Available online at www.sciencedirect.com

ScienceDirect

Energy Procedia 81 (2015) 30 – 39

Energy Procedia

69th Conference of the Italian Thermal Engineering Association, ATI 2014

Optimization of load allocation strategy of a multi-source energy system by means of dynamic programming

Agostino Gambarotta(a), Mirko Morini(a), Nicola Pompini(a) * , Pier Ruggero Spina(b)

[a] *Dipartimento di Ingegneria Industriale, Università degli Studi di Parma, strada Parco Area delle Scienze 181/A, 43124 Parma, Italy*
[b] *Dipartimento di Ingegneria, Università degli Studi di Ferrara, via Saragat 1, 44122 Ferrara, Italy*

Abstract

Multi-source systems for the fulfillment of electric, thermal and cooling demand of a building can be based on different technologies (e.g. solar photovoltaic, solar heating, cogeneration, heat pump, absorption chiller) which use renewable, partially renewable and fossil energy sources. The main issues of these kinds of multi-source systems are (i) the allocation strategy which allows the division of the energy demands among the various technologies and (ii) the proper sizing of each technology.
Furthermore, these two issues proves to be deeply interrelated because, while a wiser energy demand allocation strategy can lead to significant reductions in primary energy consumption, the definition itself of an optimal allocation strategy strongly depends on the actual sizing of the employed technologies. Thus the problem of optimizing the sizing of each technology cannot be separated from the definition of an optimal control strategy. For this purpose a model of a multi-source energy system, previously developed and implemented in the Matlab® environment, has been considered. The model takes account of the load profiles for electricity, heating and cooling for a whole year and the performance of the energy systems are modelled through a systemic approach. A dynamic programming algorithm is therefore employed in order to obtain an optimal control strategy for the energy demand allocation during the winter period. While the resulting control strategy is non-causal and therefore not suitable for the implementation on a real-time application, it allows the definition of a benchmark on the maximum primary energy savings achievable with a specific sizing solution. This result is therefore very helpful both in comparing different solutions and in subsequently define a proper causal control strategy. Finally, the model is applied to the case of a thirteen-floors tower composed of a two-floor shopping mall at ground level and eleven floors used as offices.

Peer-review under responsibility of the Scientific Committee of ATI 2014

* Corresponding author. Tel.: +39-0521-905172.
E-mail address: nicola.pompini@studenti.unipr.it

Peer-review under responsibility of the Scientific Committee of ATI 2014
doi:10.1016/j.egypro.2015.12.056

Keywords: multi-source energy plant; CHP; DP optimization; energy-scheduling

Nomenclature

A	gross area
AB	auxiliary boiler
ASHP	air source heat pump
c	coefficient
CHP	combined heat power
COP	coefficient of Performance
DP	dynamic programming
E	energy
f	energy conversion factor
GSHP	ground source heat pump
k	time variable
load	ratio between actual power and nominal power
P	power
PE	primary energy consumption
PV	photovoltaic
STH	solar heating
STORAGE	storage
t	time
u	input
x	state
XSHP	generic source heat pump
y	output
η	efficiency
π	control policy

Subscripts

available	available space in the storage
demand	demand
diss	dissipation
el	electric
fuel	fuel
in	entering
max	maximum
min	minimum
nom	nominal
out	outgoing
request	to be fulfilled
sent	sent to the grid
startup	start up
taken	taken from the grid
th	thermal
unused	not used for demand fulfilment

1. Introduction

One of the strategies for the reduction of primary energy consumption in buildings is the improvement of the energy efficiency of the energy plant. In recent years, growing attention has been placed on multi-source energy plants to fulfill building energy demands. Many examples can be found in literature. Corrado and Fabrizio [1] studied the combination of wood boiler, condensing boiler, heat pump and solar energy (both thermal and photovoltaic). Sontag and Lange [2] combined cogeneration with solar energy and wind energy. Trillat et al. [3] combined CHP with an absorption chiller and desiccant cooling. Lee et al. [4] studied an integrated renewable system composed of solar heating, solar photovoltaic, ground source heat pumps, electric chillers and gas-fired boilers.

In such a scenario of available energy sources (both fossil and renewable) and technologies (e.g. CHP, wind power, solar photovoltaic, solar thermal collector and heat pumps), it is necessary to define methods and guidelines that help to configure and manage a complex system in order to optimize the exploitation of fossil and renewable sources in terms of environmental impact and economic performance [5,6]. When evaluating the performances of a multi-energy system, the issue of an optimal energy scheduling arises, as the same energy demand could be satisfied by different combinations of the available sources. Since different components have different efficiencies, depending on the operating condition, and given the opportunity to store part of the produced energy for future utilization, it is clear that different control policies could lead to substantial changes to system overall performances.

In this study, the choice between different plant control solutions will be made on the basis of an energy-based criterion. In particular, the minimization of primary energy consumption is the only criterion applied, since the cost of technologies and the actual tariff and incentive scenarios depend on the specific country, and economic considerations may not lead to optimal solutions in terms of primary energy consumption.

For the optimal control policy definition, a Dynamic Programming algorithm (DP), which can provide a non-causal solution to a nonlinear optimization problem, is adopted. Dynamic Programming algorithm, introduced by Bellman in 1957 [7], has been applied to different scientific and engineering problems. The Dynamic Programming algorithm provides an optimal controller only if all future disturbance and reference inputs are known, which implies that the solution is not causal [8]. Nevertheless, this solution is still very useful, as it provides a benchmark to which all other causal controllers could be compared and offers a reference track for the development of simpler heuristic rule-based controllers. For these reasons, this method has been chosen.

Regarding energy system applications, the DP algorithm is widely used in dealings with multi-source energy plants. Marano et al. [9] applied a DP method to the optimal management of a hybrid power plant with wind turbines, photovoltaic panels and compressed air energy storage; Bianchi et al. [10] used DP for managing wind variability with pumped hydro storage and gas turbines and Facci et al. [11] optimized a CHCP system operation strategy by means of DP algorithm.

In this paper, a building case study and a previously-developed model of a multi-source system are briefly introduced. Following this, the procedure for defining the optimal policy for minimizing primary energy consumption with a specific size integrated multi-source energy system is developed. Finally, the results of the optimal control strategy are compared to those of a simpler rule-based heuristic strategy.

2. Load profiles and environmental conditions

The building under consideration is a thirteen-floor tower composed of: (i) the basement, used as warehousing and a garage, (ii) the ground floor and the 1st floor used as a commercial area and (iii) the 2nd ÷12th floors intended for office use. The building construction is expected to be situated in a northern Italian city in climatic zone E (the heating period runs from October 15th to April 15th). The monthly energy demands for heating and hot water have been obtained through the software for stationary simulations EdilClimaEC700® and then distributed over the hours (Δt=1 hour) by means of non-dimensional profiles which take into consideration the type of user. The results are shown in Fig. 1.

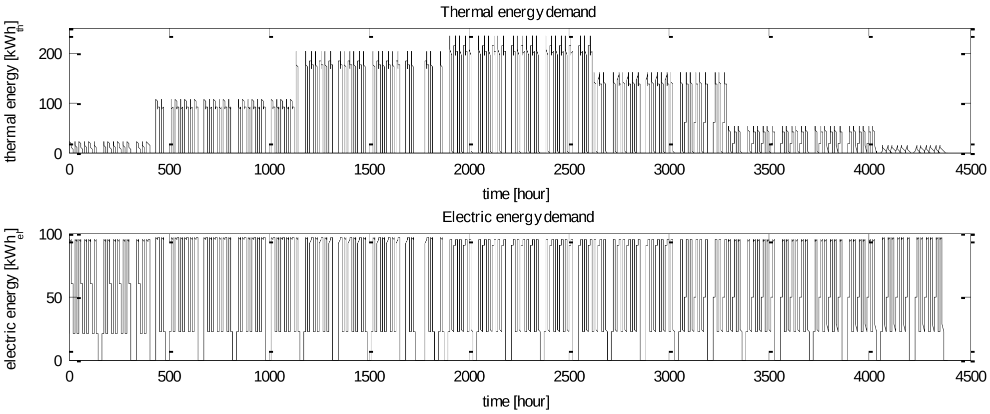

Fig. 1. Thermal and electric load profiles.

3. Energy systems

In order to meet the energy demands (in terms of electric and thermal energy) of the residential building, the following technologies are taken into consideration:

- solar photovoltaic panels (PV) and thermal collectors (STH);
- a cogenerator (CHP), based on a micro gas turbine;
- ground source (GSHP) and air source (ASHP) heat pumps;
- a condensation auxiliary boiler (AB);
- an energy storage.

These technologies are simulated by using a systemic approach, outlined in detail in [12] which makes use of performance parameters (e.g. electric efficiency and overall efficiency for the CHP system, COP for the heat pump, radiant energy conversion efficiency for solar panels, etc.). Performance parameters vary with external conditions and machine loads. The sizes of the various technologies obtained from the procedure outlined in [12] applied to the load profiles are summarized in Table 1.

A discrete time dependent representation of the system (Eq. 1), giving a correlation between system states, outputs and inputs, is adopted for the formulation of the optimization problem. Two states (x) are identified, namely the stored energy and the cogenerator operating condition (on-off). Four inputs for controlling the system (u) are identified, namely the load shares (i.e. representing the fraction of maximum machine energy output) of CHP, GSHP, ASHP and STORAGE. Finally, a strong non-linear dependency between external conditions and system behaviour is outlined by the presence of the k-th index (representing the generic hour of operation) inside the correlation f and g.

$$\begin{cases} x(k+1) = f(x(k), u(k), k) \\ y(k) = g(x(k), u(k), k) \end{cases} \tag{1}$$

In the following, some basic correlations between energy output and system conditions are outlined for every machine for the sake of the formulation of the optimization problem.

3.1. Photovoltaic system (PV)

The total efficiency of the photovoltaic system takes into account the efficiency of the photovoltaic panel (correlated to external conditions) and the inverter efficiency. Through UNI 10349, the monthly average daily values of the solar radiation on the horizontal plane and the total average monthly air temperature for the building location are determined. As there is no device for controlling the outlet power (energy), the latter depends solely on radiant power and electric efficiency, both correlated only with external conditions:

$$E_{\text{el,PV}}(k) = P_{\text{el,PV}}(k) \cdot \Delta t \tag{2}$$

3.2. Solar heating system (STH)

The efficiency of the solar heating system is estimated from technical standards in reference to external conditions. Again, no device is set for controlling outlet thermal power, leading to:

$$E_{\text{th,STH}}(k) = P_{\text{th,STH}}(k) \cdot \Delta t \tag{3}$$

If thermal energy demand is lower than $E_{\text{th,STH}}$, the excess of energy is sent to the energy storage or, if the storage is full it is considered dissipated.

3.3. Cogenerator (CHP)

A micro gas turbine (MGT) is chosen as the CHP technology. The electric efficiency $\eta_{\text{el,CHP}}$ is defined as the ratio between CHP-produced electrical energy and CHP-consumed fuel primary energy, while the thermal efficiency $\eta_{\text{th,CHP}}$ is defined as the ratio between CHP-produced thermal energy and CHP-consumed fuel primary energy. Variation of the external air temperature has an influence on the performance of the CHP. Even the load variation (u_{CHP}) affects the performance of the CHP. A linear variation of performances is assumed for both cases. The minimum CHP thermal load permitted is assumed equal to 80 %. A penalty for CHP start-up ($E_{\text{CHP,start up}}$) is added, corresponding to the fuel consumed in 10 minutes at nominal power. A binary state (x_{CHP}, [0 1]), describing the CHP condition at the start of the *k*-th hour (on-off), is introduced for the evaluation of a start-up event. The considerations above lead to equations:

$$E_{\text{th,CHP}}(k) = u_{\text{CHP}} \cdot P_{\text{th,CHP,max}}(k) \cdot \Delta t \tag{4}$$

$$E_{\text{el,CHP}}(k) = \eta_{\text{el,CHP}}(u_{\text{CHP}}, k) \cdot \frac{E_{\text{th,CHP}}(u_{\text{CHP}}, k)}{\eta_{\text{th,CHP}}(u_{\text{CHP}}, k)} \tag{5}$$

Table 1. Sizes of employed technologies.

Technology	Parameter	Unit of measurement	Value
PV	A	[m^2]	325
STH	A	[m^2]	3
CHP	$P_{\text{el.CHP,nom}}$	[kW$_{\text{el}}$]	100.0
	$\eta_{\text{el,CHP,nom}}$	[-]	0.29
	$P_{\text{th,CHP,nom}}$	[kW$_{\text{th}}$]	195.1
	$\eta_{\text{th,CHP,nom}}$	[-]	0.57
GSHP	$P_{\text{th,GSHP,nom}}$	[kW$_{\text{th}}$]	40.0
	$\text{COP}_{\text{GSHP,nom}}$	[-]	3.35
ASHP	$P_{\text{th,ASHP,nom}}$	[kW$_{\text{th}}$]	100.0
	$\text{COP}_{\text{ASHP,nom}}$	[-]	2.80
AB	$P_{\text{th,AB,nom}}$	[kW$_{\text{th}}$]	233.7
	$\eta_{\text{th,AB,nom}}$	[-]	1.06
STORAGE	E_{max}	[kWh$_{\text{th}}$]	393.3

$$E_{\mathrm{fuel,CHP}}(k)=\frac{E_{\mathrm{th,CHP}}(u_{\mathrm{CHP}},k)}{\eta_{\mathrm{th,CHP}}(u_{\mathrm{CHP}},k)}+E_{\mathrm{CHP,startup}}(x_{\mathrm{CHP}},u_{\mathrm{CHP}},k) \tag{6}$$

$$E_{\mathrm{CHP,startup}}(k)=\eta_{\mathrm{th,CHP,nom}}(k)\cdot P_{\mathrm{th,CHP,nom}}(k)\cdot\frac{\Delta t}{6}; \quad if\ x_{\mathrm{CHP}}=0\ and\ u_{\mathrm{CHP}}\neq 0 \tag{7}$$

The state is updated following the rule:

$$x_{\mathrm{CHP}}(k+1)=\begin{cases}1 & if\ u_{\mathrm{CHP}}(k)\neq 0\\ 0 & if\ u_{\mathrm{CHP}}(k)=0\end{cases} \tag{8}$$

As with STH, any excess energy produced is sent to the storage or, if the storage is full, dissipated.

3.4. Heat pumps (GSHP and ASHP)

Two different types of reversible heat pump are modelled: a ground source heat pump (GSHP) and an air source heat pump (ASHP). Both are defined by the nominal thermal power. Nominal performances of heat pumps are related to the temperature of the external heat exchanger (i.e. assumed equal to the hourly calculated ambient temperature for the ASHP and equal to the annual average of the ambient temperature for the GSHP) and to the temperature of the heat exchanger inside the building. For both heat pumps, it is assumed that load variation affects the performance of the heat pump according to UNI 11300. For the generic source heat pump (XSHP), the following correlations are applied:

$$E_{\mathrm{th,XSHP}}(k)=u_{\mathrm{XSHP}}\cdot P_{\mathrm{th,XSHP,max}}(k)\cdot\Delta t \tag{9}$$

$$E_{\mathrm{el,XSHP}}(k)=\frac{E_{\mathrm{th,XSHP}}(k)}{COP_{\mathrm{XSHP}}(k)} \tag{10}$$

In this case, any excess energy produced is dissipated, i.e. no storage for heat pumps is available.

3.5. Heat storage

Both STH and CHP are coupled with a hot water storage tank with an appropriate volume (e.g. for the CHP storage equivalent hours [13] are equal to 2 kWh/kW). The stored energy corresponds to the second state (x_{STORAGE}). At each hour, part of the stored energy can be employed to fulfil thermal energy demand:

$$E_{\mathrm{th,STORAGE,out}}(k)=u_{\mathrm{STORAGE}}\cdot x_{\mathrm{STORAGE}}(k) \tag{11}$$

On the other hand, the storage can be refilled by the excess energy produced by STH or CHP, given that the maximum energy capacity is not exceeded.

$$E_{\mathrm{th,STORAGE,in}}(k)=\min\left(E_{\mathrm{th,STORAGE,available}},\,E_{\mathrm{th,STH,unused}}(k)+E_{\mathrm{th,CHP,unused}}(k)\right) \tag{12}$$

Dissipation is included in the heat storage model, corresponding to a loss of a constant share of the actual stored energy, in the hypothesis that the heat dissipation is proportional to the heat exchange area and therefore to the quantity of stored fluid (energy). The state is updated as follows:

$$x_{\mathrm{STORAGE}}(k+1)=\left(1-c_{\mathrm{diss}}\right)\cdot\left(x_{\mathrm{STORAGE}}(k)+E_{\mathrm{th,STORAGE,in}}(k)-E_{\mathrm{th,STORAGE,out}}(k)\right) \tag{13}$$

where the dissipation coefficient (c_{diss}) is assumed equal to 0.5 % in accordance with values found in literature [14].

3.6. Auxiliary boiler

An auxiliary condensing boiler (AB) powered by natural gas with the aim of producing the heating energy not fulfilled by the other systems is considered. A linear variation of performances is assumed in correspondence to load variation of the auxiliary boiler. The correlation between primary fuel energy and thermal power can be expressed through:

$$E_{\text{fuel, AB}}(k) = \eta_{\text{th,AB}}(k) \cdot E_{\text{th,AB}}(k) \tag{14}$$

The share of heat demand covered by the auxiliary boiler corresponds to the residual demand not fulfilled by the other systems.

3.7. Energy balance

Of all the possible combinations of input vectors, only those which guarantee the balance between produced and employed energy are considered feasible. For this reason, two equations are added as a boundary to the problem, namely the conservation of thermal and electric energy. From the conservation of thermal energy comes the energy required from the auxiliary boiler:

$$\begin{aligned} E_{\text{th,AB,request}}(k) = E_{\text{th,demand}}(k) - (E_{\text{th,STH}}(k) + E_{\text{th,CHP}}(u,k) + E_{\text{th,ASHP}}(u,k) + \\ + E_{\text{th,GSHP}}(u,k) + E_{\text{th,STORAGE,out}}) \end{aligned} \tag{15}$$

If $E_{\text{th,AB,request}} < 0$ there is an excess of energy production which is stored if possible and dissipated otherwise; if $E_{\text{th,AB,request}} > E_{\text{th,AB,max}}$ the energy requested from the boiler exceeds its maximum and the combination of states and input is infeasible.

From electric energy conservation the energy required from the electricity grid is calculated as follows:

$$E_{\text{el,request}}(k) = E_{\text{el,demand}}(k) - E_{\text{el,PV}}(k) - E_{\text{el,CHP}}(u,k) + E_{\text{el,ASHP}}(u,k) + E_{\text{el,GSHP}}(u,k) \tag{16}$$

If $E_{\text{el,request}} > 0$ the electricity is withdrawn from the grid, otherwise it is sent.

4. Optimal policy evaluation

The object of the optimization is the energy scheduling, i.e. the share of the total energy demand (both electrical and thermal) that has to be fulfilled by the single machine at every working hour. The energy scheduling can be represented in generic terms as a control policy $\pi=\{\boldsymbol{u}_0,\boldsymbol{u}_1, \cdots ,\boldsymbol{u}_{N-1}\}$, where $\boldsymbol{u}_k=[u_{\text{CHP}}(k),\ u_{\text{GSHP}}(k),\ u_{\text{ASHP}}(k),\ u_{\text{STORAGE}}(k)]$. The objective function to be minimized is identified in the overall prime energy consumption (PE), defined as:

$$PE_{\pi}(x_0) = \sum_{k=0}^{N-1}\left(E_{\text{fuel,CHP}}(x,u,k) + E_{\text{fuel,AB}}(x,u,k) + f_{\text{el,taken}} \cdot E_{\text{el,taken}}(x,u,k) - f_{\text{el,sent}} \cdot E_{\text{el,sent}}(x,u,k)\right) \tag{17}$$

The optimal control policy π^* corresponds to:

$$\pi^* = \arg\min_{\pi\in\Pi} PE_{\pi}(x_0) \tag{18}$$

The evaluation of the optimal control policy is carried out by means of a DP algorithm which exploits the fact that, given an optimal control policy π^*, the truncated policy $\{\boldsymbol{u}_i,\ \boldsymbol{u}_{i+1},\cdots,\boldsymbol{u}_{N-1}\}$ is still optimal for the “tail sub-problem”. The principle of optimality suggests that an optimal policy can therefore be constructed in a piecemeal fashion, first constructing an optimal policy for the "tail sub-problem" involving the last stage, then extending the policy to the "tail sub-problem" involving the last two stages, and continuing in this manner backwards until an optimal policy for the entire problem is constructed [7]. The implementation of the DP algorithm is done by means of a Matlab® function developed by [15] which requires a formulation of the problem in a discrete state-space representation as in Eq. 1.

For the definition of the problem, the algorithm requires the definition of the constraints and a discretization of each state and input.

The state of the cogenerator (x_{CHP}) is already discrete [0 1], while the energy of the storage is discretized in ten equally spaced values in the range [0 $E_{th,STORAGE,max}$)]. Inputs are also discretized in ten values, nine of which are equally spaced inside the range [$load_{min}$ $load_{max}$] and the tenth corresponding to no action taken (load=0). Initial states are set so that the cogenerator is off and the storage is empty, while no constraint on final states is set.

5. Results

The results obtained from DP are compared to those obtained following a heuristic strategy presented in [12]. This strategy is based on the following switch-on priority for the machines:

0. Storage;
1. Solar heating collectors, which can also feed the storage;
2. CHP, which can also feed the storage;
3. Ground source heat pump;
4. Air source heat pump;
5. Auxiliary boiler.

The storage operates by accumulating thermal energy when heat production is higher than demand, and in each hour it is the first system that contributes to the heat demand fulfilment. The switch-on logic of the systems is outlined as follows. The *i*-th machine switches on if the energy demand, which is the actual user demand for i = {0, 3, 4, 5} and, if necessary, the user demand increased by the empty space in the storage, is higher than or equal to its energy production at the minimum load. If the energy demand is higher than that at maximum load, the energy production of *i*-th machine is equal to the maximum energy production, and the residual energy demand is given as input for the next machine.

In Fig. 2, the evolution of primary energy consumption over the whole winter period is represented for both cases (heuristic and DP). The adoption of an optimized control strategy leads to a reduction of 8.4 %. It is interesting to note that the reduction rate is nearly constant, suggesting that it could be correlated not to isolated decisions but to a different handling of energy production. In fact, from Figs 3 and 4, it can be inferred that one of the key aspects for a better energy scheduling lies in the interaction between energy storage and CHP. It can be seen that the control policy coming from DP manages to keep the CHP production to the maximum (Fig. 4), storing any eventual excess. This stored energy is then employed only twice or three times a day (Fig. 3), when it is enough to fulfil the whole energy demand. This policy has the effect of avoiding both the operating of CHP at part load and the unnecessary start-up of the component. Figure 3 also shows that, with DP optimal policy, both the auxiliary boiler (AB) and the ground source heat pump (GSHP) are employed, contrary to what happens using heuristic rules, where both components are unused. It can be inferred that, as before, in order to avoid disadvantageous CHP operating conditions (low loads or frequent start-up), if the stored energy is not sufficient to fulfil the whole energy demand, the adoption of alternative energy sources should be pursued.

Another key aspect of the optimal policy can be inferred from Fig. 5. In fact, it can be noticed that, following the proposed heuristic rules, the energy kept in storage is always considerably higher than in the optimal solution. As the thermal dissipation is supposed proportional to the amount of the stored energy, it follows that energy losses are considerably lower following DP solution (7.0 MWh in the rule-based case versus 2.9 MWh in DP case).

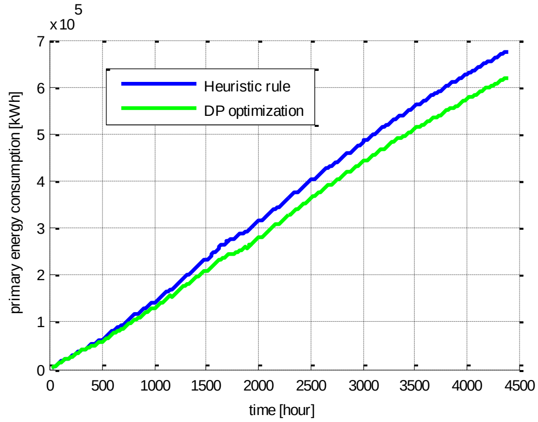

Fig. 2. Primary energy consumption.

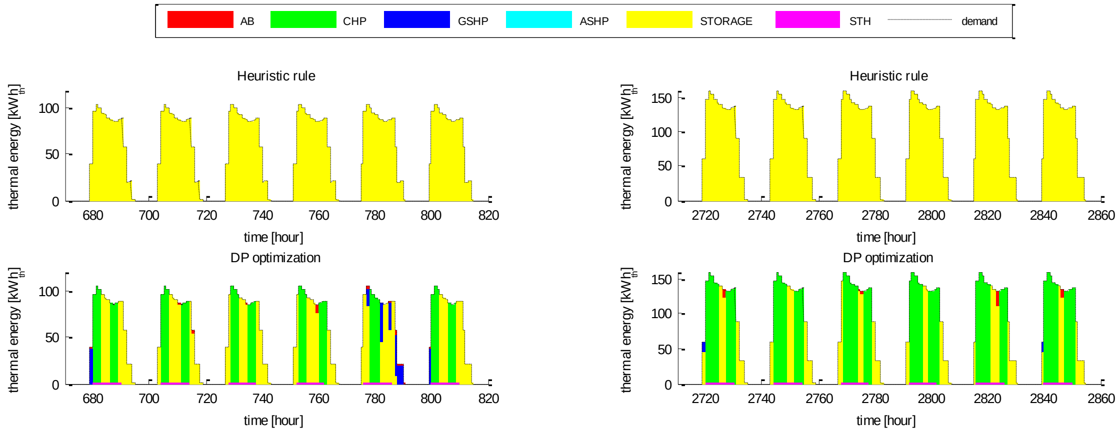

Fig. 3. Highlights of thermal energy demand fulfillment for two typical weeks (November and February).

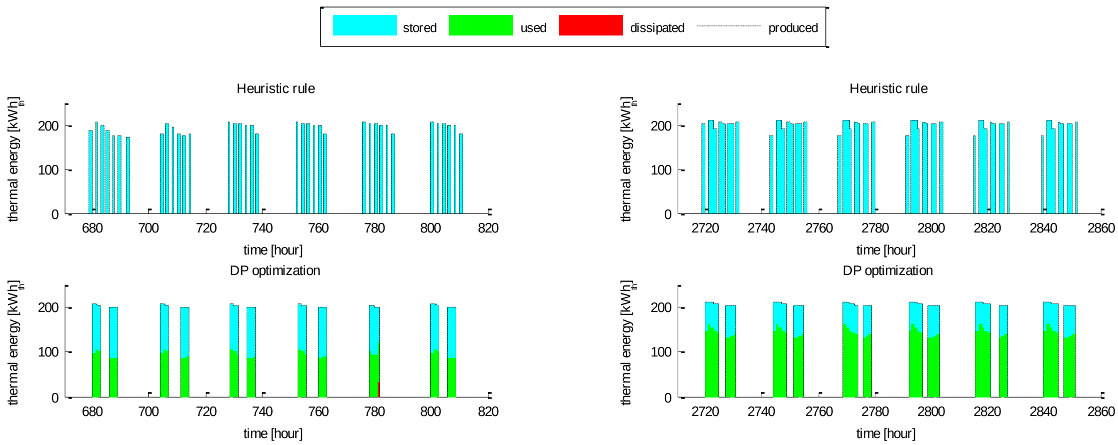

Fig. 4. Highlights of CHP thermal energy production for two typical weeks (November and February).

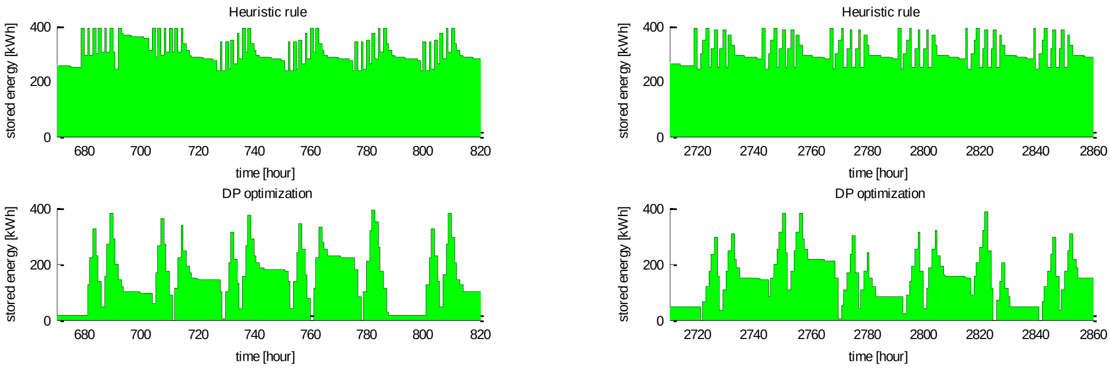

Fig. 5. Highlights of stored energy for two typical weeks (November and February).

6. Conclusions

In this paper a dynamic programming algorithm has been applied to a model of a multi-source energy system in order to obtain an optimal control strategy for the energy demand allocation during the heating period. The model has been applied to the case of a thirteen-floor tower composed of a two-floor shopping mall and eleven floors used as offices. The optimized control strategy showed a reduction of 8.4 % in primary energy consumptions with respect to the basic strategy. This is achieved mainly (i) by keeping the CHP at full load (i.e. avoiding efficiency reduction due to load modulation) and (ii) by reducing the stored energy and, therefore, its dissipation.

References

[1] Fabrizio E. *et al.*, "A model to design and optimize multi-energy systems in buildings at the design concept stage", *Renewable Energy*, Vol. 35 (2010), pp. 644-65.

[2] Sontag R., Lange A., "Cost effectiveness of decentralized energy supply system taking solar and wind utilization plants into account", *Renewable Energy*, Vol. 28 (2003), pp. 1865-1880.

[3] Trillat-Berdal V. *et al.*, "Experimental study of ground-coupled heat pump combined with thermal solar collectors", *Energy and Buildings*, Vol. 38 (2006), pp. 1477-1484.

[4] Lee K.H. *et al.*, "Preliminary determination of optimal size for renewable energy resources in building using RETScreen", *Energy*, Vol. 47 (2012), pp. 83-96.

[5] Baños R. *et al.*, "Optimization methods applied to renewable and sustainable energy: A review", *Renewable and Sustainable Energy Reviews*, Vol. 15 (2011), pp. 1753-1766.

[6] Bazmi A.A., Zahedi G., "Sustainable energy systems: Role of optimization modeling techniques in power generation and supply-A review", *Renewable and Sustainable Energy Reviews*, Vol. 15 (2011), pp. 3480–3500.

[7] Bellman R.E., *Dynamic Programming*, Princeton University Press, NJ, 1957.

[8] Bertsekas D., *Dynamic Programming and optimal control*, 3rd ed. Belmont, Massachusetts; Athena Scientific, 2005.

[9] Marano V. *et al.*, "Application of dynamic programming to the optimal management of a hybrid power plant with wind turbines, photovoltaic panels and compressed air energy storage", *Applied Energy*, Vol. 97 (2012), pp. 849–859.

[10] Bianchi M. *et al.*, "Managing wind variability with pumped hydro storage and gas turbines", *Energy Procedia*, Vol. 45 (2014), pp. 22-31.

[11] Facci A.L. *et al.*, "Optimization of CHCP (combined heat power and cooling) systems operation strategy using dynamic programming", *Energy*, Vol. 66 (2014), pp. 387–400.

[12] Barbieri E.S. *et al.*, "Optimal Sizing of a Multi-Source Energy Plant for Power Heat and Cooling Generation", *Applied Thermal Engineering* (article in press) 10.1016/j.applthermaleng.2013.11.022 (2013).

[13] Barbieri E.S. *et al.*, "Influence of the thermal energy storage on the profitability of micro-CHP system for residential building applications", *Applied Energy*, Vol. 97 (2012), pp. 714-722.

[14] Steen D. *et al.*, "Modeling of thermal storage systems in MILP distributed energy resource models", *Applied Energy* (article in press) 10.1016/j.apenergy.2014.07.036 (2014).

[15] Sundstrom O., Guzzella L., "A Generic Dynamic Programming Matlab Function", In Proceedings of the 18th IEEE International Conference on Control Applications, pp. 1625-1630, Saint Petersburg, Russia, 2009.