Computational hydromagnetic mixed convective heat and mass transfer through a porous medium in a non-uniformly heated vertical channel with heat sources and dissipation

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A B S T R A C T

In this paper we discuss the combined effect of convective Heat and mass transfer on hydromagnetic electrically conducting viscous, incompressible fluid through a porous medium in a vertical channel bounded by flat walls. A uniform magnetic field of strength $H_0$ is applied transverse to the bounding walls. Assuming the magnetic Reynolds number to be small, we neglect the induced magnetic field in comparison to the applied field. A non-uniform temperature is imposed on the walls and the concentration on these walls is taken to be constant. The viscous dissipation is taken into account in the energy equation. Assuming the slope of the boundary temperature to be small we solve the governing momentum, energy and diffusion equations by a perturbation technique. The velocity, temperature, shear stress and the rate of heat transfer have been analysed for different variations of the governing parameters. The dissipative effects on the flow, heat and mass transfer are clearly seen.

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1. Introduction

Flows which arise due to the interaction of the gravitational force and density differences caused by the simultaneous diffusion of thermal energy and chemical species have many applications in geophysics and engineering. Such thermal and mass diffusion plays a dominant role in a number of technological and engineering systems. Obviously an understanding of this transport process is desirable in order to effectively control the overall transport characteristics. The combined effect of thermal and mass diffusion in channel flows has been studied in recent times [1].

This problem of combined buoyancy driven thermal and mass diffusion has been studied in parallel plate geometries by a few authors in recent times, Lai [2], Poulikakos [1], Mehta and Nanda Kumar [3] and Angirasa et al. [4]. The Volumetric heat generation has been assumed to be constant [5,6].

On the other hand, Barletta [7] has pointed out that relevant effect of viscous dissipation on the temperature profiles and on the Nusselt numbers may occur in the fully developed forced convection in tubes. In view of this, several authors, notably Soudalgekar and Pop [8], Barletta [7,9], El-hakeing [10], Yesilata [11], Schio [12] and Israel et al. [13] have studied the effect of viscous dissipation on the convective flows past infinite vertical plates and through vertical channels and Ducts. The effect of viscous dissipation on natural convection has been studied for some different cases, including the natural convection from a horizontal cylinder.

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The effect of viscous dissipation has been studied by Nakayama and Pop [14] for a steady free convection boundary layer over non-isothermal bodies of an arbitrary shape embedded in a porous media. They used an integral method to show that the viscous dissipation results in lowering the level of the heat transfer rate from the body.

2. Formulation of the problem

We analyse the steady motion of viscous, electrically conducting incompressible fluid through a porous medium in a vertical channel bounded by flat walls which are maintained at a non-uniform wall temperature in the presence of a constant heat source, and the concentration on these walls are taken to be constant. A uniform magnetic field of strength \( H_0 \) is applied transverse to the walls. The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. The viscous, Darcy dissipations and the Joule heating are taken into account in the energy equation. Also the kinematic viscosity \( \nu \) and the thermal conductivity \( k \) are treated as constants. We choose a rectangular Cartesian system \((x, y)\) with \( x\)-axis in the vertical direction and the \( y\)-axis normal to the walls. The walls of the channel are at \( y = \pm L \). The equations governing the steady hydromagnetic flow, heat and mass transfer are in the equilibrium state

\[
0 = -\frac{\partial p_e}{\partial x} - \rho g \quad \text{(2.1)}
\]

where \( p = p_e + p_D \), \( p_0 \) being the hydrodynamic pressure.

The flow is maintained by a constant volume flux for which a characteristic velocity is defined as

\[
Q = \frac{1}{2L} \int_{-L}^{L} ud y. \quad \text{(2.2)}
\]

The boundary conditions for the velocity and temperature fields are

\[
u = 0, \quad v = 0 \quad \text{on} \quad y = \pm L \quad \text{(2.3)}
\]

\[
T - T_e = \gamma \left( \frac{\delta x}{L} \right) \quad \text{on} \quad y = \pm L \quad \text{(2.4)}
\]

\[
C = C_1 \quad \text{on} \quad y = -L \quad \text{(2.5)}
\]

\[
C = C_2 \quad \text{on} \quad y = +L. \quad \text{(2.6)}
\]

\( \gamma \) is chosen to be twice differentiable function, \( \delta \) is a small parameter characterizing the slope of the temperature variation on the boundary.

In view of the continuity equation we define the stream function \( \psi \) as

\[
u = -\psi_y, \quad v = \psi_x, \quad \text{(2.7)}
\]

The equation governing the flow in terms of \( \psi \) are

\[
\left( \frac{\partial \psi}{\partial x} \frac{\partial (\nabla^2 \psi)}{\partial y} - \frac{\partial \psi}{\partial y} \frac{\partial (\nabla^2 \psi)}{\partial x} \right) = \nu \left( \left( \frac{\partial^2 \psi}{\partial x^2} \right) + \left( \frac{\partial^2 \psi}{\partial y^2} \right) \right) - \beta g \frac{\partial T}{\partial y} - \beta^* \frac{\partial C}{\partial y} - M^2 \frac{\partial^2 \psi}{\partial y^2}
\]

\[
- \left( \frac{\nu}{k} \right) \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) \quad \text{(2.8)}
\]

\[
\rho_e C_p \left( \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial y} \right) = \lambda \nabla^2 \theta + Q + \mu \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 + \left( \frac{\partial^2 \psi}{\partial x^2} \right)^2 + \left( \frac{\partial^2 \psi}{\partial y \partial x} \right)^2 + \left( \frac{\partial \psi}{\partial y} \right)^2 \quad \text{(2.9)}
\]

\[
\left( \frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y} \right) = D_1 \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right). \quad \text{(2.10)}
\]

Introducing the non-dimensional variables in (2.8)-(2.10) as

\[
(x', y') = \left( \frac{x}{L}, \frac{y}{L} \right), \quad (u', v') = \left( \frac{u}{U}, \frac{v}{U} \right), \quad \theta = \frac{T - T_e}{\Delta T_e}, \quad C' = \frac{C - C_1}{C_2 - C_1}, \quad p' = \frac{p_0}{\rho_e U^2}, \quad y' = \frac{y}{\Delta T_e} \quad \text{(2.11)}
\]

(under the equilibrium state \( \Delta T_e = T_e(L) - T_e(-L) = \frac{\Theta L^2}{k} \))

the governing equations in the non-dimensional form (after dropping the dashes) are

\[
R \frac{\partial (\psi, \nabla^2 \psi)}{\partial (x, y)} = \nabla^4 \psi + \left( \frac{G}{R} (\beta_2 + N \alpha y) \right) - D^{-1} \nabla^2 \psi - M^2 \frac{\partial^2 \psi}{\partial y^2} \quad \text{(2.12)}
\]
and the energy diffusion equations in the non-dimensional form are

\[
PR \left( \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right) = \nabla^2 \theta + 1 + \left( \frac{PR^2 E_c}{G} \right) \left( \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 + \left( \frac{\partial^2 \psi}{\partial x^2} + D^{-1} \left( \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} \right)^2 \right) \right)
\]

(2.13)

\[
R \frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y} = \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right)
\]

(2.14)

where

\[
R = \frac{UL}{\nu} \text{ (Reynolds number)}
\]

\[
G = \frac{\beta g L^3}{\nu} \text{ (Grashof number)}
\]

\[
P = \frac{\mu c_p k}{\nu} \text{ (Prandtl number)}
\]

\[
D^{-1} = \frac{L}{\kappa} \text{ (Darcy parameter)}
\]

\[
E_c = \frac{\beta g L^3}{C_p} \text{ (Eckert number)}
\]

\[
M^2 = \frac{\sigma \mu^2 e^2 H^2}{\rho_0 L^2} \text{ (Hartmann number)}
\]

\[
N = \frac{\beta^* \Delta C \beta^* \Delta T}{\rho} \text{ (Buoyancy number)}
\]

The corresponding boundary conditions are

\[
\psi(+1) - \psi(-1) = 1 \tag{2.15}
\]

\[
\frac{\partial \psi}{\partial x} = 0, \quad \frac{\partial \psi}{\partial y} = 0 \quad \text{at } y = \pm 1 \tag{2.16}
\]

\[
\theta(x, y) = f(\delta x) \quad \text{on } y = \pm 1 \tag{2.17}
\]

\[
C = 0 \quad \text{on } y = -1 \tag{2.18}
\]

\[
C = 1 \quad \text{on } y = 1 \tag{2.19}
\]

\[
\frac{\partial \theta}{\partial y} = 0, \quad \frac{\partial C}{\partial y} = 0 \quad \text{at } y = 0. \tag{2.20}
\]

The value of \( \psi \) on the boundary assumes the constant volumetric flow is consistent with the hypothesis (2.1). Also the wall temperature varies in the axial direction in accordance with the prescribed arbitrary function \( \gamma(x) \).

3. Analysis of the flow

The main aim of the analysis is to discuss the perturbations created over a combined free and forced convection flow due to a non-uniform slowly varying temperature being imposed on the boundaries. We introduce the transformation \( \bar{x} = \delta x \).

With this transformation the Eqs. (2.12)–(2.14) reduce to

\[
R \delta \frac{\partial (\psi, \bar{F}^2 \psi)}{\partial (x, y)} = F^4 \psi + \frac{G}{R} \left( \theta_y + NC_y \right) - D^{-1} F^2 \psi - M^2 \frac{\partial^2 \psi}{\partial y^2} \tag{3.1}
\]

and the energy & diffusion equations in the non-dimensional form are

\[
PR \left( \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right) = F^2 \theta + 1 + \left( \frac{PR^2 E_c}{G} \right) \left( \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 + \left( \frac{\partial^2 \psi}{\partial x^2} + D^{-1} \left( \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} \right)^2 \right) \right) \]

\[
+ \left( D^{-1} + M^2 \right) \left( \delta^2 \left( \frac{\partial \psi}{\partial x} \right)^2 + \left( \frac{\partial \psi}{\partial y} \right)^2 \right) \tag{3.2}
\]

\[
\delta R \frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y} = F^2 C \tag{3.3}
\]

where

\[
F^2 = \delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}
\]

for small values of the slope \( \delta \), the flow develops slowly with an axial gradient of order \( \delta \) and hence we take \( \frac{\partial^2}{\partial x^2} \approx O(1) \).

We follow the perturbation scheme and analyse through the first order as a regular perturbation problem at finite values of \( R, G, P, Sc \) and \( D^{-1} \).
Introducing the asymptotic expansions

\[ \psi(x, y) = \psi_0(x, y) + \delta \psi_1(x, y) + \delta^2 \psi_2(x, y) + \cdots \]

\[ \theta(x, y) = \theta_0(x, y) + \delta \theta_1(x, y) + \delta^2 \theta_2(x, y) + \cdots \]  
(3.4)

\[ C(x, y) = C_0(x, y) + \delta C_1(x, y) + \delta^2 C_2(x, y) + \cdots . \]

On substituting (3.4) in (3.1)–(3.3) and separating the like powers of \( \delta \), the equations and respective conditions to the zeroth order are

\[ \psi_{0,yy} - M_1^2 \psi_{0,yy} = -\frac{G}{R} (\theta_{0,y} + NC_{0,y}) \]  
(3.5)

\[ \psi_{0,yy} = -\frac{PR^2 Ec}{G} \psi_{0,yy} - \frac{PM_1^2 Ec}{G} \psi_{0,y} \]  
(3.6)

\[ C_{0,yy} = 0 \]  
(3.7)

with

\[ \psi_0(+1) - \psi(-1) = 1, \]  
(3.8)

\[ \psi_{0,y} = 0, \psi_{0,x} = 0 \text{ at } y = \pm 1 \]  
(3.9)

\[ \theta_0(\pm 1) = \gamma(x) \text{ at } y = \pm 1 \]  
(3.10)

\[ C_0(-1) = 0, \quad C_0(+1) = 1 \]  
(3.11)

and to the first order are

\[ \psi_{1,yy} - M_1^2 \psi_{1,yy} = -\frac{G}{R} (\theta_{1,y} + NC_{1,y}) + R \left( \psi_{0,y} \psi_{0,yy} - \psi_{0,x} \psi_{0,yy} \right) \]  
(3.12)

\[ \theta_{1,y} = PR \left( \psi_{0,x} \theta_{0,y} - \psi_{0,y} \theta_{0,x} \right) - \frac{PR Ec}{G} \left( R^2 \psi_{1,yy}^2 + M_1^2 \psi_{1,y}^2 \right) \]  
(3.13)

\[ C_{1,yy} = RSc \left( \psi_{0,y} C_{0,x} - \psi_{0,x} C_{0,y} \right) \]  
(3.14)

\[ \psi_{1(+1)} - \psi_{1(-1)} = 0 \]  
(3.15)

\[ \psi_{1,y} = 0, \quad \psi_{1,x} = 0 \text{ at } y = \pm 1 \]  
(3.16)

\[ \theta_{1}(\pm 1) = 0 \text{ at } y = \pm 1 \]  
(3.17)

\[ C_{0}(-1) = 0, \quad C_{0}(+1) = 0. \]  
(3.18)

Assuming \( Ec \ll 1 \) to be small we take the asymptotic expansions as

\[ \psi_0(x, y) = \psi_{00}(x, y) + Ec \psi_{01}(x, y) + \cdots \]

\[ \psi_1(x, y) = \psi_{10}(x, y) + Ec \psi_{11}(x, y) + \cdots \]

\[ \theta_0(x, y) = \theta_{00}(x, y) + Ec \theta_{01}(x, y) + \cdots \]

\[ \theta_1(x, y) = \theta_{10}(x, y) + Ec \theta_{11}(x, y) + \cdots \]  
(3.19)

\[ C_0(x, y) = C_{00}(x, y) + Ec C_{01}(x, y) + \cdots \]

\[ C_1(x, y) = C_{10}(x, y) + Ec C_{11}(x, y) + \cdots . \]

Substituting the expansions (3.19) in Eqs. (3.5)–(3.13) and (3.15)–(3.18) and separating the like powers of \( Ec \) we get the following equations

\[ \theta_{00,yy} = -1, \quad \theta_{00}(\pm 1) = f(\pm 1) \]  
(3.20)

\[ C_{00,yy} = 0, \quad C_{00}(-1) = 0, \quad C_{00}(+1) = 1 \]  
(3.21)

\[ \psi_{00,yy} - M_1^2 \psi_{00,yy} = -\frac{G}{R} (\theta_{00,y} + NC_{00,y}) \]  
(3.22)

\[ \psi_{00}(+1) - \psi_{00}(-1) = 1, \quad \psi_{00,y} = 0, \quad \psi_{00,x} = 0 \text{ at } y = \pm 1 \]  
(3.23)

\[ \theta_{01,yy} = -\frac{PM_1^2}{G} \psi_{01,yy}^2 - \frac{PR^2}{G} \psi_{00,yy}^2, \quad \theta_{01}(\pm 1) = 0 \]  
(3.24)

\[ C_{01,yy} = 0, \quad C_{01}(\pm 1) = 0 \]  
(3.25)

\[ \psi_{01,yy} - M_1^2 \psi_{01,yy} = -\frac{G}{R} (\theta_{01,y} + NC_{01,y}) \]  
(3.26)
4. Shear stress and Nusselt number

The shear stress on the channel walls is given by
\[
\tau = \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)_{y=\pm 1}
\]
which in the non- dimensional form reduces to
\[
\tau = \left( \frac{\tau}{\mu u} \right) = (\psi_{yy} - \delta^2 \psi_{xx})
\]
\[
= [\psi_{00,yy} + Ec \psi_{01,yy} + \delta \psi_{10,yy} + Ec \psi_{11,yy} + O(\delta^2)]_{y=\pm 1}
\]
and the corresponding expressions are
\[
(\tau)_{y=+1} = d_3 + Ec d_4 + \delta d_5 + O(\delta^2)
\]
\[
(\tau)_{y=-1} = d_6 + Ec d_7 + \delta d_8 + O(\delta^2).
\]
The local rate of heat transfer coefficient (Nusselt number Nu) on the walls has been calculated using the formula
\[
Nu = \frac{1}{\theta_m - \theta_w} \left( \frac{\partial \theta}{\partial y} \right)_{y=\pm 1}
\]
and the corresponding expressions are
\[
(Nu)_{y=+1} = \frac{(d_{10} + \delta(d_{11} + d_{12}))}{(d_6 - \gamma(x) + \delta d_9)}
\]
\[
(Nu)_{y=-1} = \frac{(-d_{10} + \delta(d_{12} - d_{11}))}{(d_8 - \gamma(x) + \delta d_6)}
\]
where \(d_3-d_{12}\) are constants.

5. Discussion of the numerical results

The aim of the analysis is to discuss the effect of dissipation on the convective heat and mass transfer through a porous medium confined in a vertical channel, bounded by vertical channel on whose walls a non uniform temperature is maintained and we take the Prandtl number \(P = 0.71\). The non-linear coupled governing the flow have been solved by using the perturbation technique. The effect of different parameters governing the flow, heat and mass transfer on the velocity, temperature distributions is analysed and exhibited in Figs. 1–5. When the molecular buoyancy force dominates over the thermal buoyancy force, the region of reversal flow enlarges when the buoyancy forces act in the same direction, while no such reversal flow appears when the buoyancy forces (Fig. 1).

It is found that in a given porous medium, reversal flow occurs in the region abutting the left boundary and the size of the region increases marginally with an increase in \(\alpha\) also \(|u|\) reduces marginally with \(\alpha\) (Fig. 2).

The variation of \(v\) with buoyancy ratio \(N\) shows that when the buoyancy forces act in the same direction, the transverse velocity in the left region is directional towards the midregion while the reversal effect is noticed when they act in opposite
Fig. 1. \( u \) with \( N, G = 3 \times 10^3, D^{-1} = 2 \times 10^3, \alpha = 1 \).

Fig. 2. \( u \) with \( \alpha \).

directions. Also \(|v|\) increase with \( N \) when they act in the same direction and enhances with \(|N|\) when they act in opposite directions (Fig. 3).

An increase in the amplitude of the non-uniform boundary temperature enhances \(|v|\) in the left region and reduces it in the right region (Fig. 4).
The variation of $\theta$ with $N$ shows that when the molecular buoyancy force dominates over the thermal buoyancy force $\theta$ enhances in the right region and reduces in the left region when the buoyancy forces are in the same directions. When the forces act in opposite directions we find a reversed effect in $\theta$ (Fig. 5).

The shear stress and the rate of heat transfer at $y = -1$ have been evaluated for different variations in the governing parameters $G$, $N$, and $\alpha$, are exhibited in Tables 1 and 3.
Table 1
Shear stress ($\tau$) at $y = -1$, $P = 0.71$.

<table>
<thead>
<tr>
<th>$G \times 10^3$</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>5.6024</td>
<td>9.5721</td>
<td>4.4206</td>
<td>-4.9631</td>
</tr>
<tr>
<td>3</td>
<td>37.2518</td>
<td>69.5978</td>
<td>023.5043</td>
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<td>184.144</td>
<td>-56.2323</td>
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</tr>
<tr>
<td>-1</td>
<td>1.2248</td>
<td>4.0676</td>
<td>1.0189</td>
<td>-0.4858</td>
</tr>
<tr>
<td>-3</td>
<td>24.1191</td>
<td>53.0845</td>
<td>-7.1857</td>
<td>-17.8419</td>
</tr>
<tr>
<td>-5</td>
<td>74.2854</td>
<td>156.6227</td>
<td>-29.0346</td>
<td>-57.0315</td>
</tr>
</tbody>
</table>

Table 2
Average Nusselt number ($Nu$) at $y = -1$, $P = 0.71$.

<table>
<thead>
<tr>
<th>$G \times 10^3$</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.1168</td>
<td>0.0628</td>
<td>-0.1552</td>
<td>-0.1414</td>
</tr>
<tr>
<td>3</td>
<td>0.0147</td>
<td>0.0059</td>
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<td>0.0012</td>
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</tr>
<tr>
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<td>0.1973</td>
<td>0.7497</td>
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<tr>
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<tr>
<td>-5</td>
<td>0.0123</td>
<td>0.0072</td>
<td>-0.0195</td>
<td>-0.0087</td>
</tr>
</tbody>
</table>

Table 3
Shear stress ($\tau$) at $y = -1$, $P = 0.71$.

<table>
<thead>
<tr>
<th>$G \times 10^3$</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
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<td>10</td>
<td>6.5821</td>
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<td>2.4023</td>
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<td>57.9311</td>
<td>31.8577</td>
<td>-5.6009</td>
</tr>
</tbody>
</table>

We observe that the stress on $y = 1$ is positive for $N > 0$ and negative for $N < 0$. When the molecular buoyancy force dominates over the thermal buoyancy force the magnitude of $\tau$ enhances when the buoyancy forces either act in the same
Table 4
Average Nusselt number (Nu) at y = −1, P = 0.71.

<table>
<thead>
<tr>
<th>G \ τ</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^3</td>
<td>0.1168</td>
<td>0.1145</td>
<td>0.1114</td>
<td>0.1051</td>
</tr>
<tr>
<td>3 × 10^3</td>
<td>0.0147</td>
<td>0.0132</td>
<td>0.0112</td>
<td>0.0073</td>
</tr>
<tr>
<td>5 × 10^3</td>
<td>0.0047</td>
<td>0.0038</td>
<td>0.0023</td>
<td>−0.0012</td>
</tr>
<tr>
<td>−10^3</td>
<td>0.6711</td>
<td>0.7245</td>
<td>0.8162</td>
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</tr>
<tr>
<td>−3 × 10^3</td>
<td>0.0343</td>
<td>0.0366</td>
<td>0.0403</td>
<td>0.0463</td>
</tr>
<tr>
<td>−5 × 10^3</td>
<td>0.0123</td>
<td>0.0136</td>
<td>0.0154</td>
<td>0.0186</td>
</tr>
</tbody>
</table>

direction or in opposite directions (Table 1). It is found that Nu decreases in magnitude with an increase in |N| (> 0). Irrespective of the directions of the two buoyancy forces (Table 2).

The variation of stress with the amplitude α of the boundary temperature indicates that for α ≤ 1.0, τ is positive and for higher |α| ≥ 1.5, |τ| is negative. |τ| decreases with an increase in the amplitude α (Table 3).

The average Nusselt number which measures the rate of heat transfer at y = −1 is exhibited in Tables 2 and 4 for variations in G, N and α.

The rate of heat transfer experiences a reduction with an increase in the amplitude α of the boundary temperature (Table 4).

References