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Book reviews

Contents

Szegő's Theorem and its Descendants: Spectral Theory for L^2 Perturbations of Orthogonal Polynomials, Barry Simon. Princeton University Press, Cloth (2010). 720 pp., hardcover, ISBN 978-1-4008-3705-2, \$ 110.00

Introduction

This book is a gem in the usual sense of the word, but it is also a book about gems of spectral theory according to the terminology of the author: Barry Simon calls a theorem a gem, if it describes a class of spectral data and a class of objects so that an object is in the second class if and only if its spectral data lie in the first class.

In classical harmonic analysis, it is easy to identify some gems: the theorem of Bochner states that the Fourier transformation $\mathcal{F}$ establishes a one-to-one correspondence between positive finite measures and continuous positive definite functions on the real line. The theorems of Paley–Wiener and Plancherel are other well-known examples of gems. Riemann–Lebesgue's lemma is not a gem, because although it asserts that the Fourier transform of an integrable function belongs to $C_0(\mathbb{R})$, there are continuous functions vanishing at infinity which do not belong to the range $\mathcal{F}(L^1(\mathbb{R}))$.

The starting point of the book under review is a theorem of Szegő from 1915.

Theorem 1. *Let $w \in L^1_+(\mathbb{T})$ have the Fourier coefficients*

$$c_k = \int e^{-ik\theta} w(e^{i\theta}) \frac{d\theta}{2\pi}, \quad k \in \mathbb{Z}$$

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and Toeplitz determinants $D_n(w) = \det(c_{j-k}), 0 \le j, k \le n,\ n = 0, 1, \ldots$. *Then*

$$\lim_{n\to\infty} D_n(w)^{1/n} = \exp\left(\int \log\, w(\theta)\frac{d\theta}{2\pi}\right). \tag{1}$$

Szegő, an undergraduate at the time, thereby answered a question posed by the eight years older Pólya.

This theorem and its descendants constitute important chapters in the theory of orthogonal polynomials on the unit circle (OPUC) and on the real line (OPRL). In the present form Szegő's theorem is not a gem, but as the monograph shows, many gems are related to it.

OPUCs have recently been treated in the impressive work of the author [6,7], while OPRLs are the subject of the not less impressive work by Ismail; see [3]. The theory of special functions plays a much bigger role in Ismail's book than in Simon's books.

Let us consider a probability measure μ on the unit circle $\mathbb{T}$ with infinite support and assume that we have the following splitting $\mu = wm + \mu_s$ in an absolutely continuous and singular part with respect to normalized Haar measure $m = d\theta/(2\pi)$ on $\mathbb{T}$. Let Φ_n be the monic orthogonal polynomial of degree n with respect to μ. Then, there is a sequence of complex constants α_n with $|\alpha_n| < 1$ such that

$$\Phi_{n+1}(z) = z\Phi_n(z) - \overline{\alpha_n}\, \Phi_n^*(z), \tag{2}$$

where $\Phi_n^*(z) = \sum_{j=0}^n \overline{c_{n-j}} z^j$ if $\Phi_n(z) = \sum_{j=0}^n c_j z^j$.

Today, the coefficients α_n are called Verblunsky coefficients, and Verblunsky [9] proved the following version of Szegő's theorem.

Theorem 2. *For* μ *as above,*

$$\prod_{n=0}^{\infty}(1 - |\alpha_n|^2) = \exp\left(\int \log\, w(\theta)\frac{d\theta}{2\pi}\right). \tag{3}$$

This leads to the following gem.

Corollary 3. *For a probability measure* μ *as above with infinite support we have*

$$\sum_{n=0}^{\infty} |\alpha_n|^2 < \infty \iff \int \log\, w(\theta)\frac{d\theta}{2\pi} > -\infty. \tag{4}$$

In the theory of OPRLs we start with a probability measure ρ on the real line with moments of any order and infinite support. The corresponding monic orthogonal polynomials (P_n) satisfy a three term recurrence relation of the form

$$xP_n(x) = P_{n+1}(x) + b_{n+1}P_n(x) + a_n^2 P_{n-1}(x), \quad n \ge 0, \tag{5}$$

where $P_{-1} = 0$ and $a_n > 0, b_n \in \mathbb{R}, n \ge 1$. The Jacobi parameters a_n, b_n give rise to the Jacobi matrix

$$J = \begin{pmatrix} b_1 & a_1 & 0 & \cdots \\ a_1 & b_2 & a_2 & \cdots \\ 0 & a_2 & b_3 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix},$$

which is the matrix of the multiplication operator $p(x) \to xp(x)$ with respect to the orthonormal basis $p_n = P_n/\|P_n\|$.

The Jacobi parameters are the analogues of the Verblunsky coefficients. Favard's theorem can be stated that any Jacobi matrix, i.e., any sequence $(a_n, b_n) \in (0, \infty) \times \mathbb{R}$ comes from a probability measure ρ as above. However, ρ need not be uniquely determined—it is only determined up to moment equivalence: two probability measures ρ, ν with the same moment sequence

$$s_n = \int x^n \, d\rho(x) = \int x^n \, d\nu(x), \quad n = 0, 1, \ldots$$

lead to the same family of monic orthogonal polynomials and hence to the same Jacobi parameters. A classical gem is that ρ has bounded support (and then it is determined by its moments) if and only if the sequence (a_n, b_n) is bounded, which is the same as J being a bounded operator on ℓ^2.

There is a simple way of transferring results about OPUCs to results about OPRLs on bounded intervals. It is connected to the conformal bijection of the unit disk $\mathbb{D}$ onto $\mathbb{C} \cup \{\infty\} \setminus [-2, 2]$ given by $z \to z + z^{-1}$ extended to the boundary as $e^{i\theta} \to 2\cos\theta$.

Using this approach the above Corollary 3 can be transferred to the following Shohat–Nevai theorem:

Theorem 4. *Let ρ be a positive measure supported by $[-2, 2]$ and let $\rho = f(x)dx + \rho_s$ be the splitting in absolutely continuous and singular parts. Then*

$$\int_{-2}^{2} \frac{\log f(x)}{\sqrt{4 - x^2}} \, dx > -\infty$$

if and only if

$$\limsup_{n\to\infty} a_1 \ldots a_n > 0.$$

If these conditions hold, then $\lim_{n\to\infty} a_1 \ldots a_n$ exists and is positive and finite. Furthermore, the following limits exist in $\mathbb{R}$ for $N \to \infty$

$$\sum_{n=1}^{N} (a_n - 1)^2 + b_n^2, \qquad \sum_{n=1}^{N} (a_n - 1), \qquad \sum_{n=1}^{N} b_n.$$

Although this is a very nice theorem it is not a gem in the sense of Simon, because the equivalent conditions are based on the additional assumption that the support of ρ is contained in $[-2, 2]$.

A related gem is obtained by Killip and Simon in their Annals paper [4].

To explain this theorem we denote by J_0 the so-called free Jacobi matrix, whose Jacobi parameters are $a_n = 1, b_n = 0, \; n \geq 1$ and the corresponding orthogonality measure is

$$\rho_0 = \frac{\sqrt{4 - x^2}}{2\pi} \, dx,$$

which leads to the Chebyshev polynomials of the second kind transferred from $[-1, 1]$ to $[-2, 2]$.

Theorem 5. *Let (a_n, b_n) be the Jacobi parameters of a Jacobi matrix J. Then*

$$\sum_{n=1}^{\infty} (a_n - 1)^2 + b_n^2 < \infty \tag{6}$$

if and only if

(i) *J is a bounded operator with the same essential spectrum as* J_0 $(= [-2, 2])$,

(ii) *the eigenvalues E_n of J outside* $[-2, 2]$ *satisfy*

$$\sum_n \operatorname{dist}(E_n, [-2, 2])^{3/2} < \infty,$$

(iii)

$$\int_{-2}^{2} \operatorname{dist}(x, \mathbb{R} \setminus [-2, 2])^{1/2} \log f(x)\, dx > -\infty,$$

where $\rho = f(x)dx + \rho_s$ is the splitting in absolutely continuous and singular parts of the unique orthogonality measure for J.

Overview of the content.

The book (650 pages) contains 10 chapters:

1. Gems of Spectral Theory
2. Szegő's Theorem
3. The Killip–Simon Theorem: Szegő for OPRL
4. Sum rules and Consequences for Matrix Orthogonal Polynomials
5. Periodic OPRL
6. Toda Flows and Symplectic Structures
7. Right Limits
8. Szegő and Killip–Simon Theorems for Periodic OPRL
9. Szegő's Theorem for Finite Gap OPRL
10. A.C. Spectrum for Bethe–Cayley Trees

Chapter 1 is an overview of the content and it introduces the main themes as described in Section 1. Chapters 2 and 3 develop the various aspects of Szegő's theorem and the Killip–Simon theorem.

The free case is just a normalization of the case, where the Jacobi parameters are constant. A deep and important extension of the constant case occurs if the Jacobi parameters are periodic, say with period $p \geq 2$:

$$a_{n+p} = a_n, \qquad b_{n+p} = b_p, \quad n \geq 1.$$

In this case the essential spectrum of the Jacobi matrix will be a disjoint union of compact intervals. The material in Chapters 5, 6 and 8 is connected with analyzing Szegő-like theorems for OPRL. Meromorphic functions on hyperelliptic surfaces and almost periodic functions $f : \mathbb{Z} \to \mathbb{R}$ are given self-contained treatments along the way.

The book was planned as notes for the Milton Brockett Porter Lectures at Rice University in 2006, but in the process of writing, more material has been added for the sake of completeness. In addition, a collaboration with two Caltech postdocs, Jacob S. Christiansen and Maxim Zinchenko created lots of new material, which has lead to three papers and the present Chapter 9. See the survey paper [1] from the meeting celebrating Guillermo López Lagomasino.

It is not possible to discuss in detail the material of all the chapters, so I will concentrate on Chapter 9 about finite gap OPRLs.

Now the compact interval $[-2, 2]$ is replaced by the union

$$\mathfrak{e} = \bigcup_{j=1}^{\ell+1} [\alpha_j, \beta_j], \quad \alpha_1 < \beta_1 < \alpha_2 < \cdots < \beta_{\ell+1} \tag{7}$$

of $\ell + 1$ disjoint compact intervals, so we can think of $\mathfrak{e}$ as a compact set with ℓ gaps. As mentioned above, the periodic case leads to essential spectra of the form (7), but the periodicity imposes extra conditions on $\mathfrak{e}$.

The author sets out to study spectral properties of Jacobi matrices associated with measures ρ supported by $\mathfrak{e}$ except for perhaps countably many mass points.

In this case, Szegő's map $z \to z + 1/z$ has to be replaced by an analytic function

$$\mathbf{x} : \mathbb{D} \to \mathbb{C} \cup \{\infty\} \setminus \mathfrak{e},$$

which is no-longer one-to-one if $\ell > 0$. This is obtained by considering $\mathbb{D}$ as the universal covering of the non-simply connected right-hand side, and there appears a Fuchsian group Γ of fractional linear transformations of the unit disk such that for $z, w \in \mathbb{D}$

$$\mathbf{x}(z) = \mathbf{x}(w) \Leftrightarrow \exists \gamma \in \Gamma \quad \text{so that } w = \gamma(z).$$

This brings orthogonal polynomials in connection with classical parts of complex analysis and potential theory and reminds me of my years of study in Copenhagen in the late 1960s, when Werner Fenchel lectured about discontinuous groups of isometries in the hyperbolic plane in an attempt to finish the famous Fenchel–Nielsen manuscript, which eventually appeared in 2003 with the help of my colleague Asmus Schmidt, 15 years after Fenchel passed away; see [2].

The reader should not despair because almost everything needed is explained in Chapter 9: linear fractional transformations are studied at length, so are the special ones mapping the unit disk onto itself. Simon has chosen to call the latter Möbius transformations although this concept is usually the same as fractional linear transformations. The universal covering and its group of deck transformations are explained, and a complete proof of the uniformization theorem in the appropriate context is given. This group of deck transformations appears as the Fuchsian group Γ.

The idea of introducing this classical material into finite gap spectral theory was done in work by Sodin, Yuditskii and Peherstorfer; see [8,5].

Christiansen–Simon–Zinchenko's version of what they call the "Szegő–Shohat–Nevai theorem for finite gap sets" states the following.

Theorem 6. *Let $\mathfrak{e}$ be a finite gap set as in Eq. (7) and let $\rho = f(x)\,dx + \rho_s$ be the usual splitting of a finite measure ρ with Jacobi parameters (a_n, b_n) and Jacobi matrix J such that the essential spectrum of J is a subset of $\mathfrak{e}$. Suppose further that*

$$\sum_n \operatorname{dist}(E_n, \mathfrak{e})^{1/2} < \infty,$$

where the sum is over the points E_n in the spectrum of J outside the essential spectrum.

Then

$$\int \frac{\log f(x)}{\operatorname{dist}(x, \mathbb{R} \setminus \mathfrak{e})^{1/2}}\, dx > -\infty \Longleftrightarrow \limsup_{n\to\infty} \frac{a_1 \ldots a_n}{C(\mathfrak{e})^n} > 0.$$

Here $C(\mathfrak{e})$ is the logarithmic capacity of $\mathfrak{e}$.

We still have to wait for a version (a gem) of the Killip–Simon Theorem 5 in this context.

Many more results are completely proved: Remling's theorem, the Denisov–Rakhmanov–Remling theorem, Lubinsky's work on Christoffel–Darboux kernels, just to mention a few recent results.

Holomorphic functions mapping the upper half plane to itself, a wonderful subject close to the heart of the reviewer, is treated with new and interesting details. These functions have many names, Pick functions, Nevanlinna functions, Herglotz functions. All three names refer to important work of these authors—Simon has chosen the last name. Herglotz functions are important because the Cauchy or Stieltjes transform of a positive measure ρ on the real line

$$m(z) = \int \frac{d\rho(x)}{x-z}, \quad z \in \mathbb{C} \setminus \mathbb{R}$$

is a Herglotz function.

All sections end with some very useful remarks and historical notes.

Concluding remarks.

Simon can write books faster than most people can read them. The quality is very high and the level of scholarship is enormous. I only spotted few misprints: the references to some formula numbers are wrong, a logarithm is missing in Theorem 9.1.1, the invertible linear map T on page 479 maps $\mathbb{C}^2 \setminus \{0\}$ to itself, $\{\infty\}$ has nothing to do here—small misprints that do not shake the reader. One thing I missed, however, is an index of symbols, otherwise the book has lists of subjects and authors, and one can see the page numbers to which a book or paper refer.

The book is recommended to everyone who wants to broaden his or her knowledge about recent developments in orthogonal polynomials. It is a pleasure to see that many areas of mathematics are tied together.

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Wavelet Theory. An Elementary Approach with Applications, David K. Ruch, Patrick J. van Fleet. Wiley Interscience (2009). 480 pp., 10 illus. hardcover, ISBN 978-0-470-38840-2

Decomposing a given object into its primary or essential components lies in the core of Mathematics. This was the driving force also behind the work of J.B.J. Fourier when he tried to describe functions as sums of sines and cosines in his classic treatise *Théorie Analytique de la Chaleur*. Since then, the Fourier (series) theory has become an intrinsic part of several branches of mathematics and of its applications to experimental sciences.

The natural need of analyzing transient data has led the researchers to look for new tools that could provide some information about the change of scale when they observed decomposable events in nature, such as fingerprints or self-similar behavior in iterated processes. They especially needed "bases" of elements with compact support (observe that the sine and cosine functions are not). This leads to a special class of functions, called wavelets (that can be traced back to Haar in the early 20th century as singular examples), which provides a satisfactory answer to the scale problem.

The book by Ruch and van Fleet represents a very self-contained account about the use of this relatively new tool in Mathematics. Probably, it is not the most appropriate text for researchers interested only in a straightforward introduction to this topic; however, the authors provide a very thorough book from the students' point of view.

The monograph contains almost four hundred and fifty exercises distributed in nine Chapters and one Appendix (dedicated to a particular coding). Moreover, 19 out of 63 references come from research papers. All this reassures in the idea that this book was conceived as a textbook. The authors leave us also an indication of the possible ways of using this book in courses. The main criterion for choosing either one is, of course, the level of background of the students. Students without previous knowledge of the Fourier theory will require a two-semester course for an appropriate coverage of the text material. In the case of a standard graduate course, assuming the acquaintance with the definitions and results on Fourier theory, Chapters 5–9 are an excellent content for a semester course in wavelet theory.

The book is divided into the following way: Chapters 1 and 2 are essential, but introductory. They are not the core of the book but collect the necessary tools from a previous course in analysis. The reader could omit these chapters with a risk of missing an important piece of notation that is used through the whole book, as well as some background material on the complex plane $\mathbb{C}$ and the space of square-summable functions $L^2(\mathbb{R})$. Chapter 1 contains the basics of projections on Hilbert spaces. Many proofs are left to the reader, which is a good way to (self)test the understanding of the material. In Chapter 2, we can find a very thorough treatment of both Fourier series and Fourier transform, with the wavelet transform in mind for subsequent chapters.

Chapter 3 is devoted to the Haar spaces generated basically by piecewise constant functions. They are very useful for introducing the Haar wavelet spaces and the discrete Haar wavelet transform.

Chapter 4 is intended to be a collection of examples that show how the previously introduced tools can be used, with a special attention to computer software.

A generalization of the ideas of Haar spaces leads to the multiresolution analysis: this is basically a family of nested subspaces satisfying the theoretical properties defining the Haar spaces. This is the content of Chapter 5.

Chapter 6 cannot be omitted regardless the choice of material you propose for any course on wavelets. The authors recommend that "(it) should be covered if at all possible". Here, they introduce the so-called Daubechies (scaling) functions. As in the rest of the book, this chapter has plenty of examples and exercises that will help the student to succeeding.

Chapter 7 is intended as a “nice change of pace (after chapters 5 and 6)”. The content is devoted to study, in greater detail, the discrete Daubechies wavelet transform (in one and two dimensions), and to apply it to image compression, noise reduction and image segmentation. A special effort is put to deal with the standard difficulties with truncation for the decomposition and reconstruction formulas.

In Chapter 8, the authors build a (biorthogonal) structure called a dual multiresolution analysis that generalizes orthogonality and provides the solution to some shortcomings of the Daubechies wavelets. Riesz bases are a very helpful tool for this generalization.

Chapter 9 provides an alternative wavelet decomposition method, the wavelet packets. They are more complex from a computational point of view but find many applications, just to mention the fingerprint compression method used by the FBI.

The appendix is actually a part of another book by the second author. Readers interested in following the course via internet or finding further resources can visit the Wavelets Webpage at the University of St. Thomas at http://www.stthomas.edu/wavelets.

I am totally convinced that students will find here a very interesting textbook to introduce them in the vast field of wavelets and their applications.

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Generalized Bessel Functions of the First Kind, Á. Baricz. Lecture Notes in Mathematics, Vol. 1194. Springer Verlag, Berlin (2010). xii+200 pp., paperback, ISBN 978-3-642-12229-3

The Bessel functions are very useful in many contexts of mathematics, applied mathematics and engineering and it is essential to study their properties by different points of view. This monograph offers a unified treatment of different properties of Bessel and hypergeometric functions.

The author presents the generalized Bessel functions of the first kind by the point of view of the complex analysis and of the classical analysis.

The book consists of three chapters. The first contains a brief outline of Bessel functions. In particular, the generalized Bessel functions are defined and some of their properties are discussed, such as *recursive formulas*, *differentiation formulas*, *integral representations*.

The starting point of the investigations is the linear differential equation

$$z^2 w''(z) + bzw'(z) + \left(cz^2 + d\right) w(z) = 0, \quad b, c \in \mathbb{C} \tag{1}$$

where

$$d = d_1\nu^2 + d_2\nu + d_3, \quad d_1, d_2, d_3, \nu \in \mathbb{C}.$$

The solution of Eq. (1) can be written in the form

$$w(z) = z^\nu \sum_{n\geq 0} a_n z^n, \tag{2}$$

where $a_n \in \mathbb{C}$, for all $n \geq 0$.

Using a recurrence relation between the coefficient a_n and a_{n-2}, in the power series (2), Eq. (1) can be written in the form

$$z^2 w''(z) + bzw'(z) + \left[cz^2 - \nu^2 + (1-b)\nu\right] w(z) = 0. \tag{3}$$

Eq. (3) generalizes the Bessel equation, the modified Bessel equation, the spherical Bessel equation, the modified spherical Bessel equation. According to a definition by the author, any solution of (3) is called a *generalized Bessel function of order* ν.

The particular solution

$$w_\nu(z) = \sum_{n\geq 0} \frac{(-c)^n}{n!\Gamma\left(\nu + n + \frac{b+1}{2}\right)} \left(\frac{z}{2}\right)^{2n+\nu}, \quad z \in \mathbb{C}$$

is called the *generalized Bessel function of the first kind of order* ν.

Chapter 2 contains geometric properties of generalized Bessel functions. In particular, sufficient conditions of the univalence and convexity involving generalized Bessel functions associated with Hardy space and a monotonicity property of generalized and normalized Bessel functions of the first kind, are also presented.

Chapter 3 is the most substantial of the book. Here the Author presents several functional inequalities for generalized Bessel functions of the first kind, Gaussian and Kummer hypergeometric functions, power series with positive coefficients, ratios of generalized Bessel functions and other functions.

A special emphasis is given to properties of the zeros $j_{\nu,k}$ of Bessel functions. Many known inequalities are presented. These are based on the monotonicity (concavity–convexity) properties of $j_{\nu,k}$ with respect to k or with respect to ν. Unfortunately, here the references are not completely correct. For example, the inequality

$$j_{\nu,n+1} - j_{\nu,n} \geq \pi, \quad |\nu| \geq 1/2, n = 1, 2, \ldots,$$

attributed to Deano et al., was partially proved by A. Laforgia, (*Sugli zeri delle funzioni di Bessel*, Calcolo, 1980, vol. 17, n. 3, 211–220) and by Gori Nocoló–Amati, Laforgia, Moldoon (*Higher monotonicity properties and inequalities for zeros of Bessel functions*, Proc. Am. Math. Soc., 1991, vol. 112, n. 2, 513–520). Also the property $\frac{d}{d\nu} j_{\nu,n} > 1$, for all $\nu > -1$, attributed to Ifantis and Siafarikas was proved by Elbert–Gatteschi–Laforgia (*On the concavity of zeros of Bessel functions*, Applicable Analysis, 1983, vol. 16, 261–278).

This book is a very useful tool for mathematicians, physicists, engineers and anyone works in special functions and their zeros.

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Ramanujan's Lost Notebook. Part II, G.E. Andrews, B.C. Berndt. Cambridge University Press, Cambridge (2008). 288 pp., paperback, ISBN 978-0-521-09061-2, $ 37.00

This book is the collaborative work of three mathematical geniuses, George Andrews, Bruce Berndt, and Srinivasa Ramanujan. It is the second volume of what is broadly counted as Ramanujan's lost notebook, a volume with the unique style of not simply interpreting the entries from *The Lost Notebook and Other Unpublished Papers* [4] but mostly providing deep theories around each particular instance.

The importance of the scientific heritage of Ramanujan is not at all in question. The latest examples of its high influence on contemporary mathematics are the significant achievements in understanding the mock theta functions [3,6] from Ramanujan's last letter to Hardy, and their remarkable applications in number theory and combinatorics.

A fancy term that could characterize Ramanujan's work is "experimental mathematics", due to his exceptional intuitive ability to see patterns where others could hardly expect them. Andrews and Berndt have succeeded in the enormous task of adding rigor to Ramanujan's observations and making his contributions accessible to the public. *Ramanujan's Lost Notebook. Part II* is not simply a collection of entries but a valuable resource for learning fascinating identities and the related proof techniques, as well as remarkable inspiration for further discoveries. This second volume on the lost notebook contains 16 chapters and covers two themes, q-series and Eisenstein series. The first seven chapters are mainly devoted to q-series identities from the core of the original lost notebook: Heine's transformation and other identities for q-hypergeometric and q-bilateral series, triple and quintuple product identities, identities of the Rogers–Ramanujan type and Bailey pairs, partial theta functions, and generalized modular relations. The following three chapters discuss identities for the classical theta functions

$$\varphi(q) = \sum_{n=-\infty}^{\infty} q^{n^2}, \qquad \psi(q) = \sum_{n=0}^{\infty} q^{n(n+1)/2},$$

and more generally

$$\begin{aligned} f(a,b) &= \sum_{n=-\infty}^{\infty} a^{n(n+1)/2} b^{n(n-1)/2} \\ &= \prod_{k=1}^{\infty} (1+a^k b^{k-1})(1+a^{k-1}b^k)(1-a^k b^k), \quad |ab|<1, \end{aligned} \tag{4}$$

and related q-series, Ramanujan's cubic class invariant and elliptic functions. The final six chapters feature the Eisenstein series

$$P(q) = 1 - 24\sum_{n=1}^{\infty} \sigma_1(n) q^n, \qquad Q(q) = 1 + 240\sum_{n=1}^{\infty} \sigma_3(n) q^n,$$
$$R(q) = 1 - 504\sum_{n=1}^{\infty} \sigma_5(n) q^n,$$

where $\sigma_k(n) = \sum_{d|n} d^k$, with much of the material originating in Ramanujan's letters to Hardy from Fitzroy House and Matlock House during his last two years in England.

I find it best to give a taste of the book by providing some particular instances.

Entry 3.6.4 on p. 77 makes the reader familiar with the identity

$$\varphi(q)\left(2\sum_{n=-\infty}^{\infty} \frac{q^{n^2+n}}{1+q^{2n}}\right) - 8\psi(q^2)\left(\sum_{n=1}^{\infty} \frac{q^{n^2}}{1+q^{2n-1}}\right) = \varphi^3(-q)$$

relating the theta functions and (bilateral) q-series. The identity is the special case $k = 1$ of the more general formula, Entry 3.6.5 on p. 78,

$$\sum_{j=0}^{2k-1} (-1)^j \left(\sum_{m=-\infty}^{\infty} q^{km^2+jm}\right) \sum_{n=-\infty}^{\infty} \frac{q^{(2kn+j+1)(2kn+j)/2-kn^2-jn}}{1+q^{2kn+j}} = \varphi^2(-q)\varphi(-q^k).$$

This generalization, although predicted by Ramanujan, did not appear in his notes. As Andrews and Berndt write before formulating the entry, "If we carefully examine the proof [of Entry 3.6.4], we see that the same method of proof yields the following theorem, which we state as an entry, because it most likely is what Ramanujan had in mind".

A different example of reconstructing an identity that is "what Ramanujan had in mind" but did not provide enough terms of, is Entry 4.3.7 on p. 91: for complex numbers a and $b \neq 0$,

$$\sum_{n=0}^{\infty}(-1)^n \frac{(a^2q^2/b;q^2)_n b^n q^{n^2+n}}{(q^2;q^2)_n(-aq;q)_{2n}} = \frac{(bq^2;q^2)_\infty}{(a^2q^2;q^2)_\infty} \sum_{n=0}^{\infty} \frac{(aq;q)_n(a^2q^2/b;q^2)_n(1-a^2q^{4n+2})a^n b^n q^{n(5n+1)/2}}{(q;q)_n(bq^2;q^2)_n}.$$

Here the standard q-notation $(a;q)_0 = 1$ and $(a;q)_n = \prod_{j=1}^{n}(1-aq^{j-1})$ for $n = 1, 2, \ldots$ (including $n = \infty$ whenever $|q| < 1$) is used.

Entry 6.6.1 on p. 147 states that for any complex $a \neq 0$,

$$\sum_{n=0}^{\infty} \frac{(q^{n+1};q)_n q^n}{(-aq;q)_n(-q/a;q)_n} = (1+a)\sum_{n=0}^{\infty}(-1)^n a^n q^{n(n+1)} - \frac{a}{(-aq;q)_\infty(-q/a;q)_\infty}\sum_{n=0}^{\infty}(-1)^n a^{3n} q^{3n^2+2n}(1+aq^{2n+1}).$$

The entry is "just the tip of the iceberg" (in Warnaar's words [5]). The whole of Section 6.6 is devoted to explaining some parts of 'Warnaar's theory' [5], which starts with a remarkable generalization (Theorem 6.6.1 on p. 142) of the Jacobi triple product identity (4): for any complex numbers a and b,

$$(q;q)_\infty(a;q)_\infty(b;q)_\infty \sum_{n=0}^{\infty} \frac{(ab/q;q)_{2n} q^n}{(q;q)_n(a;q)_n(b;q)_n(ab;q)_n} = 1 + \sum_{n=1}^{\infty}(-1)^n a^n q^{n(n-1)/2} + \sum_{n=1}^{\infty}(-1)^n b^n q^{n(n-1)/2}.$$

The reader may also enjoy the history of discovering the identity and its proofs.

In Chapter 7, besides many cute identities with applications to partitions, we find Entry 7.4.1 on p. 168:

$$\sum_{n=0}^{\infty} \frac{(-1)^n q^{n(n+1)/2}}{(-q;q)_n^2} = \sum_{n=0}^{\infty} \frac{(-q)^{n(n+1)/2}}{(-q^2;q^2)_n} - 2\sum_{n=1}^{\infty} \frac{(-1)^n q^{2n^2}}{(-q;q^2)_{2n}},$$

$$\varphi(-q)\sum_{n=0}^{\infty} \frac{q^{n(n+1)/2}}{(q^2;q^2)_n} = \sum_{n=0}^{\infty} \frac{(-q)^{n(n+1)/2}}{(-q^2;q^2)_n} + 2\sum_{n=1}^{\infty} \frac{(-1)^n q^{2n^2}}{(-q;q^2)_{2n}}.$$

The challenge here is to find a proof which is different from the one given by Andrews [1] already in 1984.

Another type of identity relying on a parameterization of q-identities by elliptic integrals is presented in Chapter 10. The fact that the identities are not apparent becomes clear after following the proof by Duke in [2] where he shows that they are equivalent to representations of Eisenstein series of negative weight via hypergeometric series. Two particularly simple instances from p. 233 (see also Proposition 11.8.2 and Corollary 11.8.1 on p. 297) are

$$\sum_{n=0}^{\infty} \frac{(2n+1)^2}{\cosh((2n+1)\pi/2)} = \frac{\pi^{3/2}}{2\sqrt{2}\Gamma^6(3/4)} \quad \text{and}$$

$$\sum_{n=0}^{\infty} \frac{(2n+1)^2}{\cosh^2((2n+1)\pi/2)} = \frac{Q(e^{-2\pi})}{9} = \frac{\pi^2}{12\Gamma^8(3/4)}.$$

In turn, these evaluations are related to representations for the coefficients of certain quotients of Eisenstein series. The corresponding identities are given in Chapter 11 for the series

$$\frac{1}{Q(q)}, \qquad \frac{Q(q)}{R(q)}, \qquad \frac{P(q)}{R(q)}, \qquad \frac{P^2(q)}{R(q)}, \quad \text{and} \quad \frac{P(q)}{Q(q)}.$$

Several chapters in the book indicate numerous relations between theta functions (also in signature 3) and the Eisenstein series $P(q)$, $Q(q)$, and $R(q)$ which are further applied to compute the values of singular moduli. The story 'culminates' in Chapter 15 where these values are used to construct Ramanujan's famous series for $1/\pi$. Prototypes of such formulas are identities like

$$\frac{6\sqrt{n}}{\pi} = P(e^{-2\pi/\sqrt{n}}) + nP(e^{-2\pi\sqrt{n}});$$

the values of $P(q)$ can be expressed via a hypergeometric function (of order 3) and its derivative. The details on how this is done as well as some explicit implications like

$$\frac{16}{\pi} = \sum_{k=0}^{\infty} (42k+5)\binom{2k}{k}^3 \frac{1}{2^{12k}}$$

(Entry 15.6.2 on p. 381) can be found in Section 15.6.

The volume comprises in total $\lfloor 100\pi \rfloor$ entries, excellent reading for both professional and amateur mathematicians. Undoubtedly, the reader will enjoy the book.

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Skew-Orthogonal Polynomials and Random Matrix Theory, S. Ghosh. Chapman & Hall/CRC (2008). 704 pp., ISBN 978-1-58488-956-4, $ 100.00

Random Matrix Theory is concerned with spectral, combinatorial and other aspects of large $N \times N$ matrices with random variables as matrix elements. In the case where their distribution has certain invariance properties, in particular under unitary, orthogonal or symplectic transformations, their study can be reduced to typically N-fold integrals over the matrix eigenvalues. This is the starting point for this short book from the CRM monograph series. It is devoted to compute correlations of eigenvalues using orthogonal polynomials (OP) for the unitary, and skew-orthogonal polynomials (SOP) for the orthogonal and symplectic ensembles. Particular focus is put on how to generalize results valid for classical OP such as the three-step recurrence, and its consequence for their kernel, the so-called Christoffel–Darboux formula (CD).

The author reviews some classical results originating from Gaussian matrices as well as mainly his own approach to SOP for non-Gaussian potentials, both for finite-N and an asymptotic analysis. A generalization of CD is offered, that depends explicitly on the degree of the potential in the weight function, and various aspects and consequences are discussed, in particular for universality.

The author chooses a most explicit, computational approach, with many details spelled out for all symmetry classes, including the dependence on the parity of N. Throughout the chapters examples (and exercises) are given for Hermite, Laguerre, and Jacobi (S)OP, as well as for a weight with an even quartic potential. It is probably this feature that I find most useful for the reader, in particular as the Jacobi and quartic potential case are not covered in Mehta's standard textbook. On the other hand, I would have hoped that the reader would be given a more explicit, at least qualitative idea of the alternative and rigorous universal proofs for the orthogonal and symplectic ensembles by Deift and others, including their precursors in the (incomplete) physics literature. Another topic that is absent is SOP in the complex plane, which has seen much development in the past decade.

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Matrices, Moments and Quadrature with Applications, G.H. Golub, G. Meurant. Cambridge University Press, Cambridge, UK (2009). 470 pp., hardcover, ISBN 978-0-691-14341-5, $ 99.00

In their seminal paper, introducing the conjugate gradient method, Hestenes and Stiefel [3] made it clear already that their method was intimately connected to finite sequences of orthogonal polynomials for a discrete measure, corresponding continued fractions, and Gaussian quadrature. Soon after, in a series of papers collected in [4], Rutishauser stressed the additional connections to the Lanczos algorithm, to certain Hankel matrices that are referred to as moment matrices here, and to his newly found qd algorithm. Working on these topics remained popular in the 1960s and 1970s, though the relevant continued fractions were with time replaced by the (equivalent) series of their convergents, called Padé approximants. One of the highlights was the development by Golub and Welsh [2] of an elegant numerical method for constructing Gaussian quadrature formulas, which was based on an earlier found result requiring the eigenvalues and the first components of the eigenvectors of a Jacobi matrix. This is the tridiagonal matrix containing the recurrence coefficients for the related sequence of orthogonal polynomials; it should not be confused with the Jacobian matrix containing the first partial derivatives of a function of several variables. Ever since this discovery Gene Golub had a special interest in the application of moment matrices, Jacobi matrices, Gaussian quadrature, and the related Lanczos algorithm to solving particular problems in numerical linear algebra. The number of connections and results that have been detected over the following decades is staggering, and many of them can be found in papers that are coauthored by Golub. So it became his wish to collect these results – his and others – in a book. He was lucky to find in Gérard Meurant a coauthor who helped him not only find further results along these lines, but also put together a well readable survey of an enormous pile of material that has been accumulated and is reflected by the 353 references of this book. In a great effort, Gérard Meurant completed the joint work alone after Gene Golub's sudden death in November 2007.

The book is made up of two parts, Theory and Applications. Part 1 on Theory has the nine chapters: 1. Introduction, 2. Orthogonal polynomials, 3. Properties of tridiagonal matrices, 4. The Lanczos and conjugate gradient algorithms, 5. Computation of the Jacobi matrices, 6. Gauss quadrature, 7. Bounds for bilinear forms $u^T f(A)v$, 8. Extensions to nonsymmetric matrices, and 9. Solving secular equations. But not all chapters have equal weight. Of course, Chapters 2 and 3 are fundamental. Chapters 5 and 6 consume even more space, nearly 30 pages each. In particular, Chapter 5 contains long sections on the inverse eigenvalue problem for Jacobi matrices and on the modifications of weight functions, two topics where Gene Golub has coauthored a large number of papers. And Chapter 6 deals with every modification of Gauss quadrature one can think of. In contrast, Chapters 7 and 8 take only five and four pages, respectively.

Part 2 consists of six chapters: 10. Examples of Gauss quadrature rules, 11. Bounds and estimates for elements of $f(A)$, 12. Estimates of norms of errors in the conjugate gradient algorithm, 13. Least squares problems, 14. Total least squares, and 15. Discrete ill-posed problems. This second part differs from the first by the inclusion of many numerical results. In particular, its first chapter contains nearly 20 pages of examples for various ways of computing all kinds of Gauss quadrature formulas, and a few additional pages on inverse eigenvalue problems. The remaining five chapters are different in that they also present accounts of the theory for their topics. Indeed the last three chapters on least squares problems, total least squares, and discrete ill-posed problems provide the reader with nice short introductions into these highly useful areas of numerical linear algebra.

Let me finally come to an assessment of this book. The circle of ideas including orthogonal polynomials, tridiagonal matrices, Gaussian quadrature rules and the Lanczos and conjugate gradient algorithms has long been known as one of the most fascinating nets of connections in numerical analysis. To describe not only these connections, but also the many related algorithms making implicit use of these connections, is an enormous task. The authors must be admired for having been able to describe or allude to so many things on just 363 pages, of which some 100 pages are mainly filled with tables and figures of numerical results. In fact, although the book is well readable, the presentation is mostly quite brief, and understanding all the math requires some extra work with pad and pencil and looking up some references. On the other hand, the authors present or sometimes only sketch most elegant proofs. Necessarily, the notation is often a bit complicated, but the authors avoid to bother with details like index ranges for every formula, which the reader can readily deduce.

Some people may want to read this book as an introduction to a fascinating area of numerical linear algebra; they should be prepared that the red carpet is not fully rolled out, but some work is required to understand the details. Others may want to look up some connections and some additional references; for these the book is most valuable. Yet others might want to consider the book as a basis for student seminars, where the task is to read a chapter, to work out missing details, and to present the whole story; for this, the book will be a perfect source. Less advisable, I think, would be to use the book as a normal textbook; for most students there will be too much to fill in.

Are there any shortcomings? One thing that may puzzle readers is that the notation for the elements of the Jacobi matrices, which are fundamental for this book, is different in Chapters 2, 3, 4, 5, and 6! A different matter is that things one might look for are not covered. In fact, the mentioned circle of ideas contains further closely related elements: most importantly, Padé approximations (or the corresponding special continued fractions), model order reduction in systems and control theory, and perhaps also fast Hankel solvers. The book could have profited from chapters on these topics. They are, for example, well treated in the book by Bultheel and van Barel [1], which, however, is partly written on a higher level of abstraction allowing matrix elements from arbitrary fields or even rings.

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Meshfree approximation methods with Matlab, G.E. Fasshauer. Interdisciplinary Mathematical Science, Vol. 6. World Scientific (2007). 520 pp., (with CD-ROM), hardcover, ISBN 978-981-270-634-8, $ 99.00

Meshfree methods started to become popular in the eighties (of the XX century). The key difference to finite difference/element/volume methods is that the domain of interest needs not to be partitioned into elementary subdomains ("discretized"). We need only to assign nodes or centers of approximation, and eventually, to choose some shape parameters appropriately. If the approximation problem is isotropic in nature, the situation can be simplified further by considering a composition of a univariate function with a vector norm of the argument, turning the problem into a radially symmetric one, in many senses, one-dimensional. There are also reported drawbacks of meshfree methods: their higher computational cost and some instability that certain meshfree methods exhibit.

Probably the best known references for radial basis functions (RBF) are the monographs [1, 2] that contain the rigorous theory. However, both are targeted essentially at mathematicians and researchers in the RBF approximation. However, more and more practitioners become interested in these topics, and the book of Fasshauer intends to close this gap. Being a non-specialist in the field, I tested the book on myself, motivated by my recent interest in the mathematical modeling in ophthalmology. Hence, this is a rather biased review from an intended customer.

The monograph is comprised of 45 short chapters (I found the length of each chapter quite convenient), and as the author explains in the introduction, "forty-seven Matlab programs, one Maple program, one hundred figures, more than fifty tables, and more than five hundred references". I usually agree with Alice, (*"what is the use of a book (...) without pictures or conversation?"*), and I definitely enjoyed the figures. I also enjoyed the programs. As the title indicates, Matlab plays a central role in the whole exposition. A typical logical unit of the book contains a few chapters with the theoretical background followed by its implementation in Matlab. Elegance and programming style are present along the book. For instance, a typical interpolation matrix for the RBF has the form $(\varphi(\|Q_i - Q_j\|))_{i,j}$, where $\varphi : [0, +\infty) \to \mathbb{R}$ is a univariate function and Q_j are the nodes. Taking advantage of the vector manipulation capabilities of Matlab, the author constructs in the very first program of the book the function `DistanceMatrix`, in charge of creating $(\|Q_i - Q_j\|)_{i,j}$, and which becomes a building block to be further evaluated in the appropriate φ. This makes many programs much more transparent.

The focus of the monograph is made on two methods: the RBF and the moving least squares (MLS). In the first 5 chapters we get to the core of the unisolvence theory for the RBF interpolation, covering the needed background from positive definite and completely monotone functions. The interpolation with polynomial precision is covered in Chapters 6–9, followed by the compactly supported RBF.

Chapters 13–15 introduce the main tools for the convergence analysis: the concept of native spaces and the techniques for error bound estimates. This part ends with the discussion of the "trade-off principles" (e.g., for higher accuracy we need to decrease the separation distance, which typically yields a considerable increase in the condition number of the interpolation matrix) and some numerical experiments (that the reader can perform using the scripts included), illustrating the approximation order results and discussing techniques for an appropriate selection of the scaling parameter.

Optimality of the RBF interpolation (Chapter 18) allows us to introduce the least squares approximation with RBF. Here the lack of strong results reflects essentially the state of the art in the theory.

Chapters 22–27 deal with the MLS (both techniques and error bounds), which is another important ingredient of the monograph. This is in a certain sense an alternative to the RBF approximation, where instead of solving a single large system of linear equations we need to solve a large amount of small linear systems. These methods appeared in the approximation theory literature in paper [3], and the name reflects their standard interpretation in terms of the weighted least squares where the weights depend on the point where the function (or data) is approximated. Shepard's method is probably the best known MLS technique due to its simplicity.

Another block of the book introduces us to several ideas (FFT for RBF evaluation, partition of unity, fixed level residual iteration, Hermite interpolation, etc.). Finally, Chapters 38–45 treat the applications of the RBF in the solution of differential equations via the RBF collocation, pseudospectral and Galerkin methods.

In the appendices we find a compendium of useful facts and programs, as well as a catalog of the RBF (classified by their support and their character).

As I mentioned, there are very few proofs in this book, except for the most straightforward or illuminating. For further mathematical details the reader is frequently referred to [2], so those interested in both the theory and the implementation should definitely get the kit Fasshauer–Wendland. However, if you seek, as it was in my case, some fast and practical introduction to the meshfree approximation ideas, from which, as a bonus, you can even borrow some Matlab scripts and functions, either to solve the problem you are interested in or to perform numerical experiments, then this book is just for you.

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Other proceedings and compilations

Differential Algebra, Complex Analysis and Orthogonal Polynomials, P.B. Acosta-Humánez, Francisco Marcellán, eds. Contemporary Mathematics, Vol. 509. AMS, Philadelphia (RI) (2010). 231 pp., softcover, ISBN 978-0-8218-4886-9

FROM THE AMS WEBSITE: This volume represents the 2007–2008 Jairo Charris Seminar in Algebra and Analysis on Differential Algebra, Complex Analysis and Orthogonal Polynomials, which was held at the Universidad Sergio Arboleda in Bogot, Colombia.

It provides the state of the art in the theory of Integrable Dynamical Systems based on such approaches as Differential Galois Theory and Lie Groups as well as some recent developments in the theory of multivariable and q-orthogonal polynomials, weak Hilbert's 16th Problem, Singularity Theory, Tournaments in flag manifolds, and spaces of bounded analytic functions on the unit circle.

The reader will also find survey presentations, an account of recent developments, and the exposition of new trends in the areas of Differential Galois Theory, Integrable Dynamical Systems, Orthogonal Polynomials and Special Functions, and Bloch–Bergman classes of analytic functions from a theoretical and an applied perspective.

The contributions present new results and methods, as well as applications and open problems, to foster interest in research in these areas.

This book is published in cooperation with Instituto de Matemáticas y sus Aplicaciones (IMA).

Recent Trends in Orthogonal Polynomials and Approximation Theory, J. Arvesú, F. Marcellán, A. Martínez-Finkelshtein, eds. Contemporary Mathematics, Vol. 507, AMS, Philadelphia (RI) (2010). 298 pp., softcover, ISBN 978-0-8218-4803-6

FROM THE AMS WEBSITE: This volume contains invited lectures and selected contributions from the International Workshop on Orthogonal Polynomials and Approximation Theory, held at Universidad Carlos III de Madrid on September 8–12, 2008, and which honored Guillermo López Lagomasino on his 60th birthday.

This book presents the state of the art in the theory of Orthogonal Polynomials and Rational Approximation with a special emphasis on their applications in random matrices, integrable systems, and numerical quadrature. New results and methods are presented in the papers as well as a careful choice of open problems, which can foster interest in research in these mathematical areas. This volume also includes a brief account of the scientific contributions by Guillermo López Lagomasino.

Contents:

- F. Marcellán and A. Martínez-Finkelshtein, “Guillermo López Lagomasino: mathematical life”
- B. de la Calle Ysern, “A walk through approximation theory”
- L. Baratchart and M. Yattselev, “Asymptotic uniqueness of best rational approximants to complex Cauchy transforms in L^2 of the circle”
- L. Garza and F. Marcellán, “Quadrature rules on the unit circle. A survey”
- A. Ibort, P. Linares, and J.G. Llavona, “On the multilinear trigonometric problem of moments”
- A.B.J. Kuijlaars. “Multiple orthogonal polynomial ensembles”
- E. Levin and D.S. Lubinsky, “Some equivalent formulations of universality limits in the bulk”
- A. López Garca, “Greedy energy points with external fields”
- A. Martínez-Finkelshtein and E.A. Rakhmanov, “On asymptotic behavior of Heine–Stieltjes and Van Vleck polynomials”
- E.B. Saff, “Remarks on relative asymptotics for general orthogonal polynomials”
- B. Simon, “Fine structure of the zeros of orthogonal polynomials: a progress report”
- H. Stahl, “A potential-theoretic problem connected with complex orthogonality”
- W. Van Assche, “Orthogonal polynomials and approximation theory: some open problems”

Approximation Algorithms for Complex Systems, Proceedings of the 6th International Conference on Algorithms for Approximation, Ambleside, UK, 31st August–4th September 2009, E.H. Georgoulis, A. Iske, and J. Levesley, eds. Springer Proceedings in Mathematics, Vol. 3, Springer Verlag (2011). 307 pp., ISBN 978-3-642-16875-8, $ 149.00

FROM THE SPRINGER VERLAG’S WEBSITE: This book collects up-to-date papers from world experts in a broad variety of relevant applications of approximation theory, including dynamical systems, multiscale modeling of fluid flow, metrology, and geometric modeling to mention a few. The 14 papers in this volume document modern trends in approximation through recent theoretical developments, important computational aspects and multidisciplinary applications. The book is arranged in seven invited surveys, followed by seven contributed research papers. The surveys of the first seven chapters are addressing the following relevant topics: emergent behavior in large electrical networks, algorithms for multivariate piecewise constant approximation, anisotropic triangulation methods in adaptive image approximation, form assessment in coordinate metrology, discontinuous Galerkin methods for linear problems, a numerical analyst’s view of the lattice Boltzmann method, approximation of probability measures on manifolds. Moreover, the diverse contributed papers of the remaining seven chapters reflect recent developments in approximation theory, approximation practice and their applications. Graduate students who wish to discover the state of the art in a number of important directions of approximation algorithms will find this a valuable volume. Established researchers from statisticians to fluid modelers will find interesting new approaches to solving familiar but challenging problems. This book grew out of the sixth in the conference series on “Algorithms for Approximation”, which took place from 31st August to September 4th 2009 in Ambleside in the Lake District of the United Kingdom.

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