Single particle scattering used for characterization of suspended sediments.

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Abstract

The aim of this paper is to develop a theoretical model for description of ultrasound scattering from irregularly shaped individual particles. Investigations of sediment transport by use of ultrasound scattering technique demand a fundamental understanding of scattering by individual, irregularly shaped particles. Regularly shaped particles are frequently spheres, while irregularly shaped particles can be symmetrically particles with surface roughness or with angular facets and edges. A cube and a rough sphere have been used in the studies behind this paper. Laboratory experiments have been used for verification of theoretical and numerical results.

Keywords: Scattering; sediment particles; surface roughness; irregular shape; suspensions;

1. Introduction

The use of ultrasonic methods for estimation of suspended sediment concentrations closed to the seafloor has been suggested long ago [1]. Two ultrasonic based methods have in particular been developed for sediment transportation studies. Ultrasonic (back)scattering [2] and attenuation spectroscopy [3]. In the case of use of attenuation spectroscopy the particle parameters like size, number, shape and elasticity influence the attenuation spectra of ultrasound signals propagating in suspensions of sediments as the attenuation is mostly caused by scattering.

Backscattering methods have been developed and applied to field studies of suspended sediments in more cases than attenuation spectroscopic methods. The implementation of backscattering measurements with high accuracy and proper interpretation of the experimental results demands the existence of realistic models for the scattering, and there is still a lack of fundamental knowledge in this area. A model consisting of rigid, moveable, non-interactive
quartz spheres for description of suspensions of sand particles still remains the most advanced theoretical model, which gives the best fit, however, not yet satisfactory, to experimental results. Approaches using the modified form function as a “high-pass model” yield a first attempt to describe the process of single particle scattering for particles of non-spherical shapes [4]. A realistic estimation of the concentration and size distribution of suspended particles from backscattering measurements can only be obtained after a realistic definition of the backscattering form function \( |F_{sc}| \) as a function of frequency and particle shape. Based on resonance scattering theory [5], a solution for plane wave scattering by an elastic sphere with a smooth surface can be obtained in terms of the form function [6].

\[
P_{sc,0} = (P_o a f_o^o / 2 r) \exp (i k r), \text{ where } f_o^o = -2/(ka)\sum_{n=1}^{\infty} (2n-1) \sin \eta_n \exp(-i \eta_n) P_n (\cos \theta)
\]

where \( a \) is the radius of a sphere without surface roughness, \( f_o^o \) is used to indicate the form function related to scattering by spheres without surface roughness, \( \eta_n \) is the phase shift of the \( n \)’th partial wave related to the material qualities of the sphere, \( P_o \) is the pressure amplitude in the incident wave, \( r \) is the distance (considered to be large) from the coordinate origin to the field point, and \( P_n(\cos \theta) \) is the \( n \)’th order Legendre polynomial. From (1) follows, that the process of scattering by spheres involves two key aspects; the scattering directivity patterns and the backscattering spectra. If the \( ka \)-value is specified, the change of the form function with the angle of scattering forms the directivity patterns. And for \( \theta = \pi \), a curve of the backscattering spectrum versus the \( ka \)-value is obtained.

The directivity patterns show the scattering ability in different directions, and the backscattering spectrum shows the scattering ability in different \( ka \) regions. The backscattered signal from a sphere, \( P_s \), within the beam of a transducer can be calculated from the experimentally measured parameters [4] as:

\[
P_s = a P_o r_o [D^2(\theta) / 2r^2] f_o \exp (-2ar)
\]

where \( P_o \) is the transmitted pressure at range \( r_o \), \( r \) is the distance to the particle; \( D^2(\theta) \) is the transducer directivity function, \( \theta \) denotes the angle between the direction to the particle and the acoustic axis; \( a \) is the particle radius; \( \alpha \) is the attenuation coefficient due to absorption in water; and \( f_o \) is the farfield form function.

2. Scattering from cubes

The experiments were carried out in a laboratory test tank of dimensions: \( 2 \times 0.5 \times 0.5 \) m. The transducers used were a Panametric A301S (300 – 900 kHz) and a Reson TC3012 (900-1200 kHz). The backscattered signals from the target were used to determine the form function by use of (2). The transducers were transmitting pulses of 75 \( \mu \)s duration, repeated 10 times per second. The backscattered signals were amplified, filtered and averaged to improve the signal/noise ratio, and were analyzed by use of a digital oscilloscope. The targets were placed in the farfield of the transducer, normally at \( r = 0.5 \) m, and were suspended in a 350 \( \mu \)m diameter nylon wire. The materials of the scatterers studied was steel and their shapes were cubes and spheres with characteristic dimensions to cover the \( ka \)-values between 5 and 30. The material qualities of the steel (manufacturer information) were: density – 7850 kg/m\(^3\); compressional velocity – 5960 m/s, shear velocity – 3235 m/s. Figure 1 shows experimental results of the form function for a steel cube for various angles of incidence, 0\(^o\) (normal incidence) to 5\(^o\). At normal incidence Figure 1 shows that the form function is increasing with \( ka \), with resonance occurring at certain \( ka \)-values. For angles of incidence at 2.5\(^o\) and 5\(^o\), the amplitude of the form function drops significantly, although the resonance structure remains generally unchanged. Only the values of the minima become smaller.

3. Scattering from a rough sphere

Based on the resonance scattering theory for elastic spheres [5], a perturbation approach developed for wave scattering by a planer, rough surface is applied to the study of plane wave scattering by elastic spheres with surface roughness. The \( rms \)-roughness height \( h \) of the spherical surface is measured as percentage of the radius \( a \) of the sphere. A Gaussian distribution for the surface roughness height is assumed. The basic solution to the scattered field is given in terms of the form function. By changing the surface roughness from \( h/a = 0\% \) (corresponding to a smooth surface) to \( h/a = 10\% \) (corresponding to a sphere with a very rough surface), the scattered ultrasonic
pressure field was analyzed in 3 different ways. (1) By use of the scattering directivity patterns for constant $ka$-value; (2) by use of the scattering spectrum as a function of $ka$ for a certain direction of the scattered ultrasonic field; (3) by use of the total scattering cross-section, which shows the change of the scattering ability of a target (here a sphere) via the $ka$-value for different $rms$ roughness heights. Cast iron was selected for case studies in order to compare the numerical results with the results of the laboratory experiments performed. If the roughness of an object is taken into consideration, the scattered field will be strongly affected. At high-frequency conditions, the insonified part of the rough surface can be treated as a rough surface with a smooth spherical reference surface. When roughnesses are present on the surface of a sphere, and if the surface roughness heights $h$ are small compared with the radius of curvature of its reference surface $a$, a perturbation theory can be applied to the study of the ultrasonic wave scattering from the rough surface [6].

![Figure 1](image1.png) Scattering from the steel cube for several angles of incidence. Experimental results.

![Figure 2](image2.png) Backscattering from a rough sphere. Form functions versus $ka$ for different roughnesses. Numerical results.

By use of the form function definition in Expression (1), the contribution of the surface roughness to the scattered field can be expressed in terms of the form function [6]:

$$<p_{sc}> = \frac{P_o a f_o \exp(ikr)}{2r} \text{ where } f_o = (i k a) (k^2 h^2) \int_0^\infty \left[ e^{2ikA} D A \exp(-ik a \mu - k^2 h^2 A^2/2) \sin \mu d^2 \right] (3)$$

and where the parameter $D$ is given by:

$$D = \sum_{n=0}^\infty \{ (2n+1) \hat{\ell} j_n''(ka) + c_n \hat{i} y_n''(ka) \} P_n(\cos \theta_o) \quad (4)$$

Material parameters used in the calculations are: for water; velocity of sound – 1496 m/s; density – 998 kg/m$^3$, and for the steel spheres (ball bearing ball): compressional wave velocity – 5790 m/s; shear wave velocity – 3100 m/s; and density – 7900 kg/m$^3$. Due to the surface roughness, parts of the incident ultrasonic energy will be redistributed over all field points, and the scattering spectrum, the directivity patterns and even the total scattering cross section will be different from those produced by spheres without surface roughness. Figure 2 shows the form function for scattering from the rough steel sphere with different $h/a$ parameters, calculated based on Expressions (3) and (4).

Apart from the steel ball, cast iron spheres with different surface roughnesses were used as scattering targets placed in the farfield of the transducers. The surface roughness of the sphere was produced by gluing sand particles to the sphere surface. The roughness degree, $h/a$, was controlled by the average dimensions of the sand particles, and the surface profile of the rough surface was measured by use of a laser interferometer. The roughness height...
distribution was found to be close to Gaussian. The backscattered signals from the spheres were measured by use of a PVDF needle hydrophone and were used to determine the form function.

The experimental procedure was controlled by measuring the backscattering from a smooth stainless-steel sphere and the results were compared with results from numerical simulations. An excellent agreement was found. Backscattering measurements by rough cast iron spheres with radius $a = 25$ mm, were carried out in the test tank and the results are given in Figure 3, compared with numerical simulation data obtained by use of the perturbation procedure. A good agreement between the measured and the numerically calculated results can be seen from Figure 3, which shows the applicability of the simulation procedure for scattering from rough spheres. The solution gives corrections to the scattered ultrasonic pressure field in terms of the form function. The changes of the backscattered field due to surface roughness are dependent on the $h/a$- and the $ka$-values as well as on the scattering angle.

![Figure 3. Measured form function compared with numerical simulations for a rough cast iron sphere.](image)

4. Conclusions

Models for scattering of ultrasonic waves from non-spherical particles have been studied numerically and experimentally in order to develop and to verify the concept of the “orientation average form function”, which may be applied to the interpretation of backscattered experimental data from an arbitrary shaped target as for instance a sediment particle. The first step in the direction of creating a general approach to the ultrasonic scattering by irregularly shaped particles has been demonstrated in this paper by use of two models: (1) a spherical particle with irregular (random) surface roughness, and (2) a symmetrical particle composed of angular facets and edges (a cube). Numerical results have been verified by experimental investigations, and the developed approach contributes to a better understanding of the process of ultrasonic scattering by arbitrary shaped bodies.

References