathfrak{s}_{\text{mon}} = \ln 7$  and this factor corresponds to the number of trajectories of the same length  $L$ .

In this Letter we extend the exploration of the anatomy of the geometrically defined fluctuations to the case of vortices. Namely, we study the action density both on the plaquettes belonging to the vortex and on the adjacent plaquettes as a function of the lattice spacing  $a$ . Our main result is that the excess

<sup>2</sup> Throughout this Letter we consider only the case of so-called indirect maximal center gauge, see, e.g., Ref. [11].

<sup>3</sup> A caveat here is that we are interpreting the measurements on the presently available lattices only and consider appearance of negative powers of the lattice spacing  $a$  as a sign of ultraviolet divergence. The actual limit  $a \rightarrow 0$  can be different if the observed pattern changes at smaller  $a$ .

of the action density on the vortex is independent on the lattice spacing if expressed in the lattice units:

$$\langle S_{\text{vort}} \rangle - \langle S_{\text{vac}} \rangle = 0.540 \pm 0.004 \quad (\text{lattice units}), \quad (10)$$

where  $\langle S_{\text{vort}} \rangle$  is the average value of the non-Abelian action density on the plaquettes belonging to the vortex and  $\langle S_{\text{vac}} \rangle$  is the plaquette action averaged over the whole lattice. Note that the action excess for a particular value of  $\beta = 2.4$  was first measured in Refs. [13,14] and we agree with these results. Our main new point is the measurement of the action of the vortices as function of the lattice spacing  $a$ . We also calculate the excess of the action on the plaquettes which are nearest to the P-vortex worldsheet. It turns out that this quantity is more or less consistent with zero, see next section.

## 2. Numerical results

We have performed our calculations in pure  $SU(2)$  lattice gauge theory for  $2.35 \leq \beta \leq 2.6$ . The lattice spacing  $a$  is fixed using the standard values [10] of the lattice string tension, which in physical units is  $\sqrt{\sigma} = 440$  MeV. At each value of  $\beta$  we have considered 20 statistically independent configurations generated on symmetric  $L^4$  lattices. The lattice size was  $L = 16$  at  $\beta = 2.35$ ,  $L = 24$  for  $\beta = 2.4, 2.45, 2.5$  and  $L = 28$  at  $\beta = 2.55, 2.6$ . The indirect maximal center gauge [11] was employed, and the definition of the gauge is given above.

To fix the maximal Abelian and the maximal center gauges we have used the simulated annealing algorithm [12]. For maximal Abelian gauge 20 gauge copies of each  $SU(2)$  field configuration were considered and the simulated annealing algorithm was applied to each copy. For  $U(1)$  gauge fixing only the configuration which corresponds to the maximal value of the functional (3) was considered. Furthermore, only one gauge copy of the Abelian configuration was taken into account for fixing maximal center gauge, since we checked that P-vortex density varies by less than 1% for various  $U(1)$  gauge copies.

First, we discuss the P-vortex density,  $\rho = \langle N_{\text{PV}} / (6L^4 a^2) \rangle$ , where  $N_{\text{PV}}$  is the number of plaquettes occupied by P-vortices. The dependence of  $\rho$  on the lattice spacing is shown on the Fig. 1. Note that all

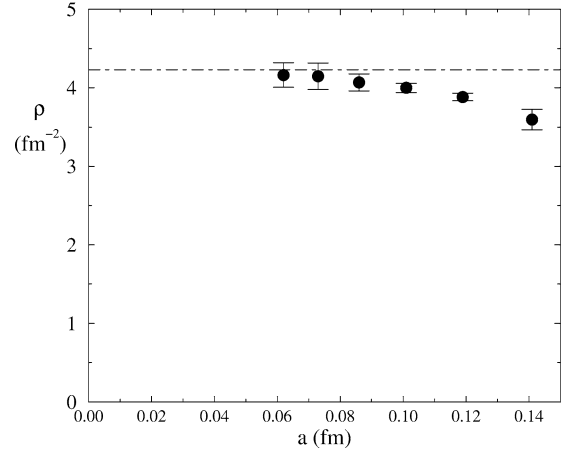


Fig. 1. The density of P-vortices vs. lattice spacing.

quantities are in physical units. It is clearly seen that  $\rho$  tends to the limit (7) as  $a \rightarrow 0$ .

Next we consider the average action density,  $S_{\text{PV}}$ , on the plaquettes dual to those forming P-vortices (we refer to these plaquettes as ‘P-vortex plaquettes’ below). It occurs that it is much larger than the average plaquette action,  $S_{\text{vac}} = \beta(1 - \langle \text{Tr} U_P \rangle / 2)$ . The dependence of the difference  $S_{\text{PV}} - S_{\text{vac}}$  on the lattice spacing is shown on Fig. 2 by circles.

In order to probe the internal structure of the vortices we measured the average action density near P-vortex worldsheet. In more details, we have studied two types of the nearest plaquettes: the first type, ‘side plaquettes’, lie in the same plane as the P-vortex plaquette and have a common link with it; the second type, ‘closest plaquettes’, have a common link with the P-vortex plaquette, but are perpendicular to it. The two types of the plaquettes are depicted in Fig. 3. The corresponding excess of the action is shown on Fig. 2 by the up and down triangles.

Moreover, as first observed in [13,14], the vortices and monopoles are strongly correlated with each other for  $\beta = 2.4$ . We confirm the strong correlation of the monopoles and vortices for other values of  $\beta$ . Moreover, we measure the fraction of  $S_{\text{PV}}$  which is due to the monopoles. We define  $S_{\text{PV}}^{\text{mon}}$  as the average action density on the P-vortex plaquettes which have a common link with a monopole trajectory. It turns out that the quantity  $S_{\text{PV}}^{\text{mon}} - S_{\text{vac}}$  (shown on Fig. 2 by squares) is even larger than  $S_{\text{PV}} - S_{\text{vac}}$  implying that indeed a large fraction of the vortex

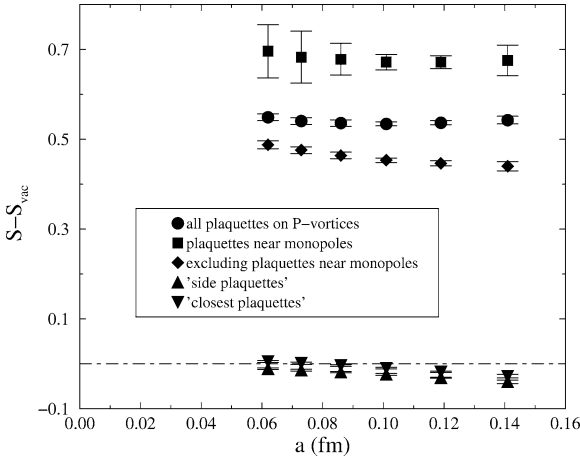


Fig. 2. The excess of the action on and around P-vortices.

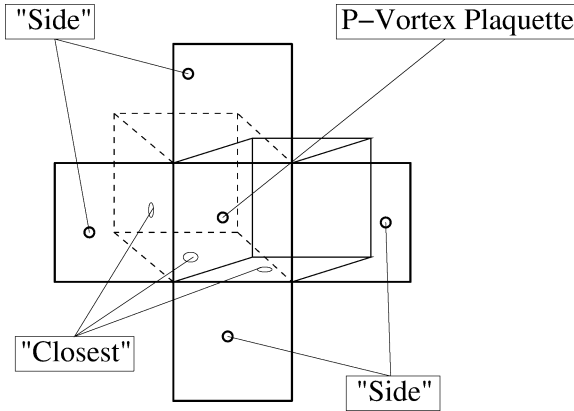


Fig. 3. Two types of the nearest to P-vortex plaquettes (see the text).

action is due to the Abelian monopoles. If we exclude the P-vortex plaquettes which touch the monopole trajectory, the corresponding average action is lower than  $S_{PV}$  (diamonds on the Fig. 2).

### 3. Discussions

We see that at presently available lattices the vortices appear as infinitely thin objects with no sign of any internal structure.<sup>4</sup> Our measurements allow to

conclude that the vortex thickness is

$$R_{\text{vort}} \lesssim 0.06 \text{ fm}, \quad (11)$$

where 0.06 fm is the smallest lattice spacing used in our simulations. Note that a similar estimate of the monopole size was obtained in [7].

Taken at face value, the lattice data imply that we are dealing with infinitely thin (and in this sense “fundamental”) strings which populate the vacuum. Assuming that at the ultraviolet scale the surfaces can be considered independent of the rest of the vacuum, the ultraviolet divergence of the action is to be canceled by a corresponding entropy factor:

$$|\mathcal{S}_{\text{vort}} - S_{\text{vort}}| = |\mathfrak{s}_{\text{vort}} - s_{\text{vort}}| \cdot A/a^2 \sim \Lambda_{\text{QCD}}^2 \cdot A, \quad (12)$$

similar to the case of the monopoles [9].

It is worth emphasizing that we define thickness of the vortex in terms of the distribution of the non-Abelian action. One can define the vortex thickness in terms of the flux carried by the vortex. Then the vortex seems not to be localized to the cutoff scale. The corresponding discussion can be found in Refs. [3, 11].

From the theoretical point of view, interpretation of the results obtained represents a challenge. Indeed, if one introduces random surfaces on the lattice with action proportional to the area they appear unstable with respect to the decay into branched polymers (see, e.g., Ref. [15] for review). In other words, the model of random surfaces collapses in fact to the theory of single non-interacting scalar particle. Note that this remark applies to the random surfaces with limited genus. The genus of the percolating P-vortices, on the other hand, grows with the lattice volume. However, this growth is associated with the distances of the order  $\Lambda_{\text{QCD}}^{-1}$  [16] while the instability mentioned above develops at the ultraviolet scale, that is at the scale of the lattice spacing  $a$ .

The nearest extension of the bosonic string is the inclusion of an extrinsic curvature term [17]. Namely, adding the curvature term one can get a fine tuned surfaces. Moreover, such a model is successful as a phenomenological statistical description of strings in four dimensions<sup>5</sup> [18].

<sup>4</sup> See, however, footnote 3 for a reservation.

<sup>5</sup> Consideration of bosonic strings with extrinsic curvature as a model for P-vortices can be found in Ref. [19].

A key to understanding the structure of the thin vortices could be provided by the observation of the strong correlation between the vortices and monopoles (which are originally defined as independent geometrical objects). In particular, basing on the fact that the monopole-associated plaquettes are “hotter” than the average (see Section 2) one is tempted to assume that the monopoles are associated with the ‘creases’ of the P-vortex worldsheet and correspond to the extrinsic curvature term in the P-vortex action.

To summarize, we have observed surfaces whose thickness is smaller than the presently available resolution,  $a$  and whose area scales in the physical units. Moreover, the thickness is defined in terms of the original non-Abelian action. The coexistence of the two scales, that is  $a$  and  $\Lambda_{\text{QCD}}$  can be called fine tuning. A remarkable feature of the surfaces is that they are associated also with the monopole trajectories. In turn the monopoles condense and in this sense correspond to the tachyonic mode in the field-theoretical language. Therefore, we can say that there are indications that in case of the four-dimensional gluodynamics the tachyonic mode is confined to a two-dimensional surface.

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