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# Ant colony optimization algorithm applied to ship steering control

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## Abstract

The article describes the application of an ant algorithm to optimize parameters of the ship course controller, based on the algorithm of PID control. The ant algorithm is a method of combinatorial optimization, which utilizes the pattern of ants search for the shortest path from the nest to the place where the food is located. The procedure of parameter tuning for the ship course controller was applied to the case when the controller was changing the course of the ship and the integral action was turned off. Tuned parameters of the ship course controller are evaluated by the ant colony algorithm, which makes use of the course error based objective function and a given rudder deflection. The results were compared with equivalent results obtained using a genetic algorithm. Moreover, the effectiveness of PID controller parameter tuning was assessed using the ant colony optimization algorithm.

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*Keywords:* Ant colony optimization algorithm; genetic algorithm; artificial intelligence; PID controller; ship control.

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## 1. Introduction

Ship control systems have been an active area of research since 1911, when Elmer Sperry installed his first mechanical autopilot for automatic ship steering. First autopilots were purely mechanical systems and performed only simple control actions: the rudder deflection was proportional to the course error. To prevent an oscillatory action, small gain values had to be chosen, which made the autopilot to be useful for maintaining the ship at a fixed course in cases where the required accuracy was very small. The introduction of the PID algorithm by Nicolas Minorsky<sup>12</sup>, greatly improved potential capabilities of achieving better control quality and

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for many years all autopilots were based only on this algorithm. The replacement of mechanical devices with electronics have made it possible to reduce the weight of autopilots.

Autopilots which are presently used on vessels typically base on the PID algorithm. The measured course of the ship is compared with the required (set) course, and the resultant error is used as the input signal for the controller. The obtained output signal is passed to the servo steering gear which changes in a relevant manner the rudder blade deflection angle. Two tasks are required from the automatic ship course control system. The first of them is to change the ship course when the ship moves along the set trajectory and in this case the task should be performed quickly and accurately. This is particularly important when the maneuver is performed on waters with increased traffic or on restricted water areas. The second task is to steer and maintain the ship at a certain fixed course and in this case its role is to minimize the rudder activity and so-called *yaw effects*, with further aim to reduce fuel consumption. In order to fulfill these two different tasks at the same time usually two different control structures are used separately depending on the task<sup>7</sup>.

The quality of work of the ship course control system is closely related to the values of the parameters of the used controller. Unfortunately, PID autopilots are difficult to tune manually because of large number of possible combinations of settings and the lack of clear relations between their values and operational requirements or environmental changes. Therefore, the search for new methods to determine the optimal values of the PID controller settings is still an active area of research<sup>2,14</sup>. Tests are being run on various new concepts including modern techniques, among others, in the field of artificial intelligence<sup>11</sup>. In this article, an ant colony algorithm has been tested to optimize parameters of the ship course controller.

## 2. Mathematical model of the ship

The steering of the ship is defined in the absolute coordinate system  $X_o, Y_o$ , while the ship motion is described in the relative coordinate system  $(x, y)$  which is fixed to the ship. The motion of the ship is shown in Figure 1. The control system described in the article concerned ship course changing manoeuvres in which the controller parameter is the ship course  $\psi$ , and the controlling parameter is the rudder angle deflection  $\delta$ .

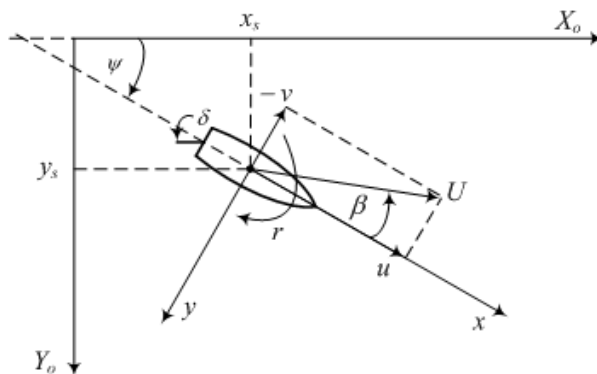


Fig. 1. Variables used to describe the motion in horizontal plane,  $\psi$  – yaw angle,  $\delta$  – rudder angle,  $(x_s, y_s)$  – ship position coordinates,  $(u, v)$  – body fixed linear velocities (surge, sway),  $r$  – yawing rate,  $\beta$  – slideship.

### 2.1. The model of Nomoto

For small rudder angles ( $|\delta| < 5^\circ$ ), the transfer function between the rudder angle  $\delta$  and the yawing rate  $r$  of a surface ship can be described by the linear model of Nomoto<sup>13</sup>. The Nomoto's 2<sup>nd</sup> order model is written as:

$$\frac{r(s)}{\delta(s)} = \frac{K(1+T_3s)}{(1+T_1s)(1+T_2s)} \quad (1)$$

where  $s$  is used to denote the Laplace operator,  $K$  is the gain constant, and  $T_i$  ( $i = 1, 2, 3$ ) are three time constants. The yaw angle  $\psi(t)$  is related to the yaw rate  $r(t)$  by the following relation:

$$\dot{\psi}(t) = r(t) \quad (2)$$

## 2.2. Forward speed effects

The influence of the forward speed  $U$  on the model parameters can be removed by using non-dimensional quantities. Let  $L$  be the length of the hull. Hence, the gain and the time constants can be made dimensionless with respect to speed variations by applying the transformations:

$$K = K^*(U/L), \quad T_i = T_i^*(L/U), \quad i = 1, 2, 3. \quad (3)$$

where  $K^*$  and  $T_i^*$  are dimensionless constants<sup>1</sup>. In the article the tanker is examined in the fully loaded state, when the ship is with cargo (liquid). In this case the non-dimensional model parameters are<sup>16</sup>.

$$K^* = 0.83, \quad T_1^* = -2.88, \quad T_2^* = 0.38, \quad T_3^* = 1.07 \quad (4)$$

The Nomoto model parameters (1), were determined for the speed  $U = 5$  m/s. The length of the tanker is  $L = 350$  m.

## 2.3. The model of Bech and Wagner-Smith

The linear ship steering equations of motion can be modified to describe large rudder angles and course-unstable ships by simply adding a nonlinear maneuvering characteristic to the Nomoto's 2<sup>nd</sup> order model. Bech and Wagner-Smith propose the following model<sup>3</sup>

$$T_1T_2\ddot{\psi} + (T_1 + T_2)\dot{\psi} + KH_B(\dot{\psi}) = K(T_3\dot{\delta} + \delta) \quad (5)$$

where the function  $H_B(\dot{\psi})$  describes the nonlinear maneuvering characteristic produced by Bech's reverse spiral maneuver, that is<sup>6</sup>:

$$H_B(\dot{\psi}) = b_3\dot{\psi}^3 + b_1\dot{\psi} \quad (6)$$

where  $b_3$  and  $b_1$  are real constants taking the values  $b_3 = b_1 = 1$  in the model.

## 2.4. The steering machine system

The model of the dynamic characteristics of the ship was complemented by the model of the steering machine system based on the results of the work<sup>15</sup>, which was schematically shown in Figure 2. This model consists of two electrohydraulic steering subsystems: the telemotor position servo and the rudder servo actuator. The input of the steering machine originates from the autopilot and is called the commanded rudder

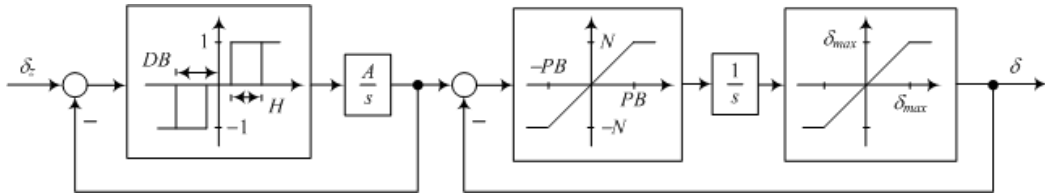


Fig. 2. Block diagram of the steering machine.

angle ( $\delta_c$ ), while the output is the actual rudder angle ( $\delta$ ). The values of the parameters for the steering machine system are the following<sup>15</sup>:

$$A = 4, \quad DB = 1, \quad H = 0.8, \quad PB = 7, \quad N = 5, \quad \delta_{\max} = 35 \tag{7}$$

### 3. Ship course control

The article considers the PID course-changing controller of the ship in which the integral action is turned off. This controller bears the name of the PD controller and has the following transfer function

$$G_{PD}(s) = \frac{\delta_c(s)}{e_\psi(s)} = K_p(1 + T_D s) = K_p + K_D s \tag{8}$$

where the tuned parameters were: the proportional gain  $K_p$  and the derivative gain  $K_D$ . The block diagram of the control system used for the simulation tests is shown in Figure 3.

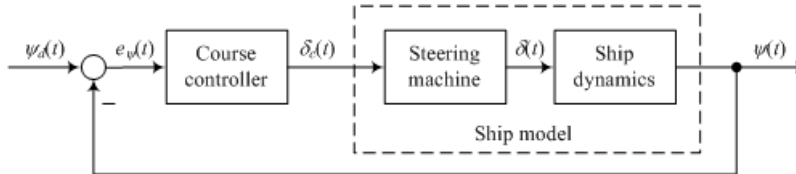


Fig. 3. Block diagram of the ship course control system.

#### 3.1. Performance index

A measure of the control quality of the ship course controller was evaluated using the integral absolute error (IAE) which bases on the course error and the rudder angle. This error can be defined in its discrete form as:

$$J_E = \frac{1}{N} \sum_{i=1}^N \left( \left| \Delta \psi_i \right| + \lambda \left| \delta_i \right| \right) \tag{9}$$

where  $N$  is the integer number of iterations in control simulations,  $\lambda$  is the scale factor (in the examined case  $\lambda = 0.1$ ),  $\Delta \psi_i$  is the  $i$ -th course error determined by subtracting the obtained course  $\psi_i$  from its desired value  $\psi_{di}$ , while  $\delta_i$  is the  $i$ -th rudder deflection angle. The ant colony optimization algorithm minimizes the value of

the function (9) by minimizing both the course error  $\Delta\psi$  and the rudder angle  $\delta$ . The component connected with the rudder angle is scaled to have a magnitude similar to that of the course error.

#### 4. Ant colony optimization algorithm (ACO)

The ant colony algorithm is a simulation program which allows to solve optimization problems and was inspired by the way the colony of ants living in forest environments searches for a path to food<sup>5</sup>. Each ant is looking for path independently, in a space of possible solutions, and leaves on its path some information about the resulting solution bearing the name of pheromone. Ants leave more pheromone on better solution components, which thereby become more and more recognizable and may be chosen in the future more frequently. At the beginning, during the initialization of the ant colony algorithm all possible solutions receive the same amount of pheromone. With the development of calculations, better solutions receive more and more of pheromone and thus the ant colony algorithm finds the optimal solution<sup>10</sup>.

##### 4.1. Optimization of the ship course controller

The PD algorithm (8) was selected as a ship course changing controller. For each parameter of the examined controller ( $K^1 = K_p, K^2 = K_D$ ), a set  $J$  of possible candidates for the solution was created in the range from minimum ( $K_{min}^i$ ) to maximum ( $K_{max}^i$ ). For simplicity, a uniform distribution between those boundaries was applied<sup>4</sup>.

$$K_1^i = K_{min}^i, \quad K_2^i = K_1^i + \frac{K_{max}^i - K_{min}^i}{J-1}, \quad \dots, \quad K_J^i = K_{max}^i \tag{10}$$

Therefore, for each parameter  $K^i$  the following set of potential values of the optimal solution is obtained.

$$K^i \in \{K_1^i, K_2^i, \dots, K_J^i\} \tag{11}$$

The limiting values ( $K_{min}^i, K_{max}^i$ ) for each of tunable parameters are collected in Table 1. This allocation method allows to assign  $J$  possible values to each tunable parameter of vector  $\mathbf{K} = [K^1, K^2]^T$ . Graphical representation of the optimized problem is shown in Figure 4, where two tunable parameters were distributed in two vectors.

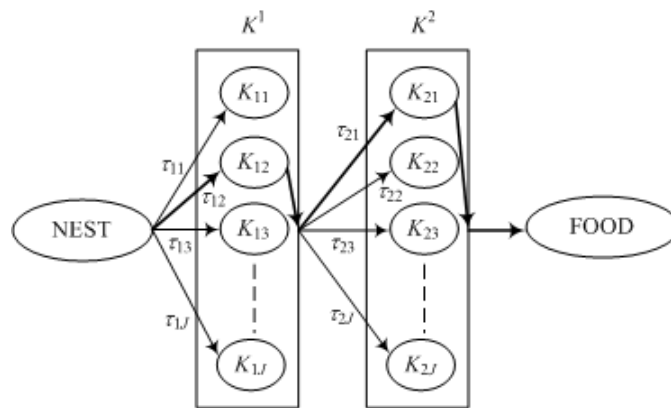


Fig. 4. Graphical representation of the optimization problem with the use of ACO.

Table 1. Limits of the optimized parameters of ship course controller.

Parameter	Min	Max
$K^1 = K_P$	0	1
$K^2 = K_D$	0	1000

Each possible value is represented by one node. Solving problem lies in finding the best combination of parameters, which will minimize the evaluation function (9). A single ant's path consists of a combination of PD controller parameters. Starting its path from a nest, the ant passes through vectors which indicate further  $K_P$  and  $K_D$  gains and eventually reaches the food source.

#### 4.2. The rule of parameter selection by ants

For each set, which describes a single parameter, the node visited by an ant is selected as the value of this parameter. The choice of a parameter value is based on the quantity of the pheromone located between the vectors of parameters. The pheromone matrix  $\tau_{ij}$  for the considered PD controller is of the size  $2 \times J$ ,  $i = 1, 2$  and  $j = 1, 2, \dots, J$ . At the beginning, the pheromone matrix  $\tau_{ij}$  is initialized with a certain constant initial value.

$$\tau_{ij} = \frac{1}{J_E^0} \quad (12)$$

where  $J_E^0$  is a random initial value of the performance index. In order to determine the initial value of the performance index  $J_E$ , firstly the values of parameters  $K_P$  and  $K_D$  were randomly selected, then the operation of the control system was simulated and on this basis the value of the initial performance index  $J_E^0$  was determined. When the ant  $k$  approaches a vector which contains a set of values of the parameter  $K^j$  it has to select a node through which it will pass this vector. This selection is based on probability values which are determined for the ant  $k$  localized at the output of the previous vector  $K^i$  and the nodes  $j$  located in the next vector  $K^j$ , calculated as follows:

$$p_{ij}(t) = \frac{\tau_{ij}^\alpha(t)}{\sum_{j=1}^N \tau_{ij}^\alpha(t)} \quad (13)$$

where  $\alpha$  is a positive constant used to gain the pheromone influence. After determining the probabilities of edge passing  $p_{ij}$ , another node is randomly selected using the roulette wheel method. The greater the value of the link probability  $p_{ij}$ , the greater the chance of choosing a particular node. When a single ant  $k$  passes the entire path from the nest to the food source then, basing on the values of the PD controller parameters selected on the path, the operation of the system shown in Figure 3 is simulated. The next step is to evaluate the quality of the control results by calculating the performance index  $J_E$  described by formula (9) and to update pheromone values on all paths  $\tau_{ij}$  passed by the ant  $k$ . This update is performed based on the following relation:

$$\tau_{ij}(t) \leftarrow \tau_{ij}(t) + \frac{Q}{J_E^k}, \quad k = 1, 2, \dots, M. \quad (14)$$

where  $M$  is the ants' population,  $Q$  is a parameter which scales the amount of pheromone left by a single ant ( $Q = 0.2$ ). The update described by the formula (14) is carried out for each ant  $k = 1, 2, \dots, M$ .

When all ants composing the population pass their paths from the nest to the food source, then a check is performed to see whether there was an improvement of the best value of the indicator  $J_E^*$  in the current iteration of optimization, and the pheromone matrix  $\tau_{ij}$  is updated based on the following formula:

$$\tau_{ij}(t+1) = (1 - \rho)\tau_{ij}(t) + \frac{1}{J_E^*} \quad (15)$$

where  $J_E^*$  is the best value performance index obtained in all iterations and used to update the best path in the current iteration,  $\rho \in \{0, \dots, 1\}$  is the rate of pheromone trail evaporation. The evaporation is added to the algorithm in order to discourage the ants from choosing the already found best paths and thus avoid premature convergence to suboptimal solution<sup>9</sup>.

## 5. Genetic algorithm

The essence of the genetic algorithm was taken from the theory of evolution, which was formulated in 1859 by Charles Darwin. It is understood as the process of changes taking place in living organisms as a result of their interaction with the environment by using the phenomenon of natural selection and inheritance. Currently, genetic algorithms have become one of the most popular methods of optimization<sup>8</sup>.

In order to determine the initial population, chromosomes are generated randomly by the method: bit by bit. The length of the chromosomes depends on the number of coded parameters and their precision  $n$ , all this being done based on the following formula

$$(k_{\max} - k_{\min}) \cdot 10^{n_i} \leq 2^{m_i} - 1 \quad k = 1, 2. \quad (16)$$

where  $n_i$  is the number of decimal places describing the accuracy and  $m_i$  is the number of bits on which the tunable parameter is going to be encoded. The chromosome which consists of particular bits of the controller parameters is decoded. The decimal value of each parameter is calculated on the basis of the following formula

$$k = k_{\min} + \text{decimal}(1010 \dots 011_2) \frac{(k_{\max} - k_{\min})}{2^{m_i} - 1} \quad k = 1, 2. \quad (17)$$

where the decimal  $(1010 \dots 011_2)$  is equal to the decimal value of the binary string.

Quality of operation of the control system with tuned parameters of the controller is assessed with the use of the integral quality index, described by formula (9). On this basis, each chromosome is assigned the corresponding value of the fitness function.

Selection of chromosomes consists in choosing the chromosomes which will participate in the creation of descendants to the next generation on the basis of the chosen values of the adaptation function. In the algorithm the so called roulette method was applied, which owes its name to an analogy to draw with the use of the roulette wheel<sup>8</sup>.

Genetic operations are used to generate a new population and include activities such as crossover and mutation. The application of genetic operators to the chromosomes selected by the selection method results in the formation of a new population, i.e. the population of descendants of previously selected parents. The first step of the crossover is the selection of pairs of chromosomes from the parent's population. At this stage, the chromosomes from the parent's population are associated in pairs. Crossing process is not performed on all

pairs of the population and depends on the adopted probability  $p_c$ . The mutation is made to the individual bits in accordance with a predetermined probability of mutation  $p_m$ . All bits in all chromosomes of the population have a chance to mutate<sup>8</sup>.

## 6. Simulation study

The ant optimization algorithm and the genetic algorithm described in the article were implemented in computing environment of Matlab/Simulink. The block diagram of ship steering on course, shown in Figure 3, was implemented in Simulink. The parameters of the course controller were tuned with the use of programs written in Matlab, which contained the ant optimization and genetic algorithms.

The tested algorithms required setting of several parameters. For both algorithms, the same range of changes of the tuned PD controller parameters were set (see Table 1). Then, for the ant colony algorithm these ranges were distributed over  $J = 1000$  possible values (Fig. 4). The assumed number of ants was  $M = 10$  and the maximum number of iterations was  $I_{max} = 20$ . The pheromone influence gain was assumed equal to  $\alpha = 3$ , and the indicator describing the evaporation rate was  $\rho = 0.05$ . At the same time for the genetic algorithm, the adopted population consisted of 8 chromosomes, with the probability of crossover  $p_c = 0.5$ , and the probability of mutation  $p_m = 0.01$ . The overall chromosome length for the PD controller consisting of two tuned parameters  $K_P$  and  $K_D$  was 24 bits.  $K_P$  parameter was determined with an accuracy of two decimal places and was encoded in 10 bits, while the parameter  $K_D$  was determined with an accuracy of one decimal place and was encoded in 14 bits. For the genetic algorithm the maximum number of generations was  $I_{max} = 100$ .

For both tested algorithms, the evaluated gains  $K_P$  and  $K_D$  were set in the control system of the course of the ship (Fig. 3). Then the step response simulation was performed, in which the set ship course was changed by about 40 deg. Sample results of the step response of the system are shown in figure 5. The time duration of the response was  $t_{max} = 600$ s. In the next step, on the basis of the results shown in Figure 5, the quality indicator  $J_E$  (9) was determined, which was then used as the measure of quality of the performed simulation of the control system operation.

In addition, for every test trials given in Tables 2 and 3, time indicators of quality, which are defined on the basis of the step response and include such quantities as the rise time ( $t_R$ ), the maximum overshoot ( $M_p$ ) and the settling time ( $t_s$ ), were determined. In the reported case the above indicators were defined as follows: the rise time ( $t_R$ ) was determined as the time interval in which the step response changes from 10% to 90% of the steady value ( $\psi_{ss}$ ), the maximum overshoot ( $M_p$ ) was determined from the formula  $M_p = 100(\psi_{max} - \psi_{ss})/\psi_{ss}$ , and

Table 2. Results of tuning of PD controller settings (8) using the ant optimization algorithm

No	$K_P$	$K_D$	$J_E$	$t_R$ (s)	$M_p$ (s)	$t_s$ (s)
1.	7.83	617.6	9.4064	146.7	2.71	487.2
2.	9.22	673.7	9.2720	141.3	2.42	463.3
3.	9.57	795.8	9.4785	158.9	1.37	455.6
4.	7.55	574.6	9.3817	141.5	3.29	486.7
5.	6.69	553.5	9.5298	148.4	3.32	508.1
6.	8.97	594.6	9.2258	129.3	3.90	455.4
7.	8.44	550.5	9.3129	126.4	5.13	456.5
8.	9.99	672.7	9.1728	132.7	2.83	448.2
9.	9.82	627.6	9.2425	126.3	4.57	444.4
10.	9.44	623.6	9.2193	129.4	3.77	451.0



Table 3. Results of tuning of PD controller settings (8) using the genetic algorithm

No	$K_P$	$K_D$	$J_E$	$t_R(s)$	$M_p(s)$	$t_s(s)$
1.	7.14	497.5	9.3724	130.5	4.86	477.3
2.	5.62	416.8	9.6127	132.0	6.37	499.8
3.	8.05	542.9	9.2898	129.3	4.41	464.7
4.	8.59	594.9	9.2479	133.4	3.41	464.3
5.	6.24	453.5	9.4912	132.1	5.49	491.7
6.	9.35	633.9	9.1984	132.5	3.14	455.0
7.	7.97	544.9	9.2879	130.6	4.20	467.6
8.	8.67	594.3	9.2411	132.4	3.52	462.5
9.	9.91	672.8	9.1779	133.7	2.74	449.7
10.	8.79	595.2	9.2286	131.3	3.59	459.6

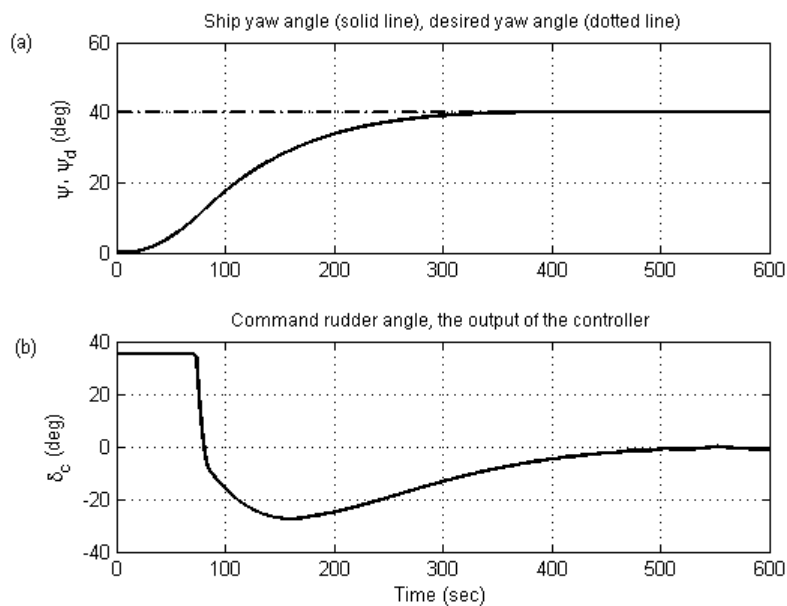


Fig. 5. Step response of the ship course control system: (a) required and measured heading, (b) commanded rudder deflection.

the settling time ( $t_s$ ) was determined as the time after which the output signal  $\psi$  reached the steady state which differed from the nominal value by 1%.

## 7. Conclusions

The article discusses the process of search for a combination of two parameters of PID controller which optimizes the operation of the ship's control system on the course. Direct search for appropriate values of these parameters among all possible combinations would be a very time-consuming task and therefore the ant colony

optimization algorithm was applied. The research has revealed that the ant colony algorithm converged very fast and in most cases the final solution was determined within ten initial iterations.

In order to obtain data for comparison, a well-known classical genetic algorithm was tested on the same control system to search for the optimal adjustment of the ship course controller's parameters. In this case longer time of determination of final result was recorded for the assumed parameters of the genetic algorithm, and therefore it was necessary to use 100 generations in a single run of algorithm application.

Each tested algorithm was run 10 times, and the selected results of tuning of PD controller's parameters have been collected in Tables 2 and 3. For the ant colony algorithm tests the minimum of the quality index  $J_E$  was obtained in test no. 8, while for the genetic algorithm - in test no. 9, and the obtained minimal value of this index began to differ only at the third decimal place.

In summary, we can conclude that the tested ant colony algorithm proved to be a very effective tool for optimization of ship course controller parameters and can be successfully used to solve more complex optimization problems.

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