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Radiative heat transfer of variable viscosity and thermal conductivity effects on inclined magnetic field with dissipation in a non-Darcy medium

S.O. Salawu*, M.S. Dada

Department of Mathematics, University of Ilorin, Ilorin, Nigeria

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Abstract

The study of thermal radiative heat transfer of an electrically conducting fluid over a continuously stretching sheet in the presence of a uniform inclined magnetic field with dissipation in a porous medium is investigated for power-law variation in the sheet temperature. The fluid viscosity and thermal conductivity are assumed to vary as a function of temperature. The governing partial differential equations of the model are reduced to a system of coupled non-linear ordinary differential equations by applying similarity variables and then solved numerically using shooting technique with fourth-order Runge–Kutta method. The results for Skin friction and Nusselt numbers are presented and discussed.

Keywords: Radiation; Variable viscosity; Dissipation; Thermal conductivity; Non-Darcy medium

1. Introduction

Studies on heat transfer in boundary layers over continuously moving or stretching sheet on free convection flow induced by the simultaneous actions of uniform inclined magnetic field and buoyancy forces resulting from variable viscosity and thermal conductivity is gaining attention due to its interesting applications in glass fibre, metal extrusion, materials handling conveyors, plastic and rubber manufacturing.

In few of these applications, Saleh et al. [1] considered heat and mass transfer in MHD visco-elastic fluid flow through a porous medium over a stretching sheet with chemical reaction. Krishnendu [2] studied heat transfer in boundary layer stagnation-point flow towards a shrinking sheet with non-uniform heat flux. It was reported that, the direct variation and inverse variation of heat flux along the sheet have completely different effects on the temperature distribution. Makinde [3] carried out analysis on heat and mass transfer by MHD mixed convection stagnation point flow toward a vertical plate embedded in a highly porous medium with radiation and internal heat generation and Siti et al. [4] investigated hydromagnetic boundary layer flow over stretching surface with thermal radiation. It was

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* Corresponding author.
E-mail addresses: kunlesalawu@yahoo.com (S.O. Salawu), dadamsa@gmail.com (M.S. Dada).

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Nomenclature

x:	Distance along the sheet, m
y:	Distance perpendicular to the sheet, m
u:	Non Darcian Velocity component in x-direction, m s^{-1}
v:	Non Darcian Velocity component in y-direction, m s^{-1}
g:	Acceleration due to gravity, m s^{-2}
B_0:	Magnetic induction, Wb m^{-2}
q^r:	Radiative heat flux, W m^{-2}
Q:	Heat source/sink, W
T:	Fluid temperature, K
C_p:	Specific heat at constant pressure, J kg^{-1} K^{-1}
k:	Thermal conductivity, W m^{-1} K^{-1}
u_w:	Fluid velocity at the wall, m s^{-1}
K^*:	Permeability of the porous medium
A, b, m:	Prescribed constants
H_a:	Hartmann number
D_a:	Darcy number
G_r:	Thermal Grashof number
E_c:	Eckert number
R:	Radiation parameter
P_r:	Prandtl number
f:	Dimensionless stream function

Greek symbols

ν:	Kinematic viscosity, m^2 s^{-1}
α:	Angle of inclination of the magnet, deg
μ:	Viscosity, kg m^{-1} s^{-1}
ρ_∞:	Free stream density, kg m^{-3}
β_T:	Thermal expansion coefficient, K^{-1}
σ:	Electric Conductivity, mΩ m^{-1}
ψ:	Stream function, m^{-2} s^{-1}
η:	Similarity variable
ϕ:	Viscosity parameter
λ:	Heat source/sink parameter
φ:	Porous media inertia coefficient

noticed that the magnetic parameter decreases the skin friction coefficient thus reduces the momentum boundary layer thickness. Mohebujjaman et al. [5] considered MHD heat transfer mixed convection flow along a vertical stretching sheet in the presence of magnetic field with heat generation while Dulal [6] investigated heat and mass transfer in stagnation-point flow in viscous fluid over a stretching vertical sheet by considering buoyancy force and thermal radiation. Singh & Makinde [7] analyzed computational dynamics of MHD free convection flow along an inclined plate with Newtonian heating in the presence of volumetric heat generation. All the above investigations were carried out on fluid flow without considering viscosity and thermal conductivity as a function of temperature.

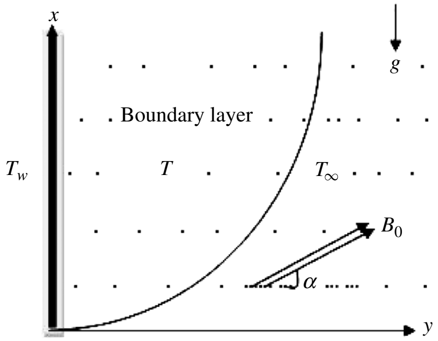
The pioneering work of sakiadis [8] considered momentum transfer for a flow over a continuously moving plate in quiescent fluid; Mureithi et al. [9] studied boundary layer flow over a moving surface in a fluid with temperature-dependent viscosity. Makinde et al. [10] reported on the MHD variable viscosity reacting flow over a convectively heated plate in a porous medium with thermophoresis and radiative heat transfer. It was found that skin friction is lower and Nusselt number is higher when the viscosity is temperature dependent. Abel & Mahesha [11] investigated heat transfer in MHD viscoelastic fluid flow over a stretching sheet with variable thermal conductivity, non-uniform heat source and radiation. From the analysis, it was found that the effect of radiation is to accelerate the heat transfer. Thus radiation should be at its minimum in order to facilitate the cooling process. Makinde [12] studied the effect of variable viscosity on thermal boundary layer over a permeable flat plate with radiation and a convective surface boundary condition. It was noticed that thermal boundary layer thickens with a rise in the local temperature as the viscous dissipation, wall injection, and convective heating each intensifies, but decreases with increasing suction and thermal radiation.

A reasonable interest has been shown on the study of variable viscosity and thermal conductivity effects on flow. Heat transfer processes are of major importance in controlling heat transfer in the production of quality product as it depends on the heat controlling factor, the space technology and high temperature processes. As a result, Makinde & Ogulu [13] studied the effect of thermal radiation on the heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate permeated by a transverse magnetic field. Gitima [14] examined the effect of variable viscosity and thermal conductivity of micropolar fluid in a porous channel in presence of magnetic field. To describe the radiative heat flux in the energy equation Rosseland diffusion approximation was used. Ali et al. [15] reported on natural convection radiation interaction in boundary layer flow over semi infinite horizontal surface. Hazarika et al. [16] investigated the effects of variable viscosity and thermal conductivity on MHD flow past a vertical plate. It was observed that the velocity profile increases with the decreases of thermal conductivity parameter while Dulal & Hiranmoy [17] investigated the effects of temperature-dependent viscosity and variable thermal conductivity on MHD non-Darcy mixed convective diffusion of species over a stretching sheet. Hunegnaw & Naikoti [18] examined MHD effects on heat transfer over stretching sheet embedded in porous medium with variable viscosity, viscous dissipation and heat source/sink.

Following the above studies, most of the authors neglected the influence of inclined magnetic field and buoyancy force on a temperature dependent viscosity and thermal conductivity with the power-law variation in the sheet temperature. Therefore, the present study examining temperature dependent viscosity and thermal conductivity over a stretching sheet with variable surface temperature subjected to buoyancy force and inclined uniform magnetic field in a non-Darcy porous medium with radiation and viscous dissipation. However, it is known that these physical properties can change significantly with temperature. When the effects of variable viscosity and thermal conductivity are taken into account, the flow characteristics are significantly changed compared to constant physical properties. Hence, in the problem under consideration, the viscosity and thermal conductivity have been assumed to be inverse linear function of temperature.

2. Mathematical formulation

A free convective heat transfer of two dimensional flow of an electrically conducting, steady, viscous, laminar and incompressible fluid flow above a heated stretching sheet with radiation and dissipation in a porous medium under the influence of uniform inclined magnetic field, buoyancy force, variable viscosity and thermal conductivity are considered. The flow is assumed to be in the x-direction with y-axis normal to it. The magnetic field of uniform strength B_0 is introduced at angle α lying in the range $0 \prec \alpha \prec \frac{\pi}{2}$ in the direction of the flow. The stretching sheet is fixed by two equal and opposite forces introduced along the x-axis. The plate is maintained at the temperature T_w and free stream temperature T_∞ respectively. The geometry and governing equations are:



Geometry of the problem

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\mu}{\rho_\infty K^*}u-Cu^2-\frac{1}{\rho_\infty}\sigma B_0^2 u\sin^2\alpha+g\beta_T\left(T-T_\infty\right) \tag{2}$$

$$\rho_\infty C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}-q^r\right)+\mu\left(\frac{\partial u}{\partial y}\right)^2+Q\left(T-T_\infty\right) \tag{3}$$

subject to the following boundary conditions:

$$\begin{aligned}&u=u_w(=bx),\qquad v=0,\qquad T=T_w(=T_\infty+Ax^m)\quad \text{at } y=0\\&u=0,\qquad T=T_\infty\quad \text{as } y\to\infty.\end{aligned} \tag{4}$$

Using Rosseland diffusion approximation [19,20] for radiation

$$q^r=-\frac{4\sigma}{3\delta}\frac{\partial T^4}{\partial y} \tag{5}$$

where σ and δ are the Stefan–Boltzmann and the mean absorption coefficient respectively, assume the temperature difference within the flow are sufficiently small such that T^4 may be expressed as a linear function of temperature, using Taylor series to expand T^4 about the free stream T_∞ and neglecting higher order terms, this gives the approximation

$$T^4\cong 4T_\infty^3T-3T_\infty^4. \tag{6}$$

Using Eq. (6), Eq. (5) can be express as.

$$\frac{\partial q^r}{\partial y}=-\frac{16\sigma T_\infty^3}{3\delta}\frac{\partial^2 T}{\partial y^2}. \tag{7}$$

The viscosity is assumed to vary as a reciprocal of a linear function of temperature [21].

$$\begin{aligned}&\frac{1}{\mu}=\frac{1}{\mu_\infty}[1+\gamma(T-T_\infty)]\\&\text{or}\quad \frac{1}{\mu}=s(T-T_r)\end{aligned} \tag{8}$$

where $s=\frac{\gamma}{\mu_\infty}$ and $T_r=T_\infty-\frac{1}{\gamma}$.

Both s and T_r are constants which depend on the reference state and the thermal property of the fluid, where $s\succ 0$ for liquids and $s\prec 0$ for gases.

The fluid thermal conductivity, k, is assumed to vary as a linear function of temperature in the form [22],

$$k=k_\infty(1+n\theta) \tag{9}$$

where $n=\frac{(k_w-k_\infty)}{k_\infty}$, is the thermal conductivity parameter.

Introducing the stream function $\psi(x,y)$ with similarity transforms

$$\psi=(b\nu)^{\frac{1}{2}}xf(\eta),\qquad \eta=\left(\frac{b}{\nu}\right)^{\frac{1}{2}}y \tag{10}$$

where the velocity components and temperature respectively become

$$u=\frac{\partial\psi}{\partial y}=bxf'(\eta),\qquad v=-\frac{\partial\psi}{\partial x}=-(b\nu)^{\frac{1}{2}}f(\eta)\quad\text{and}\quad\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}. \tag{11}$$

Using Eqs. (7)–(11) in the governing equations, the continuity equation is satisfied while Eqs. (2) and (3) become

$$f'''-\frac{\theta-\phi}{\phi}ff''-\frac{1}{\theta-\phi}\theta'f''+\frac{\theta-\phi}{\phi}(1+\varphi)f'^2-D_af'+\frac{\theta-\phi}{\phi}H_a^2\sin^2\alpha f'-\frac{\theta-\phi}{\phi}G_r\theta=0 \tag{12}$$

Table 1
Comparison of $\theta'(0)$ for $K_1 = 0$, $H_a = 0$, $n = 0$, $\phi \longrightarrow -\infty$, $E_c = 0$, $R = 0$, $G_r = 0$, $D_a = 0$, $\varphi = 0$ for various values of m, λ and P_r.

λ	m	P_r	Grubka and Bobba [23]	Ahmed [24]	Present results
0	0	0.7	–	−0.456052	−0.456049
0	0	1.0	−0.5820	−0.582229	−0.582225
0	0	10.0	−2.3080	−2.308000	−2.307950
−1.0	2	5.0	–	−4.028226	−4.028228

$$\left(1+\frac{4}{3}R\right)(1+n\theta)\theta'' - P_r E_c \frac{\phi}{\theta-\phi}(f'')^2 + n(\theta')^2 + P_r f\theta' - P_r(mf'-\lambda)\theta = 0. \tag{13}$$

The corresponding boundary conditions are.

$$\begin{aligned} &f'(0) = 1, \qquad f(0) = 0, \qquad \theta(0) = 1 \\ &f'(\infty) = 0, \qquad \theta(\infty) = 0 \end{aligned} \tag{14}$$

where $\lambda = \frac{Q}{\rho_\infty C_p b}$, $H_a^2 = \frac{\sigma B_0^2}{\rho_\infty b}$, $D_a = \frac{\nu}{K^* b}$, $\varphi = Cx$, $G_r = \frac{g\beta_T(T_w - T_\infty)}{b^2 x}$, $E_c = \frac{u_w^2}{C_p(T_w - T_\infty)}$, $R = \frac{4\sigma T_\infty}{\delta k_\infty}$, $P_r = \frac{\mu_\infty C_p}{k_\infty}$, $\phi = \frac{T_r - T_\infty}{T_w - T_\infty} = -\frac{1}{\gamma(T_w - T_\infty)}$ and $\frac{1}{\mu} = s(T - T_r)$ takes the form $\mu = \frac{\mu_\infty}{(1-\theta\phi^{-1})}$.

The largest variation in the fluid viscosity from its free stream value μ_∞, occur at the surface of the plate when $\mu = \frac{\mu_\infty}{(1-\phi^{-1})}$ where ϕ is negative for liquids and positive for gases. From the expansion, as $-\phi \to \infty$, $\mu \to \mu_\infty$, that is the viscosity variation in the boundary layer is negligible while as $-\phi \to 0$ the viscosity variation increases significantly.

The physical parameters of interest for this flow are the local skin friction C_f and the Nusselt number N_u given respectively as:

$$C_f = \frac{\tau_w}{\rho_\infty u_w^2/2}, \qquad Nu = \frac{xq_w}{k_\infty(T_w - T_\infty)} \tag{15}$$

τ_w and q_w are respectively given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \qquad q_w = k\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{16}$$

Using similarity transforms in Eqs. (10) and (11), as well as variable viscosity at the plate surface $\mu = \frac{\mu_\infty}{(1-\phi^{-1})}$ and thermal conductivity at the plate surface $k = k_\infty(1+n)$, the local skin friction and Nusselt number respectively becomes

$$C_f = \frac{2\frac{\mu_\infty}{(1-\phi^{-1})} b\left(\frac{bx^2}{\nu}\right)^{1/2}}{\mu_\infty b\left(\frac{bx^2}{\nu}\right)} f''(0), \qquad Nu = \frac{k_\infty(1+n)\left(\frac{bx^2}{\nu}\right)^{1/2}(T_w - T_\infty)}{k_\infty(T_w - T_\infty)}\theta'(0). \tag{17}$$

Hence, the set of Eqs. (17) reduces to,

$$Re_x^{\frac{1}{2}} C_f = \frac{2\phi}{\phi - 1} f''(0) \tag{18}$$

$$Nu_x Re_x^{-\frac{1}{2}} = (1+n)\theta'(0) \tag{19}$$

where $Re_x = \frac{u_w x}{\nu}$ is the Reynolds number.

3. Results and discussion

The coupled non-linear differential equations along with the boundary conditions of Eqs. (12)–(14) are solved numerically using shooting technique with fourth-order Runge–Kutta integration algorithm. In the numerical solution,

Table 2
Comparison of f'' and $\theta'(0)$ for $K_1 = 0$, $H_a = 0.01$, $\alpha = 90^0$, $E_c = 0.2$, $R = 0$, $G_r = 0$, $\phi = -1$, $m = 1$, $\lambda = 0.2$ and $P_r = 3$ for variation in n.

n	Ahmed [24]		Present results
	f''	θ'	
0.0	1.54911	–	−1.549106
0.1	1.54343	–	−1.543431
0.0	–	1.44416	−1.444162
0.1	–	1.34861	−1.348612

Table 3
Variation of $f''(0)$ and $\theta'(0)$ at the sheet with H_a, α, ϕ, E_c, m and n parameters.

PP	Values	$f''(0)$	$\theta'(0)$	PP	Values	$f''(0)$	$\theta'(0)$
Ha	1	−1.59419	−1.33337	E_c	0.0005	−3.03273	−1.05530
	2	−1.94224	−1.19810		0.035	−3.02659	−0.99919
	3	−2.43065	−0.99570		0.07	−3.02044	−0.94271
	4	−2.99824	−0.73662		0.5	−2.95052	−0.28148
α	15^0	−1.97298	−1.18584	m	0.5	−2.93932	−0.38291
	30^0	−2.99824	−0.73662		1.0	−2.99824	−0.73662
	45^0	−4.02276	−0.19831		2.0	−3.09174	−1.31752
	60^0	−4.84173	0.28576		3.0	−3.16521	−1.79632
ϕ	−0.5	−3.62337	−0.68965	n	0.007	−3.00831	−0.78044
	−1.0	−2.99824	−0.73662		0.2	−2.98849	−0.69589
	−2.0	−2.61146	−0.77019		0.45	−2.96797	−0.61493
	7.0	−1.95308	−0.83592		1.0	−2.93590	−0.49814

a check was made to confirm that smoothness conditions at the edge of the boundary layer were satisfied. Calculations were carried out for different values of the parameters. The following default parameter values are adopted for computation: $G_r = m = \varphi = 1$, $n = 0.1$, $\phi = -1$, $R = 0.01$, $\lambda = E_c = k = D_a = 0.2$, $P_r = 3$, $H_a = 4$ and $\alpha = 30^0$.

Tables 1 and 2 show the numerical results, which illustrate the effect of physical parameters on the flow field and heat transfer aspects of the present study compared with the existing studied.

Table 3 represents the numerical results, which represent the effect of some physical parameters on the flow with heat transfer aspect of the investigation. It shows that an increase in the value of the parameters H_a, α and m decreases the skin friction and causes increase in the temperature gradient at the wall while increase in the value of ϕ, E_c and n causes corresponding increase in the skin friction and the temperature gradient at the wall but ϕ decrease the temperature gradient at the wall.

Fig. 1 shows the effects of imposition of magnetic field on the fluid flow. Increases in the values of H_a exerted a retarding force on the fluid velocity and make it warmer as it moves along the plate by causing decrease in the velocity profiles. This resulted in decreasing skin friction parameter and increasing the temperature gradient.

Figs. 2 and 3 represent the influence of inclination of the magnetic field on the velocity and temperature respectively. It is noticed that an increase in the angle of inclination of the magnetic field increases the intensity of the buoyancy force and consequently the driving force to the fluid flow decreases as a result, velocity boundary layer thickness reduces and causes decrease in the velocity profiles while the temperature boundary layer thicker and causes increase in the temperature distribution.

The effect of Grashof numbers G_r for heat transfer on the velocity profiles is presented in Fig. 4. It is seen that an increase in the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer accelerate the velocity of the flow field. Thus, heat transfer has a strong influence on the flow field.

Figs. 5 and 6 illustrate the effects of porosity parameter D_a and inertial parameter φ on the velocity profiles. It is observed that the velocity decreases as the porosity and inertial parameter increases, this is because the wall of the surface provides an additional resistance to the fluid flow mechanism which causes the fluid to move at a retarded rate.

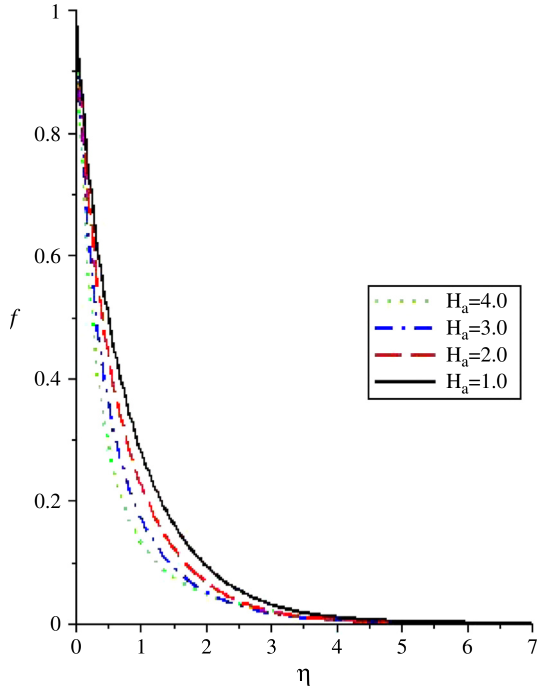


Fig. 1. Velocity profiles for different values of H_a.

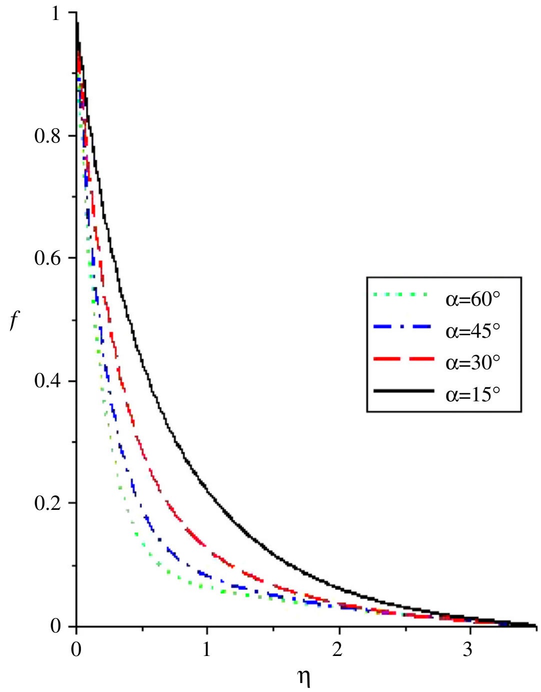


Fig. 2. Velocity profiles for different values of α.

The effect of viscosity parameter on the velocity and temperature profiles are represented in Figs. 7 and 8. Increase in the value of ϕ increases the velocity profile because the skin friction parameter increases and while the temperature profiles decreases as viscosity parameter increases due to thinning of the thermal boundary layer.

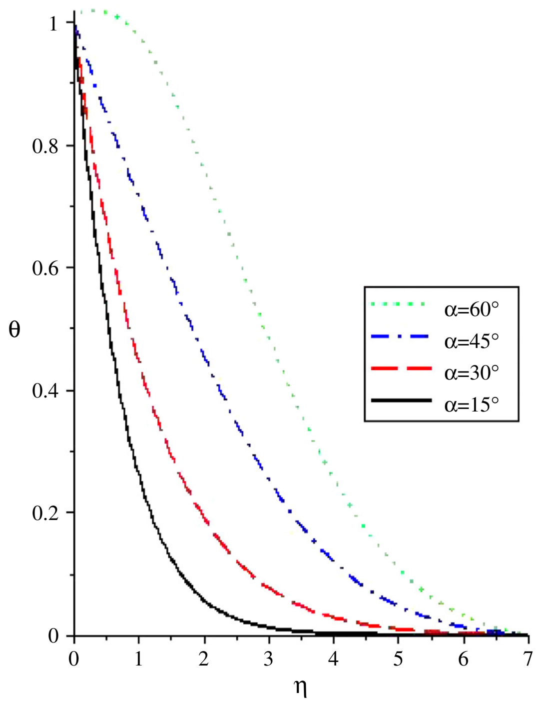


Fig. 3. Temperature profiles for different values of α.

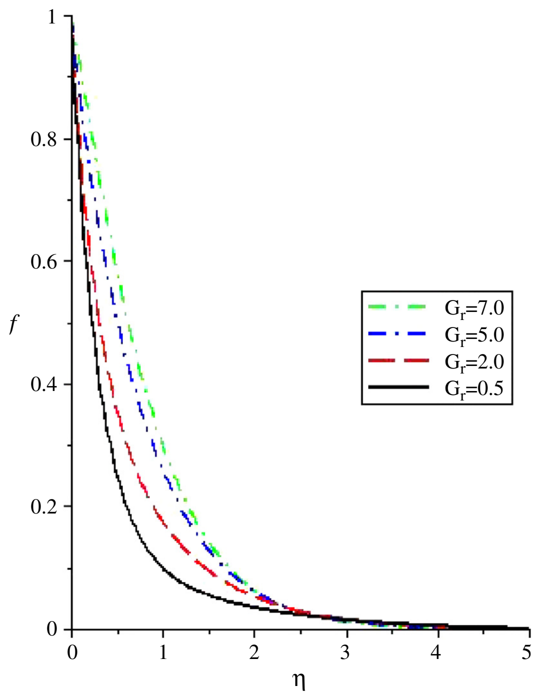


Fig. 4. Velocity profiles for different values of G_r.

The influence of heat source parameter λ and the thermal conductivity parameter n on the heat transfer are shown in Figs. 9 and 10. It is clearly seen from the profiles that an increases in the heat source and thermal conductivity parameter affected the boundary layer to generate energy which causes the temperature to increase.

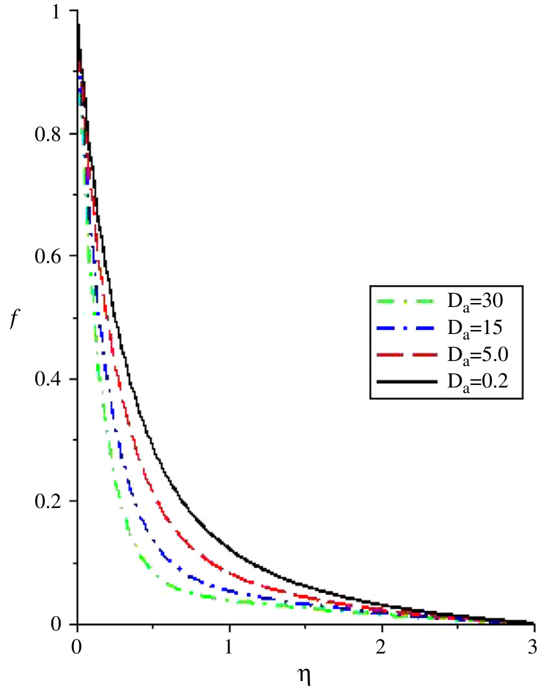


Fig. 5. Velocity profiles for different values of D_a.

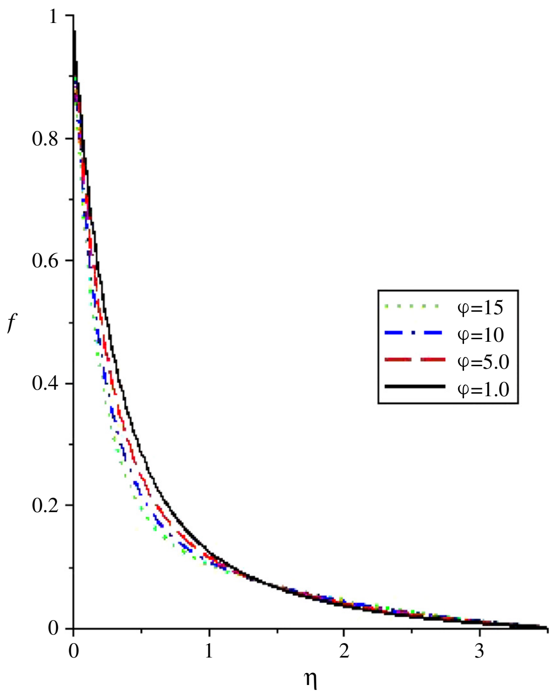


Fig. 6. Velocity profiles for different values of φ.

Fig. 11 shows the temperature profiles with variation in the values of m. It is noticed that an increase in values m causes the temperature profiles to increase. Heat flows from the stretching sheet into the ambient medium when $m \succ 0$,

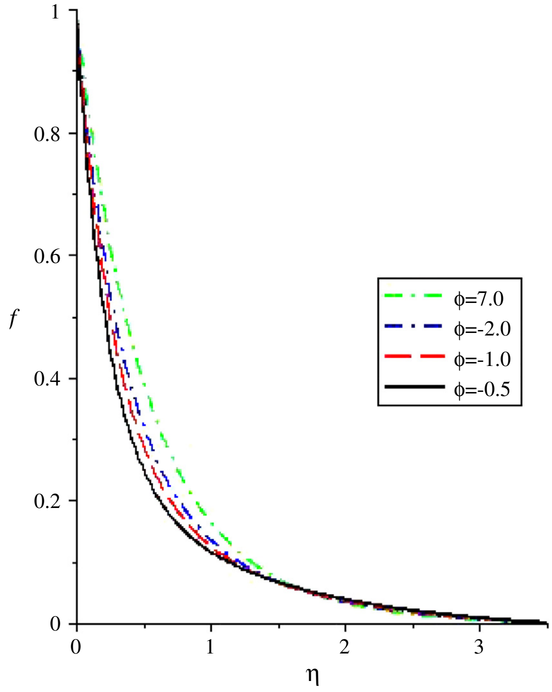


Fig. 7. Velocity profiles for different values of ϕ.

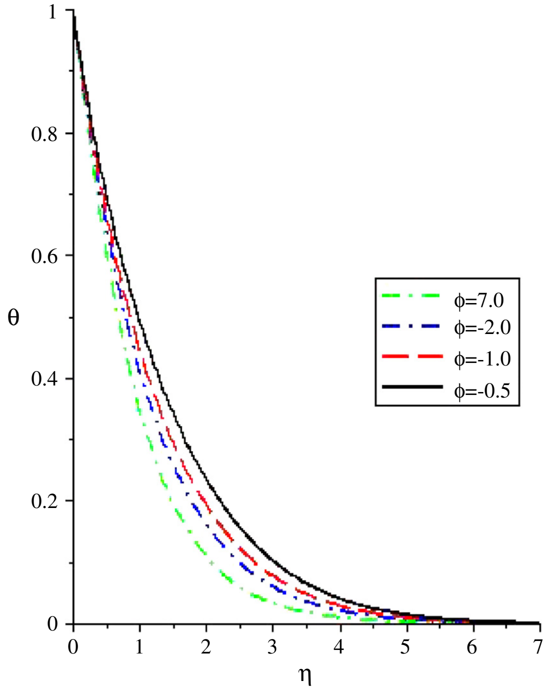


Fig. 8. Temperature profiles for different values of ϕ.

otherwise it flows from the ambient medium to the stretching sheet. Overshoot in the temperature is experienced because fluid particle moves from the heated wall temperature to where the wall temperature is lower.

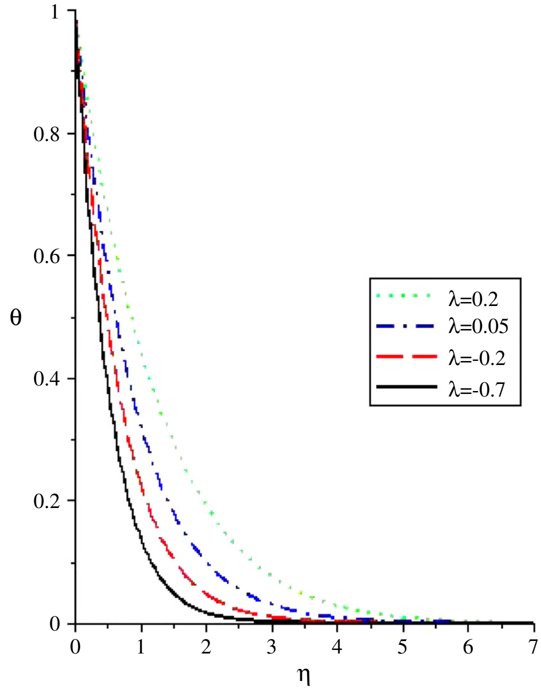


Fig. 9. Temperature profiles for different values of λ.

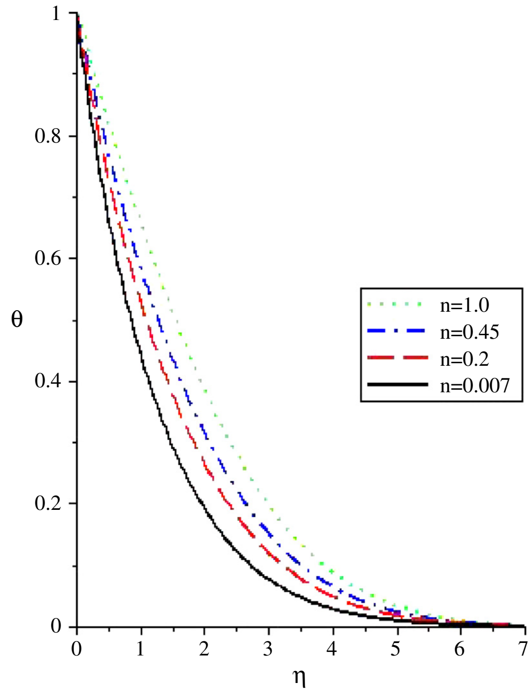


Fig. 10. Temperature profiles for different values of n.

Fig. 12 demonstrates the temperature profiles with various values of Prandtl number. The prandtl number represents the ratio of momentum diffusivity to thermal diffusivity which justify the fact that an increase P_r causes decrease

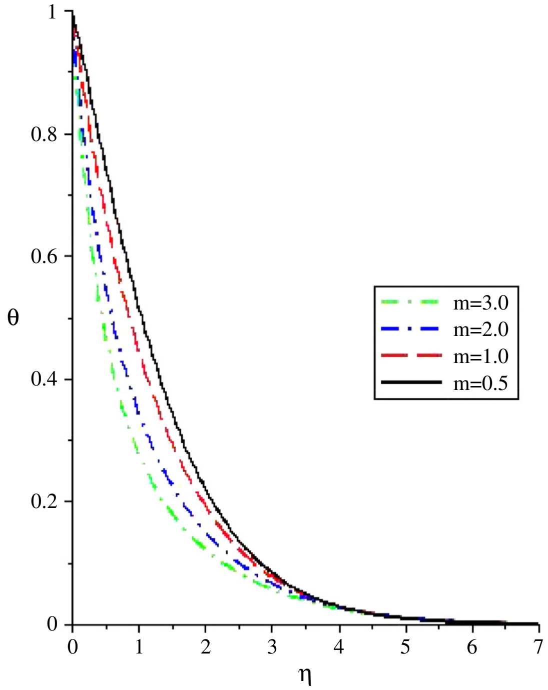


Fig. 11. Temperature profiles for different values of m.

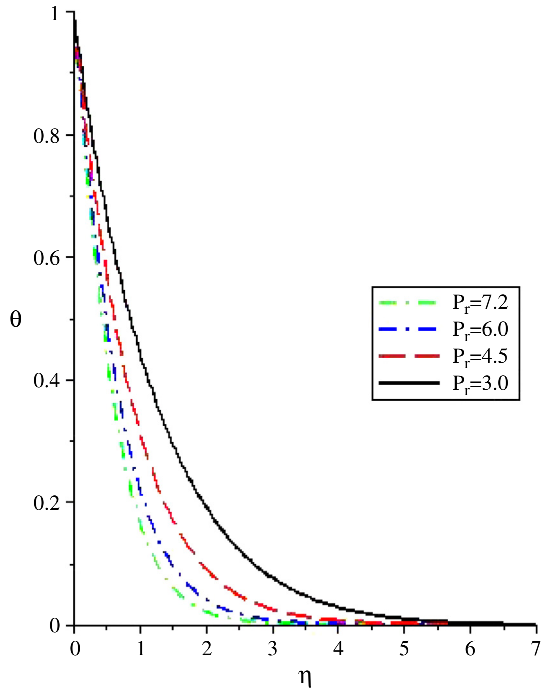


Fig. 12. Temperature profiles for different values of P_r.

in temperature profiles as shown in Fig. 12. This is because the boundary layer thinner and reduces the average temperature within the boundary layer.

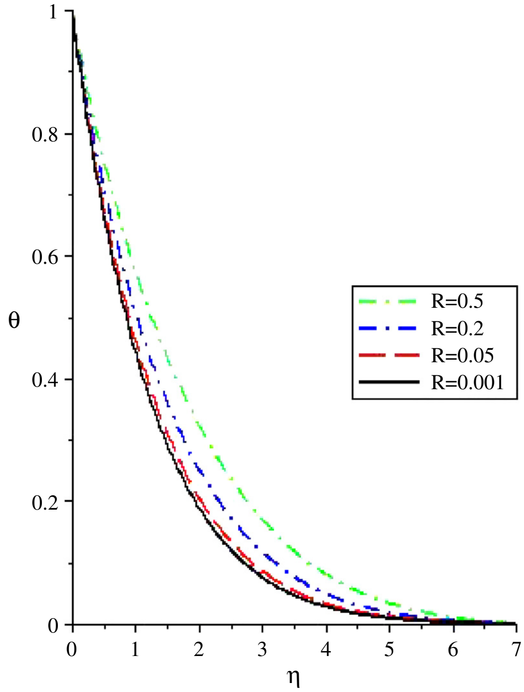


Fig. 13. Temperature profiles for different values of R.

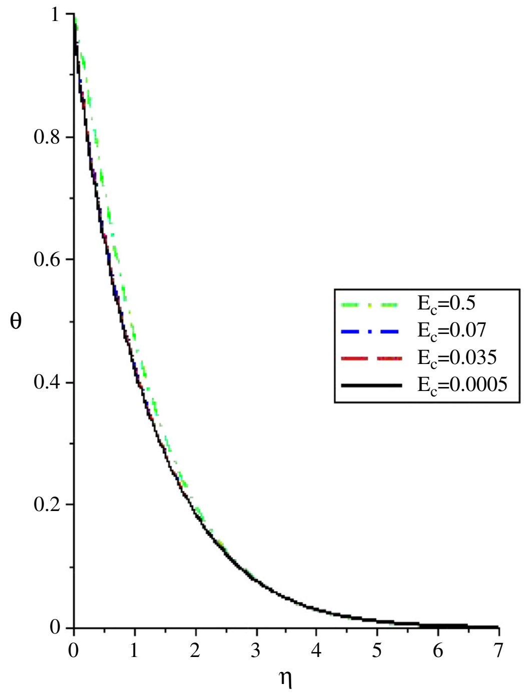


Fig. 14. Temperature profiles for different values of E_c.

The effect of radiation on the temperature profiles is illustrated in Fig. 13. It is observed that as the value of R increases, there is corresponding increase in the temperature profiles this results in an increase in the thermal boundary layer thickness. Also, the influence of dissipation function parameter is illustrated in Fig. 14. It is evidence that an increase in Eckert number enhance the temperature at any point, this can be notice in Fig. 14 as the temperature profiles increases.

4. Conclusion

The effects of variable viscosity and thermal conductivity on radiative heat transfer with inclined magnetic field and dissipation in a Darcy medium are investigated. From the numerical results, it is seen that, an increase in the values of H_a, α, D_a and φ, retarded the motion of the fluid and causes decrease in the velocity profiles. G_r and ϕ increase the skin friction and decrease the wall temperature gradient, as a result, the velocity profiles increases. Increase in α, λ, n, R and E_c increase the thermal boundary layer thickness by causing increase in the temperature profiles while increase in ϕ, m and P_r reduces the thermal boundary layer and causes heat to diffuse out of the system and thereby decreases the temperature profiles.

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