# $B_{s, d} \rightarrow \mu^{+} \mu^{-}$in technicolor model with scalars 

Zhaohua Xiong ${ }^{\text {a,b, }}$ Jin Min Yang ${ }^{\text {c }}$<br>${ }^{\text {a }}$ CCAST (World Laboratory), PO Box 8730, Beijing 100080, PR China<br>${ }^{\text {b }}$ Institute of High Energy Physics, Academia Sinica, Beijing 100039, PR China<br>${ }^{\text {c }}$ Institute of Theoretical Physics, Academia Sinica, Beijing 100080, PR China

Received 15 August 2002; received in revised form 11 September 2002; accepted 16 September 2002
Editor: T. Yanagida


#### Abstract

Rare decays $B_{s, d} \rightarrow \mu^{+} \mu^{-}$are evaluated in technicolor model with scalars. $R_{b}$ is revisited to constrain the model parameter space. It is found that restriction on $f / f^{\prime}$ arising from $R_{b}$ which was not considered in previous studies requires $f / f^{\prime}$ no larger than 1.9 at $95 \%$ confidence level, implying no significantly enhancement for $\operatorname{Br}\left(B_{s, d} \rightarrow \mu^{+} \mu^{-}\right)$from neutral scalars in the model. However, the branching ratio of $B_{s} \rightarrow \mu^{+} \mu^{-}$can still be enhanced by a factor of 5 relative to the standard model prediction. With the value of $f / f^{\prime} \lesssim 1.9$, an upgraded Tevatron with an integrated luminosity $20 \mathrm{fb}^{-1}$ will be sensitive to enhancement of $B_{s} \rightarrow \mu^{+} \mu^{-}$in this model provided that neutral scalar mass $m_{\sigma}$ is below 580 GeV .


© 2002 Elsevier Science B.V. Open access under CC BY license.
PACS: 12.60.Nz; 13.20.Hw; 13.38.Dg

## 1. Introduction

The flavor-changing neutral-current $B$-meson rare decays play an important role for testing the Standard Model (SM) at loop level and probing new physics beyond the SM. Among these decays, $B_{s, d} \rightarrow \mu^{+} \mu^{-}$ are of special interest due to their relative cleanliness and good sensitivity to new physics.

There are numerous speculations on the possible forms of new physics, among which supersymmetry and technicolor are the two typical different frameworks. Both frameworks are well motivated. As a lowenergy effective theory, the technicolor model with

[^0]scalars introduces additional scalars to connect the technicolor condensate to the ordinary fermions [1]. The phenomenology of this model has been considered extensively in the literature [1-8]. It has been found that this model does not produce unacceptably large contributions to neutral meson mixings or to the electroweak $S$ and $T$ parameters [1,2]. On the other hand, this model does predict potentially visible contributions to b-physics observables such as $R_{b}$ [6] and the rate of various rare $B$-meson decays [6-8].

Studies [9] showed that the processes $B_{s, d} \rightarrow$ $\mu^{+} \mu^{-}$are sensitive to supersymmetry. In this Letter we will extend our previous studies [ 8,10 ] and evaluate the branching ratio of $B_{s, d} \rightarrow \mu^{+} \mu^{-}$in the technicolor model with scalars. First we will present a brief description of the model, then give the analytical cal-
culations for $B_{s, d} \rightarrow \mu^{+} \mu^{-}$. We will focus our attention on the neutral scalars contributions, which are likely to be sizable because, as shown in our following analysis, they will be enhanced by a factor $\left(f / f^{\prime}\right)^{4}$ as the parameter $f / f^{\prime}$ gets large. Before performing the numerical calculations, we examine the current bounds on this model from a variety of experiments, especially the latest measurements of $R_{b}$ [11]. Since the theoretical expression for $R_{b}$ used in constraining the model parameter space [6] seems not right, we will recalculate the contributions to $R_{b}$ from the scalars in this model. We find the constraint from $R_{b}$ is still strongest as indicated in [12], compared with those from the direct searches for neutral and charged scalars [13], $B^{0}-\bar{B}^{0}$ mixing, $b \rightarrow s \gamma$ [14] as well as the muon anomalous magnetic moment [15]. Further, we evaluate restriction on $f / f^{\prime}$ arising from $R_{b}$ which was not considered in previous studies. Subject to the current bounds, the numerical results are presented in Section 5. Finally, the conclusion is assigned in Section 6 .

## 2. The technicolor model with scalars

In this section we will briefly discuss the technicolor model with scalars and give the relevant Lagrangians which are needed in our calculations. More details of the model have been described in Refs. [1,2].

The model embraces the full SM gauge structure and all SM fermions which are technicolor singlets. It has a minimal $S U(N)$ technicolor sector, with two techniflavors that transform as a left-handed doublet and two right-handed singlets under $S U(2)_{W}$,
$T_{L}=\binom{p}{m}_{L}, p_{R}, m_{R}$
with weak hypercharges $Y\left(T_{L}\right)=0, Y\left(p_{R}\right)=1$, and $Y\left(m_{R}\right)=-1$. All of the fermions couple to a weak scalar doublet $\phi$ to which both the ordinary fermions and technifermions are coupled. This scalar's purpose is to couple the technifermion condensate to the ordinary fermions and thereby generate fermion masses. If we write the matrix form of the scalar doublet as
$\Phi=\left[\begin{array}{cc}\bar{\phi}^{0} & \phi^{+} \\ -\phi^{-} & \phi^{0}\end{array}\right] \equiv \frac{\left(\sigma+f^{\prime}\right)}{\sqrt{2}} \Sigma^{\prime}$,
and adopt the non-linear representation $\Sigma=\exp \left(\frac{2 i \Pi}{f}\right)$ and $\Sigma^{\prime}=\exp \left(\frac{2 i \Pi^{\prime}}{f^{\prime}}\right)$ for technipions, with fields in $\Pi$ and $\Pi^{\prime}$ representing the pseudoscalar bound states of the technifermions $p$ and $m$, then the kinetic terms for the scalar fields are given by

$$
\begin{align*}
\mathcal{L}_{\text {K.E. }}= & \frac{1}{2} \partial_{\mu} \sigma \partial^{\mu} \sigma+\frac{1}{4} f^{2} \operatorname{Tr}\left(D_{\mu} \Sigma^{\dagger} D^{\mu} \Sigma\right) \\
& +\frac{1}{4}\left(\sigma+f^{\prime}\right)^{2} \operatorname{Tr}\left(D_{\mu} \Sigma^{\prime} D^{\mu} \Sigma^{\prime}\right) . \tag{3}
\end{align*}
$$

Here $D^{\mu}\left(D^{\prime \mu}\right)$ denote the $S U(2)_{L} \times S U(2)_{R}$ covariant derivatives, $\sigma$ is an isosinglet scalar field, $f$ and $f^{\prime}$ are the technipion decay constant and the effective vacuum expectation value (VEV), respectively.

As mixing between $\Pi$ and $\Pi^{\prime}$ occurs, $\pi_{a}$ and $\pi_{p}$ are formed with $\pi_{a}$ becoming the longitudinal component of the $W$ and $Z$, and $\pi_{p}$ remaining in the low-energy theory as an isotriplet of physical scalars. From Eq. (3) one can obtain the correct gauge boson masses providing that $f^{2}+f^{\prime 2}=v^{2}$ with the electroweak scale $v=246 \mathrm{GeV}$.

Additionally, the contributions to scalar potential generated by the technicolor interactions should be included in this model. The simplest term one can construct is
$\mathcal{L}_{T}=c_{1} 4 \pi f^{3} \operatorname{Tr}\left[\Phi\left(\begin{array}{cc}h_{+} & 0 \\ 0 & h_{-}\end{array}\right) \Sigma^{\dagger}\right]+$ h.c.,
where $c_{1}$ is a coefficient of order unity, $h_{+}$and $h_{-}$ are the Yukawa couplings of scalars to $p$ and $m$. From Eq. (4) the mass of the charged scalar at lowest order is obtained as
$m_{\pi_{p}}^{2}=2 c_{1} \sqrt{2} \frac{4 \pi f}{f^{\prime}} v^{2} h$
with $h \equiv\left(h_{+}+h_{-}\right) / 2$. To absorb the largest ColemanWeinberg radiative corrections [16] for the $\sigma$ field which affect the phenomenology of the charged scalar, the shifted scalar mass $\widetilde{M}_{\phi}$ and coupling $\tilde{\lambda}$ are determined by
$\widetilde{M}_{\phi}^{2} f^{\prime}+\frac{\tilde{\lambda}}{2} f^{\prime 3}=8 \sqrt{2} c_{1} \pi h f^{3}$.
Therefore, the mass of the scalar $\sigma$ can be expressed as
$m_{\sigma}^{2}=\widetilde{M}_{\phi}^{2}+\frac{2}{3 \pi^{2}}\left[6\left(\frac{m_{t}}{f^{\prime}}\right)^{4}+N h^{4}\right] f^{\prime 2}$
in limit (i) where the shifted $\phi^{4}$ coupling $\tilde{\lambda}$ is small and can be neglected and
$m_{\sigma}^{2}=\frac{3}{2} \tilde{\lambda} f^{\prime 2}-\frac{1}{4 \pi^{2}}\left[6\left(\frac{m_{t}}{f^{\prime}}\right)^{4}+N h^{4}\right] f^{\prime 2}$
in limit (ii) where the shifted mass of the scalar doublet $\phi, \widetilde{M}_{\phi}$ is small and can be neglected. The advantage of this model is at the lowest order, only two independent parameters in the limits (i) and (ii) are needed to describe the phenomenology. We choose ( $h, m_{\sigma}$ ) as physical parameters and assume $N=4$ and $c_{1}=1$ in numerical calculations.

## 3. Calculations

We start the calculation by writing down the effective Hamiltonian describing the process $B_{q} \rightarrow \mu^{+} \mu^{-}$ ( $q=s, d$ )

$$
\begin{align*}
& \mathcal{M}=\frac{\alpha G_{F}}{\sqrt{2} \pi} V_{t b} V_{t q}^{*} \\
& \times\left\{\begin{array}{l}
-2 C_{7}^{\mathrm{eff}} \frac{m_{b}}{p^{2}} \bar{q} i \sigma_{\mu \nu} \nu^{\nu} P_{R} b \\
\\
\\
+C_{9}^{\mathrm{efff}} \bar{q} \gamma_{\mu} P_{L} b \bar{\mu} \gamma^{\mu} \mu \\
\\
\end{array} \quad C_{10} \bar{q} \gamma_{\mu} P_{L} b \bar{\mu} \gamma^{\mu} \gamma_{5} \mu\right. \\
&\left.+C_{Q_{1}} \bar{q} P_{R} b \bar{\mu} \mu+C_{Q_{2}} \bar{q} P_{R} b \bar{\mu} \gamma_{5} \mu\right\}
\end{align*}
$$

where $P_{R, L}=\frac{1}{2}\left(1 \pm \gamma_{5}\right), p$ is the momentum transfer. Operators $\mathcal{O}_{7,9,10}$ which correspond to the first three Wilson coefficients are the same as those given in [17] and $\mathcal{Q}_{1,2}$ corresponding to the last two are the additional operators arising from the neutral scalars exchange diagrams [18].

Using the effective Hamiltonian and
$\langle 0| \bar{q} \gamma_{\mu} \gamma_{5} b\left|B_{q}\right\rangle=-f_{B_{q}} p_{\mu}$,
$\langle 0| \bar{q} \gamma_{5} b\left|B_{q}\right\rangle=-f_{B_{q}} m_{B_{q}}$,
$\langle 0| \bar{q} \sigma_{\mu \nu}\left(1+\gamma_{5}\right) b\left|B_{q}\right\rangle=0$,
we find that only operator $\mathcal{O}_{10}$ and $\mathcal{Q}_{1,2}$ contribute to process $B_{q} \rightarrow \mu^{+} \mu^{-}$with the decay rate given by

$$
\begin{align*}
& \Gamma\left(B_{q} \rightarrow \mu^{+} \mu^{-}\right) \\
& \quad=\frac{\alpha^{2} G_{F}^{2}}{64 \pi^{3}}\left|V_{t b} V_{t q}^{*}\right|^{2} f_{B_{q}}^{2} m_{B_{q}}^{3} \\
& \quad \times\left[C_{Q_{1}}^{2}+\left(C_{Q_{2}}+\frac{2 m_{\mu}}{m_{B_{q}}} C_{10}\right)^{2}\right] \tag{11}
\end{align*}
$$

For convenience, we write down the branching fractions numerically

$$
\begin{align*}
& \operatorname{Br}\left(B_{d} \rightarrow \mu^{+} \mu^{-}\right) \\
&= 3.8 \times 10^{-9}\left[\frac{\tau_{B_{d}}}{1.65 \mathrm{ps}}\right]\left[\frac{f_{B_{d}}}{210 \mathrm{MeV}}\right]^{2} \\
& \times\left|\frac{V_{t d}}{0.008}\right|^{2}\left[\frac{m_{B_{d}}}{5.28 \mathrm{GeV}}\right]^{3} \\
& \quad \times\left[C_{Q_{1}}^{2}+\left(C_{Q_{2}}+2 \frac{m_{\mu}}{m_{B_{d}}} C_{10}\right)^{2}\right], \\
& \operatorname{Br}\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right) \\
&= 1.2 \times 10^{-7}\left[\frac{\tau_{B_{s}}}{1.49 \mathrm{ps}}\right]\left[\frac{f_{B_{s}}}{245 \mathrm{MeV}}\right]^{2} \\
& \quad \times\left|\frac{V_{t s}}{0.04}\right|^{2}\left[\frac{m_{B_{s}}}{5.37 \mathrm{GeV}}\right]^{3} \\
& \times\left[C_{Q_{1}}^{2}+\left(C_{Q_{2}}+2 \frac{m_{\mu}}{m_{B_{s}}} C_{10}\right)^{2}\right], \tag{12}
\end{align*}
$$

where $\tau_{B_{q}}$ and $f_{B_{q}}$ are the $B_{q}$ lifetime and decay constant, respectively.

In the technicolor model with scalars, the additional contributions arise from the scalars. The contributions of the charged scalar $\pi_{p}^{ \pm}$with gauge boson $Z, \gamma$ exchanges to the Wilson coefficients $C_{10}$ at $m_{W}$ scale have been calculated by using Feynman rules derived from Eqs. (3), (4) and given by [7,8]

$$
\begin{align*}
& C_{10}\left(m_{W}\right)_{\mathrm{TC}} \\
& \left.\qquad \begin{array}{l}
=\frac{x_{W}}{\sin ^{2} \theta_{W}}\left(\frac{f}{f^{\prime}}\right)^{2}\left[-\frac{x_{\pi_{p}}}{8\left(x_{\pi_{p}}-1\right)}\right. \\
\\
\end{array} \quad+\frac{x_{\pi_{p}}}{8\left(x_{\pi}-1\right)^{2}} \ln x_{\pi_{p}}\right]
\end{align*}
$$

where $\theta_{W}$ is the Weinberg angle and $x_{i}=m_{t}^{2} / m_{i}^{2}$. As for the contributions arising from the neutral scalars exchanges, when only the leading terms in large $f / f^{\prime}$
limit kept, they can be expressed as [8]

$$
\begin{aligned}
& C_{Q_{1}}\left(m_{W}\right)_{\mathrm{TC}} \\
& = \\
& =-\frac{x_{W}}{\sin ^{2} \theta_{W}}\left(\frac{f}{f^{\prime}}\right)^{4} \frac{m_{b} m_{\mu}}{m_{\sigma}^{2}} \\
& \quad \times\left[\frac{4 x_{\pi_{p}}^{2}-7 x_{\pi_{p}}+1}{16\left(x_{\pi_{p}}-1\right)^{2}}-\frac{x_{\pi_{p}}^{2}-2 x_{\pi_{p}}}{8\left(x_{\pi_{p}}-1\right)^{3}} \ln x_{\pi_{p}}\right],
\end{aligned}
$$

$C_{Q_{2}}\left(m_{W}\right)_{\text {TC }}$

$$
\begin{align*}
= & -\frac{x_{W}}{\sin ^{2} \theta_{W}}\left(\frac{f}{f^{\prime}}\right)^{4} \frac{m_{b} m_{\mu}}{m_{\pi_{p}}^{2}} \\
& \times\left[\frac{x_{\pi_{p}}+1}{8\left(x_{\pi_{p}}-1\right)}-\frac{x_{\pi_{p}}}{4\left(x_{\pi_{p}}-1\right)^{2}} \ln x_{\pi_{p}}\right] . \tag{14}
\end{align*}
$$

From Eqs. (12)-(14) we find that (1) both the contributions arising from the neutral scalar exchange $C_{Q_{1,2}}$ and gauge boson exchange $C_{10}$ are subject to helicity suppression, (2) the contributions arising from the neutral scalar exchanges are proportional to $\left(f / f^{\prime}\right)^{4}$, while those from the gauge bosons exchanges proportional to $\left(f / f^{\prime}\right)^{2}$. So for a sufficiently large $f / f^{\prime}$, the contributions of neutral scalar exchanges are relatively enhanced and may become comparable with those from the gauge boson exchanges.

The Wilson coefficients at the lower scale of about $m_{b}$ can be evaluated down from $m_{W}$ scale by using the renormalization group equation. At leading order, the Wilson coefficients are $[17,18]$
$C_{10}\left(m_{b}\right)=C_{10}\left(m_{W}\right)$,
$C_{Q_{i}}\left(m_{b}\right)=\eta^{-\gamma_{Q} / \beta_{0}} C_{Q_{i}}\left(m_{W}\right)$,
where $\beta_{0}=11-2 n_{f} / 3, \eta=\alpha_{s}\left(m_{b}\right) / \alpha_{s}\left(m_{W}\right)$ and $\gamma_{Q}=-4$ is the anomalous dimension of $\bar{q} P_{R} b$.

## 4. Constraints from $\boldsymbol{R}_{\boldsymbol{b}}$

Before presenting the numerical results, let us consider the current bounds on technicolor with scalars from a variety of experiments, especially the measurement of $R_{b}$. Using the Feynman rules in Ref. [8], one can easily find that the contributions from neutral scalars are negligible compared with those from charged scalars which appear in Fig. 1, and the bottom mass-dependent terms in $R_{b}$ can also be omitted


Fig. 1. Charged scalars diagrams contributing to $Z b \bar{b}$.
safely. In these approximations the additional contribution in the technicolor with scalars is obtained as
$\delta R_{b}=R_{b}^{\mathrm{SM}}\left(1-R_{b}^{\mathrm{SM}}\right) \Delta^{\mathrm{TC}}$
with

$$
\begin{align*}
& \Delta^{\mathrm{TC}}=\left(\frac{f}{f^{\prime}}\right)^{2} \frac{\alpha}{4 \pi \sin ^{2} \theta_{W}} \frac{m_{t}^{2}}{m_{W}^{2}} \frac{v_{b L}}{v_{b L}^{2}+v_{b R}^{2}} \\
& \times\left\{v_{b L} B_{1}\right. \\
& \quad+v_{t R}\left[m_{Z}^{2}\left(C_{22}^{a}-C_{23}^{a}\right)+2 C_{24}^{a}-\frac{1}{2}\right] \\
&\left.\quad-2 v_{t L} m_{t}^{2} C_{0}^{a}-\cos 2 \theta_{W} C_{24}^{b}\right\} . \tag{18}
\end{align*}
$$

Here $B_{1}=B_{1}\left(-p_{1}, m_{t}, m_{\pi_{p}}\right), C_{0, i j}^{a}=C_{0, i j}\left(p_{1},-P\right.$, $\left.m_{\pi_{p}}, m_{t}, m_{t}\right)$ and $C_{24}^{b}=C_{24}\left(-p_{1}, P, m_{t}, m_{\pi_{p}}, m_{\pi_{p}}\right)$, with $p_{1}\left(p_{2}\right)$ and $P$ denoting the four-momentum of $b(\bar{b})$ and $Z$ boson, respectively, are the Feynman loop integral functions and their expressions can be found in [19]. The coupling constants $v_{q L}$ and $v_{q R}$ are given by

$$
\begin{align*}
& v_{q L}=T_{3}^{q}-e_{q} \sin ^{2} \theta_{W}, \\
& v_{q R}=-e_{q} \sin ^{2} \theta_{W} . \tag{19}
\end{align*}
$$

Our explicit expressions are not in agreement with those used in [6] where the results obtained in the framework of the two-Higgs doublet model (THMD) [20] were adopted directly. We checked the calculations and confirmed our results.

The current measurement of $R_{b}$ reported by the LEP is $R_{b}^{\text {expt }}=0.21646 \pm 0.00065$ [11]. Comparing with the SM value $R_{b}^{S M}=0.21573 \pm 0.0002$, we obtained the constraints in $h$ versus $m_{\sigma}$ plane shown in Figs. 2 and 3. Although our explicit expression for $R_{b}$ is different from that used in [12], a comparison of Fig. 2, Fig. 3 with Fig. 1 in Ref. [12] suggests that there is not a qualitative change in the results plotted.


Fig. 2. Constraints on technicolor with scalars in limit (i). The allowed parameter space is the shaded region bounded by the contours $m_{\sigma}=114 \mathrm{GeV}$, $\delta R_{b}$ ( $R_{b}$ line) and $h f^{\prime}=4 \pi f$. The current bound from the searches for charged scalars $m_{\pi_{p}}=79 \mathrm{GeV}$ is shown along with the reference curves $m_{\pi_{p}}=m_{t}-m_{b}$, $m_{\pi_{p}}=1 \mathrm{TeV}$. The constraint from $B^{0}-\bar{B}^{0}$ mixing is labeled " $B$ line".

Our numerical results show that the constraint on $f / f^{\prime}$ from $R_{b}$ is quite stringent, i.e., the ratio of $f / f^{\prime}$ must be smaller than 1.9 at $95 \%$ C.L., implying that the neutral scalars will not give dominate contributions to the processes of $B_{s, d} \rightarrow \mu^{+} \mu^{-}$. Since previous studies did not comment on any restriction on $f / f^{\prime}$ arising from $R_{b}$, this is a new and interesting conclusion.

In Figs. 2 and 3 we also display the bounds from $B^{0}-\bar{B}^{0}$ mixing and from the limits of Higgs masses [12]. In technicolor theories where the charged scalars couple to fermions in a similar pattern as in type-I two-Higgs doublet model, the strongest limit $m_{\pi_{D}^{ \pm}} \geqslant 79 \mathrm{GeV}$ has been obtained directly from LEP experiments [13]. On the other hand, the LEP collaborations [13] have placed a 95\% C.L. lower limit on the SM Higgs boson $M_{H}^{0} \geqslant 113.5 \mathrm{GeV}$ from searching for the process $e^{+} e^{-} \rightarrow Z^{*} \rightarrow Z H^{0}$. Although the limit on technicolor scalars may differ from that on $M_{H}^{0}$, in practice, the contour $m_{\sigma}=$


Fig. 3. Constraints on technicolor with scalars in limit (ii). The allowed parameter space is the shaded region bounded by the contours $m_{\sigma}=114 \mathrm{GeV}$ and $\delta R_{b}$ ( $R_{b}$ line). Other bound curves are the same as Fig. 2.

114 GeV can serve as an approximate boundary to the experimentally allowed region [2,12]. Note that the chiral Lagrangian analysis break down only constrain on the parameter space in limit (i) [6], the area above and to left of $h f^{\prime}=4 \pi f$ line is excluded because the technifermion current masses are no longer small compared to the chiral symmetry breaking scale. For references, we also plotted the contours $m_{\pi_{p}}=m_{t}-$ $m_{b}$ and $m_{\pi_{p}}=1 \mathrm{TeV}$. If the top quark does not decay to $\pi_{p}^{+} b$, the areas outside of $m_{\pi_{p}}=m_{t}-m_{b}$ curve is excluded in Fig. 2. Similar situation occurs to $m_{\pi_{p}}=1 \mathrm{TeV}$ curve in Fig. 3 if all scalar masses are restricted to the sub- TeV regime. In contrast to these, the excluded parameter space are the areas inside of $m_{\pi}=m_{t}-m_{b}$ curve in limit (ii) and $m_{\pi_{p}}=1 \mathrm{TeV}$ curve in limit (i).

The constraint from $b \rightarrow s \gamma$ is close to that from $B^{0}-\bar{B}^{0}$ mixing $[7,8,21,22]$, which are weaker than those from $R_{b}$ [6]. As for the constraints from the measurement of $g_{\mu}-2$, our previous study [23] showed that if the deviation of the E821 experiment result [15] and SM prediction $\Delta a_{\mu} \equiv a_{\mu}^{\exp }-a_{\mu}^{\mathrm{SM}}=$ $(43 \pm 16) \times 10^{-10}$ persists, it would severely constrain
the technicolor models because the technicolor models can hardly provide such a large contribution. However, over the last year the theoretical prediction of $a_{\mu}$ in the SM has undergone a significant revision due to the change in sign of the light hadronic correction, which leads to only a $1.6 \sigma$ deviation from the SM [24], yielding no more useful limits on this model.

## 5. Numerical results

Bearing the constraints on technicolor with scalars in mind, and for the same values of $m_{\pi_{p}}$ and $f / f^{\prime}$, the allowed value of $m_{\sigma}$ is generally smaller in limit (i), from Eq. (14) one can infer easily that the additional contributions to $B_{s, d} \rightarrow \mu^{+} \mu^{-}$in limit (i) will be larger than those in limit (ii). Furthermore, as can be seen from the numerical coefficients in Eq. (12), the decay rate of $B_{s}$ is significantly larger than $B_{d}$ due primarily to the relative size of $\left|V_{t s}\right|$ to $\left|V_{t d}\right|$. We thus take the $B_{s}$ decay in limit (i) as an example to show the numerical results.

The experimental bound on $B_{s} \rightarrow \mu^{+} \mu^{-}$comes from the CDF [25]
$\operatorname{Br}\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right)<2.6 \times 10^{-6}$
at $95 \%$ C.L. with the corresponding integrated luminosity about $100 \mathrm{pb}^{-1}$, while the SM prediction
$\operatorname{Br}\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right)=4.0 \times 10^{-9}$
is obtained by taking the central values for all inputs in Eq. (12). The branching ratio of $B_{s} \rightarrow \mu^{+} \mu^{-}$as a function of $m_{\sigma}$ is displayed in Fig. 4 for various values of $f / f^{\prime}$. The $2 \sigma$ bounds at the upgraded Tevatron with 10 and $20 \mathrm{fb}^{-1}$ are also plotted under the assumption that the background for this decay is negligible. The corresponding expected sensitivity can be reach a branching ratio of $1.3 \times 10^{-8}$ and $6.5 \times$ $10^{-9}$ (dash-dotted lines), respectively. We see that the $R_{b}$ constraint $f / f^{\prime} \leqslant 1.9$ at $95 \%$ C.L. shown in Fig. 4 is the strongest bound. Comparatively, the current upper bound on $\operatorname{Br}\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right)$from CDF [25] is much weaker, which only excludes a small region with large $f / f^{\prime}$. Under the constraint $f / f^{\prime} \leqslant 1.9$, the enhancement factor for the branching ratio in the technicolor model can still be up to 5 . The upgraded Tevatron with $20 \mathrm{fb}^{-1}$ will be sensitive to


Fig. 4. $\operatorname{Br}\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right)$as a function of $m_{\sigma}$ for $f / f^{\prime}=$ $1,1.9,5,10$ (dash lines) in limit (i). The current $2 \sigma$ upper bound [25] (dotted line), the SM prediction (solid line) as well as the expected sensitivity of the upgraded Tevatron with 10 and $20 \mathrm{fb}^{-1}$ (the dash-dotted lines) are also shown.
enhancements of $B_{s} \rightarrow \mu^{+} \mu^{-}$in this model provided that $m_{\sigma}$ is below 580 GeV .

## 6. Conclusions

We have evaluated the decays $B_{s, d} \rightarrow \mu^{+} \mu^{-}$in the technicolor model with scalars, taking into account various experimental constraints, especially $R_{b}$, on the model parameter space. We first examined the restriction on $f / f^{\prime}$ arising from $R_{b}$ which that previous study did not consider. We found that large $f / f^{\prime}$, which might cause significantly enhancement for $\operatorname{Br}\left(B_{s, d} \rightarrow\right.$ $\mu^{+} \mu^{-}$) from neutral scalars in the model, has been excluded by the constraints from $R_{b}$. Nevertheless, under the renewed $R_{b}$ constraint, the branching ratio of $B_{s} \rightarrow \mu^{+} \mu^{-}$can still be enhanced by a factor of 5 relative to the SM prediction. With the maximum allowed value of $f / f^{\prime} \sim 1.9$ from $R_{b}$, the upgraded Tevatron
with $20 \mathrm{fb}^{-1}$ will be sensitive to enhancements of $B_{s} \rightarrow \mu^{+} \mu^{-}$in this model provided that $m_{\sigma}$ is below 580 GeV . Since the theoretical uncertainties, which primarily come from the B-meson decay constants and CKM matrix elements, will be reduced in the on-going B-physics experiments and the lattice calculations, the processes $B_{s, d} \rightarrow \mu^{+} \mu^{-}$will promise to be a good probe of new physics.

## References

[1] E.H. Simmons, Nucl. Phys. B 312 (1989) 253.
[2] C.D. Carone, H. Georgi, Phys. Rev. D 49 (1994) 1427.
[3] R.S. Chivukula, S.B. Selipsky, E.H. Simmons, Phys. Rev. Lett. 69 (1992) 575.
[4] S. Samuel, Nucl. Phys. B 347 (1990) 625;
M. Dine, A. Kagan, S. Samuel, Phys. Lett. B 243 (1990) 250;
A. Kagan, S. Samuel, Phys. Lett. B 252 (1990) 605.
[5] N. Evans, Phys. Lett. B 331 (1994) 378; C.D. Carone, E.H. Simmons, Nucl. Phys. B 397 (1993) 591; C.D. Carone, M. Golden, Phys. Rev. D 49 (1994) 6211.
[6] C.D. Carone, E.H. Simmons, Y. Su, Phys. Lett. B 344 (1995) 287.
[7] Y. Su, Phys. Rev. D 56 (1997) 335.
[8] Z. Xiong, H. Chen, L. Lu, Nucl. Phys. B 561 (1999) 3; Z.H. Xiong, J.M. Yang, Nucl. Phys. B 602 (2001) 289.
[9] For examples, see S.R. Choudhury, N. Gaur, Phys. Lett. B 451 (1999) 86;
K.S. Babu, C. Koda, Phys. Rev. Lett. 84 (2000) 228;
P.H. Chankowski, L. Slawianowska, Phys. Rev. D 63 (2001) 054012;
C. Bobeth, T. Ewerth, F. Küger, J. Urban, Phys. Rev. D 64 (2001) 074014;
G. D'Ambrosio, G.F. Giudice, G. Isidori, A. Strumia, hepph/0207036;
C.-S. Huang, W. Liao, Q.-S. Yan, S.-H. Zhu, Phys. Rev. D 64 (2001) 05992;
Z.H. Xiong, J.M. Yang, Nucl. Phys. B 628 (2002) 193;
G. Isidori, A. Retico, JHEP 0111 (2001) 001;
A.J. Buras, P.H. Chankowshi, J. Rosiek, L. Slawianowska, hepph/0207241;
A. Dedes, H.K. Dreiner, U. Nierste, Phys. Rev. Lett. 87 (2001) 251804;
A. Dedes, H.K. Dreiner, U. Nierste, P. Richardson, hepph/0207026.
[10] G.R. Lu, et al., Phys. Rev. D 54 (1996) 5647; G.R. Lu, et al., Z. Phys. C 74 (1997) 355; G.R. Lu, Z. Xiong, Y.G. Cao, Nucl. Phys. B 487 (1997) 43; G.R. Lu, Z. Xiong, X.L. Wang, J.S. Huang, J. Phys. G 24 (1998) 745.
[11] T. Kawamoto, hep-ex/0105032; E. Tournefier, hep-ex/0105091; J. Drees, hep-ex/0110077.
[12] V. Hemmige, E.H. Simmons, Phys. Lett. B 518 (2001) 72.
[13] D.E. Groom, et al., Eur. Phys. J. C 15 (2000) 1.
[14] ALEPH Collaboration, Phys. Lett. B 429 (1998) 429; M. Nakao, Proceedings, ICHEP 2000, Osaka, Japan; T. Coan, Proceedings, ICHEP 2000, Osaka, Japan.
[15] Mug-2 Collaboration, H.N. Brown, et al., Phys. Rev. Lett. 86 (2001) 2227.
[16] S. Coleman, E. Weinberg, Phys. Rev. D 7 (1973) 1888.
[17] B. Grinstein, R. Springer, M.B. Wise, Phys. Lett. B 202 (1988) 138;
A. Ali, G. Hiller, L.T. Handoko, T. Morozumi, Phys. Rev. D 55 (1997) 4105;
C.S. Kim, T. Morozumi, A.I. Sanda, Phys. Rev. D 56 (1997) 7240;
S. Fukae, C.S. Kim, T. Morozumi, T. Yoshikawa, Phys. Rev. D 59 (1999) 074013.
[18] Y.B. Dai, C.S. Huang, H.W. Huang, Phys. Lett. B 390 (1997) 257.
[19] G. 't Hooft, M. Veltman, Nucl. Phys. B 153 (1979) 365; G. Passarino, M. Veltman, Nucl. Phys. B 160 (1979) 151.
[20] M. Boulare, D. Finnell, Phys. Rev. D 44 (1991) 2054.
[21] B. Grinstein, R. Springer, M.B. Wise, Nucl. Phys. B 339 (1990) 269.
[22] T.G. Rizzo, Phys. Rev. D 38 (1988) 820;
W.S. Hou, R.S. Willey, Phys. Lett. B 202 (1988) 591;
C.Q. Geng, J.N. Ng, Phys. Rev. D 38 (1988) 2858;
V. Barger, J.L. Hewett, R. Phillips, Phys. Rev. D 41 (1990) 3421.
[23] Z.H. Xiong, J.M. Yang, Phys. Lett. B 508 (2001) 295.
[24] M. Knecht, A. Nyffeler, Phys. Rev. D 65 (2002) 073034; M. Knecht, A. Nyffeler, M. Perrottet, E. de Rafael, Phys. Rev. Lett. 88 (2002) 071802;
M. Hayakawa, T. Kinoshita, hep-ph/0112102;
I. Blokland, A. Czarnecki, K. Melnikov, Phys. Rev. Lett. 88 (2002) 071803;
J. Bijnens, E. Pallante, J. Prades, Nucl. Phys. B 626 (2002) 410.
[25] CDF Collaboration, F. Abe, et al., Phys. Rev. D 57 (1998) 3811.


[^0]:    E-mail address: xiongzh@mail.ihep.ac.cn (Z. Xiong).

