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# Extrema bounds for the soft Pomeron intercept

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#### Abstract

By using an extended Regge parametrization and taking into account the discrepancies in the high-energy pp and  $\bar{p}p$  total cross section data in both accelerator and cosmic-ray regions, we estimate extrema bounds for the soft Pomeron intercept. First we consider two ensembles of data with either the CDF or the E710 and E811 results for  $\sigma_{tot}^{\bar{p}p}$  at 1.8 TeV, from which we obtain the bounds 1.102 and 1.081, respectively. These ensembles are then combined with the highest and lowest estimations for  $\sigma_{tot}^{pp}$  from cosmic-ray experiments (6–40 TeV), leading to the upper and lower bounds 1.109 and 1.082, respectively. The effects of simultaneous fits to  $\sigma_{tot}$  and  $\rho$ , individual fits to  $\sigma_{tot}$ , and the influence of the subtraction constant in the dispersion relations are also presented. Our global results favor the E710 and E811 data.

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### 1. Introduction

Analytic models for hadron–hadron scattering are characterized by simple parametrizations for the forward amplitude F and the use of dispersion relation techniques to study the total cross section  $\sigma_{tot}$  and the  $\rho$  parameter (the ratio of the real to the imaginary part of the amplitude),

$$\sigma_{\text{tot}}(s) = \frac{\text{Im } F(s, t=0)}{s},$$

$$\rho = \frac{\text{Re } F(s, t=0)}{\text{Im } F(s, t=0)},$$
(1)

where t is the four-momentum transfer squared and s the center-of-mass energy squared.

In a recent work several aspects concerning the application of the analytic models to pp and  $\bar{p}p$  elastic scattering have been studied [1]. In particular, in the case of the Donnachie–Landshoff parametrization, investigation of discrepant estimations for the total cross sections from cosmic-ray experiments allowed to infer an upper bound for the soft Pomeron intercept, namely,  $1 + \epsilon = 1.094$ . In addition, the effects of global vs. individual fits to  $\sigma_{tot}$  and  $\rho$ , and the effects of the subtraction constant in the dispersion relations have also been analyzed and discussed.

In this Letter we extend the previous analysis in several ways, with focus on new upper/lower bounds for the soft Pomeron intercept: (1) we investigate the effect of discrepant values for  $\sigma_{tot}$  from accelerator experiments at 1.8 TeV, by selecting different ensembles of data that include either the highest (CDF) or the lowest (E710/E811) results; (2) these ensembles

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are then combined with discrepant estimations for  $\sigma_{tot}$ from cosmic-ray experiments, now using as the lowest estimations the results by Block, Halzen, and Stanev; (3) we use here, as a framework, an extended parametrization with non-degenerate C = +1 and C = -1meson trajectories. As in the previous analysis we also present the effects of individual and global fits to  $\sigma_{tot}$ and  $\rho$ , and the effect of the subtraction constant. Since the soft Pomeron exchange dominates the high energy behavior of the total cross sections and the *pp* and  $\bar{p}p$  scattering correspond to the highest energy interval with available data (including information from cosmic-ray experiments), we shall limit our analysis to these processes.

The Letter is organized as follows. In Section 2 the essential formula of the analytic approach with the extended Regge parametrization are presented. In Section 3 the discrepancies at both accelerator and cosmic-ray domains are reviewed, and the fit results through four different ensembles of experimental information are presented. The conclusions and some final remarks are the contents of Section 4.

### 2. Extended Regge parametrization

The forward effective Regge amplitude introduced by Donnachie and Landshoff has two contributions, one from a single Pomeron and the other from secondary Reggeons exchanges [2]. The model assumes degeneracies between the secondary Reggeons, imposing a common intercept for the C = +1 ( $a_2$ ,  $f_2$ ) and the C = -1 ( $\omega$ ,  $\rho$ ) trajectories. This was the parametrization adopted in the previous paper [1].

Although the original fits by Donnachie and Landshoff have been performed only to the  $\sigma_{tot}$  data, more recent analysis, treating global fits to  $\sigma_{tot}$  and  $\rho$ , have indicated that the best results are obtained with non-degenerate meson trajectories [3,4]. In this case the forward scattering amplitude is decomposed into three Reggeon exchanges,  $F(s) = F_{\mathbb{P}}(s) + F_{a_2/f_2}(s) + \tau F_{\omega/\rho}(s)$ , where the first term represents the exchange of a single Pomeron, the other two the secondary Reggeons and  $\tau = +1$  (-1) for pp ( $\bar{p}p$ ) amplitudes. Using the notation  $\alpha_{\mathbb{P}}(0) = 1 + \epsilon$ ,  $\alpha_{+}(0) = 1 - \eta_{+}$  and  $\alpha_{-}(0) = 1 - \eta_{-}$  for the intercepts of the Pomeron and the C = +1 and C = -1 trajectories, respectively, the total cross sections, Eq. (1), for pp and  $\bar{p}p$ 

interactions are written as

$$\sigma_{\rm tot}(s) = Xs^{\epsilon} + Y_{+}s^{-\eta_{+}} + \tau Y_{-}s^{-\eta_{-}}.$$
 (2)

The connection with the  $\rho$  parameter is obtained by means of dispersion relations and for the above parametrization convergence is ensured by using analyticity relations with one subtraction. Defining  $2F_{\pm} \equiv F_{pp} \pm F_{\bar{p}p}$  these relations read [1]

$$\operatorname{Re} F_{+}(s) = K + s \tan\left[\frac{\pi}{2} \frac{d}{d \ln s}\right] \frac{\operatorname{Im} F_{+}(s)}{s},$$
  
$$\operatorname{Re} F_{-}(s) = \tan\left[\frac{\pi}{2} \frac{d}{d \ln s}\right] \operatorname{Im} F_{-}(s),$$
(3)

where *K* is the subtraction constant. Within this formalism, Eqs. (1)–(3) lead to the following connection between  $\rho(s)$  and  $\sigma_{\text{tot}}(s)$ :

$$\rho(s)\sigma_{\text{tot}}(s) = \frac{K}{s} + Xs^{\epsilon} \tan\left(\frac{\pi\epsilon}{2}\right)$$
$$-Y_{+}s^{-\eta_{+}} \tan\left(\frac{\pi\eta_{+}}{2}\right)$$
$$+\tau Y_{-}s^{-\eta_{-}} \cot\left(\frac{\pi\eta_{-}}{2}\right).$$

### 3. Discrepancies, strategies and fitting results

The experimental information on pp and  $\bar{p}p$  total cross sections at the highest energies are characterized by discrepant results. As is well known, in the accelerator region, the conflit concerns the results for  $\sigma_{\rm tot}^{\bar{p}p}$  at  $\sqrt{s} = 1.8$  TeV reported by the CDF Collaboration [5] and those reported by the E710 [6] and the E811 [7,8] Collaborations (Fig. 1). In the cosmic-ray region, 6 TeV  $<\sqrt{s} \le 40$  TeV, the discrepancies are due to both experimental and theoretical uncertainties in the determination of  $\sigma_{tot}^{pp}$  from *p*-air cross sections. The situation has been recently reviewed in detail in our previous paper [1], where a complete list of references, numerical results and discussions are presented. As showed there, the highest predictions for  $\sigma_{tot}^{pp}$  concern the result by Gaisser, Sukhatme, and Yodh [9] together with those by Nikolaev [10]. In the other extreme, the lowest values come from the results by Block, Halzen, and Stanev [11]. These extrema estimations are displayed in Fig. 1 (numerical values may be found in Ref. [1]).

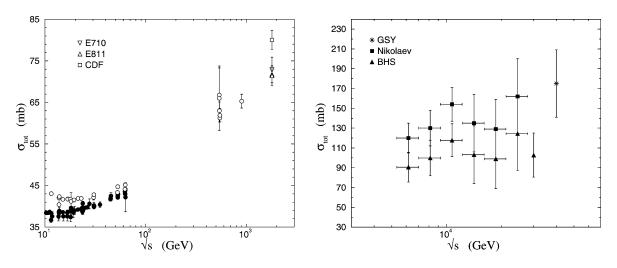


Fig. 1. The *pp* (black symbols) and  $\bar{p}p$  (white symbols) total cross section above  $\sqrt{s} = 10$  GeV from accelerator experiments (left) and estimations of *pp* total cross section from cosmic-ray experiments (right) by Gaisser, Sukhatme, and Yodh (GSY) [9], Nikolaev [10], and Block, Halzen, and Stanev (BHS) [11,12].

Although, in principle, all available data in the accelerator region could be used, it should be stressed that the difference between the CDF and the E710/E811 results involves two standard deviations [7]. This strong disagreement certainly indicates the possibility of distinct scenarios for the rise of the total cross section and consequently for the value of the Pomeron intercept. Moreover, despite the large error bars in the extracted values of  $\sigma_{tot}^{pp}$  from cosmic-ray experiments, the discrepancies also presented can corroborate the distinction between the different scenarios. It is expected that answers to these questions will be provided by the new values for  $\sigma_{tot}$  and  $\rho$  coming from the BNL RHIC, the Fermilab Tevatron-run II and the CERN LHC.

Based on these facts, we consider important at the moment to investigate these experimental discrepancies and examine its consequences in terms of extrema bounds for the Pomeron intercept.

Since recent analysis showed that the parameters of Regge fits are stable for a cutoff  $\sqrt{s} \sim 9$  GeV [13], in what follows we consider experimental data on  $\sigma_{\text{tot}}$  and  $\rho$  above  $\sqrt{s} = 10$  GeV. We use the data sets compiled and analyzed by the Particle Data Group [14], to which we add the new E811 data on  $\sigma_{\text{tot}}$  and p at 1.8 TeV [8]. The statistic and systematic errors have been added in quadrature.

In order to investigate the effects of the discrepancies in a quantitative way, we select different ensembles of  $\sigma_{tot}$  data that include all the  $\rho$  results above 10 GeV. First we only consider accelerator data in two ensembles with the following notation: ensemble I:  $\sigma_{tot}^{pp}$  and  $\sigma_{tot}^{\bar{p}p}$  data ( $10 \leq \sqrt{s} \leq 900$  GeV) + CDF datum ( $\sqrt{s} = 1.8$  TeV); ensemble II:  $\sigma_{tot}^{pp}$  and  $\sigma_{tot}^{\bar{p}p}$  data ( $10 \leq \sqrt{s} \leq 900$  GeV) + E710/E811 data ( $\sqrt{s} = 1.8$  TeV). Ensemble I represents the faster increase scenario for the rise of  $\sigma_{tot}$  from accelerator data and ensemble II the slowest one. These ensembles are then combined with the highest and lowest estimations for  $\sigma_{tot}^{pp}$  from cosmic-ray experiments, namely, the Nikolaev and Gaisser, Sukhatme, and Yodh (NGSY) results and the Block, Halzen, and Stanev (BHS) results, respectively. These new ensembles are denoted by ensemble I + NGSY and ensemble II + BHS.

As in the previous paper, we consider both individual fits to  $\sigma_{tot}$ , and simultaneous fits to  $\sigma_{tot}$  and  $\rho$ , either in the case where the subtraction constant is considered as a free fit parameter or assuming K = 0 in Eq. (3). The fits have been performed with the program CERN-MINUIT and the errors in the fit parameters correspond to an increase of the  $\chi^2$  by one unit.

In the case of accelerator data only the fit results for  $\sigma_{tot}$  and  $\rho$  with ensembles I and II are displayed in Table 1 and Figs. 2, 3 and 4. The results concerning the combination of these ensembles with the estimations from cosmic-ray experiments, namely, ensembles I + NGSY and II + BHS, are shown in Table 2 and Figs. 5, 6 and 7. In the last two cases we present

Table 1

Individual and global fits to  $\sigma_{tot}$  and  $\rho$  with ensemble I (CDF datum) and ensemble II (E710/E811 data) and the subtraction constant K = 0 or as a free fit parameter

Fit:	Individual $\sigma_{tot}$		Global $\sigma_{\text{tot}}$ and $\rho$ with $K = 0$		Global $\sigma_{\text{tot}}$ and $\rho$ with K free	
Ensemble:	Ι	II	Ι	II	Ι	II
$\epsilon$	$0.096 \pm 0.005$	$0.085\pm0.004$	$0.098 \pm 0.004$	$0.090 \pm 0.003$	$0.095\pm0.005$	$0.085 \pm 0.003$
X (mb)	$18 \pm 1$	$20 \pm 1$	$18 \pm 1$	$19 \pm 1$	$19 \pm 1$	$21 \pm 1$
$\eta_{\pm}$	$0.31 \pm 0.04$	$0.38\pm0.04$	$0.32\pm0.02$	$0.35 \pm 0.02$	$0.35 \pm 0.04$	$0.41\pm0.04$
$Y_+$ (mb)	$55\pm5$	$62 \pm 8$	$56 \pm 3$	$58 \pm 3$	$62 \pm 7$	$71\pm8$
$\eta_{-}$	$0.42 \pm 0.04$	$0.42 \pm 0.04$	$0.53\pm0.02$	$0.53\pm0.02$	$0.52\pm0.02$	$0.52\pm0.02$
$Y_{-}$ (mb)	$-17 \pm 4$	$-17 \pm 4$	$-30 \pm 4$	$-30 \pm 4$	$-29 \pm 4$	$-29 \pm 4$
K	-	-	0	0	$74 \pm 61$	$136 \pm 64$
No. DOF	87	89	147	149	146	148
$\chi^2$ /DOF	0.95	0.94	1.08	1.10	1.07	1.07

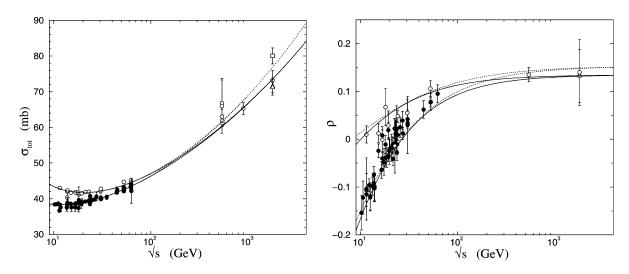


Fig. 2. Fits to pp (black symbols) and  $\bar{p}p$  (white symbols) total cross section data from ensembles I (dotted curves) and II (solid curves) and the corresponding predictions for  $\rho(s)$  with K = 0.

the curves and experimental information in the region from 500 GeV to 50 TeV, since the results are the same at lower energies, as can be seen in the corresponding results for  $\rho(s)$ .

### 4. Conclusions and final remarks

In this analysis we have used the experimental information presently available in the accelerator domain, including the recent E811 results on of  $\sigma_{tot}^{\bar{p}p}$  and  $\rho$  at 1.8 TeV, and also the highest and lowest estimations for  $\sigma_{tot}^{pp}$  from cosmic-ray experiments. From Table 1 (only accelerator data), we may infer the following upper and lower values for the Pomeron intercept:  $\alpha_{\mathbb{P}}^{\text{upper}}(0) = 1.098 \pm 0.004$  (global fits to ensemble I, with K = 0) and  $\alpha_{\mathbb{P}}^{\text{lower}}(0) = 1.085 \pm$ 0.004 (individual fit to  $\sigma_{\text{tot}}$  from ensemble II), with bounds 1.102 and 1.081, respectively.

Adding the cosmic-ray information, Table 2, we infer  $\alpha_{\mathbb{P}}^{\text{upper}}(0) = 1.104 \pm 0.005$  (individual fit to  $\sigma_{\text{tot}}$  from ensemble I + NGSY) and  $\alpha_{\mathbb{P}}^{\text{lower}}(0) = 1.085 \pm 0.003$  (global fits to ensemble II + BHS, and *K* as a free fit parameter or individual fit to  $\sigma_{\text{tot}}$  from this ensemble), with bounds 1.109 and 1.082, respectively.

Our approach, and the above values and bounds, may be compared with some representative results ob-

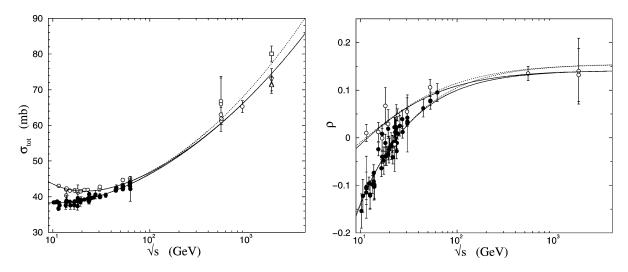


Fig. 3. Simultaneous fits to  $\sigma_{tot}(s)$  and  $\rho(s)$  data from ensembles I (dotted curves) and II (solid curves), with K = 0.

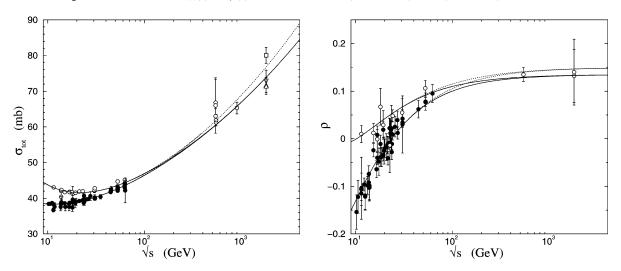


Fig. 4. Simultaneous fits to  $\sigma_{tot}(s)$  and  $\rho(s)$  data from ensembles I (dotted curves) and II (solid curves), with K as a free fit parameter.

tained by other authors, which are displayed in Table 3 and are reviewed in what follows. The fits by Donnachie and Landshoff (DL) [2] have been performed to only pp and  $\bar{p}p$  total cross section data, above 10 GeV and with the E710 result at 1.8 TeV. The CDF Collaboration (CDF), based on their result for the of  $\sigma_{tot}^{\bar{p}p}$  obtained a higher value for the intercept [5]. Further analyses, through the extended Regge parametrization, included also the E811 result. In the work by Cudell, Kang, and Kim (CKK) [3] only pp and  $\bar{p}p$  data above 10 GeV have been fitted. The analysis by Covolan, Montanha and Goulianos (CMG) [4] (using both a Born level and eikonal parametrizations) involved global fits to pp,  $\bar{p}p$ ,  $\pi^{\pm}p$  and  $k^{\pm}p$  at  $\sqrt{s} \ge 6$  GeV. The COMPETE Collaboration (COMPETE) [13] treated simultaneous fits to  $\sigma_{tot}$  and  $\rho$  in global fits to pp,  $\bar{p}p$ , meson-p,  $\gamma p$  and  $\gamma \gamma$  above 9 GeV. All these results concerned only accelerator data. In Ref. [1], Ávila, Luna and Menon (ALM) included also some cosmic-ray estimations for the of  $\sigma_{tot}^{pp}$  and made use of the original DL parametrization. It is also shown in Table 3 a recent theoretical result by Janik (Janik) [15] through a non-perturbative approach and using the AdS/CFT correspondence.

Table 2 Individual and global fits to  $\sigma_{\text{tot}}$  and  $\rho$  with ensemble I + NGSY and ensemble II + BHS and the subtraction constant K = 0 or as a free fit parameter

Fit:	Individual $\sigma_{tot}$		Global $\sigma_{\text{tot}}$ and $\rho$ with $K = 0$		Global $\sigma_{\text{tot}}$ and $\rho$ with K free	
Ensemble:	I + NGSY	II + BHS	I + NGSY	II + BHS	I + NGSY	II + BHS
$\epsilon$	$0.104\pm0.005$	$0.085\pm0.003$	$0.102\pm0.004$	$0.089 \pm 0.003$	$0.100\pm0.004$	$0.085\pm0.003$
X (mb)	$16 \pm 1$	$20 \pm 1$	$17 \pm 1$	$19 \pm 1$	$17 \pm 1$	$21 \pm 1$
$\eta_+$	$0.28\pm0.03$	$0.38\pm0.04$	$0.30 \pm 0.02$	$0.35\pm0.02$	$0.32\pm0.03$	$0.41 \pm 0.04$
$Y_{+}$ (mb)	$51 \pm 4$	$62 \pm 7$	$55 \pm 3$	$58 \pm 3$	$58 \pm 5$	$71 \pm 9$
$\eta_{-}$	$0.42\pm0.04$	$0.42\pm0.04$	$0.52\pm0.02$	$0.53\pm0.02$	$0.52\pm0.02$	$0.52\pm0.03$
$Y_{-}$ (mb)	$-17 \pm 4$	$-17 \pm 4$	$-29 \pm 4$	$-30 \pm 4$	$-29 \pm 4$	$-29 \pm 4$
K	_	_	0	0	$41 \pm 52$	$135\pm68$
No. DOF	94	96	154	156	153	155
$\chi^2/DOF$	1.01	0.89	1.11	1.06	1.11	1.03

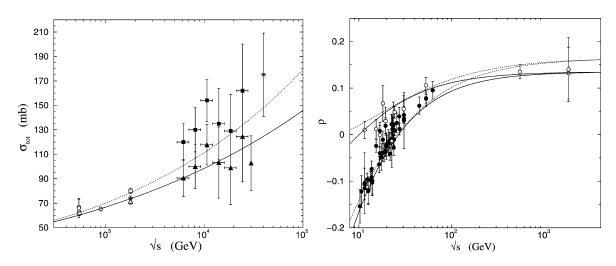


Fig. 5. Fits to pp and  $\bar{p}p$  total cross section data from ensembles I + NGSY (dotted curves) and II + BHS (solid curves) and the corresponding predictions for  $\rho(s)$  with K = 0.

In this Letter we have presented all the possible fits to pp and  $\bar{p}p$  data, above 10 GeV, through the extended Regge parametrization, exploring the contrasting data and the faster and the slower increase scenarios for the rise of the total cross section, allowed by the experimental information presently available.

From Table 3, our results exclude the values for the Pomeron intercept obtained by CMG (in the case of the eikonal parametrization), the lower bounds by ALM and Janik and the mean value by the CDF Collaboration. The DL result is barely compatible with our lower limit. It should be noted that, if the same ensemble is fitted, the introduction of non-degenerate trajectories result in a slightly increase of the Pomeron intercept. For example, fit to all the accelerator data Table 3

Some representative values, bounds and limits for the soft Pomeron intercept and those obtained in this work

	$\epsilon = \alpha_{\mathbb{P}}(0) - 1$	Bounds/Limits
DL [2]	0.0808	_
CDF [5]	$0.112 \pm 0.013$	-
CKK [3]	$0.096^{+0.012}_{-0.009}$	_
CMG [4]	$0.104 \pm 0.002$ (Born)	-
	$0.122 \pm 0.002$ (Eikonal)	-
COMPETE [13]	$0.093 \pm 0.002$	-
ALM [1]	_	0.0790-0.0940
Janik [15]	_	0.0729-0.083
This work	$0.085 \pm 0.004$ (lower)	0.081
	$0.104\pm0.005~(\text{upper})$	0.109

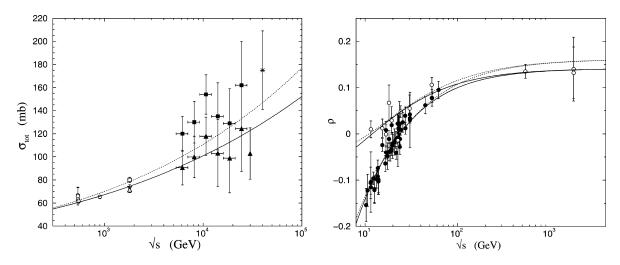


Fig. 6. Simultaneous fits to  $\sigma_{tot}(s)$  and  $\rho(s)$  data from ensembles I + NGSY (dotted curves) and II + BHS (solid curves), with K = 0.

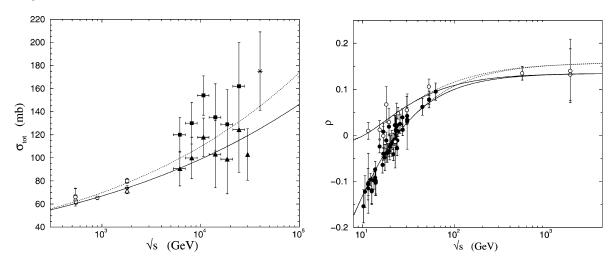


Fig. 7. Simultaneous fits to  $\sigma_{tot}(s)$  and  $\rho(s)$  data from ensembles I + NGSY (dotted curves for pp and dashed for  $\bar{p}p$ ) and II + BHS (solid curves for pp and dot dashed for  $\bar{p}p$ ), with K as a free fit parameter.

above 10 GeV (including the CDF and the E710/E811 values), leads to  $\epsilon = 0.086 \pm 0.003$  and  $\epsilon = 0.089 \pm 0.004$ , in the cases of degenerate (DL) and non-degenerate parametrizations, respectively.

From Figs. 2–7, we see that in all the cases investigated the  $\rho$  parameter is better described with ensembles I and I + BHS, a result that is also roughly supported by the  $\chi^2$ /DOF (Tables 1 and 2). We understand that this picture favors the E710/E811 results. This conclusion is contrary to that obtained by CMC [4] and more recently by the COMPETE Collaboration [16]. As a next step it may be important to investigate the consequences of the above extrema bounds in fittings to *p*-mesons,  $p\gamma$ , and  $\gamma\gamma$  scattering, with focus in the ratio of strengths of the Pomeron exchange (quark counting and factorization).

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