To sum up, this book was written with particular applications in mind, and it will no doubt find uses among specialists in differential equations. Otherwise, the wealth of detail and the precision of the error estimates in it go beyond what is generally available in book or monograph form and commend the work to a more general audience.

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M. Holschneider, *Wavelets: An Analysis Tool*, Oxford Mathematical Monographs, Clarendon, Oxford, 1995, xiii + 423 pp.

This monograph provides a solid introduction to the theory of wavelets. Unlike many other introductory texts, which put an emphasis on the discrete wavelet transform, the main focus of this book is the continuous wavelet transform, in the first chapter formally defined as

$$W[g, s](b, a) = \int_{-\infty}^{+\infty} \frac{1}{a}\, \bar{g}\left(\frac{t-b}{a}\right) s(t)\, dt.$$

This transforms a function s over the real line to a function $W[g, s]$ over the upper half-plane $\mathbb{H} = \{(b, a) \mid b \in \mathbb{R},\ a > 0\}$. The wavelet g is assumed to be localized in time and henceforth the wavelet coefficients $W[g, s](b, a)$ analyze the function s at position b with scale a. A formula involving another wavelet h reconstructs a function over the real line from a function over the upper half-plane. The formula leads to a stable inversion of the wavelet transform if the pair g, h satisfies a specified condition. This condition reduces to the wavelet admissibility condition in the case where $g = h$. Results of this type are proved where the functions to be analyzed (reconstructed) are taken from L^2 spaces or spaces of distributions. The analyzing (reconstructing) wavelets are required to be elements of L^2 spaces or spaces of highly localized and regular functions, respectively.

In the second chapter, the author studies (partial) reconstruction of functions over the real line from the wavelet transform on subsets of the upper half-plane $\mathbb{H}$. As an application of the Poisson summation formula, a wavelet analysis over the one-dimensional torus is constructed. It is also demonstrated that the reconstruction of a function from its wavelet coefficients on certain grids in the upper half-plane requires the wavelets under consideration to induce Bessel sequences or frames. The subject of Chapter 3 is multiresolution analysis of several L^2 spaces. Compactly supported orthonormal wavelet bases are constructed from Lagrange interpolation spaces, hence proving a well-known result by Daubechies. In the fourth chapter the connection between local regularity and pointwise differentiability of functions and the behavior of its wavelet transform at small scales is discussed. In this manner, the continuous wavelet transform is used as an analysis tool to study the regularity of a typical trajectory of a Brownian motion, the Riemann–Weierstrass function, and certain dynamical systems. In Chapter 5, the wavelet transform over locally compact groups is treated. Apart from its theoretical value, this chapter also puts the results of the first three chapters in a broader perspective. The chapter ends with an interesting example, the inversion of the Radon transform over the two-dimensional plane. The sixth and last chapter of the book introduces Banach spaces that are characterized by the localization of the wavelet coefficients over the half-plane. In this context, Calderòn–Zygmund operators are discussed and the author refers to the books of Meyer and Coifman for further reading.

The book contains a large number of interesting topics which could not all be mentioned in this review. The text is enriched with instructive, nontrivial examples and illustrations. The volume would increase in value as a reference book if the discussion of the literature references were more systematic and if the index were more exhaustive. The book is accessible to the nonspecialist, but requires close reading at certain points.

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S. J. Gardiner, *Harmonic Approximation*, London Mathematical Society Lecture Notes Series **221**, Cambridge Univ. Press, Cambridge UK, 1995, xiii + 132 pp.

Weierstrass' theorem (1885) that every continuous function on a compact interval can be approximated uniformly by polynomials is now just over a century old. This result can be considered as one of the foundations of polynomial approximation. Possible generalizations are approximation with more general analytic functions and approximation on more general compact sets of the complex plane. Curiously, Runge published in the same year his approximation theorem in a more general setting: if Ω is an open set in the complex plane and $K \subset \Omega$ a compact subset such that $\Omega \backslash K$ has no components which are relatively compact in Ω, then every function f which is analytic (holomorphic) in a neighborhood of K can be approximated uniformly by analytic functions on Ω. This result should be considered as a typical example of holomorphic approximation. Further research in this area is due to Carleman (1927), who showed that for every continuous function $f\colon \mathbb{R} \to \mathbb{C}$ and for every error function $\varepsilon\colon \mathbb{R} \to (0, 1]$ one can find an analytic function g such that $|f-g| \leqslant \varepsilon$ on $\mathbb{R}$. Later, Mergelyan (1952) showed that a continuous function f on a compact set $K \subset \mathbb{C}$, which is analytic on the interior of K, can be approximated uniformly on K by entire functions (and thus also by polynomials) if and only if $\mathbb{C} \backslash K$ is connected. All these results deal with approximation on $\mathbb{R}$ or $\mathbb{C}$. If one replaces analytic functions by harmonic functions and if one looks for approximations on the n-dimensional Euclidean space $\mathbb{R}^n$, then one arrives at *harmonic approximation*, the theme of this book. The starting point of this kind of approximation is the following result by Walsh (1929): if $K \subset \mathbb{R}^n$ is a compact set such that $\mathbb{R}^n \backslash K$ is connected, then every function which is harmonic on an open set containing K can be approximated uniformly by a harmonic polynomial.

This booklet gives a systematic account of harmonic approximation. The theorems by Runge, Mergelyan, Carleman, and Walsh, which we mentioned earlier, are continuously used as an illustration, inspiration, and motivation for the results in harmonic approximation. At the end of each chapter the author gives appropriate credit and references for the results which were given in the chapter, and in addition some more recent developments are pointed out, many of which are by the author.

The first chapter deals with local harmonic approximation using functions which are harmonic on a neighborhood of the compact set on which one wants to approximate. The notion of *thin sets* is very important for this (comparable with the importance of connectedness for holomorphic approximation) and therefore this notion is briefly explained in a preliminary chapter. Then the harmonic analogues of the theorems of Runge and Mergelyan are given. In the second chapter the emphasis is on fusion of harmonic functions, which is a generalization of fusion of rational functions as described by Roth: for every pair of disjoint compact sets K_1 and K_2 in $\mathbb{C}^* = \mathbb{C} \cup \{\infty\}$, there exists a constant C such that for every pair of rational functions r_1, r_2 for which $|r_1 - r_2| < \varepsilon$ on a compact set $K \subset \mathbb{C}^*$ there exists a rational function r with $|r - r_1| < C\varepsilon$ on $K_1 \cup K$ and $|r - r_2| < C\varepsilon$ on $K_2 \cup K$. Chapter 3 starts with Arakelyan's generalization (1968) of Mergelyan's theorem. Then the corresponding result for harmonic approximation is given (Theorem 3.19). During this analysis a more general harmonic version of Runge's theorem from the first chapter is obtained. Carleman's generalization of