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On de Sitter vacua in type IIA orientifold compactifications

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Abstract

This Letter discusses the orientifold projection of the quantum corrections to type IIA strings compactified on rigid Calabi–Yau threefolds. It is shown that $N = 2$ membrane instanton effects give a holomorphic contribution to the superpotential, while the perturbative corrections enter into the Kähler potential. At the level of the scalar potential the corrections to the Kähler potential give rise to a positive energy contribution similar to adding anti-D3-branes in the KKLT scenario. This provides a natural mechanism to lift an AdS vacuum to a meta-stable dS vacuum.

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1. Introduction

Solving the problem of moduli stabilization is of primary importance for deriving (semi-)realistic vacua from string theory compactifications. Building on the work of KKLT [1], subsequent investigations, mostly in the context of orientifold compactifications of the type IIB string and its F-theory dual [2], but also for the type IIA string [3–7], have given evidence that suitable combinations of background fluxes and non-perturbative effects like instantons are capable of fixing all string moduli in an AdS vacuum. Astronomical observations, however, indicate that our observable universe has a small positive cosmological constant [8], suggesting that one should search for string theory vacua with such a property, i.e., dS vacua. In the context of the type IIB string, KKLT outlined the construction of such vacua by first using background fluxes and non-perturbative effects to stabilize all moduli in an AdS vacuum. Subsequently, a positive energy contribution in form of a $\overline{\text{D3}}$-brane was added to lift the AdS vacuum to a meta-stable dS vacuum. After stabilizing all moduli except for the volume modulus σ of the compactification their potential reads [1]:

$$V_{\text{KKLT}} = \frac{aA\mathrm{e}^{-a\sigma}}{2\sigma^2}\left(W_0 + \frac{a}{3}\sigma A\mathrm{e}^{-a\sigma} + A\mathrm{e}^{-a\sigma}\right) + \frac{D}{\sigma^3}. \tag{1}$$

Here the terms in brackets originate from fluxes, instanton corrections and gluino condensation, while the last term proportional to D corresponds to the positive energy contribution which was added by hand. The first term of the potential (1) vanishes when switching off the non-perturbative effects (setting $A = 0$). In this case the type IIB potential (without $\overline{\text{D3}}$-brane contribution) is independent of the volume modulus, which is the known no-scale structure of IIB orientifold compactifications. Stabilizing all (geometric) moduli in this framework then requires the inclusion of non-perturbative effects as discussed above. Furthermore, for $D > 0$ the parameters a, A, W_0 can be chosen such that V_{KKLT} has a meta-stable dS vacuum.

In this Letter we focus on orientifold projections of type IIA string theory compactified on a rigid Calabi–Yau threefold (CY_3). Here it is possible to stabilize all (geometric) moduli in an AdS vacuum using classical fluxes only [5,6]. To our knowledge, the uplift of these vacua to meta-stable dS vacua has not

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yet been performed; the aim of this Letter is to derive a mechanism for such an uplift.

Starting from the perturbative and membrane instanton corrections to the universal hypermultiplet arising from compactifications of the type IIA string on a rigid CY_3 [9], we perform the orientifold projection along the lines of [10]. The result agrees with the standard folklore that the Kähler potential (describing the projected hypermultiplet sector) receives perturbative corrections, while the leading instanton corrections yield a holomorphic contribution to the $N=1$ superpotential. Secondly, we study the scalar potential arising from a space–time filling 4-form flux together with the leading instanton contributions. It will turn out that, at the level of the scalar potential, the perturbative corrections to the Kähler potential give rise to a positive energy contribution, analogous to adding an $\overline{\mathrm{D3}}$-brane in the potential (1). Therefore, these corrections provide a natural mechanism which can lift an AdS vacuum to a meta-stable dS vacuum.[1]

2. The $N=1$ orientifold model

In the following we will consider $N=1$ supergravity coupled to a single chiral multiplet. Our model describes a subsector of an orientifold projection of type IIA strings on a rigid CY_3. Before the projection, the $N=2$ supergravity effective action comprises the universal hypermultiplet (containing the dilaton) and $h_{1,1}$ vector multiplets. The combined scalar target space is the product of a (four-dimensional) quaternion-Kähler (QK) space and a special Kähler manifold parametrized by the complexified Kähler moduli.

After projecting to $N=1$, the QK space becomes a (two-dimensional) Kähler space parametrized by a single chiral multiplet containing the dilaton ϕ and RR scalar v. Classically, they define the complex structure

$$z = 2\mathrm{e}^{\phi/2} + \mathrm{i}v. \tag{2}$$

In our conventions the asymptotic value of the dilaton is related to the string coupling constant by $\mathrm{e}^{-\phi_\infty/2} = g_s$. The special Kähler manifold remains special Kähler, but of lower dimension [10]. Classically, the total scalar manifold retains the product structure inherited from the $N=2$ theory. Consistency of the orientifold projection requires that the tadpole for the RR 7-form potential, arising from the presence of O6-planes, is suitably canceled. This can be done by adding D6-branes [5]. In the following, we consider the sector of the universal hypermultiplet only, assuming that the Kähler moduli have already been stabilized (presumably by 4-form fluxes F_4 [5]).

The potential for the chiral scalar z is then given by

$$V = \mathrm{e}^{\mathcal{K}}\big[\mathcal{K}^{z\bar{z}} D_z W \bar{D}_{\bar{z}} \bar{W} - 3|W|^2\big], \tag{3}$$

where $\mathcal{K}(z,\bar{z})$ is the Kähler potential, $\mathcal{K}^{z\bar{z}} = (\partial_z\partial_{\bar{z}}\mathcal{K})^{-1}$ the inverse Kähler metric, $W(z)$ the holomorphic superpotential, and $D_z W = \partial_z W + W\partial_z\mathcal{K}$. We consider a Kähler potential of the form

$$\mathcal{K} = -2\ln\big[(z+\bar{z})^2 - 16c\big]. \tag{4}$$

Here, the constant $c \in \mathbb{R}$ describes a loop correction, since it is suppressed by powers of g_s. Furthermore, we consider the superpotential

$$W = 8W_0 + \bar{A}\mathrm{e}^{-z}, \tag{5}$$

with two constant parameters W_0 and A. We can assume without loss of generality that W_0 is real, while A is taken to be complex. The constant term in W arises from background fluxes, while the exponential term can be understood as a membrane instanton correction.

We now investigate the properties of the scalar potential $V(z,\bar{z})$ in the presence of a non-vanishing c. It turns out to be convenient to express the potential in terms of

$$z = 2\sqrt{r+c} + \mathrm{i}v, \tag{6}$$

where we denote $r = \mathrm{e}^{\phi}$ and require $r + c \geqslant 0$. As we show below, this is the one-loop corrected complex structure inherited from the $N=2$ theory.[2] We can then compute the potential and find

$$V = \frac{W_0^2}{4r^2} + \frac{W_0}{8r^{3/2}}\big(\bar{A}\mathrm{e}^{-z} + A\mathrm{e}^{-\bar{z}}\big) - \frac{cW_0^2}{r^2(r+2c)} + \cdots, \tag{7}$$

where r should be understood as a function of z and $\bar{z}$ via the relation (6). We have included only the leading instanton contribution to V; subleading terms proportional to $\mathrm{e}^{-(z+\bar{z})}$, which belong to the 2-instanton sector not taken into account in the superpotential (5), have been dropped for consistency. Moreover, we have expanded the r-dependent prefactor of the 1-instanton term for large $r > -c$ and kept only the leading power.

The last term in V is induced by the constant c appearing in the Kähler potential (4). Comparing to (1), we find that for $c < 0$ this term has the same effect as adding a $\overline{\mathrm{D3}}$-brane in the proposal of KKLT. Including this modification therefore provides a way of realizing the positive energy contribution required for lifting an AdS vacuum to a meta-stable dS vacuum. Indeed, we find that for $c < 0$ the potential (7) has a range of values of A for which z can be stabilized in a meta-stable de Sitter vacuum.[3] This follows from the observation that, as we show in the next section, the potential precisely corresponds to the one analyzed in [9], where details and figures showing this dS vacuum can be found.

3. Rigid Calabi–Yau compactifications

Ref. [9] studied perturbative and membrane instanton corrections to the universal hypermultiplet arising from the compactification of type IIA strings on a rigid CY_3. It was shown that these quantum corrections can be described within the

[1] In the context of type IIB orientifold compactifications a similar lifting due to α' corrections to the Kähler potential has been discussed in [11].

[2] Quantum corrections to complex structures also appear in three-dimensional gauge theories with four supercharges [12,13].

[3] For $c \geqslant 0$ this lifting does not work, and we can stabilize z in an AdS vacuum only.

framework of Przanowski–Tod metrics [14,15], i.e., the most general four-dimensional QK metrics with (at least) one Killing vector.[4] In local coordinates r, u, v, t they read[5]

$$\mathrm{d}s^2 = \frac{1}{2r^2}\left[f\,\mathrm{d}r^2 + f\mathrm{e}^h\left(\mathrm{d}u^2 + \mathrm{d}v^2\right) + f^{-1}(\mathrm{d}t + \Theta)^2\right]. \tag{8}$$

The metric is determined in terms of one scalar function $h(r, u, v)$, which is subject to the three-dimensional Toda equation

$$\left(\partial_u^2 + \partial_v^2\right)h + \partial_r^2 \mathrm{e}^h = 0. \tag{9}$$

The function $f(r, u, v)$ is related to h through

$$f = -\frac{3}{\Lambda}(2 - r\partial_r h), \tag{10}$$

while the 1-form $\Theta(r, u, v) = \Theta_r\,\mathrm{d}r + \Theta_u\,\mathrm{d}u + \Theta_v\,\mathrm{d}v$ is a solution to the equation

$$\mathrm{d}\Theta = (\partial_u f\,\mathrm{d}v - \partial_v f\,\mathrm{d}u) \wedge \mathrm{d}r + \partial_r\left(f\mathrm{e}^h\right)\mathrm{d}u \wedge \mathrm{d}v. \tag{11}$$

The metric has a manifest isometry that acts as a shift of the coordinate t. Λ in (10) is the target space cosmological constant; for the universal hypermultiplet we have $R_{AB} = -3G_{AB}$, thus $\Lambda = -3$.

The coordinates r, u, v, t can be identified with the fields in the universal hypermultiplet coming from string theory via

$$t = \sigma, \qquad r = \mathrm{e}^\phi, \qquad u = \chi, \qquad v = \varphi. \tag{12}$$

Here, χ and φ are the RR scalars originating from the dimensional reduction of the ten-dimensional 3-form $\hat{C}_3$, ϕ is the dilaton, and σ denotes the axion arising from the dualization of the 4-dimensional part of the NS 2-form $\hat{B}_2$. With this identification the universal hypermultiplet including the one-loop correction of [16,17] is described by [9]

$$\mathrm{e}^h = r + c, \qquad f = \frac{r + 2c}{r + c}, \qquad \Theta = u\,\mathrm{d}v. \tag{13}$$

For $c = 0$ this solution gives the classical moduli space $\mathrm{SU}(1,2)/\mathrm{U}(2)$ of the universal hypermultiplet. The one-loop correction is determined through

$$c = -\frac{4\zeta(2)\chi(X)}{(2\pi)^3} = -\frac{1}{6\pi}(h_{1,1} - h_{1,2}), \tag{14}$$

where $h_{1,1}$ and $h_{1,2}$ are the Hodge numbers of the CY threefold X on which the type IIA string has been compactified. For rigid CY's, where $h_{1,2} = 0$, we have that $c < 0$.[6]

[4] This Killing vector is present in the absence of fivebrane instantons.

[5] Note that, in contrast to [9] where $\kappa^2 = 1/2$, we here work with the more standard conventions in which $\kappa^2 = 1$. Compared to [9] this results in a rescaling of the scalar field metric by a factor 1/2, while the scalar potential is multiplied by an overall factor of 1/4. We refer to Appendix A of [9] for details.

[6] We expect that the numerical value of c obtained for a CY_3 orientifold differs from this $N = 2$ result, since projecting out states from the spectrum will affect the resulting one-loop contribution. The sign of c, however, should not change.

Furthermore, it was shown in [9] that, upon including the leading membrane instanton corrections, e^h takes the form

$$\mathrm{e}^h = r + c + \frac{1}{2}r^{-m_1/2}\left(A_1\mathrm{e}^{\mathrm{i}v} + B_1\mathrm{e}^{-\mathrm{i}u} + \mathrm{c.c.}\right)\mathrm{e}^{-2\sqrt{r+c}} + \cdots, \tag{15}$$

where m_1 is an undetermined integer coefficient which has to satisfy $m_1 \geqslant -2$, and A_1, B_1 are complex constants. Ref. [9] then explained that this expansion can be completed to a full solution of the Toda equation (9) and that it reproduces the leading order membrane instanton corrections to the four-hyperino couplings predicted by string theory [18]. Turning on a space–time filling F_4-form flux then induces a scalar potential in the $N = 2$ theory corresponding to gauging the shift symmetry of the axion. Upon including the perturbative and leading order corrections in the one-instanton sector, the resulting scalar potential reads [9]

$$V = \frac{e_0^2}{4r^2} - \frac{ce_0^2}{r^2(r + 2c)} - e_0^2 r^{-(m_1+5)/2}\mathrm{e}^{-2\sqrt{r+c}}\,\mathrm{Re}\left(A_1\mathrm{e}^{\mathrm{i}v} + B_1\mathrm{e}^{-\mathrm{i}u}\right) + \cdots. \tag{16}$$

4. The orientifold projection

We now apply the orientifold projection to the universal hypermultiplet discussed above, thereby truncating the theory to an $N = 1$ supergravity Lagrangian. According to [10], this projection amounts to eliminating the axion σ and the RR scalar u. Using the tensor multiplet description given in Appendix B of [9], the truncated line element becomes

$$\mathrm{d}s^2 = \frac{f}{2r^2}\left(\mathrm{d}r^2 + \mathrm{e}^h\,\mathrm{d}v^2\right). \tag{17}$$

Substituting the solution (15) and taking into account the leading instanton term only then yields

$$\mathrm{d}s^2 = \frac{r + 2c}{2r^2(r + c)}\left[\mathrm{d}r^2 + (r + c)\,\mathrm{d}v^2\right] + \frac{1}{2}r^{-(m_1+5)/2}\,\mathrm{Re}\left(A_1\mathrm{e}^{-2\sqrt{r+c}+\mathrm{i}v}\right)\left[\mathrm{d}r^2 + r\,\mathrm{d}v^2\right] + \cdots. \tag{18}$$

Here the first line arises from the perturbative corrections (13), while the second line displays the leading instanton contribution. $N = 1$ supergravity requires this metric to be Kähler. At the perturbative level, this is easy to verify by defining the complex structures as in (6) and substituting it into the metric

$$\mathrm{d}s^2 = 2\mathcal{K}_{z\bar{z}}\,\mathrm{d}z\,\mathrm{d}\bar{z}, \tag{19}$$

with Kähler potential given by (4). If we now define the complex structure

$$z = 2\sqrt{r + c} + \mathrm{i}v - \frac{1}{2}\bar{A}_1 r^{-(m_1+1)/2}\mathrm{e}^{-(2\sqrt{r+c}+\mathrm{i}v)}, \tag{20}$$

we find that the leading order one-instanton contribution in (18) is *still* captured by the Kähler potential (4). Hence, the Kähler potential receives perturbative corrections stemming from the one-loop correction encoded by c only; the leading membrane instanton corrections can be absorbed in a coordinate transformation.[7]

The scalar potential (7) of the $N = 1$ theory arises from setting the u-dependent terms in (16) to zero. At the perturbative level, this is immediately clear after identifying $W_0 = e_0$.

Comparing the leading order instanton correction is slightly more involved, because one has to take into account the instanton correction to the complex structure given by (20). As one can verify, the leading order instanton correction to the complex structure contributes to the potential (7) at subleading order only. Hence, the potentials (7) and (16) coincide upon identifying

$$A = -4e_0 A_1, \qquad m_1 = -2, \tag{21}$$

i.e., if m_1 takes its lowest possible value allowed by the Toda equation. This shows that the leading $N = 2$ membrane instanton correction provides a holomorphic contribution to the $N = 1$ superpotential.

It would be interesting to see if the lifting mechanism discussed in this Letter also applies to more realistic orientifold models such as the ones discussed in [6] or compactifications on G_2 manifolds [3].

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