Research on flexural stiffness reduction factor of reinforced concrete column with equiaxial + shaped section

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Abstract

The reduced stiffness method had been adopted to evaluate the material nonlinearity characteristics of reinforced concrete structures in compliance with concrete structure standards of the United States, New Zealand and Canada. Concrete structure design code in China also accepts the reduced stiffness method as a supplement of the second-order effects. However, the concrete structure with specially shaped columns code of China still use amplified coefficients of eccentricity to consider nonlinearity characteristics of reinforced concrete structure with special shaped columns. Based on the numerical integral method, a flexural stiffness reduction factor is proposed to consider characteristics of material nonlinearity and geometrical nonlinearity of reinforced concrete columns with equiaxial + shaped section.

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1. Introduction

The reduced stiffness method had been adopted to consider material nonlinearity characteristics of reinforced concrete structures in concrete structure standards of the United States (ACI, 2008), New Zealand (NZS, 2006) and Canada (CSA, 2004). The concrete structure design code of China (Ministry of Construction, 2010) also

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accepts the reduced stiffness method as a supplementary method of considering the second-order effects problem. But they advise that this method should only be used when necessary. However, concrete structure with specially shaped columns code of China (Ministry of Construction, 2006) still use amplified coefficients of eccentricity to consider nonlinearity characteristics of reinforced concrete structure with special shaped columns. Flexural stiffness reduction factor of reinforced concrete columns with special shaped section considering characters of material nonlinearity and geometrical nonlinearity is lacking corresponding research.

In order to simulate the nonlinear full response analysis of reinforced concrete columns with special-shaped section which experience biaxial bending and compression, a computer program (\(M - \Phi\) program) was created by using the numerical integral method using FORTRAN language. Based on the \(M - \Phi\) program, a load and lateral displacement relationship program (\(P - \Delta\) program) was created by using the numerical integral method, and it was verified by the existed test (Huang et al., 2008). It is concluded that the theoretical values of the lateral displacement are consistent with the test values of the lateral displacement, and the method is reliable and can be safely used for the analysis of reinforced concrete columns with special shaped section. Furthermore, a calculating flexural stiffness reduction factor program was also created by using the numerical integral method. Based on the \(M - \Phi\) program and \(P - \Delta\) program, reinforced concrete columns with equiaxial shaped section are analyzed under loading angle of 0°, which is the weakest loading angle of the section (Wang et al., 1999).

As a result, a flexural stiffness reduction factor is proposed to consider characteristics of material nonlinearity and geometrical nonlinearity of reinforced concrete columns with equiaxial shaped section.

2. Principle of analysis

2.1. Basic behavior assumptions

In order to simulate the nonlinear full response analysis of reinforced concrete columns with special shaped section which experience biaxial bending and compression, a computer program was created by using the numerical integral method and the following assumptions:

- Plane sections before bending remain plane sections after bending.
- The tensile strength of concrete is neglected, and the stress-strain curve for concrete in compression reference the kent-park model (Kent et al., 1971), as shown in Figure 1a, whose curve characteristics are as follows:

\[
\sigma_c = \begin{cases} 
AB : k_f' \left[ \frac{2\varepsilon_c}{\varepsilon_{co}} - \left( \frac{\varepsilon_c}{\varepsilon_{co}} \right)^2 \right], & \varepsilon_c \leq \varepsilon_{co} \\
BC : k_f' \left[ 1 - Z(\varepsilon_c - \varepsilon_{co}) \right], & \varepsilon_c > \varepsilon_{co}
\end{cases}
\]  

(1)

Where, \(\varepsilon_{co} = 0.002k\), \(\varepsilon_{cu} = 0.002k + 0.15/Z\), \(k = 1 + \rho_s f_{sh}/f_c'\), and \(Z = \frac{0.5}{\frac{3 + 0.29 f_c'}{145 f_c' - 1000} + 0.75 \rho_s \frac{f_c'}{f_{sh}} - 0.002k}\).

The peak stress \(\sigma_{peak} = k_f'\) happens at \(\varepsilon_{peak} = 0.002k\), \(\varepsilon_{cu} = \) the extreme compression strain of concrete, which is 0.01424, \(\varepsilon_c = \) longitudinal strain in concrete, \(f_c' = \) longitudinal stress in concrete (MPa), \(f_c' = \) concrete compressive cylinder strength (unconfined) (MPa), \(f_c' = 0.8 f_{cu}\), \(f_{cu} = \) concrete compressive cube strength (MPa), \(k_f' = \) the peak concrete stress, \(f_{sh} = \) yield strength of hoop reinforcement (MPa), \(\rho_s = \) ratio of volume of
hoop reinforcement to volume of concrete core measured to outside of the hoops (mm), \( h'' \)=width of concrete core measured to outside of the peripheral hoop (mm), and \( s_h \)=center to center spacing of hoop sets (mm).

- The stress-strain curve for the steel is taken, as shown in Figure 1b, the usual bilinear relationship.
- The spalling from cover concrete is always neglected.
- The section as shown in Figure 2a is divided into a number of small fiber elements. The concrete element stress distribution is also divided into a number of small fiber elements.
- Concrete shrinkage and creep are ignored.

![Fig. 1. (a) stress-strain curve for concrete; (b) stress-strain curve for steel](image)

2.2. Programming principle

The moment-curvature relationships of reinforced concrete columns with special shaped section under biaxial bending and compression can be obtained by adding curvature step-by-step under axial load \( N \) and loading angle \( \alpha \) (\( \alpha \) is shown in Figure 2a and its positive direction is counterclockwise). The specific procedure for adding the curvature is as follows:

- Select distance \( R \) and angle \( \theta \) (See Figure 2b). Then, distance from each reinforcing bar to the neutral axis, and distance of the centroid of each concrete unit from the neutral axis, can be calculated using:

\[
r_i = R - (x'_i \cos \theta + y'_i \sin \theta)
\]  

(2)

Where \( x'_i, y'_i \) are the coordinates of point \( i \).
To determine the section initial curvature $\Phi_0$, do the following. Based on the plane section assumption, concrete stress $\sigma_{ci}$ of point $i$ and reinforcing bar stress $\sigma_{sj}$ of point $j$ are obtained by using the strain equation $\varepsilon_i = \Phi_i r_i$ of point $i$ and stress-strain relationships for concrete and steel. Therefore, the section internal forces $N'$, $M'$ are derived through the equilibrium force condition.

Comparing obtained force $N'$ with $N$, see whether they satisfy the requirement errors. If they do not satisfy the requirement errors, then change the basic parameters of the neutral axis, recalculating until they satisfy the requirement errors. Therefore, $M'$ corresponds to the $M$ value of the reinforced concrete column under axial bending and compression when the value of curvature $\Phi$ is given.

From here, add curvature $\Phi$ and repeat steps 1~3. Obtain the values of $\Phi_u$ and $M_u$ when the maximum compressive strain of the edge in compressive region reaches the extreme strain of concrete.

2.3. Program verification

To verify the reliability of the computer program, the program of the section bearing capacity of the reinforced concrete columns with special shaped section is applied to analyze the test models in the literature (Huang et al., 2008). The theoretical values calculated using the computer program and the test values (See Figure 3).

Fig. 2. (a) Definition of loading angle $\alpha$ and fiber element; (b) calculation models

(a) specimen HT06 (b) specimen HT13
It is concluded that the theoretical values of the moment-curvature relationships agree with the test values of the moment-curvature relationships from Figure 3 and the method is reliable and can be safely used for analysis of reinforced concrete columns with special shaped sections.

3. Analysis procedures for the stiffness reduction factor

Use the following steps when analyzing the stiffness reduction factor:

- Determine the geometric parameters, sectional reinforcement, and concrete strength of the example.
- Select a different axial load level from the example to calculate the moment-curvature relation.
- Determine the force of the horizontal earthquake action, by dividing the example into \( n \) equal portions along the height (See Figure 4a), and obtaining the curvature of each section (Zhu et al., 1985).
- Calculate the lateral displacement value of the top of the column by integrating the curvatures along the height of the column (Park et al., 1975), and the moment considering second-order effects of each section.
- Obtain the curvature by considered second-order effects of each section according to the moment-curvature relation.
- Finally, obtain the reduced stiffness factor based upon the equivalent principle of bending energy.

The internal force of each element and the displacement of each node are calculated from the moment-curvature relation. The final deformation of some column stage is shown in Figure 4, which has total \( n + 1 \) nodes and is divided into \( n \) elements. The average bending stiffness \( (EI)_i \) of the \( i \) element is calculated from the moment of the element and the rotation of the element nodes. Supposing the average moment of the \( i \) element is \( M_i \), and the rotation of two ends of the element is \( \Delta \varphi_i \) and the length of the element is \( \Delta s_i \), then the average curvature can be written as:

\[
\bar{k}_i = \frac{\Delta \varphi_i}{\Delta s_i}
\]  

(3)
By the moment-curvature relation, the average bending stiffness $(\overline{EI})_i$ of the $i$ element can be written as follows:

$$\overline{EI}_i = \overline{M}_i / \overline{k}_i$$

(4)

Bending deformation energy of some column stage can be written as follows:

$$E_b = \frac{1}{2} \sum_{i=1}^{n} \overline{M}_i \Delta \varphi_i = \frac{1}{2} \sum_{i=1}^{n} (\overline{EI}_i) \overline{k}_i^2 \Delta s_i$$

(5)

Supposing the bending stiffness of some equivalent section of elastic pole is $(EI)^*$, which has the same bending deformation energy of the original column stage in the same forming process condition. It can be written as follow:

$$E_b = \frac{1}{2} \sum_{i=1}^{n} (EI)^* \overline{k}_i^2 \Delta s_i$$

(6)

$(EI)^*$ can be calculated from equation (5) and equation (6). The equivalent bending stiffness reduction factor $\alpha_e$ can be shown as following:

$$\alpha_e = \frac{(EI)^*}{EI} = \frac{\sum_{i=1}^{n} (\overline{EI}_i) \overline{k}_i^2 \Delta s_i}{EI \sum_{i=1}^{n} \overline{k}_i^2 \Delta s_i}$$

(7)

4. Calculation example analysis for the stiffness reduction factor

The calculation model column has an equiaxial + shaped section with 600mm×600mm×200mm. The effective standard length of the column is 2900.0 mm (Ministry of Construction, 2006). The concrete is C30, which has a cubic strength of 14.3N/mm² and a modulus of elasticity of $3.0 \times 10^4$ N/mm². The main steel is 12φ22 of HRB400, and has a yield strength of 360.0 N/mm² and a modulus of elasticity of $2.0 \times 10^5$ N/mm². The hoop bar is Φ10 of HPB235, spaced at 100.0 mm, and has a yield strength of 300.0 N/mm² and a modulus of elasticity of $2.0 \times 10^5$ N/mm². The thickness of concrete cover is 30 mm.

For loading angle of 0° is the weakest loading angle of the section (Wang et al., 1999), and the calculated stiffness reduction factors under the loading angle of 0° can be safely used for reinforced concrete columns with equiaxial + shaped section.

Based on the equation (7), the stiffness reduction factors of reinforced concrete columns with equiaxial + shaped section are laid out in Table 1.

<table>
<thead>
<tr>
<th>Loading Angle</th>
<th>Axial Load Level</th>
<th>Stiffness Reduction Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0.10</td>
<td>0.4445</td>
</tr>
</tbody>
</table>
0.15  0.4401
0.20  0.4311
0.25  0.4216
0.30  0.4110
0.35  0.3994
0.40  0.3745
0.45  0.2781

5. Conclusions

The following conclusions can be drawn from this study. First, the theoretical results concerning the flexural stiffness reduction factor of reinforced concrete column with equiaxial + shaped section are in agreement with the test results. Second, the axial load level does have an influence on the stiffness reduction factor. Third, the suggested stiffness reduction factor of reinforced concrete column with equiaxial + shaped section and axial load level below 0.45 has an average value of 0.4, which is calculated under the weakest loading angle of the section.

References

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