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Baryon stability on the Higgs dissolution edge: threshold corrections and suppression of baryon violation in the NMSGUT

Charanjit S. Aulakh *, Ila Garg, Charanjit K. Khosa

Department of Physics, Panjab University, Chandigarh, India

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Abstract

Superheavy threshold corrections to the matching condition between matter Yukawa couplings of the effective Minimal Supersymmetric Standard Model (MSSM) and the New Minimal Supersymmetric (SO(10)) GUT (NMSGUT) provide a novel and generic mechanism for reducing the long standing and generically problematic operator dimension 5 baryon decay rates. In suitable regions of the parameter space strong wave function renormalization of the effective MSSM Higgs doublets due to the large number of heavy fields can take the wave function renormalization of the MSSM Higgs field close to the dissolution value ($Z_{H,\bar{H}} = 0$). Rescaling to canonical kinetic terms lowers the SO(10) Yukawas required to match the MSSM fermion data. Since the same Yukawas determine the dimension 5 B violation operator coefficients, the associated rates can be suppressed to levels compatible with current limits. Including these threshold effects also relaxes the constraint $y_b - y_\tau \simeq y_s - y_\mu$ operative between **10**–**120**-plet generated tree level MSSM matter fermion Yukawas y_f. We exhibit accurate fits of the MSSM fermion mass-mixing data in terms of NMSGUT superpotential couplings and 5 independent soft Susy breaking parameters specified at $10^{16.25}$ GeV with the claimed suppression of baryon decay rates. As before, our s-spectra are of the mini split supersymmetry type with large $|A_0|, \mu, m_{H,\bar{H}} > 100$ TeV, light gauginos and normal s-hierarchy. Large A_0, μ and soft masses allow significant deviation from the canonical GUT gaugino mass ratios and ensure vacuum safety. Even without optimization, prominent candidates for BSM discovery such as the muon magnetic anomaly, $b \to s\gamma$ and leptogenesis CP violation emerge in the preferred ball park.

* Corresponding author.
E-mail address: aulakh@pu.ac.in (C.S. Aulakh).
URL: http://physics.puchd.ac.in/aulakh/ (C.S. Aulakh).

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1. Introduction

Supersymmetric Grand Unification based on the SO(10) gauge group [1] has received well deserved attention over the last 3 decades. Models proposed fall into two counter posed broad classes. The first consists of just a few models which preserve R-parity down to low energies [2–8]. It uses Higgs representations ($\overline{\mathbf{126}}$) of SO(10) that contain R-parity even SM singlets. The other large and diverse class of R-parity violating models [9] attempts to construct viable models using sets of small (dimension $d \leqslant 54$, index $S_2(d) \leqslant 12$) SO(10) representations even after sacrificing the vital distinction provided by R-parity between matter and Higgs multiplets in the first class of models. This issue is only the tip of a sharp wedge that divides the outlooks of these two schools of supersymmetric SO(10) unification and discussion of their contrasting attitudes towards fundamental questions regarding the nature of the UV completion of the MSSM is unavoidable.

The defining feature of R-parity preserving (RPP) GUTs [2–8] is use of a pair of $\mathbf{126} + \overline{\mathbf{126}}$ dimensional irreps which generate (via *renormalizable* B$-$L/R-parity even vevs) large right handed neutrino masses (small left handed triplet vevs) required by Type I (Type II) seesaw mechanism [10] for light neutrino mass. Such large irreps *cannot* arise in the massless sector of known string theory models. Thus this class of models may properly call itself "Unstrung GUTs" [11]. Following upon the proposal of [12] a great deal of attention was paid with considerable success, [13] to the issue of fitting the fermion mass and mixing data using $\mathbf{10} \oplus \overline{\mathbf{126}}$ vevs with *generic* coefficients (rather than derived in terms of GUT superpotential parameters). However when the realization of the generic coefficients in terms of actual GUT parameters was probed it was found that the fits were not feasible [6,14–16].

In direct contrast to RPP GUTs are R-parity violating (RPV) GUTs [9], which are typically "string inspired" or "string compatible", and employ **16** dimensional Higgs irreps (with B–L odd neutral components) to generate seesaw neutrino masses via "composite $\overline{\mathbf{126}}$" channels, i.e., $d > 4$ *non-renormalizable* operators thought to arise generically in the effective theory below the Planck/string scale. Out of the infinite set of possible $d > 4$ operators these models pick a convenient small subset and use their coefficients to fit data. The very absence of *any* calculation of the coefficients of such gravity/string induced operators from UV theory is taken to justify assuming them to have convenient values. Suppression of superfast B-decay and other unpleasant R-parity violating effects is accomplished by introducing -again with 'string inspiration'- suitable discrete symmetries. In contrast, RPP GUTs use only renormalizable interactions, avoid invoking *ad hoc* non-gauge symmetries and claim *parameter counting* – as opposed to field counting – minimality as their USP. In this respect they are perhaps closer in spirit to the original form of string unification where the infinite plethora of string excitations was justified by reference to the single coupling of the stringy TOE just beyond the horizon! Their neglect of the possibility that all non-renormalizable operators induced by gravity become strong, in the absence of any calculation of the coefficients of such operators, while not provably justified for now, is at least consistent with their renormalizable framework and assumptions and provides a fertile starting point for a self consistent exploration of a very complex theory which would not be illuminated by induction of an arbitrary number of new couplings.

Nevertheless the replies of RPP SO(10) model builders to perennial objections (see specially [17]) bear repetition and elaboration since the replies have evolved as the detailed structure of these very well defined and calculable models continues to be excavated [2–8,15,18–22] due to a focus maintained over 30 years: that few other models have succeeded in inspiring. Firstly use of the $\mathbf{126} + \overline{\mathbf{126}}$ pair with SO(10) indices $S_2(126) = 35$ *each makes a Landau pole in the* SO(10) *gauge coupling inevitable* at a scale Λ *within an order of magnitude above the perturbative unification scale*. We note that banning these irreps outright for large beta function would also eliminate the only other renormalizable channel for fermion mass in SO(10) namely the **120**-plet which has an index $S_2(120) = 28$. This would leave only models with a bunch of **10**-plets in consideration before even showing that such an impoverishment in structural richness is actually called for. SO(10) group theory clearly signals the importance of the $\overline{\mathbf{126}}$ dimensional representation for accommodating the most important mechanism for understanding neutrino mass seamlessly. Secondly If perturbative unification is postponed to a higher scale near to or coinciding with the Planck scale $M_p \sim 10^{18.4}$ GeV then the neglect of non-renormalizable operators suppressed only by the Planck scale is said to be unjustifiable. We have countered these objections [7,19,23,24] by arguing that detailed calculation of gauge threshold effects shows the perturbative unification scale-properly defined [19,25] – is indeed raised [7,19] towards the Planck scale. So it makes inevitable the coincidence of the SO(10) Landau pole with M_p itself. The unitarity violation arising in non-renormalizable Fermi theory determined a cutoff at the electroweak scale and required new degrees of freedom for UV completion and thus led to the discovery of the Standard Model. Similarly the Landau pole of RPP GUTs mandated by the neutrino seesaw dynamics required to assimilate convincingly the only known BSM dynamics within GUT models points to a new physical cutoff and need for a new UV completion. *The convergence of Λ and M_p points to a* origin *for gravity in the physics of strongly coupled supersymmetric* SO(10). For instance it could arise from that strongly coupled theory as an induced gravity [26] with the supersymmetric strong coupling scale $\Lambda \sim 10^{18}$ GeV setting the Newton constant much more plausibly and consistently than the original (inconsistent if non-supersymmetric [27]) proposal based on an asymptotically free gauge theory. In any case the existence of a Landau Pole at the Planck scale does not invalidate the use of a weakly coupled SO(10) GUT framework *below* that cutoff scale – where both SO(10) and gravitational couplings are small – just as it does not invalidate Fermi theory of weak interactions or chiral perturbation theory below the appropriate (internally determined) *physical* cutoffs. In short the Landau pole signals an internally determined physical cutoff of RPP SO(10) GUTs and is a potential *addition* to our physical understanding analogous to the information furnished by the breakdown of chiral at scales ∼1 GeV or Fermi perturbation theory at scales ∼50 GeV. We emphasize that in this wise RPP GUTs are no worse than the plethora of RPV SO(10) GUTs which are not only non-renormalizable, but typically assume without calculation that an infinity of operators present by consistency are negligible as also the (incalculable?) radiative corrections that should be applied.

This brings us to the related question of how restrictions to maintain perturbativity should be imposed in complex QFTs with many fields and couplings. We may keep in mind that well accepted theories like string theory and Kaluza–Klein models would fail this test utterly if a naive restriction like $g^2 < N^{-1}$ (N the number of fields) were adopted. As is well known even the QED perturbation series, in spite of giving predictions ($g - 2$ of muon, etc.) verifiable to 7 decimal places, does not, strictly speaking, converge and is only 'Borel summable'. Thus, at the stage when the quantum effects of the RPP SO(10) (which *is* at least perturbatively renormalizable in contrast to RPV models) have only begun to be calculated, demonstration of perturbative consistency can only proceed, order by order in the loop expansion, by demanding that radiative corrections to (directly or indirectly) measurable (e.g. $M_X, \alpha(M_X), \alpha_3(M_Z)$ [7,19]) or theoretically central (e.g. $Z_{f,\bar{f},H,\bar{Z}}$ in [28] and this paper) quantities should remain under control and respect basic consistency requirements such as correct sign (non-ghost) kinetic terms. Each such loop corrected element of the theory will obviously need to be checked at every loop order reached and there is no way of checking this at higher orders before the computation at lower orders. This situation is shared with other UV completions. Indeed, an important implication of our results is that theories such as string theory, before claiming consistent effective low energy models must check the threshold effects involved in specifying the light modes which mix strongly with heavy ones as we have done! What was long feared [17] and we have encountered when checking corrections to tree level fits found in [7] is that *due to the large number of fields* the wave function corrections can easily violate even basic constraints such as positivity ($Z > 0$) very badly. We therefore imposed positivity of wave function renormalization as a very effective proxy for merely numerical guess estimates for the magnitudes of perturbativity limits on couplings: because such a criterion already includes the crucial effect of the large number of fields. In fact we continued to find good fits although requiring positivity reduced the magnitude of Z by a factor of several hundred, brought the sign back to the physically acceptable one and drastically reduced the magnitude of the SO(10) couplings found! The baryon decay mechanism we advocate relies on this very (large N facilitated) limiting value being approached, i.e., $Z_{H,\bar{H}} \simeq 0$. Even if aggravated large N combinatorics at higher loops further restrict the magnitudes of SO(10) couplings they could still – by definition – yield values compatible with positivity. Finding solutions respecting $1 \gg Z \simeq 0$ for light field renormalization has *improved* our confidence in the perturbative status of the couplings so determined. We have identified parameter sets where the achievements of tree level fermion fits and gauge unification [7] are preserved even while the magnitudes of the couplings are much smaller and positivity of kinetic terms not violated (as the tree level fits were actually found to do once the huge computations we have performed became available). This confidence may well survive higher loop corrections as well unless the theory has a pathologically ill defined perturbation expansion. Our results on threshold corrected gauge unification [6,7,19] and fermion fitting [7] have rather lessened this fear by showing that the very complexity of the spectra effectively enlarges the possibilities for finding arrangements of parameters for which the feared breakdown does not take place. Our results favor the view that there is an intrinsic tendency for a “Higgs dissolution edge” to form when implementing the strange requirement of a fine tuned light MSSM Higgs pair to precipitate out of a plethora of superheavy MSSM doublets. There is no reason to preclude before computation the possibility that higher loop effects may further reduce the magnitudes of couplings required to attain the Higgs dissolution edge and thus further strengthen this growing confidence that the richness of SO(10) will dissipate the primordial fears of [17] and similarly render innocuous the threat of the nearby Landau pole. In any event the issue cannot be prejudged. Note that nothing in our interpretations of our extrapolates the small coupling calculation to a region where it is manifestly inapplicable.

Besides structural attractions, such as the automatic inclusion of the conjugate neutrino fields necessary for neutrino mass, SO(10) GUTs offer a number of other natural features. Among these are third generation Yukawa unification [29,30], automatic embedding of minimal supersymmetric left–right models, natural R-parity preservation [4] down to the weak scale and consequently natural LSP WIMP dark matter, economic and explicitly soluble symmetry breaking at the GUT scale [5], explicitly calculable superheavy spectra [18–21], interesting gauge unification threshold effects [6,7,15,19] which can lead to a natural elevation of the unification scale to near the Planck scale [7], GUT scale threshold corrections to the QCD coupling $\alpha_3(M_Z)$ of the required

[31] sign and size [32] and a deep interplay between the scales of baryon and lepton number violation as suggested by the neutrino oscillation measurements and the seesaw formulae connecting neutrino masses to the B−L breaking scale.

The fascination of the MSSM RG flow at large $\tan\beta$ stems from the tendency of third generation Yukawa couplings to converge, at the MSSM unification scale [29,30], in a manner reminiscent of gauge unification in the MSSM RG flow [33,34]. For suitably large $\tan\beta$ and for close to central input values of SM fermion couplings at the Susy breaking scale $M_S \sim M_Z$, third generation Yukawas actually almost coincide at M_X. On the other hand, in SO(10) theories with only the simplest possible fermion mass giving (FM) Higgs content (a single **10**-plet), when all the complications of threshold effects at $M_X \sim 10^{16}$ GeV (not to speak of those at seesaw scales $M_{\bar\nu} \sim 10^7$–10^{12} GeV) are ignored, one does expect to generate boundary conditions for the gauge and Yukawa couplings that are unified gauge group wise and (third generation) flavor wise.

However, fitting the rest of the known fermion data (15 more parameters) definitely requires other fermion Higgs multiplets (more **10**-plets, **120**, $\overline{\mathbf{126}}$s, etc.). A principled position (*monoHiggism*?) with regard to the choice of FM Higgs irreps is to introduce *only one* of each irrep present in the conjugate of the direct product of fermion representations. This principle may be motivated by regarding the different Higgs representations as characteristic "FM channels" through which the fermion mass (FM) is transmitted in structurally distinguishable ways. For example the Georgi–Jarlskog mechanism distinguishes the **45** plet Higgs in SU(5) ($\overline{\mathbf{126}}$ in SO(10)) from the $\mathbf{5} + \bar{\mathbf{5}}$ (**10** in SO(10)) due to their ability to explain the quark–lepton mass relations in the second and third generations respectively. Similarly the $\overline{\mathbf{126}}$ in SO(10) is peculiarly suitable for implementing the Type I and Type II seesaw mechanisms for neutrino mass (as well as embedding the Georgi–Jarlskog mechanism: but the two functions may be incompatible [6]). If one duplicates the Higgs multiplets transforming as the *same* gauge group representation, for example by taking multiple **10**-plets in SO(10), then one abandons the quest for a structural explanation of the pattern of fermion masses in favor of "just so" solutions.

In previous work [7] we have shown that it is possible to obtain accurate fits of the complete effective MSSM fermion couplings (including neutrino mass Weinberg operator derived from Type I and Type II Seesaw masses) from the SO(10) Susy GUT specified by [2,3] the **210**, **10**, **120**, **126**, $\overline{\mathbf{126}}$ Higgs system. A very notable feature of this fit was that it was achieved by deducing that threshold corrections at M_S must play a vital role from the *cul de sac* into which the theory had apparently [35] painted itself by leaving only **10**, **120**-plets to fit charged fermion masses. The use of threshold corrections to evade the no-go of too small d, s masses found in [35] then led to the remarkable prediction, well ahead of the discovery of Higgs mass at 126 GeV, that the shierachy is *normal*, i.e., *stops are heavy* and supersymmetry is in the decoupling regime ($M_A \gg M_Z$) [36] and mini-split [37]: $A_0, \mu, m_{3/2}$ are multi or tens of TeV. All these were anathema to Susy orthodoxy in those years: now they are accepted to be required by Susy and 126 GeV light Higgs! However this notable success was faced with the uncomfortable fact that the parameters found implied [7] proton decay lifetimes $\sim 10^{28}$ yrs, i.e., at least six orders of magnitude greater than current limits.

To tackle this situation we proposed [28] that in Minimal renormalizable SO(10) theories [2,3,5] due to the large number of heavy fields running within the light field propagators entering the fermion Yukawa vertices a strong wave function renormalization is possible even in the perturbative regime. This can then radically modify the MSSM–GUT Yukawa matching conditions by suppressing the SO(10) Yukawas required to match the MSSM fermion 'data'. An preliminary calculation – with some defects – of the threshold corrections to the matching condition between

MSSM and GUT determined Yukawas was used to argue that the B-decay rate in renormalizable Susy SO(10) could be strongly suppressed. In this paper we present a complete calculation of the threshold corrections to the matter fermion and MSSM Higgs vertices. We also found realistic fits of the earlier type [7] but now fully viable inasmuch as the $d = 5$, $\Delta B \neq 0$ lifetimes can be 10^{34} yrs or more. We note that superheavy threshold corrections also relax the stringent constraint $y_b - y_\tau \simeq y_s - y_\mu$ that we found [38–40] operative at M_X in SO(10) models with a **10–120** FM Higgs system.

A detailed discussion of the historical developments, motivations and phenomenological issues related to the present work can be found in the preliminary survey in [28]. Other calculable quantities include quark and lepton flavor violation rates, muon $g - 2$ anomaly, candidate loop corrected Susy discovery spectra, leptogenesis parameters and NMSGUT based inflection point inflation (with inflaton scale set by the Type I Seesaw mass scale [41]). In this paper we mainly focus on resolving the major issue of $d = 5$, $\Delta B \neq 0$ rates.

In Section 2 we briefly review the structure of the NMSGUT [6,7] to establish the notation for presentation of our results on threshold effects in Subsection 3.1 and Appendix A. In Subsection 3.2 we present illustrative examples to underline the significance of the GUT scale threshold effects and the need to include them. In Section 4 we discuss various aspects of our fitting criteria together with threshold effects, and give a description of the tables in Appendix B in Subsection 4.1. In Section 5 we discuss exotic observables and specially the acceptable $d = 5$ operator baryon violation rates we have found. In Section 6 we summarize our conclusions and discuss which improvements in the fitting, RG flows and searches are urgently called for. Appendix A contains details of the calculation of threshold effects at M_X. In Appendix B we give two example solutions of NMSGUT parameters which fit fermion mass-mixing data and are compatible with B decay limits.

2. NMSGUT recapitulated

The NMSGUT [7] is a renormalizable globally supersymmetric SO(10) GUT whose Higgs chiral supermultiplets consist of AM (Adjoint Multiplet) type totally antisymmetric tensors: $\mathbf{210}(\Phi_{ijkl})$, $\overline{\mathbf{126}}(\overline{\boldsymbol{\Sigma}}_{ijklm})$, $\mathbf{126}(\boldsymbol{\Sigma}_{ijklm})$ $(i, j = 1, \ldots, 10)$ which break the SO(10) symmetry to the MSSM, together with fermion mass (FM) Higgs **10** ($\mathbf{H}_i$) and **120** (Θ_{ijk}). The SO(10) anti-self dual $\overline{\mathbf{126}}$ plays a dual or AM–FM role since it also enables the generation of realistic charged fermion and neutrino masses and mixings (via the Type I and/or Type II Seesaw mechanisms); three **16**-plets $\boldsymbol{\Psi}_{\boldsymbol{A}}$ $(A = 1, 2, 3)$ contain the matter including the three conjugate neutrinos $(\bar{\nu}_L^A)$. The superpotential (see [5–7,18–20] for comprehensive details) contains the mass parameters

$$m\colon\ \mathbf{210}^2; \qquad M\colon\ \mathbf{126}\cdot\overline{\mathbf{126}}; \qquad M_H\colon\ \mathbf{10}^2; \qquad m_\Theta\colon\ \mathbf{120}^2 \tag{1}$$

and trilinear couplings corresponding to the superfield chiral invariants indicated:

$$\begin{aligned}
&\lambda\colon\ \mathbf{210}^3; \qquad \eta\colon\ \mathbf{210}\cdot\mathbf{126}\cdot\overline{\mathbf{126}}; \qquad \rho\colon\ \mathbf{120}\cdot\mathbf{120}\cdot\mathbf{210}\\
&k\colon\ \mathbf{10}\cdot\mathbf{120}\cdot\mathbf{210}; \qquad \gamma\oplus\bar{\gamma}\colon\ \mathbf{10}\cdot\mathbf{210}\cdot(\mathbf{126}\oplus\overline{\mathbf{126}})\\
&\zeta\oplus\bar{\zeta}\colon\ \mathbf{120}\cdot\mathbf{210}\cdot(\mathbf{126}\oplus\overline{\mathbf{126}})
\end{aligned} \tag{2}$$

In addition one has two symmetric matrices h_{AB}, f_{AB} of Yukawa couplings of the $\mathbf{10}$, $\overline{\mathbf{126}}$ Higgs multiplets to the $\mathbf{16_A}\cdot\mathbf{16_B}$ matter bilinears and one antisymmetric matrix g_{AB} for the coupling of the **120** to $\mathbf{16_A}\cdot\mathbf{16_B}$. One of the complex symmetric matrices can be made real and diagonal by a choice of SO(10) flavor basis. Thus initially complex FM Yukawas contain 3 real and 9 complex parameters. Five overall phases (one for each Higgs), say those of $m, M, \lambda, \gamma, \bar{\gamma}$ can be set by fixing phase conventions. One (complex parameter) out of the rest of the superpotential parameters, i.e., $m, M_H, M, m_\Theta, \lambda, \eta, \rho, k, \gamma, \bar{\gamma}, \zeta, \bar{\zeta}$, say M_H, can be fixed by the fine tuning condition to keep two doublets light so that the effective theory is the MSSM. After removing unphysical phases this leaves 23 magnitudes and 15 phases as parameters: still in the lead out of any theories aspiring to do as much [5]. As explained in [5,18,19] the fine tuning fixes the Higgs fractions, i.e., the composition of the massless electroweak doublets in terms of the (6 pairs of suitable) doublet fields in the GUT.

The GUT scale vevs and therefore the mass spectrum are all expressible [5,19,20] in terms of a single complex parameter x which is a solution of the cubic equation

$$8x^3 - 15x^2 + 14x - 3 + \xi(1-x)^2 = 0 \tag{3}$$

where $\xi = \frac{\lambda M}{\eta m}$.

In our programs we find it convenient to scan over the “three for a buck” [5,6,42] parameter x and determine ξ therefrom. Then the phase of λ is adjusted to be that implied by x and the relation $\xi = \frac{\lambda M}{\eta m}$ and is not itself scanned over independently. It is a convenient fact that the **592** fields in the Higgs sector fall into precisely **26** different types of SM gauge representations which can hence be naturally labeled by the **26** letters of the English alphabet [19]. The decomposition of SO(10) in terms of the labels of its “Pati–Salam” maximal subgroup $SU(4) \times SU(2)_R \times SU(2)_L$ provided [18] a translation manual from SO(10) to unitary group labels. The complete GUT scale spectrum and couplings of this theory have been given in [7,19]. The MSSM fermion Yukawa couplings and neutrino mass (Weinberg) operator of the effective MSSM arising from this GUT after fine tuning (but before application of GUT scale threshold corrections), along with the implementation of loop corrected electroweak symmetry breaking based on a fixed value of $\tan\beta$, M_Z and the run down values of $M^2_{H,\bar{H}}$ and the threshold corrections to the matching conditions between MSSM and SM fermion Yukawa are given in [7, Appendix C].

In the NMSGUT, to enhance the light neutrino Type I seesaw masses [6,7], the conjugate (i.e., “right handed”) neutrino Majorana masses are 4 or more orders of magnitude smaller than the GUT scale due to very small $\overline{\mathbf{126}}$ couplings. Therefore for purposes of calculating the threshold corrections to the Yukawa couplings at M_X we can consistently treat the conjugate neutrinos as light particles on the same footing as the other 15 fermions of each SM family. These fermion mass formulae, after correcting for threshold effects, are to be confronted with the fermion Yukawa couplings and Weinberg neutrino mass operator (RG-extrapolated from $Q = M_Z$ to $Q = M_X^0 = 10^{16.25}$ GeV). The calculation of the change in the unification scale exponent (Δ_X) also fixes [19] the scale m of the high scale symmetry breaking [6,7]. The simultaneous requirements of a common origin for the unification-seesaw scale, gauge unification, with the right high scale and Susy breaking scale, RG threshold corrections to shift the GUT prediction of $\alpha_3(M_Z)$ down to acceptable values [32] and to lower the down and strange fermion Yukawas to a level achievable in this type of GUT [7], are very stringent. They are effective in singling out characteristic and suggestive GUT parameters (including Susy breaking parameters at M_X) which realize a fully realistic effective theory with distinctive signatures *derived* from its UV completion. We now show how the NMSGUT can successfully bypass the remaining roadblock of rapid dimension 5 proton decay which is generic to Susy GUTs.

3. GUT scale Yukawa threshold corrections

3.1. One loop threshold correction formulae

The technique of [43] for calculating high scale threshold corrections to Yukawa couplings, generalizes the Weinberg–Hall [25] method for calculating threshold corrections to gauge couplings, and has long been available but has not been exploited much; possibly due to the assumption that such effects are always negligible. In supersymmetric theories the superpotential parameters are renormalized only due to wave function correction and this is easy – if tedious – to calculate for the large number of heavy fields which couple to the light fermions and MSSM Higgs at SO(10) Yukawa and gauge vertices. The calculation involves going to a basis in which the heavy field supermultiplet mass matrices are diagonal. This basis is easily computable given the complete set of mass matrices and trilinear coupling decompositions given in [7,18,19]. For a generic heavy field type Φ (conjugate $\bar{\Phi}$) the mass terms in the superpotential diagonalize as:

$$\bar{\Phi} = U^{\Phi}\bar{\Phi}'; \qquad \Phi = V^{\Phi}\Phi' \quad \Rightarrow \quad \bar{\Phi}^T M \Phi = \bar{\Phi}'^T M_{Diag}\Phi' \tag{4}$$

The circulation of heavy supermultiplets within the one loop insertions on each of the 3 chiral superfield lines ($f_c = \bar{f}, f, H_f = H, \bar{H}$) entering the matter Yukawa vertices:

$$\mathcal{L} = \left[f_c^T Y_f f H_f\right]_F + \text{H.c.} + \cdots \tag{5}$$

implies [43] a finite wave function renormalization in the kinetic terms:

$$\mathcal{L} = \left[\sum_{A,B}\left(\bar{f}_A^{\dagger}(Z_{\bar{f}})_A^B \bar{f}_B + f_A^{\dagger}(Z_f)_A^B f_B\right) + H^{\dagger} Z_H H + \bar{H}^{\dagger} Z_{\bar{H}} \bar{H}\right]_D + \cdots \tag{6}$$

where $A, B = 1, 2, 3$ run over matter generations and $H, \bar{H}$ are the light Higgs doublets of the MSSM. The light Higgs superfields are actually mixtures of 6 Higgs doublets $h_i, \bar{h}_i, i = 1, \ldots, 6$ from the GUT multiplets:

$$H = \sum_i \alpha_i^* h_i; \qquad \bar{H} = \sum_i \bar{\alpha}_i^* \bar{h}_i \tag{7}$$

where the Higgs fractions $\alpha_i, \bar{\alpha}_i$ are components of the null eigenvectors of the Higgs doublet mass matrix $\mathcal{H}$ [5,7,18,19].

Let $U_{Z_f}, \bar{U}_{Z_{\bar{f}}}$ be the unitary matrices that diagonalize ($U^{\dagger} Z U = \Lambda_Z$) $Z_{f,\bar{f}}$ to positive definite form $\Lambda_{Z_f, Z_{\bar{f}}}$. We define a new basis to put the kinetic terms of the light matter and Higgs fields in canonical form:

$$f = U_{Z_f}\Lambda_{Z_f}^{-\frac{1}{2}}\tilde{f} = \tilde{U}_{Z_f}\tilde{f}; \qquad \bar{f} = U_{Z_{\bar{f}}}\Lambda_{Z_{\bar{f}}}^{-\frac{1}{2}}\tilde{\bar{f}} = \tilde{U}_{Z_{\bar{f}}}\tilde{\bar{f}}$$

$$H = \frac{\tilde{H}}{\sqrt{Z_H}}; \qquad \bar{H} = \frac{\tilde{\bar{H}}}{\sqrt{Z_{\bar{H}}}} \tag{8}$$

then

$$\mathcal{L} = \left[\sum_A\left(\tilde{\bar{f}}_A^{\dagger}\tilde{\bar{f}}_A + f_A^{\dagger}\tilde{f}_A\right) + \tilde{H}^{\dagger}\tilde{H} + \tilde{\bar{H}}^{\dagger}\tilde{\bar{H}}\right]_D + \left[\tilde{\bar{f}}^T \tilde{Y}_f \tilde{f}\tilde{H}_f\right]_F + \text{H.c.} + \cdots$$

$$\tilde{Y}_f = \Lambda_{Z_{\bar{f}}}^{-\frac{1}{2}} U_{Z_{\bar{f}}}^T \frac{Y_f}{\sqrt{Z_{H_f}}} U_{Z_f}\Lambda_{Z_f}^{-\frac{1}{2}} = \tilde{U}_{Z_{\bar{f}}}^T \frac{Y_f}{\sqrt{Z_{H_f}}}\tilde{U}_{Z_f} \tag{9}$$

Thus when matching to the effective MSSM it is $\tilde{Y}_f$ and not the original Y_f obtained [5,18,19] from the SO(10) Yukawas that must equal the value of the MSSM Yukawa at the matching scale.

For any light chiral field Φ_i the corrections have generic form ($Z = 1 - \mathcal{K}$):

$$\mathcal{K}_i^j = -\frac{g_{10}^2}{8\pi^2}\sum_\alpha Q_{ik}^{\alpha\,*} Q_{kj}^\alpha F(m_\alpha, m_k) + \frac{1}{32\pi^2}\sum_{kl} Y_{ikl} Y_{jkl}^* F(m_k, m_l) \tag{10}$$

where $\mathcal{L} = g_{10} Q_{ik}^\alpha \psi_i^\dagger \gamma^\mu A_\mu{}^\alpha \psi_k$ describes the generic gauge coupling of the (fermion component ψ_i of) Φ_i to a generic SO(10) *heavy* gauge boson A^α and charge Q^α ($g_{10} = g_5/\sqrt{2}$ and g_5 are the SO(10) and SU(5) gauge couplings). The generic Yukawa couplings are defined by the superpotential $W = \frac{1}{6} Y_{ijk} \Phi_i \Phi_j \Phi_k$.

When both the fields running in the loop are heavy fields the symmetric Passarino–Veltman function $F(m_1, m_2)$ should be taken to be

$$F_{12}(M_A, M_B, Q) = \frac{1}{(M_A^2 - M_B^2)}\left(M_A^2 \ln\frac{M_A^2}{Q^2} - M_B^2 \ln\frac{M_B^2}{Q^2}\right) - 1 \tag{11}$$

which reduces to just

$$F_{11}(M_A, Q) = F_{12}(M_A, 0, Q) = \ln\frac{M_A^2}{Q^2} - 1 \tag{12}$$

when one field is light ($M_B \to 0$). When one of the heavy fields in the loop has MSSM doublet type G_{321} quantum numbers $[1, 2, \pm 1]$ (so that one eigenvalue is light while the other *five* [7] are heavy) care should be taken to avoid summing over light–light loops: since that calculation belongs to the MSSM radiative corrections.

The crucial point to notice is that the SO(10) Yukawa couplings $(h, f, g)_{AB}$ also enter into the coefficients L_{ABCD}, R_{ABCD} of the $d = 5$ baryon decay operators in the effective superpotential obtained by integrating out the heavy chiral supermultiplets that mediate baryon decay (see [7,18,19] for discussion of the contributing higgsino modes and derivation of expressions):

$$W_{\mathit{eff}}^{\Delta B \neq 0} = -L_{ABCD}\left(\frac{1}{2}\epsilon Q_A Q_B Q_C L_D\right) - R_{ABCD}(\epsilon \bar{e}_A \bar{u}_B \bar{u}_C \bar{d}_D) \tag{13}$$

After the redefinition (8) to the tilde basis to make the kinetic terms canonical, $\tilde{Y}_f$ must be diagonalized to mass basis (denoted by primes) using bi-unitary $U_{\bar{f}}(N_g) \times U_f(N_g)$ kinetic term redefinitions via the unitary matrices $(U_f^{L,R})$ made up of the left and right eigenvectors of $\tilde{Y}_f$ with phases fixed by the requirement that $(U_f^L)^T \tilde{Y}_f U_f^R = \Lambda_f$ yields positive definite Λ_f:

$$\begin{aligned} W &= (\bar{f}')^T \Lambda_f f' \tilde{H}_f \\ f &= \tilde{U}_{Z_f} U_f^R f' = \tilde{U}'_f f' \\ \bar{f} &= \tilde{U}_{Z_{\bar{f}}} U_f^L \bar{f}' = \tilde{U}'_{\bar{f}} \bar{f}' \end{aligned} \tag{14}$$

As a result the $d = 5$, $\Delta B = \pm 1$ decay operator coefficients in terms of the Yukawa eigenstate basis become

$$\begin{aligned} L'_{ABCD} &= \sum_{a,b,c,d} L_{abcd} (\tilde{U}'_Q)_{aA} (\tilde{U}'_Q)_{bB} (\tilde{U}'_Q)_{cC} (\tilde{U}'_L)_{dD} \\ R'_{ABCD} &= \sum_{a,b,c,d} R_{abcd} (\tilde{U}'_{\bar{e}})_{aA} (\tilde{U}'_{\bar{u}})_{bB} (\tilde{U}'_{\bar{u}})_{cC} (\tilde{U}'_{\bar{d}})_{dD} \end{aligned} \tag{15}$$

When we search for a fit of the MSSM Yukawas in terms of the SO(10) parameters under the constraint that L'_{ABCD}, R'_{ABCD} be sufficiently suppressed (i.e., yielding proton lifetime $\tau_p > 10^{34}$ yrs) we find that the search is guided ineluctably towards those regions of SO(10) parameter space where $Z_{H,\bar{H}} \ll 1$. As a result the SO(10) Yukawa couplings required to match the MSSM become much smaller than they would be if these threshold corrections are ignored. The same SO(10) Yukawa couplings enter L'_{ABCD}, R'_{ABCD} but there is no boost derived from wave function renormalization because $d = 5$ operators have no external Higgs line. This mechanism is generically available in realistic multi-Higgs theories. It remains to be checked what is the effect on $d = 6$ B violation operators with one external Higgs line. However those operators are severely suppressed to begin with.

The decomposition of SO(10) invariant terms in the superpotential and gauge terms yields [7,18,19] a large number (∼1100) of relevant light–heavy–heavy/light SO(10) vertices. It then requires a tedious but straightforward calculation to determine the threshold corrections. The explicit expressions are given in Appendix A.

Heretofore such threshold corrections have mostly been argued to be negligible (<1%) although at least one paper [44] faced with the difficulties of literal third generation Yukawa unification has considered the possibility, without any explicit model which permitted calculation, that the third generation Yukawa unification relations must necessarily be subject to threshold corrections of up to 50%. In which case it was found that the various stratagems invoked to permit precise 3 generation Yukawa unification could become redundant. We shall see that the calculation of the GUT scale 1-loop Yukawa threshold effects in the NMSGUT can actually change the naive (i.e., pure **10**-plet) unification relations $y_t = y_b = y_\tau$ significantly.

Furthermore the **10–120**-plet fermion fits have been shown (in the absence of GUT scale threshold effects) to require a close equality $|y_b - y_\tau/(y_s - y_\mu)| \approx 1$ at M_X which is very constricting when searching for fits. The fits we exhibited in [7] were all of this type. However in the present case the fits we obtain can deviate significantly from $\frac{y_b - y_\tau}{y_s - y_\mu} \simeq 1$. Of course one should study the higher loop threshold corrections to see if the 1-loop results we find are stable. At present this task seems computationally prohibitive. However we have calculated the complete SO(10) two loop beta functions [45] using the fact that the beta functions are determined by anomalous dimensions alone. Since the two loop threshold corrections will also rely upon essentially the same type of anomalous dimensions, its may be possible to convolute the GUT scale mass spectra with our SO(10) loop sums to determine the two loop threshold corrections as well. In any case our one loop results are a necessary first step for higher loop studies. As noted before the restriction $Z > 0$ also leads to smaller couplings and to heavy spectra that are significantly less spread out than in our previous solutions.

The effect of the wave function renormalization on the relation between other GUT and MSSM parameters is also interesting and illuminating. The MSSM superpotential μ parameter is larger than the GUT μ parameter by the factor $(Z_H Z_{\bar{H}})^{-1/2}$ and the same goes for the soft Susy breaking parameter B. On the other hand, the matter sfermion soft masses are enhanced only by $Z_{\bar{f}}^{-1}$ which will be very close to 1. The soft Higgs masses will however be boosted by $Z_{H/\bar{H}}^{-1}$. It is the boosted parameters we determine in our fits and it is interesting to note (see Appendix B and [7]) that we typically find $\mu, A_0, |m_{H/\bar{H}}| \gg m_{\tilde{f}/\tilde{\bar{f}}} \gg M_{1/2}$! However the A_0 parameter does not change since the wave function enhancements are absorbed by the Yukawa coupling in terms of which it is defined ($A = A_0 \tilde{Y}$).

Finally the right handed neutrino masses $(M_{\bar{\nu}})_{AB} \sim f_{AB} \langle \bar{\sigma} \rangle$ will also change due to finite corrections to the SO(10) breaking induced mass term due to heavy field loops. However since the

Table 1
Eigenvalues of the wave function renormalization matrices Z_f for fermion lines and for MSSM Higgs ($Z_{H,\bar{H}}$) for solutions found in [7].

Solution 1			
Eigenvalues ($Z_{\bar{u}}$)	0.928326	0.930946	1.031795
Eigenvalues ($Z_{\bar{d}}$)	0.915317	0.917464	0.979132
Eigenvalues ($Z_{\bar{\nu}}$)	0.870911	0.873470	0.975019
Eigenvalues ($Z_{\bar{e}}$)	0.904179	0.908973	0.971322
Eigenvalues (Z_Q)	0.942772	0.946127	1.027745
Eigenvalues (Z_L)	0.911375	0.916329	0.997229
$Z_{\bar{H}}, Z_H$	−109.367	−193.755	
Solution 2			
Eigenvalues ($Z_{\bar{u}}$)	−7.526729	−7.416343	1.192789
Eigenvalues ($Z_{\bar{d}}$)	−7.845885	−7.738424	1.191023
Eigenvalues ($Z_{\bar{\nu}}$)	−8.830309	−8.681419	1.234923
Eigenvalues ($Z_{\bar{e}}$)	−7.880892	−7.716853	1.238144
Eigenvalues (Z_Q)	−9.203739	−9.109832	1.171956
Eigenvalues (Z_L)	−9.797736	−9.698265	1.217620
$Z_{\bar{H}}, Z_H$	−264.776	−386.534	

vev $\langle\bar{\sigma}\rangle$ is protected by the non-renormalization theorem, i.e., is fixed in terms of the parameters m, λ, M, η, and the corresponding field fluctuation is *not* a part of the low energy effective theory, the heavy loops will redefine $f_{AB} \to \tilde{f}_{AB} = (\tilde{U}_{\bar{\nu}}^T f \tilde{U}_{\bar{\nu}})_{AB}$ along with $Y^{\nu}_{AB} \to \tilde{Y}^{\nu}_{AB}$ (Eq. (9)). As a result when the right handed neutrinos $\bar{\nu}$ are integrated out the factors $\tilde{U}_{\bar{\nu}}$ actually cancel out of the Type I seesaw formula leaving only $\tilde{U}_{\nu}, Z_H^{-1/2}$ to dress the formula obtained without threshold corrections. Since $Z_{\bar{\nu}}$ is rather close to unity the effect on neutrino masses is likely to be small. We have included these factors in our calculations. This discussion also shows that we have given a complete calculation of the germane 1-loop GUT scale threshold corrections to the relation between observable gauge, Yukawa, Seesaw and B-decay couplings and GUT scale parameters.

3.2. *Necessity of including threshold effects*

To appreciate the importance of the threshold corrections at M_X for the matter fermion Yukawas it is sufficient to consider what one obtains for $Z_{f,\bar{f},H,\bar{H}}$ using parameters from the examples of tree level fits (found ignoring GUT scale threshold corrections) given in [7].

It is clear from Table 1 that neglect of the wave function corrections would be a serious error since they are easily so large as to change the sign of the effective kinetic terms! In the case of Solution 2, not only the Higgs but even the fermion line corrections can be large enough to do this! This seems to put the solutions found in [7] (as well as all previous GUTs with a Higgs structure rich enough to account for the observed charged fermion and neutrino data) in a dubious light. However we shall see that the disease contains its own cure: when the wave function corrections are correctly accounted for, and searches mounted while maintaining $Z > 0$ for all fields we are led to regions of the parameter space where not only the matter Yukawa couplings but also the other super-potential parameters are significantly lowered in magnitude: *inter alia* improving the status of the model vis a vis perturbativity. Since accounting for the effects of threshold corrections also allows us to lower the $d = 5$ operator mediated B-violation rate, it is clear that a central result of our work is that henceforth close attention must be paid to the consequences of the fact that MSSM Higgs multiplets derive from multiple GUT sources. Analyses of GUT models that neglect the multiple GUT level parentage of MSSM Higgs and the consequent drastic threshold corrections to tree level effective MSSM couplings should no longer be countenanced uncritically. Of course this warning traces back to [17], but our emphasis [7,19] has been to exploit the richness of the Quantum effects rather than a pessimistic one.

4. Realistic fits with threshold corrections included

4.1. Description of search strategy and conditions

We follow the procedure described in [7] to find sets of GUT superpotential and GUT compatible soft Susy breaking parameters which allow accurate fits of *all* the fermion masses and mixing angles. The new features are that

- We use our search programs to find fits after including the threshold effects at M_X^0.
- We include the effects of Susy thresholds on the gauge unification parameters ($\Delta_{X,G,3}$) which we earlier neglected but should not have since the sparticle spectrum we found is decoupled (large $A_0, \mu, m_0 \gg M_Z$) and quite spread out.

We impose strict unitarity in the sense that the wave function renormalization must remain positive, i.e.,

$$Z_{f,\bar{f},H,\bar{H}} > 0 \tag{16}$$

The search programs [7] do find solutions (quite far from the examples of [7] in that many couplings, such as most noticeably η undergo major changes, being driven towards smaller values) which satisfy this constraint and still provide consistent unification and accurate fits of the fermion mass data. Unless higher loop effects could somehow overcome and forbid the tendency of Z to be reduced below 1 that we found by calculating 1-loop effects, it is likely that even smaller values of the couplings will make $Z_H \simeq 0$ achievable. Then the suppression of proton decay may become even easier.

Moreover, the effectiveness of our mechanism for reducing the size of the $d = 5$ B decay operators is verified. When we conduct searches while demanding that these coefficients be suppressed strongly the search program incorporating threshold corrections succeeds in finding solutions: whereas earlier proton decay lifetimes greater than about 10^{28} yrs could not be achieved. Specifically, without the threshold corrections the generic values of the maximal absolute magnitude $Max(O^{(4)})$ of the LLLL and RRRR coefficients in the $d = 5, \Delta B \neq 0$ effective superpotential was found to be typically of order 10^{-17} GeV^{-1} corresponding to fast baryon decay rates $\sim 10^{-27}$ yr^{-1}. Our quick fix to the problem of limiting the B-decay rate while searching for accurate fermion fits is to limit ($\tilde{O}$ is the dimensionless operator in units of $|m/\lambda|$) $Max(\tilde{O}^{(4)}) < 10^{-5}$ (in dimensionful terms $Max(O^{(4)}) < 10^{-22}$ GeV^{-1}). This produces fits with proton lifetimes above 10^{34} yrs, so we work with a penalty for violating: $Max(\tilde{O}^{(4)}) < 10^{-5}$. These fits are always in regions where $Z_{H,\bar{H}}$ approach zero (from above) while $Z_{f,\bar{f}}$ suffer only minor corrections since the **16**-plet Yukawas are now suppressed. In addition to the penalty for rapid proton decay we also imposed the following conditions for acceptable fits:

1. As already explained in detail in [7] the gauge unification RG flow is constrained so that perturbation theory in the gauge coupling at unification remains valid, the unification scale

is less than M_{Planck} and the GUT threshold contributions to $\alpha_3(M_Z)$ (together with the corrections from the rather high value of the superpartner masses: see below) are in the right range [7,31,32]:

$$\begin{aligned} &-22.0 \leqslant \Delta_G \equiv \Delta\big(\alpha_G^{-1}(M_X)\big) \leqslant 25 \\ &3.0 \geqslant \Delta_X \equiv \Delta(\mathrm{Log}_{10} M_X) \geqslant -0.03 \\ &-0.0126 < \Delta_3 \equiv \Delta\alpha_3(M_Z) < -0.0122 \end{aligned} \tag{17}$$

2. We constrain the $|\mu(M_Z)|$, $|A_0(M_Z)|$ parameters to be smaller than 150 TeV. Two loop RGE flow from M_X to M_Z, *ignoring generation mixing*, was used to determine these soft Susy parameters (by imposing consistency with Susy threshold effects required for fitting $y_{d,s,b}$) since only the diagonal threshold correction formulae are available at present. This is justified in view of our limited expectations of overall accuracy of sfermion spectra which are so far uncorrected by loops. Typically these parameters emerge in the range ∼50–150 TeV while the gaugino masses M_i are driven to the lower limits imposed (since it is the ratios $\mu(M_Z)/M_i(M_Z)$, $A_0(M_Z)/M_i(M_Z)$ which control the efficacy of the large $\tan\beta$ corrections required for our purposes [7] (the search selects very small gaugino masses at M_X compatible with $M_i(M_X) \simeq 0$, since in any case the two loop running of gaugino masses, specially with large A_0, is enough to generate adequate gaugino masses)). Sfermion masses lie in the 1–50 TeV range though a few (notably the Right chiral smuon) can be lighter than a TeV. This is the price one must pay to correct the fermion Yukawas to achievable values in the NMSGUT. Large values of A_0 are often feared to lead to charge and color breaking (CCB) minimae [46] or unbounded from below (UFB) potentials [47]. However it is also established [48] that the metastable standard vacua that we are considering (with all mass squared parameters of charged or colored or sneutrino scalar fields *positive*, i.e., at a local minimum which preserves color, charge and R-parity) can well be stable on time scales of order the age of the universe (∼10 gigayears), provided $|A_0|$, μ are above about 5 TeV: as found in our fits. This is natural for the decoupled/Mini Split Susy s-spectra [36,37] we have always found since 2008.
3. In accordance with experimental constraints [49] we also constrain lightest chargino (essentially wino $\tilde{W}^{\pm}$) masses to be greater than 110 GeV. All the charged sfermions as well as the charged Higgs are constrained to lie above 110 GeV and the uncharged loop corrected Higgs (h^0) mass to be in the measured range 124 GeV $< m_{h^0} <$ 126 GeV. The Higgs masses were calculated using the 1-loop corrected electroweak symmetry breaking conditions and 1-loop effective potential using a subroutine [50] based on [51]. The large values of A_0, μ (and thus $X_t = A_t - \mu\tan\beta$, $X_b = A_b - \mu\cot\beta$) favor large masses for the light Higgs through loop corrections. It is a matter of gratification for the NMSGUT that it selected such values in 2008: long before the Higgs discovery in 2012 which abruptly promoted large A_0 values (even if not the NMSGUT!) from eccentric to fashionable and rigorous.
4. The LHC Susy searches have now arrived [52] at a fairly model independent lower limits of about 1200 GeV for the gluino mass. In models with very large A_0 and Non-Universal Higgs masses like ours the correlation between gaugino masses at low scales can deviate substantially from the standard 1 : 2 : 7 ratio common to GUT models with universal gaugino masses at the unification scale. However the scales are still grouped together so the characteristic spectrum associated with the NMSGUT finds a useful anchor in the LHC gluino limit ($M_{\tilde{G}} > 1$ TeV) which we implement via a penalty. This has the effect of not allowing LSP (bino) masses lower than about 200 GeV so that the LHC limit may be regarded as signaling

also the inability of the NMSGUT to provide a very light LSP. The friability of the standard gaugino mass ratio is also remarkable. For small A_0 this ratio is almost fixed in stone by one loop RGE and GUT mandated gaugino mass universality at M_X. However, invocation of gaugino masses generated by SO(10) variant F terms which is sometimes advocated [53] seems a too much to pay for such a freedom. Inasmuch as it is assumes hidden Supersymmetry breaking involving SO(10) Higgs multiplets can be consistently sequestered without proof, such a scenario is orthogonal to the motivation of our work. We find that the SO(10) GUT is rich enough to allow generation of variant gaugino mass ratios via $A_0 \sim 100$ TeV consistently with other demands of our model.

5. An improvement concerning the treatment of Susy threshold effects on gauge unification parameters $\alpha_3(M_Z), M_X, \alpha(M_X)$ is introduced to account for the spread out spectrum of supersymmetric masses. A weighted sum over all the Susy particles (M_{Susy}) is used in $\Delta_{\alpha_s}^{\text{Susy}}$ as given in [31].

$$\Delta_{\alpha_s}^{\text{Susy}} \approx \frac{-19\alpha_s^2}{28\pi} \ln \frac{M_{\text{Susy}}}{M_Z}$$
$$M_{\text{Susy}} = \prod_i m_i^{-\frac{5}{38}(4b_i^1 - 9.6b_i^2 + 5.6b_i^3)} \tag{18}$$
$$\Delta_X^{\text{Susy}} = \frac{1}{11.2\pi} \sum_i (b_1 - b_2) \operatorname{Log}_{10} \frac{m_i}{M_Z} \tag{19}$$
$$\Delta_G^{\text{Susy}} = \frac{1}{11.2\pi} \sum_i (6.6b_2 - b_1) \ln \frac{m_i}{M_Z} \tag{20}$$

Here b_1, b_2, b_3 are the 1-loop β function coefficient of U(1), SU(2), SU(3) in the MSSM respectively. $\Delta_{\alpha_s}^{\text{Susy}}$ can be significant so it changes the allowed range at GUT scale. We considered the following limits for $\Delta_{\alpha_s}^{\text{Susy}}$ in the search program.

$$-0.0146 < \Delta_{\alpha_s}^{\text{Susy}} < -0.0102 \tag{21}$$

4.2. Description of tables

In Tables 2–13 in Appendix B we have shown two example fits of fermion mass mixing parameters in terms of NMSGUT parameters. In Tables 2, 8 we give the complete set of NMSGUT parameters defined at the one loop unification scale $M_X^0 = 10^{16.25}$ GeV – which we always use as the GUT–MSSM matching scale – together with the values of the soft Susy breaking parameters ($m_0, m_{1/2}, A_0, B, M^2_{H,\bar{H}}$) and the superpotential parameter μ. The values of $\mu(M_X)$, $B(M_X)$ are determined by RG evolution from M_Z to M_X of the values determined by the loop corrected electro-weak symmetry breaking conditions [7,54]. Our soft supersymmetry breaking parameters are thus those of a $N = 1$ Supergravity GUT compatible scenario with different soft scalar masses allowed for different SO(10) irreps. As a result Non-Universal soft Higgs Masses (NUHM) for the light Higgs of the MSSM are justified since the light doublets are a mixture of doublets from several sources in different SO(10) irreps each of which is free to have its own soft mass. Our solutions always find negative values for these soft masses which can readily arise only if the soft masses of at least some of the originating representations are themselves negative. Another point to be noted is that $|m_{1/2}|$ is quite small (0–500 GeV) compared to other soft parameters.

Besides these parameter values of the SUGRY–NUHM NMSGUT we also give the mass spectrum of superheavy fields including the right handed neutrinos. We also report Type I and Type II neutrino seesaw masses as well as the changes($\Delta_{X,G,3}^{\mathrm{GUT/Susy}}$) in gauge unification parameters from their 1-loop MSSM values due to GUT scale and Susy breaking scale threshold corrections. The benefit of imposing $1 \gg Z > 0$, i.e., that it guides the Nelder–Mead search amoeba [55] to regions of the parameter space with a smaller spread in superheavy masses and smaller values for the non-matter superpotential couplings as well (making the spectrum and perturbation theory in the superpotential parameters more trustworthy) can be appreciated by comparing the values in Tables 2 and 8 with those in the corresponding tables of [7].

In Tables 3, 9 we give the values of the target fermion parameters (i.e., two loop RGE extrapolated, Susy threshold corrected MSSM Yukawas, mixing angles, neutrino mass squared differences and neutrino mixing angles). Their uncertainties are estimated as in [56], together with the achieved values and pulls. We obtain excellent fits with typical fractional errors O(0.1%). We also give the eigenvalues of the GUT scale Yukawa vertex threshold correction factors $Z_{f,\bar{f},H,\bar{H}}$ and "Higgs fractions" [5,7,19] $\alpha_i, \bar{\alpha}_i$ crucial for determining the fermion mass formulae [6,7,15,19]. These parameters are determined as a consequence of the GUT scale symmetry breaking and the fine tuning to preserve a light pair of MSSM Higgs doublets. They distill the influence of the SO(10) GUT on the low energy fermion physics. The reader may use them together with the formulae given in [7] to check the fits even without entering into the details of our GUT scale mass spectra. We note that the values of the $\alpha_1, \bar{\alpha}_1$ quoted were chosen real by convention (see Appendix C in the arXiv version of [7] where full expressions are given) but the phases of $V_{i1}^H \sim \alpha_i, U_{i1}^H \sim \bar{\alpha}_i$ used in the threshold correction formulae were fixed by demanding semi-positive eigenvalues for the Higgs mass matrix. Since the overall phase of the $\alpha, \bar{\alpha}$ nowhere enters our physical parameters we have let the discrepancy stand. Tables 2, 8 show the reduction in magnitude of SO(10) matter Yukawas. As a result universal corrections dominate and make the GUT scale threshold corrections to all three generations small and almost equal.

In Tables 4, 10 values of the SM masses at M_Z are compared with those of masses from the run down Yukawas achieved in the NMSGUT both before and after large $\tan\beta$ driven radiative corrections. Note that due to the inclusion of Susy threshold corrections the current experimental central value of $m_b(M_Z) = 2.9$ GeV can become acceptable (see Solution 2, Tables 9, 10) in contrast to small A_0 scenarios where the need for $m_b(M_Z) > 3.1$ GeV, i.e., more than one standard deviation away, has been a source of tension for small A_0 models [57].

In Tables 5, 11 we give values of the soft supersymmetry breaking parameters which are a crucial and remarkable output of this study since they tie the survival of the NMSGUT to a distinctive type of soft Susy spectrum with large $\mu, A_0, B > 100$ TeV and third generation sfermion masses in the 10–50 TeV range. Remarkably, and in sharp contrast to received (small $A_0, M_{H,\bar{H}}^2$) wisdom, the third s-generation is much *heavier* than the first two sgenerations, which however are themselves not very light *except* possibly for the *right chiral* sfermions particularly the smuon (see Solution 1) which can descend close to their experimental lower limits. Light smuon solutions are very interesting since they permit a significant supersymmetric contribution to the muon $g-2$ anomaly. They can also contribute to the effectiveness of the pure bino LSP (and pure wino lightest chargino and next to lightest neutralinos) as candidate dark matter by providing co-annihilation channels of the sort a light *stau* is often enlisted for in standard Susy scenarios.

Tables 6, 12 give Susy particle masses determined using two loop RGEs and without generation mixing switched on while in Tables 7, 13 give the masses with generation mixing. They are so similar as to justify the use of the diagonal values for estimating the Susy threshold corrections. For the case of the lightest sfermions however the corrections are sometimes as large as 10–30%. This again sounds a note of caution regarding the exact numerical values of the (tree level) lighter sfermion masses we obtain.

In Table 14, we collect values of B-decay rates for our example solutions. In Table 15 we give the values of the $b \to s\gamma$ branching ratio, the contribution to the muon $g-2$ anomaly, the variation in the Standard Model ρ parameter, and the value of the CP violation parameter ϵ [58] in the leptonic sector which is relevant for leptogenesis:

$$\epsilon \simeq -\frac{3M_1}{8\pi M_2}\frac{\mathrm{Im}[(Y_\nu^\dagger Y_\nu)^2]_{12}}{(Y_\nu^\dagger Y_\nu)_{11}} \tag{22}$$

We have not yet optimized our solutions with respect to flavor violation observables and limits. The overlap of the range of values seen with the range allowed by experimental constraints implies that a successful optimization is possible and highly constraining once Supersymmetric particles are observed.

5. Discussion of exotic observables

Baryon decay via $d=5$ operators is, as usual [59,60], dominated by the chargino mediated channels. The heavy sfermions help with suppressing B-decay. The dominant channels are *baryon* $\to$ *meson* + *neutrino*. We emphasize that the flavor violation required by $d=5$ B violation is supplied entirely by the rundown values of the (off diagonal) SuperCKM values determined by the fitting of the fermion Yukawas at M_X by the SO(10) light fermion Yukawa formulae [5–7,18,19]. We calculated the proton decay rates in the dominant channels using the formulae for the dimension 5 operators obtained in [7], after running them down to M_Z using 1-loop RG equations, adapting the formalism of [59,60].

Table 14 shows that we have been able to suppress the B decay rates to lie comfortably within the current limits. Thus the search criteria may even be loosened without conflict with experiment. Given enough computational resources, we could also conduct fine grained searches where B-decay rates are calculated for every trial parameter set. We note that our programs can already calculate the rates in other channels driven by gluino, neutralino, higgsino, etc., exchange. However we defer a presentation of the results for the subdominant channels till the various corrections and improvements still needed have been implemented. Our aim was to show that the NMSGUT is quite compatible with the stability of the proton to the degree it has been tested, and even beyond. Firm predictions will ensue only once the Susy spectrum is anchored in reality by a discovery of a supersymmetric particle.

We plugged our soft Susy parameters at M_Z into the SPHENO [50] routines to obtain the "flavor" violation contributions shown in Table 15. The very heavy third sgeneration masses imply acceptable rates BR($b \to s\gamma$) which are uniform over the fits. These branching ratio values are right in the center of the region (3–4×10^{-4}) $\pm$ 15% determined by measurements at CLEO, BaBar and Belle [49,61–63]. The Susy contribution to muon anomalous magnetic moment $\Delta a_\mu = \Delta(g-2)_\mu/2$ ranges from negligible to significant depending on the smuon mass. The current difference between experiment and theory for the muon magnetic moment anomaly is $\Delta a_\mu = 287(63)(49) \times 10^{-11}$ [49]. Thus our light smuon solutions give a_μ in the right range. The ρ parameter $\Delta\rho$ is also found to be severely suppressed by the decoupled spectrum of sfermions. The predicted change in the ρ parameter is so small as to be insignificant compared with the experimental uncertainties $\sim$0.001 [49]. Finally the values of the leptonic CP violation parameters ϵ, δ_{PMNS} seem to be somewhat small relative to estimates [58] in the literature but may well increase upon optimization since CP violation parameters which arise from phases are notoriously fickle. The values in Table 15 are thus in the right ball park and we may well begin to use the value of Δa_μ to discriminate between different models provided one is confident that all instabilities in the parameter determination process have been controlled by adequate attention to loop and threshold effects. At the moment however we simply note that there is no gross conflict.

The unification scale tends to be raised above M_X^0 in the NMSGUT, i.e., $\Delta_X > 0$. This is especially true once we demand that $d = 5$ operators mediating proton decay be suppressed. In fact in fits of [7] the values of Δ_X are $-0.30, 2.15$ while with threshold corrections we get (Tables 2, 8) $0.67, 0.80$. Thus we see that the unification scale (defined as the mass of the B-violating gauginos of type $X[3, 2, \pm\frac{5}{3}]$) is typically raised one order of magnitude to $\sim 10^{17}$–$10^{17.5}$ GeV. On the other hand, the correction to the inverse value of the fine structure constant (Δ_G) at the unification scale tends to make the gauge coupling at unification quite large ($\alpha_G \sim 0.2$). Both these tendencies together with the well known UV Landau pole in the SO(10) gauge RG flow due to the large gauge beta functions of the large SO(10) irreps used again point to the existence of a physical cutoff lying around $10^{17.5}$ GeV. This is close to the Planck scale where gravity is expected to become strong. Solutions with smaller α_G can at most improve the coincidence of the two scales. An ideal scenario [23,24] is that the theory is still weakly coupled enough to be well described by perturbative SO(10) at the threshold corrected unification scale $M_X \sim 10^{17.5}$ GeV, but that thereafter the Susy GUT becomes strongly coupled simultaneously with gravity. In that case the Planck scale may be identified as a physical cutoff for the Susy NMSGUT where it condenses as strongly coupled Supersymmetric gauge theory described by an appropriate SO(10) singlet supersymmetric sigma model. We envisaged [24] the possibility that gravity arises dynamically as an induced effect of the quantum fluctuations of the Susy GUT calculated in a coordinate independent framework. This may be realized as a path integral over a background metric that begins to propagate only at low energies leading to the near canonical $N = 1$ Supergravity perturbative NMSGUT as the effective theory below M_{Planck}: as we assume in this work.

6. Conclusions and outlook

This paper is the second of a series [7] devoted to evaluating the ability of the NMSGUT to fit all the known fermion mass and mixing data and be consistent with known constraints on exotic BSM processes. The ultimate aim is to develop the NMSO(10)GUT into complete and calculable theory of particle physics and particle cosmology [41] at scales below the Planck scale. In earlier papers, after developing a translation manual to rewrite field theories invariant under orthogonal groups in terms of labels of their unitary subgroups [18] as a basic enabling technique, we showed [6,7,15,19] that the theory is sufficiently simple as to allow explicit calculation of the spontaneous symmetry breaking, mass spectra and eigenstates. It allows computation of the RG flow in terms of the fundamental GUT parameters to the point where one can attempt to actually fit the low energy data, i.e., the SM parameters together with the neutrino mixing data, in its entirety. However, although successful in fitting the fermion mass data [7] and yielding distinctive and falsifiable signals regarding the required Susy spectra, the fits gave $d = 5$ operator mediated proton decay rates that are at least 6 orders of magnitude larger than the current experimental limits [49].

Faced with an apparent nullification of the previous successes we re-examined our treatment of the relation between the Higgs doublets of effective and High scale theories [28]. Our approximate treatment [28] of threshold corrections immediately showed that superheavy corrections to Higgs (and matter) kinetic terms and thus to the Yukawa couplings would inevitably play a critical role due to the large number of fields involved in dressing each line entering the effective

MSSM vertices. In fact care must be taken to maintain positivity of the kinetic terms after renormalization which is otherwise generically badly violated: in particular by the fits found earlier. In this paper we have completed and corrected the approximate treatment of [28] while maintaining positive kinetic terms. As a result we find that searches incorporating threshold corrected Yukawa couplings, and a constraint to respect B-decay limits, naturally flow to region of parameter space that have weak Yukawa couplings and $Z_{H,\bar{H}}$ close to zero and hence imply strong lowering of the required SO(10) matter Yukawa couplings. The mechanism that we have demonstrated is likely [45] to work in *any* realistic GUT since the features required are so generic and the necessity of implementation of threshold corrections while maintaining unitarity undeniable. Since its success depends on Z_H approaching zero while remaining positive rather than fine tuning to some specific parameter values our mechanism is likely to be robust against 2 and higher loop corrections. Moreover, the large wave function renormalization driven threshold/matching effects can also have notable influence on soft supersymmetry breaking parameters, enhancing μ, $M^2_{H,\bar{H}}$ relative to their GUT scale values consistent with the patterns found in our fits here and before. As such our paper yet again confirms [7,17,19] that the calculation of threshold effects should be a *sine qua non* of serious work on Grand Unified models.

In this paper our focus has been to report only the details of the calculation of the complete threshold corrections for the NMSGUT and exhibit successful fits that also respect baryon decay limits. We have also exhibited the values of the most prominent monitors of BSM viability such as estimates of a_μ, $\Gamma(b \to s\gamma)$ and found that the $d = 5$, $\Delta B \neq 0$ problem is essentially solved but there is room for optimization of other BSM parameters in future searches.

Since our theory claims to be a realistic UV completion of the MSSM a host of phenomenological issues arises. Serious consideration of these requires implementation of improvements such as using loop corrected sparticle masses, implementation of heavy neutrino thresholds, detailed and generic analysis of the RG flows in novel Susy parameter region indicated by the NMSGUT, incorporation of generation mixing flows in the soft sector, issues of safety as regards Color and Charge breaking minima, detailed BSM phenomenology, calculation of leptogenesis using the calculable leptonic CP violation, Dark matter constraints, Inflationary scenarios [41] and so on. These will be reported in the sequels.

In summary, by solving the conundrum of fast dimension 5 operator mediated B decay the NMSGUT has passed another formidable barrier to its development into a complete, calculable and falsifiable theory providing consistent UV completion to Particle Physics and Cosmology.

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Appendix A

We give below our results for the threshold corrections to the Yukawa couplings of the matter fields due to heavy fields running in a self energy loop on a line leading into the Yukawa vertex. The calculation is quite tedious but we applied various consistency checks to ensure that we had included contributions from all members of multiplets.

The corrections to the matter field lines are given by using the trilinear invariants of the matter fields in the **16**-plet to the Higgs in the **10**, **120**, $\overline{\mathbf{126}}$ irreps, and gauge fields in the **45**-plet, decomposed into MSSM irreps [6,7,18,19]. With $Z = 1 - \mathcal{K}$ in the notation of Eq. (10), K_{Φ}^{f} refers to the loop corrections on the matter (f) line in which the loop contains the heavy multiplet Φ. The corrections on the Higgs lines $\mathcal{K}_{H,\bar{H}}$ are described below. Using the formulae in Section 3 leads straightforwardly to

$$\begin{aligned}(16\pi^2)\mathcal{K}^{\bar{u}} = {} & K_{\bar{T}}^{\bar{u}} + K_{T}^{\bar{u}} + 2K_{H}^{\bar{u}} + \frac{16}{3}K_{C}^{\bar{u}} + 2K_{D}^{\bar{u}} + K_{J}^{\bar{u}} + 4K_{L}^{\bar{u}} + 4K_{\bar{K}}^{\bar{u}} + 16K_{M}^{\bar{u}} \\ & - 2g_{10}^2\big(0.05F_{11}(m_{\lambda_G}, Q) + F_{11}(m_{\lambda_J}, Q) + F_{11}(m_{\lambda_F}, Q) + 4F_{11}(m_{\lambda_X}, Q) \\ & + 2F_{11}(m_{\lambda_E}, Q)\big) \end{aligned} \tag{23}$$

$$K_{\bar{T}}^{\bar{u}} = 2\sum_{a=1}^{7}\big(\bar{h}U_{1a}^{T} - 2\bar{f}U_{2a}^{T} - \sqrt{2}i\bar{g}U_{7a}^{T}\big)^*\big(\bar{h}U_{1a}^{T} - 2\bar{f}U_{2a}^{T} + \sqrt{2}i\bar{g}U_{7a}^{T}\big)F_{11}\big(m_a^{T}, Q\big) \tag{24}$$

$$\begin{aligned} K_{T}^{\bar{u}} = {} & \sum_{a=1}^{7}\big(\bar{h}V_{1a}^{T} - 2\bar{f}V_{2a}^{T} - 2\sqrt{2}i\bar{f}V_{4a}^{T} - \sqrt{2}\bar{g}V_{6a}^{T} + \sqrt{2}i\bar{g}V_{7a}^{T}\big)^* \\ & \times \big(\bar{h}V_{1a}^{T} - 2\bar{f}V_{2a}^{T} - 2\sqrt{2}i\bar{f}V_{4a}^{T} + \sqrt{2}\bar{g}V_{6a}^{T} - \sqrt{2}i\bar{g}V_{7a}^{T}\big)F_{11}\big(m_a^{T}, Q\big) \end{aligned} \tag{25}$$

$$\begin{aligned} K_{H}^{\bar{u}} = {} & \sum_{a=2}^{6}\left(\bar{h}V_{1a}^{H} - \frac{2i}{\sqrt{3}}\bar{f}V_{2a}^{H} - \bar{g}V_{5a}^{H} + \frac{i}{\sqrt{3}}\bar{g}V_{6a}^{H}\right)^* \\ & \times \left(\bar{h}V_{1a}^{H} - \frac{2i}{\sqrt{3}}\bar{f}V_{2a}^{H} + \bar{g}V_{5a}^{H} - \frac{i}{\sqrt{3}}\bar{g}V_{6a}^{H}\right)F_{11}\big(m_a^{H}, Q\big) \end{aligned} \tag{26}$$

$$K_{C}^{\bar{u}} = \sum_{a=1}^{3}\big(-4\bar{f}V_{2a}^{C} + 2\bar{g}V_{3a}^{C}\big)^*\big(-4\bar{f}V_{2a}^{C} - 2\bar{g}V_{3a}^{C}\big)F_{11}\big(m_a^{C}, Q\big) \tag{27}$$

$$K_{D}^{\bar{u}} = \sum_{a=1}^{3}\big(-4\bar{f}V_{1a}^{D} + 2\bar{g}V_{3a}^{D}\big)^*\big(-4\bar{f}V_{1a}^{D} - 2\bar{g}V_{3a}^{D}\big)F_{11}\big(m_a^{D}, Q\big) \tag{28}$$

$$K_{J}^{\bar{u}} = \sum_{a=1, a\neq 4}^{5}\big(-4i\bar{f}V_{1a}^{J} + 2i\bar{g}V_{5a}^{J}\big)^*\big(-4i\bar{f}V_{1a}^{J} - 2i\bar{g}V_{5a}^{J}\big)F_{11}\big(m_a^{J}, Q\big) \tag{29}$$

$$K_{L}^{\bar{u}} = \sum_{a=1}^{2}\big(-2\sqrt{2}i\bar{f}V_{1a}^{L} - \sqrt{2}\bar{g}V_{2a}^{L}\big)^*\big(-2\sqrt{2}i\bar{f}V_{1a}^{L} + \sqrt{2}\bar{g}V_{2a}^{L}\big)F_{11}\big(m_a^{L}, Q\big) \tag{30}$$

$$K^{\bar{u}}_{\bar{K}} = 2\sum_{a=1}^{2}(-i\bar{g})^*(i\bar{g})\left|U^K_{2a}\right|^2 F_{11}\left(m^K_a, Q\right) \tag{31}$$

$$K^{\bar{u}}_{M} = (2i\bar{f})^*(2i\bar{f}) F_{11}\left(m^M, Q\right) \tag{32}$$

$$\begin{aligned}\left(16\pi^2\right)\mathcal{K}^{\bar{d}} = {} & K^{\bar{d}}_{\bar{T}} + K^{\bar{d}}_{T} + 2K^{\bar{d}}_{\bar{H}} + \frac{16}{3}K^{\bar{d}}_{\bar{C}} + 2K^{\bar{d}}_{E} + K^{\bar{d}}_{K} + 4K^{\bar{d}}_{L} + 4K^{\bar{d}}_{\bar{J}} + 16K^{\bar{d}}_{N} \\ & - 2g_{10}^2\left(0.45F_{11}(m_{\lambda_G}, Q) + F_{11}(m_{\lambda_J}, Q) + F_{11}(m_{\lambda_F}, Q) + 2F_{11}(m_{\lambda_X}, Q) \right. \\ & \left. + 4F_{11}(m_{\lambda_E}, Q)\right)\end{aligned} \tag{33}$$

$$K^{\bar{d}}_{\bar{T}} = 2\sum_{a=1}^{7}\left(\bar{h}U^T_{1a} - 2\bar{f}U^T_{2a} + \sqrt{2}i\bar{g}U^T_{7a}\right)^*\left(\bar{h}U^T_{1a} - 2\bar{f}U^T_{2a} - \sqrt{2}i\bar{g}U^T_{7a}\right)F_{11}\left(m^T_a, Q\right) \tag{34}$$

$$\begin{aligned}K^{\bar{d}}_{T} = {} & \sum_{a=1}^{7}\left(-\bar{h}V^T_{1a} + 2\bar{f}V^T_{2a} - 2\sqrt{2}i\bar{f}V^T_{4a} + \sqrt{2}\bar{g}V^T_{6a} + \sqrt{2}i\bar{g}V^T_{7a}\right)^* \\ & \times\left(-\bar{h}V^T_{1a} + 2\bar{f}V^T_{2a} - 2\sqrt{2}i\bar{f}V^T_{4a} - \sqrt{2}\bar{g}V^T_{6a} - \sqrt{2}i\bar{g}V^T_{7a}\right)F_{11}\left(m^T_a, Q\right)\end{aligned} \tag{35}$$

$$\begin{aligned}K^{\bar{d}}_{\bar{H}} = {} & \sum_{a=2}^{6}\left(-\bar{h}U^H_{1a} + \frac{2i}{\sqrt{3}}\bar{f}U^H_{2a} + \bar{g}U^H_{5a} - \frac{i}{\sqrt{3}}\bar{g}U^H_{6a}\right)^* \\ & \times\left(-\bar{h}U^H_{1a} + \frac{2i}{\sqrt{3}}\bar{f}U^H_{2a} - \bar{g}U^H_{5a} + \frac{i}{\sqrt{3}}\bar{g}U^H_{6a}\right)F_{11}\left(m^H_a, Q\right)\end{aligned} \tag{36}$$

$$K^{\bar{d}}_{\bar{C}} = \sum_{a=1}^{3}\left(4\bar{f}U^C_{1a} - 2\bar{g}U^C_{3a}\right)^*\left(4\bar{f}U^C_{1a} + 2\bar{g}U^C_{3a}\right)F_{11}\left(m^C_a, Q\right) \tag{37}$$

$$K^{\bar{d}}_{E} = \sum_{a=1,a\neq 5}^{6}\left(4\bar{f}V^E_{1a} - 2\bar{g}V^E_{6a}\right)^*\left(4\bar{f}V^E_{1a} + 2\bar{g}V^E_{6a}\right)F_{11}\left(m^E_a, Q\right) \tag{38}$$

$$K^{\bar{d}}_{K} = \sum_{a=1}^{2}\left(4i\bar{f}V^K_{1a} - 2i\bar{g}V^K_{2a}\right)^*\left(4i\bar{f}V^K_{1a} + 2i\bar{g}V^K_{2a}\right)F_{11}\left(m^K_a, Q\right) \tag{39}$$

$$K^{\bar{d}}_{L} = \sum_{a=1}^{2}\left(-2\sqrt{2}i\bar{f}V^L_{1a} + \sqrt{2}\bar{g}V^L_{2a}\right)^*\left(-2\sqrt{2}i\bar{f}V^L_{1a} - \sqrt{2}\bar{g}V^L_{2a}\right)F_{11}\left(m^L_a, Q\right) \tag{40}$$

$$K^{\bar{d}}_{\bar{J}} = 2\sum_{a=1,a\neq 4}^{5}(i\bar{g})^*(-i\bar{g})\left|U^J_{5a}\right|^2 F_{11}\left(m^J_a, Q\right) \tag{41}$$

$$K^{\bar{d}}_{N} = (2i\bar{f})^*(2i\bar{f}) F_{11}\left(m^N, Q\right) \tag{42}$$

$$\begin{aligned}\left(16\pi^2\right)\mathcal{K}^{\bar{e}} = {} & 3K^{\bar{e}}_{T} + 2K^{\bar{e}}_{\bar{H}} + K^{\bar{e}}_{\bar{F}} + 6K^{\bar{e}}_{\bar{D}} + 3K^{\bar{e}}_{K} + 4K^{\bar{e}}_{\bar{A}} - 2g_{10}^2\left(0.05F_{11}(m_{\lambda_G}, Q) \right. \\ & \left. + 3F_{11}(m_{\lambda_J}, Q) + F_{11}(m_{\lambda_F}, Q) + 6F_{11}(m_{\lambda_X}, Q)\right)\end{aligned} \tag{43}$$

$$K^{\bar{e}}_{T} = \sum_{a=1}^{7} \left(\bar{h} V^T_{1a} - 2\bar{f} V^T_{2a} - 2\sqrt{2} i \bar{f} V^T_{4a} + \sqrt{2}\bar{g} V^T_{6a} - \sqrt{2} i \bar{g} V^T_{7a}\right)^* \times \left(\bar{h} V^T_{1a} - 2\bar{f} V^T_{2a} - 2\sqrt{2} i \bar{f} V^T_{4a} - \sqrt{2}\bar{g} V^T_{6a} + \sqrt{2} i \bar{g} V^T_{7a}\right) F_{11}\left(m^T_a, Q\right) \quad (44)$$

$$K^{\bar{e}}_{\bar{H}} = \sum_{a=2}^{6} \left(-\bar{h} U^H_{1a} - 2\sqrt{3} i \bar{f} U^H_{2a} + \bar{g} U^H_{5a} + \sqrt{3} i \bar{g} U^H_{6a}\right)^* \times \left(-\bar{h} U^H_{1a} - 2\sqrt{3} i \bar{f} U^H_{2a} - \bar{g} U^H_{5a} - \sqrt{3} i \bar{g} U^H_{6a}\right) F_{11}\left(m^H_a, Q\right) \quad (45)$$

$$K^{\bar{e}}_{\bar{F}} = \sum_{a=1, a\neq 3}^{4} \left(4i \bar{f} U^F_{1a} - 2\bar{g} U^F_{4a}\right)^* \left(4i \bar{f} U^F_{1a} + 2\bar{g} U^F_{4a}\right) F_{11}\left(m^F_a, Q\right) \quad (46)$$

$$K^{\bar{e}}_{\bar{D}} = \sum_{a=1}^{3} \left(4\bar{f} U^D_{2a} - 2\bar{g} U^D_{3a}\right)^* \left(4\bar{f} U^D_{2a} + 2\bar{g} U^D_{3a}\right) F_{11}\left(m^D_a, Q\right) \quad (47)$$

$$K^{\bar{e}}_{K} = \sum_{a=1}^{2} \left(4i \bar{f} V^K_{1a} + 2i \bar{g} V^K_{2a}\right)^* \left(4i \bar{f} V^K_{1a} - 2i \bar{g} V^K_{2a}\right) F_{11}\left(m^K_a, Q\right) \quad (48)$$

$$K^{\bar{e}}_{\bar{A}} = (2\sqrt{2} i \bar{f})^* (2\sqrt{2} i \bar{f}) F_{11}\left(m^A, Q\right) \quad (49)$$

$$\left(16\pi^2\right)\mathcal{K}^{\bar{\nu}} = 3K^{\bar{\nu}}_{T} + 2K^{\bar{\nu}}_{H} + K^{\bar{\nu}}_{\bar{F}} + 6K^{\bar{\nu}}_{\bar{E}} + 3K^{\bar{\nu}}_{J} + 4K^{\bar{\nu}}_{\bar{G}} - 2g_{10}^2\big(1.25 F_{11}(m_{\lambda_G}, Q) + 3F_{11}(m_{\lambda_J}, Q) + F_{11}(m_{\lambda_F}, Q) + 6F_{11}(m_{\lambda_E}, Q)\big) \quad (50)$$

$$K^{\bar{\nu}}_{T} = \sum_{a=1}^{7} \left(-\bar{h} V^T_{1a} + 2\bar{f} V^T_{2a} - 2\sqrt{2} i \bar{f} V^T_{4a} - \sqrt{2}\bar{g} V^T_{6a} - \sqrt{2} i \bar{g} V^T_{7a}\right)^* \times \left(-\bar{h} V^T_{1a} + 2\bar{f} V^T_{2a} - 2\sqrt{2} i \bar{f} V^T_{4a} + \sqrt{2}\bar{g} V^T_{6a} + \sqrt{2} i \bar{g} V^T_{7a}\right) F_{11}\left(m^T_a, Q\right) \quad (51)$$

$$K^{\bar{\nu}}_{H} = \sum_{a=2}^{6} \left(\bar{h} V^H_{1a} + 2\sqrt{3} i \bar{f} V^H_{2a} - \bar{g} V^H_{5a} - \sqrt{3} i \bar{g} V^H_{6a}\right)^* \times \left(\bar{h} V^H_{1a} + 2\sqrt{3} i \bar{f} V^H_{2a} + \bar{g} V^H_{5a} + \sqrt{3} i \bar{g} V^H_{6a}\right) F_{11}\left(m^H_a, Q\right) \quad (52)$$

$$K^{\bar{\nu}}_{\bar{F}} = \sum_{a=1, a\neq 3}^{4} \left(-4i \bar{f} U^F_{1a} + 2\bar{g} U^F_{4a}\right)^* \left(-4i \bar{f} U^F_{1a} - 2\bar{g} U^F_{4a}\right) F_{11}\left(m^F_a, Q\right) \quad (53)$$

$$K^{\bar{\nu}}_{\bar{E}} = \sum_{a=1, a\neq 5}^{6} \left(-4\bar{f} U^E_{2a} + 2\bar{g} U^E_{6a}\right)^* \left(-4\bar{f} U^E_{2a} - 2\bar{g} U^E_{6a}\right) F_{11}\left(m^E_a, Q\right) \quad (54)$$

$$K^{\bar{\nu}}_{J} = \sum_{a=1, a\neq 4}^{5} \left(-4i \bar{f} V^J_{1a} - 2i \bar{g} V^J_{5a}\right)^* \left(-4i \bar{f} V^J_{1a} + 2i \bar{g} V^J_{5a}\right) F_{11}\left(m^J_a, Q\right) \quad (55)$$

$$K^{\bar{\nu}}_{\bar{G}} = \sum_{a=1}^{5} (-2\sqrt{2} i \bar{f})^* (-2\sqrt{2} i \bar{f}) \left|U^G_{5a}\right|^2 F_{11}\left(m^G_a, Q\right) \quad (56)$$

$$\begin{aligned}(16\pi^2)\mathcal{K}^u &= K^u_{\bar{T}} + K^u_T + K^u_{\bar{H}} + K^u_H + \frac{8}{3}K^u_C + \frac{8}{3}K^u_{\bar{C}} + K^u_{\bar{E}} + K^u_{\bar{D}} + 3K^u_{\bar{P}} + 12K^u_P \\ &\quad + 48K^{\bar{u}}_W + 4K^{\bar{u}}_{\bar{L}} - 2g^2_{10}\big(0.05F_{11}(m_{\lambda_G}, Q) + F_{11}(m_{\lambda_J}, Q) \\ &\quad + 3F_{11}(m_{\lambda_X}, Q) + 3F_{11}(m_{\lambda_E}, Q)\big) = (16\pi^2)\mathcal{K}^d \end{aligned} \tag{57}$$

$$K^u_{\bar{T}} = \sum_{a=1}^{7}\big(-\bar{h}U^T_{1a} - 2\bar{f}U^T_{2a} + \sqrt{2}\bar{g}U^T_{6a}\big)^*\big(-\bar{h}U^T_{1a} - 2\bar{f}U^T_{2a} - \sqrt{2}\bar{g}U^T_{6a}\big)F_{11}\big(m^T_a, Q\big) \tag{58}$$

$$K^u_T = 2\sum_{a=1}^{7}\big(\bar{h}V^T_{1a} + 2\bar{f}V^T_{2a}\big)^*\big(\bar{h}V^T_{1a} + 2\bar{f}V^T_{2a}\big)F_{11}\big(m^T_a, Q\big) \tag{59}$$

$$\begin{aligned}K^u_{\bar{H}} &= \sum_{a=2}^{6}\left(-\bar{h}U^H_{1a} + \frac{2i}{\sqrt{3}}\bar{f}U^H_{2a} - \bar{g}U^H_{5a} + \frac{i}{\sqrt{3}}\bar{g}U^H_{6a}\right)^* \\ &\quad \times \left(-\bar{h}U^H_{1a} + \frac{2i}{\sqrt{3}}\bar{f}U^H_{2a} + \bar{g}U^H_{5a} - \frac{i}{\sqrt{3}}\bar{g}U^H_{6a}\right)F_{11}\big(m^H_a, Q\big)\end{aligned} \tag{60}$$

$$\begin{aligned}K^u_H &= \sum_{a=2}^{6}\left(\bar{h}V^H_{1a} - \frac{2i}{\sqrt{3}}\bar{f}V^H_{2a} + \bar{g}V^H_{5a} - \frac{i\bar{g}}{\sqrt{3}}V^H_{6a}\right)^* \\ &\quad \times \left(\bar{h}V^H_{1a} - \frac{2i}{\sqrt{3}}\bar{f}V^H_{2a} - \bar{g}V^H_{5a} + \frac{i\bar{g}}{\sqrt{3}}V^H_{6a}\right)F_{11}\big(m^H_a, Q\big)\end{aligned} \tag{61}$$

$$K^u_C = \sum_{a=1}^{3}\big(-4\bar{f}V^C_{2a} - 2\bar{g}V^C_{3a}\big)^*\big(-4\bar{f}V^C_{2a} + 2\bar{g}V^C_{3a}\big)F_{11}\big(m^C_a, Q\big) \tag{62}$$

$$K^u_{\bar{C}} = \sum_{a=1}^{3}\big(4\bar{f}U^C_{1a} + 2\bar{g}U^C_{3a}\big)^*\big(4\bar{f}U^C_{1a} - 2\bar{g}U^C_{3a}\big)F_{11}\big(m^C_a, Q\big) \tag{63}$$

$$K^u_{\bar{E}} = \sum_{a=1, a\neq 5}^{6}\big(-4\bar{f}U^E_{2a} - 2\bar{g}U^E_{6a}\big)^*\big(-4\bar{f}U^E_{2a} + 2\bar{g}U^E_{6a}\big)F_{11}\big(m^E_a, Q\big) \tag{64}$$

$$K^u_{\bar{D}} = \sum_{a=1}^{3}\big(4\bar{f}U^D_{2a} + 2\bar{g}U^D_{3a}\big)^*\big(4\bar{f}U^D_{2a} - 2\bar{g}U^D_{3a}\big)F_{11}\big(m^D_a, Q\big) \tag{65}$$

$$K^u_{\bar{P}} = \sum_{a=1}^{2}\big(2\sqrt{2}\bar{f}U^P_{1a} - \sqrt{2}\bar{g}U^P_{2a}\big)^*\big(2\sqrt{2}\bar{f}U^P_{1a} + \sqrt{2}\bar{g}U^P_{2a}\big)F_{11}\big(m^P_a, Q\big) \tag{66}$$

$$K^u_P = 2\sum_{a=1}^{2}\left(-\frac{\bar{g}}{\sqrt{2}}\right)^*\left(\frac{\bar{g}}{\sqrt{2}}\right)\big|V^P_{2a}\big|^2F_{11}\big(m^P_a, Q\big) \tag{67}$$

$$K^u_W = (\sqrt{2}\bar{f})^*(\sqrt{2}\bar{f})F_{11}\big(m^W, Q\big) \tag{68}$$

$$K^u_{\bar{L}} = \sum_{a=1}^{2}(-\sqrt{2}\bar{g})^*(\sqrt{2}\bar{g})\big|U^L_{2a}\big|^2F_{11}\big(m^L_a, Q\big) \tag{69}$$

$$\begin{aligned}(16\pi^2)\mathcal{K}^e &= 3K^e_{\bar{T}} + K^e_{\bar{H}} + K^e_H + 3K^e_D + 3K^e_E + 9K^e_{\bar{P}} + K^e_F + 12K^e_{\bar{O}} \\ &\quad - 2g_{10}^2\big(0.45F_{11}(m_{\lambda_G}, Q) + 3F_{11}(m_{\lambda_J}, Q) + 3F_{11}(m_{\lambda_X}, Q) \\ &\quad + 3F_{11}(m_{\lambda_E}, Q)\big) = (16\pi^2)\mathcal{K}^\nu \end{aligned} \tag{70}$$

$$K^e_{\bar{T}} = \sum_{a=1}^{7}\big(-\bar{h}U^T_{1a} - 2\bar{f}U^T_{2a} - \sqrt{2}\bar{g}U^T_{6a}\big)^*\big(-\bar{h}U^T_{1a} - 2\bar{f}U^T_{2a} + \sqrt{2}\bar{g}U^T_{6a}\big)F_{11}\big(m^T_a, Q\big) \tag{71}$$

$$\begin{aligned}K^e_{\bar{H}} &= \sum_{a=2}^{6}\big(\bar{h}U^H_{1a} + 2\sqrt{3}i\bar{f}U^H_{2a} + \bar{g}U^H_{5a} + \sqrt{3}i\bar{g}U^H_{6a}\big)^* \\ &\quad \times \big(\bar{h}U^H_{1a} + 2\sqrt{3}i\bar{f}U^H_{2a} - \bar{g}U^H_{5a} - \sqrt{3}i\bar{g}U^H_{6a}\big)F_{11}\big(m^H_a, Q\big)\end{aligned} \tag{72}$$

$$\begin{aligned}K^e_H &= \sum_{a=2}^{6}\big(-\bar{h}V^H_{1a} - 2\sqrt{3}i\bar{f}V^H_{2a} - \bar{g}V^H_{5a} - i\sqrt{3}\bar{g}V^H_{6a}\big)^* \\ &\quad \times \big(-\bar{h}V^H_{1a} - 2\sqrt{3}i\bar{f}V^H_{2a} + \bar{g}V^H_{5a} + i\sqrt{3}\bar{g}V^H_{6a}\big)F_{11}\big(m^F_a, Q\big)\end{aligned} \tag{73}$$

$$K^e_D = \sum_{a=1}^{3}\big(4\bar{f}V^D_{1a} + 2\bar{g}V^D_{3a}\big)^*\big(4\bar{f}V^D_{1a} - 2\bar{g}V^D_{3a}\big)F_{11}\big(m^D_a, Q\big) \tag{74}$$

$$K^e_E = \sum_{a=1, a\neq 5}^{6}\big(-4\bar{f}V^E_{1a} - 2\bar{g}V^E_{6a}\big)^*\big(-4\bar{f}V^E_{1a} + 2\bar{g}V^E_{6a}\big)F_{11}\big(m^E_a, Q\big) \tag{75}$$

$$K^e_{\bar{P}} = \sum_{a=1}^{2}\big(2\sqrt{2}\bar{f}U^P_{1a} + \sqrt{2}\bar{g}U^P_{2a}\big)^*\big(2\sqrt{2}\bar{f}U^P_{1a} - \sqrt{2}\bar{g}U^P_{2a}\big)F_{11}\big(m^P_a, Q\big) \tag{76}$$

$$K^e_F = \sum_{a=1, a\neq 3}^{4}(-2\bar{g})^*(2\bar{g})\big|V^F_{4a}\big|^2F_{11}\big(m^F_a, Q\big) \tag{77}$$

$$K^e_{\bar{O}} = (2i\bar{f})^*(2i\bar{f})F_{11}\big(m^O, Q\big) \tag{78}$$

Here g_{10} is the SO(10) gauge coupling and

$$\bar{h} = 2\sqrt{2}h; \quad \bar{g} = 2\sqrt{2}g; \quad \bar{f} = 2\sqrt{2}f$$

The calculation for the corrections to the light Higgs doublet lines $H, \bar{H}$ is much more complicated than the matter lines since these are mixtures of pairs of doublets from the **10**, **120** (2 pairs) $\overline{\mathbf{126}}$, **126**, **210** SO(10) Higgs multiplets ($H = (V^H)^\dagger h$, $\bar{H} = (U^H)^\dagger \bar{h}$). The couplings of the GUT field doublets $h_a, \bar{h}_a, = 1, 2, \ldots, 6$ (see [7] for conventions) to various *pairs* of the 26 different MSSM irrep-types (labeled conveniently by the letters of the alphabet: see [7,19]) that occur in this theory can be easily – if tediously – worked out using the technology[18] of SO(10) decomposition via the Pati–Salam group. Amusingly there are again precisely 26 different combinations of GUT multiplets (labeled by the letter pairs for irreps which can combine to give operators that can form singlets with the MSSM $H[1, 2, 1]$ and 26 with $\bar{H}[1, 2, -1]$). Then we get

$$(16\pi^2)\mathcal{K}_H = 8K_{R\bar{C}} + 3K_{J\bar{D}} + 3K_{E\bar{J}} + 9K_{X\bar{P}} + 3K_{X\bar{T}} + 9K_{P\bar{E}} + 3K_{T\bar{E}} + 6K_{Y\bar{L}} + K_{VF} + 8K_{C\bar{Z}} + 3K_{D\bar{I}} + 24K_{Q\bar{C}} + 9K_{E\bar{U}} + 9K_{U\bar{D}} + 6K_{L\bar{B}} + 3K_{K\bar{X}} + 6K_{B\bar{M}} + 18K_{W\bar{B}} + 18K_{Y\bar{W}} + 3K_{V\bar{O}} + 6K_{N\bar{Y}} + K_{\bar{V}\bar{A}} + 3K_{HO} + 3K_{S\bar{H}} + K_{H\bar{F}} + K_{G\bar{H}} \tag{79}$$

Similarly for $\bar{H}$ we get the conjugated pairs running in the loop (unless it is a real irrep)

$$K_{R\bar{C}} = \sum_{a=1}^{d(R)}\sum_{a'=1}^{d(C)}\bigg|\bigg(\frac{i\kappa}{\sqrt{2}}V^R_{2a}U^C_{3a'} - \gamma V^R_{1a}U^C_{2a'} + \frac{\gamma}{\sqrt{2}}V^R_{2a}U^C_{2a'} - \bar{\gamma}V^R_{1a}U^C_{1a'} - \frac{\bar{\gamma}}{\sqrt{2}}V^R_{2a}U^C_{1a'}\bigg)V^H_{11} + \bigg(\frac{2\eta}{\sqrt{3}}V^R_{1a}U^C_{2a'} - \sqrt{\frac{2}{3}}\eta V^R_{2a}U^C_{2a'} + \frac{i\bar{\zeta}}{\sqrt{6}}V^R_{2a}U^C_{3a'}\bigg)V^H_{21} + \bigg(\frac{2\eta}{\sqrt{3}}V^R_{1a}U^C_{1a'} + \sqrt{\frac{2}{3}}\eta V^R_{2a}U^C_{1a'} + \frac{i\zeta}{\sqrt{6}}V^R_{2a}U^C_{3a'}\bigg)V^H_{31} + \bigg(\frac{\zeta}{\sqrt{2}}V^R_{2a}U^C_{2a'} - \frac{i\rho}{3\sqrt{2}}V^R_{2a}U^C_{3a'} + \frac{\bar{\zeta}}{\sqrt{2}}V^R_{2a}U^C_{1a'}\bigg)V^H_{51} - \bigg(\frac{i\zeta}{\sqrt{6}}V^R_{2a}U^C_{2a'} + \frac{i\bar{\zeta}}{\sqrt{6}}V^R_{2a}U^C_{1a'} - \frac{\rho}{3\sqrt{3}}V^R_{1a}U^C_{3a'}\bigg)V^H_{61}\bigg|^2 F_{12}(m^R_a, m^C_{a'}, Q) \tag{80}$$

$$K_{J\bar{D}} = \sum_{a=1}^{d(J)}\sum_{a'=1}^{d(D)}\bigg|\bigg(\gamma V^J_{2a}U^D_{1a'} - \frac{\gamma}{\sqrt{2}}V^J_{3a}U^D_{1a'} + \bar{\gamma}V^J_{2a}U^D_{2a'} + \frac{\bar{\gamma}}{\sqrt{2}}V^J_{3a}U^D_{2a'} - \frac{i\kappa}{\sqrt{2}}V^J_{3a}U^D_{3a'}\bigg)V^H_{11} + \bigg(\frac{2\eta}{\sqrt{3}}V^J_{2a}U^D_{1a'} - \sqrt{6}\eta V^J_{3a}U^D_{1a'} - \frac{2i\bar{\zeta}}{\sqrt{3}}V^J_{2a}U^D_{3a'} + \sqrt{\frac{3}{2}}i\bar{\zeta}V^J_{3a}U^D_{3a'}\bigg)V^H_{21} + \bigg(\frac{-i}{\sqrt{6}}\zeta V^J_{3a}U^D_{3a'} - \frac{2i\zeta}{\sqrt{3}}V^J_{2a}U^D_{3a'} + \frac{2\eta}{\sqrt{3}}V^J_{2a}U^D_{2a'} - \sqrt{\frac{2}{3}}\eta V^J_{3a}U^D_{2a'}\bigg)V^H_{31} - \bigg(\frac{i\rho}{3}V^J_{5a}U^D_{3a'} + 4\eta V^J_{1a}U^D_{1a'} + 2i\bar{\zeta}V^J_{1a}U^D_{3a'} + 2\bar{\zeta}V^J_{5a}U^D_{2a'}\bigg)V^H_{41} + \bigg(\frac{i\rho}{3\sqrt{2}}V^J_{3a}U^D_{3a'} - \frac{\bar{\zeta}}{\sqrt{2}}V^J_{3a}U^D_{2a'} - \frac{\zeta}{\sqrt{2}}V^J_{3a}U^D_{1a'}\bigg)V^H_{51} + \bigg(\frac{2i\zeta}{\sqrt{3}}V^J_{2a}U^D_{1a'} - \sqrt{\frac{3}{2}}i\zeta V^J_{3a}U^D_{1a'} + \frac{i\bar{\zeta}}{\sqrt{6}}V^J_{3a}U^D_{2a'} + \frac{2i\bar{\zeta}}{\sqrt{3}}V^J_{2a}U^D_{2a'} + \frac{\rho}{3\sqrt{3}}V^J_{2a}U^D_{3a'} - \frac{\sqrt{2}\rho}{3\sqrt{3}}V^J_{3a}U^D_{3a'}\bigg)V^H_{61}\bigg|^2 F_{12}(m^J_a, m^D_{a'}, Q) - 2g^2_{10}\bigg|\frac{-2i}{\sqrt{3}}\big(V^{D*}_{1a'}V^H_{21} + V^{D*}_{2a'}V^H_{31} + V^{D*}_{3a'}V^H_{61}\big)\bigg|^2 F_{12}(m_{\lambda_J}, m^D_{a'}, Q) \tag{81}$$

$$
\begin{aligned}
K_{E\bar{J}} = {} & \sum_{a=1}^{\mathrm{d}(E)}\sum_{a'=1}^{\mathrm{d}(J)}\bigg|\bigg(\gamma V^E_{2a}U^J_{2a'} + \sqrt{2}\gamma i V^E_{3a}U^J_{1a'} - \frac{\gamma}{\sqrt{2}}V^E_{2a}U^J_{3a'} + \bar{\gamma}V^E_{1a}U^J_{2a'} + \frac{\bar{\gamma}}{\sqrt{2}}V^E_{1a}U^J_{3a'} \\
& + i\kappa V^E_{4a}U^J_{5a'} - \frac{i\kappa}{\sqrt{2}}V^E_{6a}U^J_{3a'}\bigg)V^H_{11} + \bigg(\frac{2\eta}{\sqrt{3}}V^E_{2a}U^J_{2a'} + 2\sqrt{\frac{2}{3}}i\eta V^E_{3a}U^J_{1a'} \\
& + \sqrt{\frac{2}{3}}\eta V^E_{2a}U^J_{3a'} + \frac{4i\eta}{\sqrt{3}}V^E_{4a}U^J_{1a'} - \frac{i\bar{\zeta}}{\sqrt{6}}V^E_{6a}U^J_{3a'} + \frac{2i\bar{\zeta}}{\sqrt{3}}V^E_{6a}U^J_{2a'} - \frac{i\bar{\zeta}}{\sqrt{3}}V^E_{4a}U^J_{5a'}\bigg)V^H_{21} \\
& + \bigg(\frac{2i\zeta}{\sqrt{3}}V^E_{6a}U^J_{2a'} + \sqrt{\frac{3}{2}}i\zeta V^E_{6a}U^J_{3a'} - \frac{2\sqrt{2}i\zeta}{\sqrt{3}}V^E_{3a}U^J_{5a'} \\
& + \frac{i\zeta}{\sqrt{3}}V^E_{4a}U^J_{5a'} + \sqrt{6}\eta V^E_{1a}U^J_{3a'} + \frac{2\eta}{\sqrt{3}}V^E_{1a}U^J_{2a'}\bigg)V^H_{31} \\
& + \sqrt{2}i\lambda\big(2V^E_{3a}U^J_{2a'} - V^E_{4a}U^J_{3a'} - \sqrt{2}V^E_{3a}U^J_{3a'}\big)V^H_{41} + \bigg(\frac{i\rho}{3\sqrt{2}}V^E_{6a}U^J_{3a'} + \frac{i\rho}{3}V^E_{4a}U^J_{5a'} \\
& - \frac{\zeta}{\sqrt{2}}V^E_{2a}U^J_{3a'} - \sqrt{2}i\zeta V^E_{3a}U^J_{1a'} - \frac{\bar{\zeta}}{\sqrt{2}}V^E_{1a}U^J_{3a'}\bigg)V^H_{51} \\
& + \bigg(\frac{\sqrt{2}\rho}{3\sqrt{3}}V^E_{6a}U^J_{3a'} - \frac{\sqrt{2}\rho}{3\sqrt{3}}V^E_{3a}U^J_{5a'} \\
& - \frac{\rho}{3\sqrt{3}}V^E_{4a}U^J_{5a'} + \frac{\rho}{3\sqrt{3}}V^E_{6a}U^J_{2a'} + \frac{2\zeta}{\sqrt{3}}V^E_{4a}U^J_{1a'} - \frac{2i\zeta}{\sqrt{3}}V^E_{2a}U^J_{2a'} + \frac{\sqrt{2}\zeta}{\sqrt{3}}V^E_{3a}U^J_{1a'} \\
& + \frac{i\zeta}{\sqrt{6}}V^E_{2a}U^J_{3a'} - \sqrt{\frac{3}{2}}i\bar{\zeta}V^E_{1a}U^J_{3a'} - \frac{2i\bar{\zeta}}{\sqrt{3}}V^E_{1a}U^J_{2a'}\bigg)V^H_{61}\bigg|^2 F_{12}\big(m^E_a, m^J_{a'}, Q\big) \\
& - 2g^2_{10}\bigg|\frac{2i}{\sqrt{3}}U^{E*}_{2a}V^H_{21} + \frac{2i}{\sqrt{3}}U^{E*}_{1a}V^H_{31} \\
& - \sqrt{2}U^{E*}_{3a}V^H_{41} + \frac{2i}{\sqrt{3}}U^{E*}_{6a}V^H_{61}\bigg|^2 F_{12}\big(m^E_a, m_{\lambda_J}, Q\big) \\
& - 2g^2_{10}\bigg|\frac{2}{\sqrt{3}}V^{J*}_{1a'}V^H_{21} - iV^{J*}_{2a'}V^H_{41} + \frac{i}{\sqrt{2}}V^{J*}_{3a'}V^H_{41} + iV^{J*}_{5a'}V^H_{51} - \frac{1}{\sqrt{3}}V^{J*}_{5a'}V^H_{61}\bigg|^2 \\
& \times F_{12}\big(m_{\lambda_E}, m^J_{a'}, Q\big)
\end{aligned} \tag{82}
$$

$$
\begin{aligned}
K_{X\bar{P}} = {} & \sum_{a=1}^{\mathrm{d}(X)}\sum_{a'=1}^{\mathrm{d}(P)}\bigg|\bigg(\bar{\gamma}V^X_{1a}U^P_{1a'} - \frac{\kappa}{\sqrt{2}}V^X_{2a}U^P_{2a'}\bigg)V^H_{11} - \bigg(\frac{2\bar{\zeta}}{\sqrt{3}}V^X_{1a}U^P_{2a'} + \frac{\bar{\zeta}}{\sqrt{6}}V^X_{2a}U^P_{2a'}\bigg)V^H_{21} \\
& + \bigg(\frac{\zeta}{\sqrt{6}}V^X_{2a}U^P_{2a'} + \frac{2\eta}{\sqrt{3}}V^X_{1a}U^P_{1a'} - \frac{2\sqrt{2}\eta}{\sqrt{3}}V^X_{2a}U^P_{1a'}\bigg)V^H_{31} \\
& + \bigg(\frac{\rho}{3\sqrt{2}}V^X_{2a}U^P_{2a'} + \bar{\zeta}V^X_{1a}U^P_{1a'}\bigg)V^H_{51} \\
& + \frac{i}{\sqrt{3}}\bigg(\sqrt{2}\bar{\zeta}V^X_{2a}U^P_{1a'} - \bar{\zeta}V^X_{1a}U^P_{1a'} \\
& + \frac{\rho}{3}V^X_{1a}U^P_{2a'} - \frac{\rho}{3\sqrt{2}}V^X_{2a}U^P_{2a'}\bigg)V^H_{61}\bigg|^2 F_{12}\big(m^X_a, m^P_{a'}, Q\big)
\end{aligned}
$$

$$-2g_{10}^2\left|i\sqrt{\frac{2}{3}}V_{1a'}^{P*}V_{31}^H-\frac{V_{2a'}^{P*}}{\sqrt{2}}V_{51}^H+\frac{i}{\sqrt{6}}V_{2a'}^{P*}V_{61}^H\right|^2F_{12}\big(m_{\lambda_X},m_{a'}^P,Q\big) \quad (83)$$

$$K_{X\bar{T}}=\sum_{a=1}^{d(X)}\sum_{a'=1}^{d(T)}\Bigg|\bigg(\kappa V_{1a}^XU_{6a'}^T-\gamma V_{2a}^XU_{3a'}^T-i\gamma V_{1a}^XU_{4a'}^T-\bar{\gamma}V_{2a}^XU_{2a'}^T-\frac{i\kappa}{\sqrt{2}}V_{2a}^XU_{7a'}^T\bigg)V_{11}^H$$
$$+\bigg(\frac{\sqrt{2}i\bar{\gamma}}{\sqrt{3}}V_{1a}^XU_{1a'}^T-\frac{\bar{\gamma}}{\sqrt{3}}V_{2a}^XU_{1a'}^T-2\sqrt{\frac{2}{3}}\eta V_{1a}^XU_{3a'}^T-\frac{2i\eta}{\sqrt{3}}V_{1a}^XU_{4a'}^T$$
$$-2\sqrt{\frac{2}{3}}i\eta V_{2a}^XU_{4a'}^T-\frac{\bar{\zeta}}{\sqrt{3}}V_{1a}^XU_{6a'}^T-\frac{\sqrt{2}\bar{\zeta}}{\sqrt{3}}V_{2a}^XU_{6a'}^T+\frac{i\bar{\zeta}}{\sqrt{6}}V_{2a}^XU_{7a'}^T\bigg)V_{21}^H$$
$$+\bigg(2\sqrt{\frac{2}{3}}\eta V_{1a}^XU_{2a'}^T-\frac{\zeta}{\sqrt{3}}V_{1a}^XU_{6a'}^T+\frac{\sqrt{2}\zeta}{\sqrt{3}}V_{2a}^XU_{6a'}^T+\frac{2i\zeta}{\sqrt{3}}V_{1a}^XU_{7a'}^T$$
$$-\frac{i\zeta}{\sqrt{6}}V_{2a}^XU_{7a'}^T+\frac{\sqrt{2}i\gamma}{\sqrt{3}}V_{1a}^XU_{1a'}^T+\frac{\gamma}{\sqrt{3}}V_{2a}^XU_{1a'}^T\bigg)V_{31}^H$$
$$-2i\lambda\big(V_{2a}^XU_{5a'}^T+\sqrt{2}V_{1a}^XU_{5a'}^T\big)V_{41}^H$$
$$+\bigg(i\zeta V_{1a}^XU_{4a'}^T-\kappa V_{2a}^XU_{1a'}^T-\frac{i\rho}{3\sqrt{2}}V_{2a}^XU_{7a'}^T\bigg)V_{51}^H$$
$$+\bigg(\frac{\sqrt{2}i\zeta}{\sqrt{3}}V_{1a}^XU_{3a'}^T-\frac{\sqrt{2}\zeta}{\sqrt{3}}V_{2a}^XU_{4a'}^T+\frac{i\zeta}{\sqrt{3}}V_{2a}^XU_{3a'}^T-\frac{\zeta}{\sqrt{3}}V_{1a}^XU_{4a'}^T$$
$$-\frac{\sqrt{2}i}{\sqrt{3}}\bar{\zeta}V_{1a}^XU_{2a'}^T+\frac{i\bar{\zeta}}{\sqrt{3}}V_{2a}^XU_{2a'}^T+\frac{i\rho}{3\sqrt{3}}V_{1a}^XU_{6a'}^T+\frac{\rho}{3\sqrt{3}}V_{1a}^XU_{7a'}^T$$
$$+\frac{\rho}{3\sqrt{6}}V_{2a}^XU_{7a'}^T-\sqrt{\frac{2}{3}}i\kappa V_{1a}^XU_{1a'}^T\bigg)V_{61}^H\Bigg|^2F_{12}\big(m_a^X,m_{a'}^T,Q\big)$$
$$-2g_{10}^2\Bigg|-V_{1a'}^{T*}V_{11}^H-\bigg(\frac{i}{\sqrt{3}}V_{2a'}^{T*}+\sqrt{\frac{2}{3}}V_{4a'}^{T*}\bigg)V_{21}^H-\frac{i}{\sqrt{3}}V_{3a'}^{T*}V_{31}^H+iV_{5a'}^{T*}V_{41}^H$$
$$-\frac{i}{\sqrt{2}}V_{7a'}^{T*}V_{51}^H+\bigg(i\sqrt{\frac{2}{3}}V_{6a'}^{T*}+\frac{V_{7a'}^{T*}}{\sqrt{6}}\bigg)V_{61}^H\Bigg|^2F_{12}\big(m_{\lambda_X},m_{a'}^T,Q\big) \quad (84)$$

$$K_{P\bar{E}}=\sum_{a=1}^{d(P)}\sum_{a'=1}^{d(E)}\Bigg|\bigg(\gamma V_{1a}^PU_{3a'}^E-\frac{\kappa}{\sqrt{2}}V_{2a}^PU_{4a'}^E\bigg)V_{11}^H$$
$$+\bigg(\frac{2\eta}{\sqrt{3}}V_{1a}^P\big(U_{3a'}^E-\sqrt{2}U_{4a'}^E\big)+\frac{\bar{\zeta}}{\sqrt{6}}V_{2a}^PU_{4a'}^E\bigg)V_{21}^H$$
$$-\frac{\zeta}{\sqrt{3}}\bigg(2V_{2a}^PU_{3a'}^E+\frac{V_{2a}^P}{\sqrt{2}}U_{4a'}^E\bigg)V_{31}^H$$
$$+\bigg(2\sqrt{2}\eta iV_{1a}^PU_{2a'}^E-\frac{\rho V_{2a}^P}{3\sqrt{2}}U_{6a'}^E-\sqrt{2}\zeta V_{1a}^PU_{6a'}^E+\sqrt{2}i\zeta V_{2a}^PU_{1a'}^E\bigg)V_{41}^H$$
$$+\bigg(\frac{\rho}{3\sqrt{2}}V_{2a}^PU_{4a'}^E+\zeta V_{1a}^PU_{3a'}^E\bigg)V_{51}^H$$

$$+\left(\frac{i\zeta}{\sqrt{3}}V^P_{1a}U^E_{3a'}-\frac{\sqrt{2}i\zeta}{\sqrt{3}}V^P_{1a}U^E_{4a'}\right.$$
$$\left.\left.-\frac{i\rho}{3\sqrt{3}}V^P_{2a}U^E_{3a'}+\frac{i\rho}{3\sqrt{6}}V^P_{2a}U^E_{4a'}\right)V^H_{61}\right|^2F_{12}\left(m^P_a,m^E_{a'},Q\right)$$
$$-2g_{10}^2\left|-i\sqrt{\frac{2}{3}}U^{P*}_{1a}V^H_{21}-\frac{U^{P*}_{2a}}{\sqrt{2}}V^H_{51}-\frac{iU^{P*}_{2a}}{\sqrt{6}}V^H_{61}\right|^2F_{12}\left(m^P_a,m_{\lambda_E},Q\right) \tag{85}$$

$$K_{T\bar{E}}=\sum_{a=1}^{d(T)}\sum_{a'=1}^{d(E)}\left|\left(\gamma V^T_{5a}U^E_{1a'}-\gamma V^T_{3a}U^E_{4a'}-\bar{\gamma}V^T_{2a}U^E_{4a'}-\bar{\gamma}V^T_{5a}U^E_{2a'}-i\bar{\gamma}V^T_{4a}U^E_{3a'}\right.\right.$$
$$\left.+\kappa V^T_{6a}U^E_{3a'}+i\kappa V^T_{5a}U^E_{6a'}-\frac{i\kappa}{\sqrt{2}}V^T_{7a}U^E_{4a'}\right)V^H_{11}$$
$$+\left(2\sqrt{\frac{2}{3}}\eta V^T_{3a}U^E_{3a'}+2\sqrt{3}\eta V^T_{5a}U^E_{1a'}+i\bar{\gamma}\sqrt{\frac{2}{3}}V^T_{1a}U^E_{3a'}+\frac{\bar{\gamma}}{\sqrt{3}}V^T_{1a}U^E_{4a'}\right.$$
$$-\frac{\bar{\zeta}}{\sqrt{3}}V^T_{6a}U^E_{3a'}-i\sqrt{3}\bar{\zeta}V^T_{5a}U^E_{6a'}+\frac{2i\bar{\zeta}}{\sqrt{3}}V^T_{7a}U^E_{3a'}-\frac{i\bar{\zeta}}{\sqrt{6}}V^T_{7a}U^E_{4a'}$$
$$\left.+\sqrt{\frac{2}{3}}\bar{\zeta}V^T_{6a}U^E_{4a'}\right)V^H_{21}+\left(\frac{i\zeta}{\sqrt{6}}V^T_{7a}U^E_{4a'}-\frac{\zeta}{\sqrt{3}}V^T_{6a}U^E_{3a'}-\sqrt{\frac{2}{3}}\zeta V^T_{6a}U^E_{4a'}\right.$$
$$+\frac{i}{\sqrt{3}}\zeta V^T_{5a}U^E_{6a'}+\sqrt{\frac{2}{3}}i\gamma V^T_{1a}U^E_{3a'}-\frac{\gamma}{\sqrt{3}}V^T_{1a}U^E_{4a'}-2\sqrt{\frac{2}{3}}\eta V^T_{2a}U^E_{3a'}$$
$$\left.-\frac{2i\eta}{\sqrt{3}}V^T_{4a}U^E_{3a'}+\frac{2\eta}{\sqrt{3}}V^T_{5a}U^E_{2a'}-2\sqrt{\frac{2}{3}}i\eta V^T_{4a}U^E_{4a'}\right)V^H_{31}$$
$$+\left(2\eta iV^T_{3a}U^E_{2a'}-2\eta iV^T_{2a}U^E_{1a'}-2\sqrt{2}\eta V^T_{4a}U^E_{1a'}-\gamma V^T_{1a}U^E_{1a'}-\bar{\gamma}V^T_{1a}U^E_{2a'}\right.$$
$$-\kappa V^T_{1a}U^E_{6a'}-\frac{\sqrt{2}\rho}{3}V^T_{6a}U^E_{6a'}-\frac{i\rho}{3\sqrt{2}}V^T_{7a}U^E_{6a'}+\sqrt{2}\zeta iV^T_{6a}U^E_{1a'}-\zeta V^T_{3a}U^E_{6a'}$$
$$\left.+\sqrt{2}i\bar{\zeta}V^T_{6a}U^E_{2a'}+\bar{\zeta}V^T_{2a}U^E_{6a'}-\sqrt{2}i\bar{\zeta}V^T_{4a}U^E_{6a'}-\sqrt{2}\bar{\zeta}V^T_{7a}U^E_{2a'}\right)V^H_{41}$$
$$+\left(\zeta V^T_{5a}U^E_{1a'}-\kappa V^T_{1a}U^E_{4a'}-\frac{i\rho}{3}V^T_{5a}U^E_{6a'}\right.$$
$$\left.-\frac{i\rho}{3\sqrt{2}}V^T_{7a}U^E_{4a'}+i\bar{\zeta}V^T_{4a}U^E_{3a'}+\bar{\zeta}V^T_{5a}U^E_{2a'}\right)V^H_{51}$$
$$+\left(\sqrt{\frac{2}{3}}i\kappa V^T_{1a}U^E_{3a'}+\sqrt{\frac{2}{3}}i\zeta V^T_{3a}U^E_{3a'}+\sqrt{3}i\zeta V^T_{5a}U^E_{1a'}\right.$$
$$-\frac{i\zeta}{\sqrt{3}}V^T_{3a}U^E_{4a'}-\sqrt{\frac{2}{3}}i\bar{\zeta}V^T_{2a}U^E_{3a'}-\frac{i\bar{\zeta}}{\sqrt{3}}V^T_{5a}U^E_{2a'}+\sqrt{\frac{2}{3}}\bar{\zeta}V^T_{4a}U^E_{4a'}$$
$$-\frac{i\bar{\zeta}}{\sqrt{3}}V^T_{2a}U^E_{4a'}+\frac{\bar{\zeta}}{\sqrt{3}}V^T_{4a}U^E_{3a'}-\frac{i\rho}{3\sqrt{3}}V^T_{6a}U^E_{3a'}$$

$$- \frac{\rho}{3\sqrt{3}} V^T_{7a} U^E_{3a'} - \frac{\rho}{3\sqrt{6}} V^T_{7a} U^E_{4a'} + \frac{2\rho}{3\sqrt{3}} V^T_{5a} U^E_{6a'} \Big) V^H_{61} \Big|^2 F_{12}\big(m^T_a, m^E_{a'}, Q\big)$$

$$- 2g^2_{10} \Big| -U^{T*}_{1a} V^H_{11} + \frac{i}{\sqrt{3}} U^{T*}_{2a} V^H_{21} + \left(\frac{i}{\sqrt{3}} U^{T*}_{3a} + \sqrt{\frac{2}{3}} U^{T*}_{4a} \right) V^H_{31} - \frac{i}{\sqrt{2}} U^{T*}_{7a} V^H_{51}$$

$$- \left(\frac{U^{T*}_{7a}}{\sqrt{6}} + i\sqrt{\frac{2}{3}} U^{T*}_{6a} \right) V^H_{61} \Big|^2 F_{12}\big(m^T_a, m_{\lambda_E}, Q\big) \tag{86}$$

$$K_{Y\bar{L}} = \sum_{a=1}^{\mathrm{d}(L)} \Big| \big(k U^L_{2a} - i\gamma U^L_{1a}\big) V^H_{11} + \left(\frac{2i\eta}{\sqrt{3}} U^L_{1a} + \frac{\bar{\zeta}}{\sqrt{3}} U^L_{2a} \right) V^H_{21}$$

$$+ \frac{\zeta}{\sqrt{3}} U^L_{2a} V^H_{31} + i\zeta U^L_{1a} V^H_{51} + \left(\frac{\zeta}{\sqrt{3}} U^L_{1a} - \frac{i\rho}{3\sqrt{3}} U^L_{2a} \right) V^H_{61} \Big|^2 F_{12}\big(m^Y, m^L_a, Q\big) \tag{87}$$

$$K_{VF} = \sum_{a=1}^{\mathrm{d}(F)} \Big| \big(\kappa V^F_{4a} - i\gamma V^F_{1a}\big) V^H_{11} - \big(2\sqrt{3} i\eta V^F_{1a} + \sqrt{3}\bar{\zeta} V^F_{4a}\big) V^H_{21} - \sqrt{3}\zeta V^F_{4a} V^H_{31}$$

$$- 2\sqrt{3}\lambda V^F_{2a} V^H_{41} + i\zeta V^F_{1a} V^H_{51} - \left(\sqrt{3}\zeta V^F_{1a} - \frac{i\rho}{\sqrt{3}} V^F_{4a} \right) V^H_{61} \Big|^2 F_{12}\big(m^V, m^F_a, Q\big)$$

$$- 2g^2_{10} \big| i V^H_{41} \big|^2 F_{12}\big(m^V, m_{\lambda_F}, Q\big) \tag{88}$$

$$K_{C\bar{Z}} = \sum_{a=1}^{\mathrm{d}(C)} \Big| \big(\bar{\gamma} V^C_{2a} - \gamma V^C_{1a} - i\kappa V^C_{3a}\big) V^H_{11} + \left(\frac{2\eta}{\sqrt{3}} V^C_{1a} - \frac{i\bar{\zeta}}{\sqrt{3}} V^C_{3a} \right) V^H_{21}$$

$$- \left(\frac{i\zeta}{\sqrt{3}} V^C_{3a} + \frac{2\eta}{\sqrt{3}} V^C_{2a} \right) V^H_{31}$$

$$+ \left(\frac{i\rho}{3} V^C_{3a} - \zeta V^C_{1a} - \bar{\zeta} V^C_{2a} \right) V^H_{51}$$

$$+ \left(\frac{i\zeta}{\sqrt{3}} V^C_{1a} + \frac{i\bar{\zeta}}{\sqrt{3}} V^C_{2a} \right) V^H_{61} \Big|^2 F_{12}\big(m^Z, m^C_a, Q\big) \tag{89}$$

$$K_{D\bar{I}} = \sum_{a=1}^{\mathrm{d}(D)} \Big| \big(\gamma V^D_{2a} - \bar{\gamma} V^D_{1a} + i\kappa V^D_{3a}\big) V^H_{11} + \left(\frac{i\bar{\zeta}}{\sqrt{3}} V^D_{3a} - \frac{2\eta}{\sqrt{3}} V^D_{2a} \right) V^H_{21}$$

$$+ \big(-i\zeta\sqrt{3} V^D_{3a} - 2\sqrt{3}\eta V^D_{1a}\big) V^H_{31}$$

$$+ \left(\zeta V^D_{2a} - \frac{i\rho}{3} V^D_{3a} + \bar{\zeta} V^D_{1a} \right) V^H_{51}$$

$$- \frac{1}{\sqrt{3}} \left(i\zeta V^D_{2a} - 3i\bar{\zeta} V^D_{1a} + \frac{2\rho}{3} V^D_{3a} \right) V^H_{61} \Big|^2 F_{12}\big(m^I, m^D_a, Q\big) \tag{90}$$

$$K_{Q\bar{C}} = \sum_{a=1}^{\mathrm{d}(C)} \Big| \left(\frac{i\bar{\gamma}}{\sqrt{2}} U^C_{1a} - \frac{i\gamma}{\sqrt{2}} U^C_{2a} - \frac{\kappa}{\sqrt{2}} U^C_{3a} \right) V^H_{11}$$

$$+\left(\sqrt{\frac{2}{3}}i\eta U^C_{2a}-\frac{\bar\zeta}{\sqrt6}U^C_{3a}\right)V^H_{21}-\left(\frac{\zeta}{\sqrt6}U^C_{3a}+\sqrt{\frac{2}{3}}i\eta U^C_{1a}\right)V^H_{31}$$
$$+\left(\frac{i\zeta}{\sqrt2}U^C_{2a}-\frac{\rho}{3\sqrt2}U^C_{3a}+\frac{i\bar\zeta}{\sqrt2}U^C_{1a}\right)V^H_{51}$$
$$+\left(\frac{\zeta}{\sqrt6}U^C_{2a}+\frac{\bar\zeta}{\sqrt6}U^C_{1a}\right)V^H_{61}\bigg|^2F_{12}\big(m^Q,m^C_a,Q\big) \tag{91}$$

$$K_{E\bar U}=\sum_{a=1}^{\mathrm{d}(E)}\bigg|\left(\frac{i\gamma}{\sqrt2}V^E_{2a}-\frac{i\bar\gamma}{\sqrt2}V^E_{1a}+\frac{\kappa}{\sqrt2}V^E_{6a}\right)V^H_{11}+\left(\sqrt6 i\eta V^E_{2a}-\sqrt{\frac{3}{2}}\bar\zeta V^E_{6a}\right)V^H_{21}$$
$$+\left(\sqrt{\frac{2}{3}}i\eta V^E_{1a}+\frac{\zeta}{\sqrt6}V^E_{6a}\right)V^H_{31}+\big(\sqrt2\lambda V^E_{4a}-2\lambda V^E_{3a}\big)V^H_{41}$$
$$+\left(\frac{\rho}{3\sqrt2}V^E_{6a}-\frac{i\zeta}{\sqrt2}V^E_{2a}-\frac{i\bar\zeta}{\sqrt2}V^E_{1a}\right)V^H_{51}$$
$$+\left(\sqrt{\frac{3}{2}}\zeta V^E_{2a}-\frac{\bar\zeta}{\sqrt6}V^E_{1a}+\frac{\sqrt2 i\rho}{3\sqrt3}V^E_{6a}\right)V^H_{61}\bigg|^2F_{12}\big(m^U,m^E_a,Q\big)$$
$$-2g_{10}^2\left|\frac{-V^H_{41}}{\sqrt2}\right|^2F_{12}\big(m^U,m_{\lambda_E},Q\big) \tag{92}$$

$$K_{U\bar D}=\sum_{a=1}^{\mathrm{d}(D)}\bigg|\left(\frac{i\gamma}{\sqrt2}U^D_{1a}-\frac{i\bar\gamma}{\sqrt2}U^D_{2a}+\frac{\kappa}{\sqrt2}U^D_{3a}\right)V^H_{11}+\left(\frac{\bar\zeta}{\sqrt6}U^D_{3a}-\sqrt{\frac{2}{3}}i\eta U^D_{1a}\right)V^H_{21}$$
$$+\left(-\sqrt{\frac{3}{2}}\zeta U^D_{3a}-i\sqrt6\eta U^D_{2a}\right)V^H_{31}+\left(\frac{\rho}{3\sqrt2}U^D_{3a}-\frac{i\zeta}{\sqrt2}U^D_{1a}-\frac{i\bar\zeta}{\sqrt2}U^D_{2a}\right)V^H_{51}$$
$$+\left(\sqrt{\frac{3}{2}}\bar\zeta U^D_{2a}-\frac{\zeta}{\sqrt6}U^D_{1a}-\frac{\sqrt2 i\rho}{3\sqrt3}U^D_{3a}\right)V^H_{61}\bigg|^2F_{12}\big(m^U,m^D_a,Q\big) \tag{93}$$

$$K_{L\bar B}=\sum_{a=1}^{\mathrm{d}(L)}\bigg|\big(\kappa V^L_{2a}-i\bar\gamma V^L_{1a}\big)V^H_{11}+\frac{\bar\zeta}{\sqrt3}V^L_{2a}V^H_{21}+\left(\frac{\zeta}{\sqrt3}V^L_{2a}+\frac{2i\eta}{\sqrt3}V^L_{1a}\right)V^H_{31}+i\bar\zeta V^L_{1a}V^H_{51}$$
$$+\left(\frac{i\rho}{3\sqrt3}V^L_{2a}-\frac{\bar\zeta}{\sqrt3}V^L_{1a}\right)V^H_{61}\bigg|^2F_{12}\big(m^B,m^L_a,Q\big) \tag{94}$$

$$K_{K\bar X}=\sum_{a=1}^{\mathrm{d}(K)}\sum_{a'=1}^{\mathrm{d}(X)}\bigg|\big(\sqrt2 i\bar\gamma V^K_{1a}U^X_{1a'}+i\kappa V^K_{2a}U^X_{2a'}\big)V^H_{11}$$
$$+\left(\frac{i\bar\zeta}{\sqrt3}V^K_{2a}U^X_{2a'}-2\sqrt{\frac{2}{3}}i\bar\zeta V^K_{2a}U^X_{1a'}\right)V^H_{21}$$
$$+\left(2\sqrt{\frac{2}{3}}i\eta V^K_{1a}U^X_{1a'}-\frac{i\zeta}{\sqrt3}V^K_{2a}U^X_{2a'}+\frac{4i\eta}{\sqrt3}V^K_{1a}U^X_{2a'}\right)V^H_{31}$$
$$+\left(\frac{i\rho}{3}V^K_{2a}U^X_{2a'}-\sqrt2 i\bar\zeta V^K_{1a}U^X_{1a'}\right)V^H_{51}$$

$$+\left(\frac{\rho}{3}\sqrt{\frac{2}{3}}V^K_{2a}U^X_{1a'}+\frac{\rho}{3\sqrt{3}}V^K_{2a}U^X_{2a'}-\frac{2\bar\zeta}{\sqrt{3}}V^K_{1a}U^X_{2a'}-\sqrt{\frac{2}{3}}\bar\zeta V^K_{1a}U^X_{1a'}\right)V^H_{61}\bigg|^2F_{12}\big(m^K_a,m^X_{a'},Q\big)$$

$$-2g_{10}^2\left|-\frac{2U^{K*}_{1a}}{\sqrt{3}}V^H_{31}+iU^{K*}_{2a}V^H_{51}+\frac{U^{K*}_{2a}}{\sqrt{3}}V^H_{61}\right|^2F_{12}\big(m^K_a,m_{\lambda_X},Q\big) \quad (95)$$

$$K_{B\bar M}=\left|\sqrt{2}i\gamma V^H_{11}-2\sqrt{\frac{2}{3}}i\eta V^H_{21}-\sqrt{2}i\zeta V^H_{51}-\sqrt{\frac{2}{3}}\zeta V^H_{61}\right|^2F_{12}\big(m^B,m^M,Q\big) \quad (96)$$

$$K_{W\bar B}=\left|\gamma V^H_{11}-\frac{2\eta}{\sqrt{3}}V^H_{21}+\zeta V^H_{51}-\frac{i\zeta}{\sqrt{3}}V^H_{61}\right|^2F_{12}\big(m^W,m^B,Q\big) \quad (97)$$

$$K_{Y\bar W}=\left|\bar\gamma V^H_{11}-\frac{2\eta}{\sqrt{3}}V^H_{31}+\bar\zeta V^H_{51}+\frac{i\bar\zeta}{\sqrt{3}}V^H_{61}\right|^2F_{12}\big(m^Y,m^W,Q\big) \quad (98)$$

$$K_{V\bar O}=\left|\bar\gamma V^H_{11}+2\sqrt{3}\eta V^H_{31}+\bar\zeta V^H_{51}-\sqrt{3}i\bar\zeta V^H_{61}\right|^2F_{12}\big(m^V,m^O,Q\big) \quad (99)$$

$$K_{N\bar Y}=\left|\sqrt{2}i\bar\gamma V^H_{11}-2\sqrt{\frac{2}{3}}i\eta V^H_{31}-\sqrt{2}i\bar\zeta V^H_{51}+\sqrt{\frac{2}{3}}\bar\zeta V^H_{61}\right|^2F_{12}\big(m^N,m^Y,Q\big) \quad (100)$$

$$K_{\bar V\bar A}=\left|\sqrt{2}i\bar\gamma V^H_{11}+2\sqrt{6}i\eta V^H_{31}-\sqrt{2}i\bar\zeta V^H_{51}-\sqrt{6}\bar\zeta V^H_{61}\right|^2F_{12}\big(m^V,m^A,Q\big) \quad (101)$$

$$K_{HO}=\sum_{a=2}^{\mathrm{d(H)}}\big|\gamma V^H_{4a}V^H_{11}+2\sqrt{3}\eta V^H_{4a}V^H_{21}+\big(\gamma V^H_{1a}+2\sqrt{3}\eta V^H_{2a}+\zeta V^H_{5a}+\sqrt{3}i\zeta V^H_{6a}\big)V^H_{41}$$
$$+\zeta V^H_{4a}V^H_{51}+\sqrt{3}i\zeta V^H_{4a}V^H_{61}\big|^2F_{12}\big(m^O,m^H_a,Q\big)$$
$$+\big|2\gamma V^H_{41}V^H_{11}+4\sqrt{3}\eta V^H_{41}V^H_{21}+2\zeta V^H_{51}V^H_{41}+2\sqrt{3}i\zeta V^H_{61}V^H_{41}\big|^2F_{11}\big(m^O,Q\big) \quad (102)$$

$$K_{S\bar H}=\sum_{a=2}^{\mathrm{d}(H)}\bigg|\left(\frac{i\bar\gamma}{\sqrt{2}}U^H_{2a}-\frac{i\gamma}{\sqrt{2}}U^H_{3a}-\frac{\kappa}{\sqrt{2}}U^H_{6a}\right)V^H_{11}$$
$$-\left(2\sqrt{\frac{2}{3}}i\eta U^H_{3a}-\sqrt{\frac{2}{3}}\bar\zeta U^H_{6a}+\frac{i\bar\zeta}{\sqrt{2}}U^H_{5a}+\frac{i\bar\gamma}{\sqrt{2}}U^H_{1a}\right)V^H_{21}$$
$$+\left(\sqrt{\frac{2}{3}}\zeta U^H_{6a}-\frac{i\zeta}{\sqrt{2}}U^H_{5a}+\frac{i\gamma}{\sqrt{2}}U^H_{1a}+2\sqrt{\frac{2}{3}}i\eta U^H_{2a}\right)V^H_{31}$$
$$-\sqrt{6}i\lambda U^H_{4a}V^H_{41}-\left(\frac{\rho}{3\sqrt{2}}U^H_{6a}-\frac{i\bar\zeta}{\sqrt{2}}U^H_{2a}-\frac{i\zeta}{\sqrt{2}}U^H_{3a}\right)V^H_{51}$$
$$+\left(\frac{\kappa}{\sqrt{2}}U^H_{1a}-\sqrt{\frac{2}{3}}\bar\zeta U^H_{2a}-\sqrt{\frac{2}{3}}\zeta U^H_{3a}+\frac{\rho}{3\sqrt{2}}U^H_{5a}\right)V^H_{61}\bigg|^2F_{12}\big(m^S,m^H_a,Q\big)$$
$$+\bigg|\left(\frac{i\bar\gamma}{\sqrt{2}}U^H_{21}-\frac{i\gamma}{\sqrt{2}}U^H_{31}-\frac{\kappa}{\sqrt{2}}U^H_{61}\right)V^H_{11}$$

$$
\begin{aligned}
&-\left(2\sqrt{\frac{2}{3}}i\eta U_{31}^H-\sqrt{\frac{2}{3}}\bar\zeta U_{61}^H+\frac{i\bar\zeta}{\sqrt2}U_{51}^H+\frac{i\bar\gamma}{\sqrt2}U_{11}^H\right)V_{21}^H\\
&+\left(\sqrt{\frac{2}{3}}\zeta U_{61}^H-\frac{i\zeta}{\sqrt2}U_{51}^H+\frac{i\gamma}{\sqrt2}U_{11}^H+2\sqrt{\frac{2}{3}}i\eta U_{21}^H\right)V_{31}^H-\sqrt6 i\lambda U_{41}^H V_{41}^H\\
&-\left(\frac{\rho}{3\sqrt2}U_{61}^H-\frac{i\bar\zeta}{\sqrt2}U_{21}^H-\frac{i\zeta}{\sqrt2}U_{31}^H\right)V_{51}^H\\
&+\left(\frac{\kappa}{\sqrt2}U_{11}^H-\sqrt{\frac{2}{3}}\bar\zeta U_{21}^H-\sqrt{\frac{2}{3}}\zeta U_{31}^H+\frac{\rho}{3\sqrt2}U_{51}^H\right)V_{61}^H\Bigg|^2F_{11}\big(m^S,Q\big) \qquad (103)
\end{aligned}
$$

$$
\begin{aligned}
K_{H\bar F}=\sum_{a=2}^{\mathrm{d}(H)}\sum_{a'=1}^{\mathrm{d}(F)}\Bigg|&\big(\bar\gamma U_{2a'}^F V_{2a}^H-i\bar\gamma U_{1a'}^F V_{4a}^H-\gamma U_{2a'}^F V_{3a}^H+\kappa U_{4a'}^F V_{4a}^H-i\kappa U_{2a'}^F V_{6a}^H\big)V_{11}^H\\
&+\left(\bar\zeta U_{2a'}^F V_{5a}^H-\sqrt3\bar\zeta U_{4a'}^F V_{4a}^H+\frac{2i\bar\zeta}{\sqrt3}U_{2a'}^F V_{6a}^H-\frac{4\eta}{\sqrt3}U_{2a'}^F V_{3a}^H-\bar\gamma U_{2a'}^F V_{1a}^H\right)V_{21}^H\\
&+\left(\frac{4\eta}{\sqrt3}U_{2a'}^F V_{2a}^H-\sqrt3\zeta U_{4a'}^F V_{4a}^H+\frac{2i\zeta}{\sqrt3}U_{2a'}^F V_{6a}^H+\zeta U_{2a'}^F V_{5a}^H\right.\\
&\left.-2\sqrt3\eta i U_{1a'}^F V_{4a}^H+\gamma U_{2a'}^F V_{1a}^H\right)V_{31}^H\\
&+\left(2\sqrt3\eta i U_{1a'}^F V_{3a}^H+\frac{i\rho}{\sqrt3}U_{4a'}^F V_{6a}^H-i\bar\zeta U_{1a'}^F V_{5a}^H\right.\\
&-\sqrt3\bar\zeta U_{1a'}^F V_{6a}^H+i\bar\gamma U_{1a'}^F V_{1a}^H+\sqrt3\zeta U_{4a'}^F V_{3a}^H\\
&\left.-\kappa U_{4a'}^F V_{1a}^H+\sqrt3\bar\zeta U_{4a'}^F V_{2a}^H\right)V_{41}^H\\
&+\left(i\bar\zeta U_{1a'}^F V_{4a}^H-\bar\zeta U_{2a'}^F V_{2a}^H-\zeta U_{2a'}^F V_{3a}^H+\frac{i\rho}{3}U_{2a'}^F V_{6a}^H\right)V_{51}^H\\
&+\left(i\kappa U_{2a'}^F V_{1a}^H-\frac{2i\bar\zeta}{\sqrt3}U_{2a'}^F V_{2a}^H-\frac{i\rho}{\sqrt3}U_{4a'}^F V_{4a}^H-\frac{2i\zeta}{\sqrt3}U_{2a'}^F V_{3a}^H+\sqrt3\bar\zeta U_{1a'}^F V_{4a}^H\right.\\
&\left.-\frac{i\rho}{3}U_{2a'}^F V_{5a}^H\right)V_{61}^H\Bigg|^2F_{12}\big(m_a^H,m_{a'}^F,Q\big)\\
&-\sum_{a=2}^{\mathrm{d(H)}}2g_{10}^2\big|i\big(U_{1a}^{H*}V_{11}^H+U_{2a}^{H*}V_{21}^H+U_{3a}^{H*}V_{31}^H\\
&+U_{5a}^{H*}V_{51}^H+U_{6a}^{H*}V_{61}^H\big)\big|^2F_{12}\big(m_a^H,m_{\lambda_F},Q\big)\\
&-2g_{10}^2\big|i\big(U_{11}^{H*}V_{11}^H+U_{21}^{H*}V_{21}^H+U_{31}^{H*}V_{31}^H+U_{51}^{H*}V_{51}^H+U_{61}^{H*}V_{61}^H\big)\big|^2F_{11}(m_{\lambda_F},Q)
\end{aligned}
\qquad (104)
$$

$$
\begin{aligned}
K_{G\bar H}=\sum_{a=2}^{\mathrm{d}(H)}\sum_{a'=1}^{\mathrm{d}(G)}\Bigg|&\left(\frac{\gamma}{\sqrt2}V_{3a'}^G U_{3a}^H-\gamma V_{2a'}^G U_{3a}^H-\sqrt2 i\gamma V_{4a'}^G U_{4a}^H-\bar\gamma V_{2a'}^G U_{2a}^H\right.\\
&\left.-\frac{\bar\gamma}{\sqrt2}V_{3a'}^G U_{2a}^H+\frac{i\kappa}{\sqrt2}V_{3a'}^G U_{6a}^H+\kappa V_{1a'}^G U_{5a}^H\right)V_{11}^H
\end{aligned}
$$

$$+\left(2\sqrt{\frac{2}{3}}\eta V^G_{3a'}U^H_{3a}-\frac{4\eta}{\sqrt{3}}V^G_{2a'}U^H_{3a}-2\sqrt{6}i\eta V^G_{4a'}U^H_{4a}\right.$$
$$\left.-\sqrt{\frac{2}{3}}i\bar{\zeta}V^G_{3a'}U^H_{6a}+i\bar{\zeta}V^G_{1a'}U^H_{6a}-\frac{\bar{\zeta}}{\sqrt{2}}V^G_{3a'}U^H_{5a}-\bar{\gamma}V^G_{2a'}U^H_{1a}+\frac{\bar{\gamma}}{\sqrt{2}}V^G_{3a'}U^H_{1a}\right)V^H_{21}$$
$$-\left(\sqrt{\frac{2}{3}}i\zeta V^G_{3a'}U^H_{6a}+i\zeta V^G_{1a'}U^H_{6a}+\frac{\zeta}{\sqrt{2}}V^G_{3a'}U^H_{5a}+\gamma V^G_{2a'}U^H_{1a}\right.$$
$$\left.+\frac{\gamma}{\sqrt{2}}V^G_{3a'}U^H_{1a}+\frac{4\eta}{\sqrt{3}}V^G_{2a'}U^H_{2a}+2\sqrt{\frac{2}{3}}\eta V^G_{3a'}U^H_{2a}\right)V^H_{31}$$
$$+\big(\sqrt{6}\lambda V^G_{3a'}U^H_{4a}-2\sqrt{3}\lambda V^G_{2a'}U^H_{4a}-\sqrt{2}i\bar{\zeta}V^G_{5a'}U^H_{5a}-\sqrt{6}\bar{\zeta}V^G_{5a'}U^H_{6a}$$
$$+\sqrt{2}i\bar{\gamma}V^G_{5a'}U^H_{1a}+2\sqrt{6}i\eta V^G_{5a'}U^H_{3a}\big)V^H_{41}$$
$$+\left(\kappa V^G_{1a'}U^H_{1a}-\frac{i\rho}{3\sqrt{2}}V^G_{3a'}U^H_{6a}+\frac{\bar{\zeta}}{\sqrt{2}}V^G_{3a'}U^H_{2a}\right.$$
$$\left.+\frac{\zeta}{\sqrt{2}}V^G_{3a'}U^H_{3a}+\sqrt{2}i\zeta V^G_{4a'}U^H_{4a}\right)V^H_{51}$$
$$+\left(\sqrt{\frac{2}{3}}\bar{\zeta}iV^G_{3a'}U^H_{2a}+i\bar{\zeta}V^G_{1a'}U^H_{2a}-\frac{2\rho}{3\sqrt{3}}V^G_{2a'}U^H_{6a}\right.$$
$$-\frac{i\kappa}{\sqrt{2}}V^G_{3a'}U^H_{1a}+\sqrt{\frac{2}{3}}i\zeta V^G_{3a'}U^H_{3a}-i\zeta V^G_{1a'}U^H_{3a}$$
$$\left.\left.-\sqrt{6}\zeta V^G_{4a'}U^H_{4a}+\frac{i\rho}{3\sqrt{2}}V^G_{3a'}U^H_{5a}\right)V^H_{61}\right|^2F_{12}\big(m^H_a,m^G_{a'},Q\big)$$
$$-\sum_{a=2}^{\mathrm{d}(H)}2g^2_{10}\left|\frac{i}{\sqrt{5}}\big(V^{H*}_{1a}V^H_{11}+V^{H*}_{2a}V^H_{21}+V^{H*}_{3a}V^H_{31}-4V^{H*}_{4a}V^H_{41}+V^{H*}_{5a}V^H_{51}\right.$$
$$\left.+V^{H*}_{6a}V^H_{61}\big)\right|^2F_{12}\big(m^H_a,m_{\lambda_G},Q\big)$$
$$+\sum_{a'=1}^{\mathrm{d}(G)}\left|\left(\frac{\gamma}{\sqrt{2}}V^G_{3a'}U^H_{31}-\gamma V^G_{2a'}U^H_{31}-\sqrt{2}i\gamma V^G_{4a'}U^H_{41}\right.\right.$$
$$\left.-\bar{\gamma}V^G_{2a'}U^H_{21}-\frac{\bar{\gamma}}{\sqrt{2}}V^G_{3a'}U^H_{21}+\frac{i\kappa}{\sqrt{2}}V^G_{3a'}U^H_{61}+\kappa V^G_{1a'}U^H_{51}\right)V^H_{11}$$
$$+\left(2\sqrt{\frac{2}{3}}\eta V^G_{3a'}U^H_{31}-\frac{4\eta}{\sqrt{3}}V^G_{2a'}U^H_{31}-2\sqrt{6}i\eta V^G_{4a'}U^H_{41}-\sqrt{\frac{2}{3}}i\bar{\zeta}V^G_{3a'}U^H_{61}\right.$$
$$\left.+i\bar{\zeta}V^G_{1a'}U^H_{61}-\frac{\bar{\zeta}}{\sqrt{2}}V^G_{3a'}U^H_{51}-\bar{\gamma}V^G_{2a'}U^H_{11}+\frac{\bar{\gamma}}{\sqrt{2}}V^G_{3a'}U^H_{11}\right)V^H_{21}$$
$$-\left(\sqrt{\frac{2}{3}}i\zeta V^G_{3a'}U^H_{61}+i\zeta V^G_{1a'}U^H_{61}+\frac{\zeta}{\sqrt{2}}V^G_{3a'}U^H_{51}\right.$$
$$\left.+\gamma V^G_{2a'}U^H_{11}+\frac{\gamma}{\sqrt{2}}V^G_{3a'}U^H_{11}+\frac{4\eta}{\sqrt{3}}V^G_{2a'}U^H_{21}+2\sqrt{\frac{2}{3}}\eta V^G_{3a'}U^H_{21}\right)V^H_{31}$$

$$+ \big(\sqrt{6}\lambda V^G_{3a'}U^H_{41} - 2\sqrt{3}\lambda V^G_{2a'}U^H_{41} - \sqrt{2}i\bar{\zeta} V^G_{5a'}U^H_{51} - \sqrt{6}\bar{\zeta} V^G_{5a'}U^H_{61}$$
$$+ \sqrt{2}i\bar{\gamma} V^G_{5a'}U^H_{1a} + 2\sqrt{6}i\eta V^G_{5a'}U^H_{31}\big)V^H_{41}$$
$$+ \left(\kappa V^G_{1a'}U^H_{11} - \frac{i\rho}{3\sqrt{2}}V^G_{3a'}U^H_{61} + \frac{\bar{\zeta}}{\sqrt{2}}V^G_{3a'}U^H_{21}\right.$$
$$\left.+ \frac{\zeta}{\sqrt{2}}V^G_{3a'}U^H_{31} + \sqrt{2}i\zeta V^G_{4a'}U^H_{41}\right)V^H_{51}$$
$$+ \left(\sqrt{\frac{2}{3}}\bar{\zeta} i V^G_{3a'}U^H_{21} + i\bar{\zeta} V^G_{1a'}U^H_{21} - \frac{2\rho}{3\sqrt{3}}V^G_{2a'}U^H_{61}\right.$$
$$- \frac{i\kappa}{\sqrt{2}}V^G_{3a'}U^H_{11} + \sqrt{\frac{2}{3}}i\zeta V^G_{3a'}U^H_{31} - i\zeta V^G_{1a'}U^H_{31}$$
$$\left.\left. - \sqrt{6}\zeta V^G_{4a'}U^H_{41} + \frac{i\rho}{3\sqrt{2}}V^G_{3a'}U^H_{51}\right)V^H_{61}\right|^2 F_{11}\big(m^G_{a'}, Q\big)$$
$$- 2g^2_{10}\left|\frac{i}{\sqrt{5}}\big(V^{H*}_{11}V^H_{11} + V^{H*}_{21}V^H_{21} + V^{H*}_{31}V^H_{31} - 4V^{H*}_{41}V^H_{41} + V^{H*}_{51}V^H_{51}\right.$$
$$\left. + V^{H*}_{61}V^H_{61}\big)\right|^2 F_{11}(m_{\lambda_G}, Q) \tag{105}$$

For the $\bar{H}[1,2,-1]$ line we have

$$\begin{aligned}\big(16\pi^2\big)\mathcal{K}_{\bar{H}} = {}& 8K_{RC} + 3K_{D\bar{J}} + 3K_{J\bar{E}} + 9K_{P\bar{X}} + 3K_{T\bar{X}} + 9K_{E\bar{P}} \\ & + 3K_{E\bar{T}} + 6K_{L\bar{Y}} + K_{\bar{V}\bar{F}} + 8K_{Z\bar{C}} + 3K_{I\bar{D}} + 24K_{QC} + 9K_{U\bar{E}} + 9K_{D\bar{U}} \\ & + 6K_{B\bar{L}} + 3K_{X\bar{K}} + 6K_{M\bar{B}} + 18K_{B\bar{W}} + 18K_{W\bar{Y}} + 3K_{\bar{V}O} \\ & + 6K_{Y\bar{N}} + K_{VA} + 3K_{\bar{H}\bar{O}} + 3K_{SH} + K_{F\bar{H}} + K_{GH}\end{aligned} \tag{106}$$

$$K_{RC} = \sum_{a=1}^{d(R)}\sum_{a'=1}^{d(C)}\left|\left(\gamma V^R_{1a}V^C_{1a'} + \frac{\gamma}{\sqrt{2}}V^R_{2a}V^C_{1a'} + \bar{\gamma} V^R_{1a}V^C_{2a'} - \frac{\bar{\gamma}}{\sqrt{2}}V^R_{2a}V^C_{2a'} + \frac{i\kappa}{\sqrt{2}}V^R_{2a}V^C_{3a'}\right)U^H_{11}\right.$$
$$+ \left(\frac{i\bar{\zeta}}{\sqrt{6}}V^R_{2a}V^C_{3a'} - \frac{2\eta}{\sqrt{3}}V^R_{1a}V^C_{1a'} - \sqrt{\frac{2}{3}}\eta V^R_{2a}V^C_{1a'}\right)U^H_{21}$$
$$+ \left(\sqrt{\frac{2}{3}}\eta V^R_{2a}V^C_{2a'} - \frac{2\eta}{\sqrt{3}}V^R_{1a}V^C_{2a'} + \frac{i\zeta}{\sqrt{6}}V^R_{2a}V^C_{3a'}\right)U^H_{31}$$
$$+ \left(\frac{\zeta}{\sqrt{2}}V^R_{2a}V^C_{1a'} + \frac{\bar{\zeta}}{\sqrt{2}}V^R_{2a}V^C_{2a'} - \frac{i\rho}{3\sqrt{2}}V^R_{2a}V^C_{3a'}\right)U^H_{51}$$
$$\left. - \left(\frac{\rho}{3\sqrt{3}}V^R_{1a}V^C_{3a'} + \frac{i\zeta}{\sqrt{6}}V^R_{2a}V^C_{1a'} + \frac{i\bar{\zeta}}{\sqrt{6}}V^R_{2a}V^C_{2a'}\right)U^H_{61}\right|^2 F_{12}\big(m^R_a, m^C_{a'}, Q\big) \tag{107}$$

$$K_{D\bar{J}} = \sum_{a=1}^{d(D)}\sum_{a'=1}^{d(J)}\left|\left(\frac{\bar{\gamma}}{\sqrt{2}}V^D_{1a}U^J_{3a'} - \bar{\gamma} V^D_{1a}U^J_{2a'} - \gamma V^D_{2a}U^J_{2a'}\right.\right.$$
$$\left. - \frac{\gamma}{\sqrt{2}}V^D_{2a}U^J_{3a'} - \frac{i\kappa}{\sqrt{2}}V^D_{3a}U^J_{3a'}\right)U^H_{11}$$

$$
\begin{aligned}
&+\left(\sqrt{\frac{2}{3}}\eta V^D_{2a}U^J_{3a'}-\frac{2\eta}{\sqrt{3}}V^D_{2a}U^J_{2a'}-\frac{2i\bar{\zeta}}{\sqrt{3}}V^D_{3a}U^J_{2a'}-\frac{i\bar{\zeta}}{\sqrt{6}}V^D_{3a}U^J_{3a'}\right)U^H_{21}\\
&+\left(\sqrt{6}\eta V^D_{1a}U^J_{3a'}-\frac{2\eta}{\sqrt{3}}V^D_{1a}U^J_{2a'}-\frac{2i\zeta}{\sqrt{3}}V^D_{3a}U^J_{2a'}+\sqrt{\frac{3}{2}}i\zeta V^D_{3a}U^J_{3a'}\right)U^H_{31}\\
&+\left(\frac{i\rho}{3}V^D_{3a}U^J_{5a'}-4\eta V^D_{1a}U^J_{1a'}-2\zeta V^D_{2a}U^J_{5a'}+2i\zeta V^D_{3a}U^J_{1a'}\right)U^H_{41}\\
&+\left(\frac{i\rho}{3}V^D_{3a}-\zeta V^D_{2a}-\bar{\zeta}V^D_{1a}\right)\frac{U^J_{3a'}}{\sqrt{2}}U^H_{51}\\
&+\left(\frac{2i\zeta}{\sqrt{3}}V^D_{2a}U^J_{2a'}+\frac{i\zeta}{\sqrt{6}}V^D_{2a}U^J_{3a'}-\sqrt{\frac{3}{2}}i\bar{\zeta}V^D_{1a}U^J_{3a'}\right.\\
&\left.+\frac{2i\bar{\zeta}}{\sqrt{3}}V^D_{1a}U^J_{2a'}-\frac{\rho}{3\sqrt{3}}V^D_{3a}U^J_{2a'}+\frac{\sqrt{2}\rho}{3\sqrt{3}}V^D_{3a}U^J_{3a'}\right)U^H_{61}\Bigg|^2F_{12}\left(m^D_a,m^J_{a'},Q\right)\\
&-2g^2_{10}\left|\frac{2i}{\sqrt{3}}\left(U^{D*}_{2a}U^H_{21}+U^{D*}_{1a'}U^H_{31}+U^{D*}_{3a'}U^H_{61}\right)\right|^2F_{12}\left(m^D_a,m_{\lambda_J},Q\right)
\end{aligned}
\tag{108}
$$

$$
\begin{aligned}
K_{J\bar{E}}=\sum_{a=1}^{\mathrm{d}(J)}\sum_{a'=1}^{\mathrm{d}(E)}\Bigg|&\left(\frac{\bar{\gamma}}{\sqrt{2}}V^J_{3a}U^E_{2a'}-i\sqrt{2}\bar{\gamma}U^E_{3a'}V^J_{1a}-\bar{\gamma}U^E_{2a'}V^J_{2a}\right.\\
&\left.-\gamma V^J_{2a}U^E_{1a'}-\frac{\gamma}{\sqrt{2}}V^J_{3a}U^E_{1a'}-\frac{i\kappa}{\sqrt{2}}V^J_{3a}U^E_{6a'}-i\kappa V^J_{5a}U^E_{4a'}\right)U^H_{11}\\
&+\left(\sqrt{\frac{3}{2}}i\bar{\zeta}V^J_{3a}U^E_{6a'}+\frac{2i\bar{\zeta}}{\sqrt{3}}V^J_{2a}U^E_{6a'}+\frac{2\sqrt{2}i\bar{\zeta}}{\sqrt{3}}V^J_{5a}U^E_{3a'}-\frac{i\bar{\zeta}}{\sqrt{3}}V^J_{5a}U^E_{4a'}\right.\\
&\left.-\frac{2\eta}{\sqrt{3}}V^J_{2a}U^E_{1a'}-\sqrt{6}\eta V^J_{3a}U^E_{1a'}\right)U^H_{21}+\left(-\frac{i\zeta}{\sqrt{6}}V^J_{3a}U^E_{6a'}+\frac{2i\zeta}{\sqrt{3}}V^J_{2a}U^E_{6a'}\right.\\
&+\frac{i\zeta}{\sqrt{3}}V^J_{5a}U^E_{4a'}-\frac{2\eta}{\sqrt{3}}V^J_{2a}U^E_{2a'}-\sqrt{\frac{2}{3}}\eta V^J_{3a}U^E_{2a'}\\
&\left.-\frac{4i\eta}{\sqrt{3}}V^J_{1a}U^E_{4a'}-\sqrt{\frac{8}{3}}i\eta V^J_{1a}U^E_{3a'}\right)U^H_{31}\\
&+\left(2\sqrt{2}i\lambda V^J_{2a}U^E_{3a'}-\sqrt{2}i\lambda V^J_{3a}U^E_{4a'}-2i\lambda V^J_{3a}U^E_{3a'}\right)U^H_{41}\\
&+\left(\frac{i\rho}{3\sqrt{2}}V^J_{3a}U^E_{6a'}-\frac{i\rho}{3}V^J_{5a}U^E_{4a'}-\frac{\bar{\zeta}}{\sqrt{2}}V^J_{3a}U^E_{2a'}\right.\\
&\left.+\sqrt{2}i\bar{\zeta}U^E_{3a'}V^J_{1a}-\frac{\zeta}{\sqrt{2}}V^J_{3a}U^E_{1a'}\right)U^H_{51}\\
&+\left(\frac{2}{\sqrt{3}}\bar{\zeta}V^J_{1a}U^E_{4a'}+\sqrt{\frac{2}{3}}\bar{\zeta}V^J_{1a}U^E_{3a'}+\frac{i\bar{\zeta}}{\sqrt{6}}V^J_{3a}U^E_{2a'}-\frac{2i}{\sqrt{3}}\bar{\zeta}V^J_{2a}U^E_{2a'}\right.\\
&-\sqrt{\frac{3}{2}}i\zeta V^J_{3a}U^E_{1a'}-\frac{2i}{\sqrt{3}}\zeta V^J_{2a}U^E_{1a'}-\frac{\rho}{3\sqrt{3}}V^J_{5a}U^E_{4a'}\\
&\left.-\frac{\sqrt{2}\rho}{3\sqrt{3}}V^J_{5a}U^E_{3a'}-\frac{\sqrt{2}\rho}{3\sqrt{3}}V^J_{3a}U^E_{6a'}-\frac{\rho}{3\sqrt{3}}V^J_{2a}U^E_{6a'}\right)U^H_{61}\Bigg|^2F_{12}\left(m^J_a,m^E_{a'},Q\right)
\end{aligned}
$$

$$
\begin{aligned}
&-2g_{10}^2\bigg|-\frac{2i}{\sqrt{3}}V_{1a'}^{E*}U_{21}^H-\frac{2i}{\sqrt{3}}V_{2a'}^{E*}U_{31}^H\\
&-\sqrt{2}V_{3a'}^{E*}U_{41}^H-\frac{2i}{\sqrt{3}}V_{6a'}^{E*}U_{61}^H\bigg|^2F_{12}\big(m_{\lambda_J},m_{a'}^E,Q\big)\\
&-2g_{10}^2\bigg|\frac{2}{\sqrt{3}}U_{1a}^{J*}U_{31}^H-iU_{2a}^{J*}U_{41}^H+\frac{i}{\sqrt{2}}U_{3a}^{J*}U_{41}^H-iU_{5a}^{J*}U_{51}^H\\
&-\frac{1}{\sqrt{3}}U_{5a}^{J*}U_{61}^H\bigg|^2F_{12}\big(m_a^J,m_{\lambda_E},Q\big)
\end{aligned}
\tag{109}
$$

$$
\begin{aligned}
K_{P\bar{X}}=\sum_{a=1}^{d(P)}\sum_{a'=1}^{d(X)}&\bigg|\bigg(\frac{\kappa}{\sqrt{2}}V_{2a}^PU_{2a'}^X-\gamma V_{1a}^PU_{1a'}^X\bigg)U_{11}^H+\bigg(\frac{2\sqrt{2}\eta}{\sqrt{3}}V_{1a}^PU_{2a'}^X-\frac{2\eta}{\sqrt{3}}V_{1a}^PU_{1a'}^X\\
&-\frac{\bar{\zeta}}{\sqrt{6}}V_{2a}^PU_{2a'}^X\bigg)U_{21}^H+\frac{\zeta}{\sqrt{3}}V_{2a}^P\bigg(2U_{1a'}^X+\frac{U_{2a'}^X}{\sqrt{2}}\bigg)U_{31}^H\\
&-\bigg(\frac{\rho}{3\sqrt{2}}V_{2a}^PU_{2a'}^X+\zeta V_{1a}^PU_{1a'}^X\bigg)U_{51}^H\\
&+\bigg(\frac{i\sqrt{2}\zeta}{\sqrt{3}}V_{1a}^PU_{2a'}^X-\frac{i\zeta}{\sqrt{3}}V_{1a}^PU_{1a'}^X+\frac{i\rho}{3\sqrt{3}}V_{2a}^PU_{1a'}^X\\
&-\frac{i\rho}{3\sqrt{6}}V_{2a}^PU_{2a'}^X\bigg)U_{61}^H\bigg|^2F_{12}\big(m_a^P,m_{a'}^X,Q\big)\\
&-2g_{10}^2\bigg|i\sqrt{\frac{2}{3}}U_{1a}^{P*}U_{21}^H+\frac{U_{2a}^{P*}}{\sqrt{2}}U_{51}^H+\frac{i}{\sqrt{6}}U_{2a}^{P*}U_{61}^H\bigg|^2F_{12}\big(m_a^P,m_{\lambda_X},Q\big)
\end{aligned}
\tag{110}
$$

$$
\begin{aligned}
K_{T\bar{X}}=\sum_{a'=1}^{d(X)}\sum_{a=1}^{d(T)}&\bigg|\bigg(\gamma U_{2a'}^XV_{3a}^T-i\bar{\gamma}U_{1a'}^XV_{4a}^T+\bar{\gamma}U_{2a'}^XV_{2a}^T-\frac{i\kappa}{\sqrt{2}}U_{2a'}^XV_{7a}^T-\kappa U_{1a'}^XV_{6a}^T\bigg)U_{11}^H\\
&+\bigg(\frac{\bar{\zeta}}{\sqrt{3}}U_{1a'}^XV_{6a}^T+\frac{2i\bar{\zeta}}{\sqrt{3}}U_{1a'}^XV_{7a}^T-\frac{i\bar{\zeta}}{\sqrt{6}}V_{7a}^TU_{2a'}^X\\
&-\sqrt{\frac{2}{3}}\bar{\zeta}U_{2a'}^XV_{6a}^T-\frac{\sqrt{2}i\bar{\gamma}}{\sqrt{3}}U_{1a'}^XV_{1a}^T-\frac{\bar{\gamma}}{\sqrt{3}}U_{2a'}^XV_{1a}^T-\frac{4\eta}{\sqrt{6}}U_{1a'}^XV_{3a}^T\bigg)U_{21}^H\\
&+\bigg(\frac{\gamma}{\sqrt{3}}U_{2a'}^XV_{1a}^T-\frac{i\sqrt{2}\gamma}{\sqrt{3}}U_{1a'}^XV_{1a}^T+2\sqrt{\frac{2}{3}}\eta U_{1a'}^XV_{2a}^T-2\sqrt{\frac{2}{3}}i\eta U_{2a'}^XV_{4a}^T\\
&-\frac{2i\eta}{\sqrt{3}}U_{1a'}^XV_{4a}^T+\frac{\zeta}{\sqrt{3}}U_{1a'}^XV_{6a}^T+\frac{i\zeta}{\sqrt{6}}U_{2a'}^XV_{7a}^T+\frac{\sqrt{2}\zeta}{\sqrt{3}}U_{2a'}^XV_{6a}^T\bigg)U_{31}^H\\
&+2i\lambda\big(U_{2a'}^X+\sqrt{2}U_{1a'}^X\big)V_{5a}^TU_{41}^H+\bigg(\kappa U_{2a'}^XV_{1a}^T-\frac{i\rho}{3\sqrt{2}}U_{2a'}^XV_{7a}^T+i\bar{\zeta}U_{1a'}^XV_{4a}^T\bigg)U_{51}^H\\
&+\bigg(\frac{i\sqrt{2}\bar{\zeta}}{\sqrt{3}}U_{1a'}^XV_{2a}^T+\frac{\bar{\zeta}}{\sqrt{3}}U_{1a'}^XV_{4a}^T+\frac{\sqrt{2}\bar{\zeta}}{\sqrt{3}}U_{2a'}^XV_{4a}^T\\
&+\frac{i\bar{\zeta}}{\sqrt{3}}U_{2a'}^XV_{2a}^T-\sqrt{\frac{2}{3}}i\kappa U_{1a'}^XV_{1a}^T+\frac{i\zeta}{\sqrt{3}}U_{2a'}^XV_{3a}^T
\end{aligned}
$$

$$- \frac{\sqrt{2}i\zeta}{\sqrt{3}} U^X_{1a'} V^T_{3a} - \frac{\rho}{3\sqrt{3}} U^X_{1a'} V^T_{7a} - \frac{\rho}{3\sqrt{6}} U^X_{2a'} V^T_{7a}$$

$$+ \frac{i\rho}{3\sqrt{3}} U^X_{1a'} V^T_{6a} \bigg) U^H_{61} \bigg|^2 F_{12}\big(m^X_{a'}, m^T_a, Q\big)$$

$$- 2g^2_{10} \bigg| U^{T*}_{1a} U^H_{11} - \frac{i}{\sqrt{3}} U^{T*}_{2a} U^H_{21} + \bigg(\sqrt{\frac{2}{3}} U^{T*}_{4a} - \frac{iU^{T*}_{3a}}{\sqrt{3}} \bigg) U^H_{31}$$

$$- iU^{T*}_{5a} U^H_{41} - \frac{i}{\sqrt{2}} U^{T*}_{7a} U^H_{51} + \bigg(i\sqrt{\frac{2}{3}} U^{T*}_{6a} - \frac{U^{T*}_{7a}}{\sqrt{6}} \bigg) U^H_{61} \bigg|^2 F_{12}\big(m_{\lambda_X}, m^T_a, Q\big) \quad (111)$$

$$K_{E\bar{P}} = \sum_{a=1}^{\mathrm{d}(E)} \sum_{a'=1}^{\mathrm{d}(P)} \bigg| \bigg(\frac{\kappa}{\sqrt{2}} V^E_{4a} U^P_{2a'} - \bar{\gamma} V^E_{3a} U^P_{1a'} \bigg) U^H_{11} + \bigg(\frac{2\bar{\zeta}}{\sqrt{3}} V^E_{3a} U^P_{2a'} + \frac{\bar{\zeta}}{\sqrt{6}} V^E_{4a} U^P_{2a'} \bigg) U^H_{21}$$

$$+ \bigg(\frac{2\sqrt{2}\eta}{\sqrt{3}} V^E_{4a} U^P_{1a'} - \frac{2\eta}{\sqrt{3}} V^E_{3a} U^P_{1a'} - \frac{\zeta}{\sqrt{6}} V^E_{4a} U^P_{2a'} \bigg) U^H_{31}$$

$$+ \bigg(2\sqrt{2}i\eta V^E_{2a} U^P_{1a'} + \frac{\rho}{3\sqrt{2}} V^E_{6a} U^P_{2a'} + i\sqrt{2}\bar{\zeta} V^E_{1a} U^P_{2a'} + \sqrt{2}\bar{\zeta} V^E_{6a} U^P_{1a'} \bigg) U^H_{41}$$

$$- \bigg(\frac{\rho}{3\sqrt{2}} V^E_{4a} U^P_{2a'} + \bar{\zeta} V^E_{3a} U^P_{1a'} \bigg) U^H_{51}$$

$$+ \bigg(\frac{i\bar{\zeta}}{\sqrt{3}} V^E_{3a} U^P_{1a'} - \frac{i\sqrt{2}\bar{\zeta}}{\sqrt{3}} V^E_{4a} U^P_{1a'}$$

$$- \frac{i\rho}{3\sqrt{3}} V^E_{3a} U^P_{2a'} + \frac{i\rho}{3\sqrt{6}} V^E_{4a} U^P_{2a'} \bigg) U^H_{61} \bigg|^2 F_{12}\big(m^E_a, m^P_{a'}, Q\big)$$

$$- 2g^2_{10} \bigg| -i\sqrt{\frac{2}{3}} V^{P*}_{1a'} U^H_{31} + \frac{V^{P*}_{2a'}}{\sqrt{2}} U^H_{51} - \frac{i}{\sqrt{6}} V^{P*}_{2a'} U^H_{61} \bigg|^2 F_{12}\big(m_{\lambda_E}, m^P_{a'}, Q\big) \quad (112)$$

$$K_{E\bar{T}} = \sum_{a'=1}^{\mathrm{d}(T)} \sum_{a=1}^{\mathrm{d}(E)} \bigg| \bigg(\gamma U^T_{3a'} V^E_{4a} - \gamma U^T_{5a'} V^E_{2a} - i\gamma U^T_{4a'} V^E_{3a} + \bar{\gamma} U^T_{2a'} V^E_{4a} + \bar{\gamma} U^T_{5a'} V^E_{1a}$$

$$- \kappa U^T_{6a'} V^E_{3a} - i\kappa U^T_{5a'} V^E_{6a} - \frac{i\kappa}{\sqrt{2}} U^T_{7a'} V^E_{4a} \bigg) U^H_{11}$$

$$+ \bigg(\frac{\bar{\gamma}}{\sqrt{3}} U^T_{1a'} V^E_{4a} - \frac{\sqrt{2}i\bar{\gamma}}{\sqrt{3}} U^T_{1a'} V^E_{3a} + \frac{2\sqrt{2}\eta}{\sqrt{3}} U^T_{3a'} V^E_{3a} - \frac{2i\eta}{\sqrt{3}} U^T_{4a'} V^E_{3a}$$

$$+ \frac{2\eta}{\sqrt{3}} U^T_{5a'} V^E_{2a} - \frac{2\sqrt{2}i\eta}{\sqrt{3}} U^T_{4a'} V^E_{4a} + \frac{\bar{\zeta}}{\sqrt{3}} U^T_{6a'} V^E_{3a} + \frac{\sqrt{2}\bar{\zeta}}{\sqrt{3}} U^T_{6a'} V^E_{4a}$$

$$+ \frac{i\bar{\zeta}}{\sqrt{6}} U^T_{7a'} V^E_{4a} - \frac{i\bar{\zeta}}{\sqrt{3}} U^T_{5a'} V^E_{6a} \bigg) U^H_{21}$$

$$+ \bigg(2\sqrt{3}\eta U^T_{5a'} V^E_{1a} - \frac{2\sqrt{2}\eta}{\sqrt{3}} U^T_{2a'} V^E_{3a}$$

$$- \frac{\sqrt{2}i\gamma}{\sqrt{3}} U^T_{1a'} V^E_{3a} - \frac{\gamma}{\sqrt{3}} U^T_{1a'} V^E_{4a} + \frac{\zeta}{\sqrt{3}} U^T_{6a'} V^E_{3a} - \frac{\sqrt{2}\zeta}{\sqrt{3}} U^T_{6a'} V^E_{4a}$$

$$
\begin{aligned}
&+i\zeta\sqrt{3}U^T_{5a'}V^E_{6a}+\frac{2i\zeta}{\sqrt{3}}U^T_{7a'}V^E_{3a}-\frac{i\zeta}{\sqrt{6}}U^T_{7a'}V^E_{4a}\Big)U^H_{31}\\
&+\Big(2i\eta U^T_{2a'}V^E_{2a}-2i\eta U^T_{3a'}V^E_{1a}+2\sqrt{2}\eta U^T_{4a'}V^E_{1a}\\
&-\bar{\gamma}U^T_{1a'}V^E_{1a}-\gamma U^T_{1a'}V^E_{2a}+\kappa U^T_{1a'}V^E_{6a}+\frac{\sqrt{2}\rho}{3}U^T_{6a'}V^E_{6a}\\
&-\frac{i\rho}{3\sqrt{2}}U^T_{7a'}V^E_{6a}+\sqrt{2}i\zeta U^T_{6a'}V^E_{2a}-\zeta U^T_{3a'}V^E_{6a}\\
&-\sqrt{2}i\zeta U^T_{4a'}V^E_{6a}+\sqrt{2}\zeta U^T_{7a'}V^E_{2a}+\sqrt{2}i\bar{\zeta}U^T_{6a'}V^E_{1a}+\bar{\zeta}U^T_{2a'}V^E_{6a}\Big)U^H_{41}\\
&+\Big(\kappa U^T_{1a'}V^E_{4a}-\frac{i\rho}{3\sqrt{2}}U^T_{7a'}V^E_{4a}+\frac{i\rho}{3}U^T_{5a'}V^E_{6a}\\
&-\zeta U^T_{5a'}V^E_{2a}+i\zeta U^T_{4a'}V^E_{3a}-\bar{\zeta}U^T_{5a'}V^E_{1a}\Big)U^H_{51}\\
&+\Big(i\kappa\sqrt{\frac{2}{3}}U^T_{1a'}V^E_{3a}+\frac{i\zeta}{\sqrt{3}}U^T_{5a'}V^E_{2a}-\frac{\sqrt{2}\zeta}{\sqrt{3}}U^T_{4a'}V^E_{4a}-\frac{i\zeta}{\sqrt{3}}U^T_{3a'}V^E_{4a}-\frac{\zeta}{\sqrt{3}}U^T_{4a'}V^E_{3a}\\
&-\frac{\sqrt{2}i\zeta}{\sqrt{3}}U^T_{3a'}V^E_{3a}+\sqrt{\frac{2}{3}}i\bar{\zeta}U^T_{2a'}V^E_{3a}-\sqrt{3}i\bar{\zeta}U^T_{5a'}V^E_{1a}-\frac{i\bar{\zeta}}{\sqrt{3}}U^T_{2a'}V^E_{4a}\\
&+\frac{\rho}{3\sqrt{3}}\Big(U^T_{7a'}V^E_{3a}+U^T_{7a'}\frac{V^E_{4a}}{\sqrt{2}}+2U^T_{5a'}V^E_{6a}-iU^T_{6a'}V^E_{3a}\Big)\Big)U^H_{61}\Big|^2F_{12}\big(m^T_{a'},m^E_a,Q\big)\\
&-2g^2_{10}\Big|V^{T*}_{1a'}U^H_{11}+\Big(\frac{i}{\sqrt{3}}V^{T*}_{2a'}-\sqrt{\frac{2}{3}}V^{T*}_{4a'}\Big)U^H_{21}+\frac{i}{\sqrt{3}}V^{T*}_{3a'}U^H_{31}\\
&-\frac{i}{\sqrt{2}}V^{T*}_{7a'}U^H_{51}+\Big(\frac{V^{T*}_{7a'}}{\sqrt{6}}-i\sqrt{\frac{2}{3}}V^{T*}_{6a'}\Big)U^H_{61}\Big|^2F_{12}\big(m^T_{a'},m_{\lambda_E},Q\big)
\end{aligned}
\tag{113}
$$

$$
\begin{aligned}
K_{L\bar{Y}}=\sum_{a=1}^{d(L)}\Big|&-\big(i\bar{\gamma}V^L_{1a}+\kappa V^L_{2a}\big)U^H_{11}-\frac{\bar{\zeta}}{\sqrt{3}}V^L_{2a}U^H_{21}+\big(2i\eta V^L_{1a}-\zeta V^L_{2a}\big)\frac{U^H_{31}}{\sqrt{3}}+i\bar{\zeta}V^L_{1a}U^H_{51}\\
&-\Big(\frac{i\rho}{3\sqrt{3}}V^L_{2a}+\frac{\bar{\zeta}}{\sqrt{3}}V^L_{1a}\Big)U^H_{61}\Big|^2F_{12}\big(m^L_a,m_Y,Q\big)
\end{aligned}
\tag{114}
$$

$$
\begin{aligned}
K_{\bar{V}\bar{F}}=\sum_{a=1}^{d(F)}\Big|&-\big(i\bar{\gamma}U^F_{1a}+\kappa U^F_{4a}\big)U^H_{11}+\sqrt{3}\bar{\zeta}U^F_{4a}U^H_{21}+\big(\sqrt{3}\zeta U^F_{4a}-2\sqrt{3}i\eta U^F_{1a}\big)U^H_{31}\\
&+2\sqrt{3}\lambda U^F_{2a}U^H_{41}+i\bar{\zeta}U^F_{1a}U^H_{51}+\Big(\frac{i\rho}{\sqrt{3}}U^F_{4a}+\sqrt{3}\bar{\zeta}U^F_{1a}\Big)U^H_{61}\Big|^2F_{12}\big(m^V,m^F_a,Q\big)\\
&-2g^2_{10}\big|iU^H_{41}\big|^2F_{12}\big(m^V,m_{\lambda_F},Q\big)
\end{aligned}
\tag{115}
$$

$$
K_{Z\bar{C}}=\sum_{a=1}^{d(C)}\Big|\big(i\kappa U^C_{3a}+\gamma U^C_{2a}-\bar{\gamma}U^C_{1a}\big)U^H_{11}+\big(i\bar{\zeta}U^C_{3a}-2\eta U^C_{2a}\big)\frac{U^H_{21}}{\sqrt{3}}
$$

$$+ \left(i\zeta U^C_{3a} + 2\eta U^C_{1a}\right)\frac{U^H_{31}}{\sqrt{3}} + \left(\zeta U^C_{2a} - \frac{i\rho}{3}U^C_{3a} + \bar{\zeta} U^C_{1a}\right)U^H_{51}$$
$$- \frac{i}{\sqrt{3}}\left(\zeta U^C_{2a} + \bar{\zeta} U^C_{1a}\right)U^H_{61}\Big|^2 F_{12}\left(m^Z, m^C_a, Q\right) \tag{116}$$

$$K_{I\bar{D}} = \sum_{a=1}^{\mathrm{d}(D)} \Big| \left(\bar{\gamma} U^D_{2a} - i\kappa U^D_{3a} - \gamma U^D_{1a}\right)U^H_{11}$$
$$+ \sqrt{3}\left(i\bar{\zeta} U^D_{3a} - 2\eta U^D_{1a}\right)U^H_{21} + \left(-i\zeta U^D_{3a} - 2\eta U^D_{2a}\right)\frac{U^H_{31}}{\sqrt{3}}$$
$$+ \left(\frac{i\rho U^D_{3a}}{3} - \zeta U^D_{1a} - \bar{\zeta} U^D_{2a}\right)U^H_{51}$$
$$- \left(\sqrt{3} i\zeta U^D_{1a} - \frac{i\bar{\zeta} U^D_{2a}}{\sqrt{3}} + \frac{2\rho U^D_{3a}}{3\sqrt{3}}\right)U^H_{61}\Big|^2 F_{12}\left(m^I, m^D_a, Q\right) \tag{117}$$

$$K_{QC} = \sum_{a=1}^{\mathrm{d}(C)} \Big| \left(i\gamma V^C_{1a} + \kappa V^C_{3a} - i\bar{\gamma} V^C_{2a}\right)\frac{U^H_{11}}{\sqrt{2}} - \left(2i\eta V^C_{1a} + \bar{\zeta} V^C_{3a}\right)\frac{U^H_{21}}{\sqrt{6}}$$
$$+ \left(\zeta V^C_{3a} + 2i\eta V^C_{2a}\right)\frac{U^H_{31}}{\sqrt{6}} + \left(\frac{\rho}{3}V^C_{3a} - i\zeta V^C_{1a} - i\bar{\zeta} V^C_{2a}\right)\frac{U^H_{51}}{\sqrt{2}}$$
$$- \left(\zeta V^C_{1a} + \bar{\zeta} V^C_{2a}\right)\frac{U^H_{61}}{\sqrt{6}}\Big|^2 F_{12}\left(m^Q, m^C_a, Q\right) \tag{118}$$

$$K_{U\bar{E}} = \sum_{a=1}^{\mathrm{d}(E)} \Big| \left(i\bar{\gamma} U^E_{2a} - i\gamma U^E_{1a} - \kappa U^E_{6a}\right)\frac{U^H_{11}}{\sqrt{2}} + \left(2i\eta U^E_{1a} - \bar{\zeta} U^E_{6a}\right)\frac{U^H_{21}}{\sqrt{6}}$$
$$+ \left(6i\eta U^E_{2a} + 3\zeta U^E_{6a}\right)\frac{U^H_{31}}{\sqrt{6}} + \left(2\lambda U^E_{3a} - \sqrt{2}\lambda U^E_{4a}\right)U^H_{41}$$
$$+ \left(i\zeta U^E_{1a} - \frac{\rho}{3}U^E_{6a} + i\bar{\zeta} U^E_{2a}\right)\frac{U^H_{51}}{\sqrt{2}}$$
$$+ \left(\zeta U^E_{1a} - 3\bar{\zeta} U^E_{2a} + \frac{2i\rho}{3}U^E_{6a}\right)\frac{U^H_{61}}{\sqrt{6}}\Big|^2 F_{12}\left(m^U, m^E_a, Q\right)$$
$$- 2g^2_{10}\left|\frac{U^H_{41}}{\sqrt{2}}\right|^2 F_{12}\left(m^U, m_{\lambda_E}, Q\right) \tag{119}$$

$$K_{D\bar{U}} = \sum_{a=1}^{\mathrm{d}(D)} \Big| \left(i\bar{\gamma} V^D_{1a} - i\gamma V^D_{2a} - \kappa V^D_{3a}\right)\frac{U^H_{11}}{\sqrt{2}} + \left(\sqrt{\frac{3}{2}}\bar{\zeta} V^D_{3a} - \sqrt{6} i\eta V^D_{2a}\right)U^H_{21}$$
$$+ \left(-\frac{\zeta}{\sqrt{6}}V^D_{3a} - \sqrt{\frac{2}{3}} i\eta V^D_{1a}\right)U^H_{31} + \left(i\zeta V^D_{2a} - \frac{\rho}{3}V^D_{3a} + i\bar{\zeta} V^D_{1a}\right)\frac{U^H_{51}}{\sqrt{2}}$$
$$+ \left(\bar{\zeta} V^D_{1a} - 3\zeta V^D_{2a} - \frac{2i\rho}{3}V^D_{3a}\right)\frac{U^H_{61}}{\sqrt{6}}\Big|^2 F_{12}\left(m^U, m^D_a, Q\right) \tag{120}$$

$$K_{B\bar{L}} = \sum_{a=1}^{\mathrm{d}(L)} \bigg| -\big(i\gamma U^L_{1a} + \kappa U^L_{2a}\big) U^H_{11} + \big(2i\eta U^L_{1a} - \bar{\zeta} U^L_{2a}\big)\frac{U^H_{21}}{\sqrt{3}} - \zeta U^L_{2a}\frac{U^H_{31}}{\sqrt{3}} + i\zeta U^L_{1a} U^H_{51} + \left(\zeta U^L_{1a} + \frac{i\rho}{3} U^L_{2a}\right)\frac{U^H_{61}}{\sqrt{3}}\bigg|^2 F_{12}\big(m^B, m^L_a, Q\big) \tag{121}$$

$$\begin{aligned} K_{X\bar{K}} = \sum_{a=1}^{\mathrm{d}(X)} \sum_{a'=1}^{\mathrm{d}(K)} \bigg| &-\big(i\gamma\sqrt{2} V^X_{1a} U^K_{1a'} + i\kappa V^X_{2a} U^K_{2a'}\big) U^H_{11} \\ &- \left(\frac{2\sqrt{2}i\eta}{\sqrt{3}} V^X_{1a} U^K_{1a'} + \frac{4i\eta}{\sqrt{3}} V^X_{2a} U^K_{1a'} - \frac{i\bar{\zeta}}{\sqrt{3}} V^X_{2a} U^K_{2a'}\right) U^H_{21} \\ &+ \frac{i\zeta}{\sqrt{3}}\big(2\sqrt{2}V^X_{1a} - V^X_{2a}\big) U^K_{2a'} U^H_{31} + \left(\sqrt{2}i\zeta V^X_{1a} U^K_{1a'} - \frac{i\rho}{3} V^X_{2a} U^K_{2a'}\right) U^H_{51} \\ &+ \left(\frac{\rho}{3\sqrt{3}} V^X_{2a} U^K_{2a'} + \frac{\sqrt{2}\rho}{3\sqrt{3}} V^X_{1a} U^K_{2a'} \right. \\ &\left. - \frac{\sqrt{2}\zeta}{\sqrt{3}} V^X_{1a} U^K_{1a'} - \frac{2\zeta}{\sqrt{3}} V^X_{2a} U^K_{1a'}\right) U^H_{61}\bigg|^2 F_{12}\big(m^X_a, m^K_{a'}, Q\big) \\ &- 2g^2_{10}\left|\frac{-2}{\sqrt{3}} V^{K*}_{1a'} U^H_{21} - i V^{K*}_{2a'} U^H_{51} + \frac{V^{K*}_{2a'}}{\sqrt{3}} U^H_{61}\right|^2 F_{12}\big(m_{\lambda_X}, m^K_{a'}, Q\big) \end{aligned} \tag{122}$$

$$K_{M\bar{B}} = \left|-\sqrt{2}i\bar{\gamma} U^H_{11} + 2\sqrt{\frac{2}{3}} i\eta U^H_{31} + \sqrt{2}i\bar{\zeta} U^H_{51} - \sqrt{\frac{2}{3}}\bar{\zeta} U^H_{61}\right|^2 F_{12}\big(m^M, m^B, Q\big) \tag{123}$$

$$K_{B\bar{W}} = \left|-\bar{\gamma} U^H_{11} + \frac{2\eta}{\sqrt{3}} U^H_{31} - \bar{\zeta} U^H_{51} - \frac{i\bar{\zeta}}{\sqrt{3}} U^H_{61}\right|^2 F_{12}\big(m^B, m^W, Q\big) \tag{124}$$

$$K_{W\bar{Y}} = \left|-\gamma U^H_{11} + \frac{2\eta}{\sqrt{3}} U^H_{21} - \zeta U^H_{51} + \frac{i\zeta}{\sqrt{3}} U^H_{61}\right|^2 F_{12}\big(m^W, m^Y, Q\big) \tag{125}$$

$$K_{\bar{V}O} = \left|-\gamma U^H_{11} - 2\sqrt{3}\eta U^H_{21} - \zeta U^H_{51} - \sqrt{3}i\zeta U^H_{61}\right|^2 F_{12}\big(m^V, m^O, Q\big) \tag{126}$$

$$K_{Y\bar{N}} = \left|-\sqrt{2}i\gamma U^H_{11} + 2\sqrt{\frac{2}{3}} i\eta U^H_{21} + \sqrt{2}i\zeta U^H_{51} + \sqrt{\frac{2}{3}}\zeta U^H_{61}\right|^2 F_{12}\big(m^M, m^B, Q\big) \tag{127}$$

$$K_{VA} = \left|-\sqrt{2}i\gamma U^H_{11} - 2\sqrt{6}i\eta U^H_{21} + \sqrt{2}i\zeta U^H_{51} - \sqrt{6}\zeta U^H_{61}\right|^2 F_{12}\big(m^V, m^A, Q\big) \tag{128}$$

$$\begin{aligned} K_{\bar{O}\bar{H}} = \sum_{a=2}^{\mathrm{d}(H)} \big| &\bar{\gamma} U^H_{4a} U^H_{11} + 2\sqrt{3}\eta U^H_{4a} U^H_{31} + \big(2\sqrt{3}\eta U^H_{3a} + \bar{\zeta} U^H_{5a} - \sqrt{3}i\bar{\zeta} U^H_{6a} + \bar{\gamma} U^H_{1a}\big) U^H_{41} \\ &+ \bar{\zeta} U^H_{4a} U^H_{51} - \sqrt{3}i\bar{\zeta} U^H_{4a} U^H_{61}\big|^2 F_{12}\big(m^H_a, m^O, Q\big) \\ &+ \big|2\bar{\gamma} U^H_{41} U^H_{11} + 4\sqrt{3}\eta U^H_{41} U^H_{31} + 2\bar{\zeta}\big(U^H_{41} U^H_{51} - \sqrt{3}i U^H_{41} U^H_{61}\big)\big|^2 F_{11}\big(m^O, Q\big) \end{aligned} \tag{129}$$

$$K_{SH} = \sum_{a=2}^{\mathrm{d}(H)} \bigg| \big(i\gamma V^H_{3a} - i\bar{\gamma} V^H_{2a} + \kappa V^H_{6a}\big)\frac{U^H_{11}}{\sqrt{2}}$$

$$
\begin{aligned}
&+\left(2\sqrt{\frac{2}{3}}i\eta V_{3a}^H-\sqrt{\frac{2}{3}}\bar{\zeta}V_{6a}^H+\frac{i\bar{\zeta}}{\sqrt{2}}V_{5a}^H+\frac{i\bar{\gamma}}{\sqrt{2}}V_{1a}^H\right)U_{21}^H\\
&+\left(\frac{i\zeta}{\sqrt{2}}V_{5a}^H-\sqrt{\frac{2}{3}}\zeta V_{6a}^H-\frac{i\gamma}{\sqrt{2}}V_{1a}^H-2\sqrt{\frac{2}{3}}i\eta V_{2a}^H\right)U_{31}^H-\sqrt{6}i\lambda V_{4a}^H U_{41}^H\\
&+\left(\frac{\rho}{3}V_{6a}^H-i\zeta V_{3a}^H-i\bar{\zeta}V_{2a}^H\right)\frac{U_{51}^H}{\sqrt{2}}\\
&+\left(\sqrt{\frac{2}{3}}\bar{\zeta}V_{2a}^H-\frac{\kappa}{\sqrt{2}}V_{1a}^H+\sqrt{\frac{2}{3}}\zeta V_{3a}^H-\frac{\rho}{3\sqrt{2}}V_{5a}^H\right)U_{61}^H\bigg|^2 F_{12}\big(m_a^H,m^S,Q\big)\\
&+\bigg|\big(i\gamma V_{31}^H-i\bar{\gamma}V_{21}^H+\kappa V_{61}^H\big)\frac{U_{11}^H}{\sqrt{2}}\\
&+\left(2\sqrt{\frac{2}{3}}i\eta V_{31}^H-\sqrt{\frac{2}{3}}\bar{\zeta}V_{61}^H+\frac{i\bar{\zeta}}{\sqrt{2}}V_{51}^H+\frac{i\bar{\gamma}}{\sqrt{2}}V_{11}^H\right)U_{21}^H\\
&+\left(\frac{i\zeta}{\sqrt{2}}V_{51}^H-\sqrt{\frac{2}{3}}\zeta V_{61}^H-\frac{i\gamma}{\sqrt{2}}V_{11}^H-2\sqrt{\frac{2}{3}}i\eta V_{21}^H\right)U_{31}^H-\sqrt{6}i\lambda V_{41}^H U_{41}^H\\
&+\left(\frac{\rho}{3}V_{61}^H-i\zeta V_{31}^H-i\bar{\zeta}V_{21}^H\right)\frac{U_{51}^H}{\sqrt{2}}\\
&+\left(\sqrt{\frac{2}{3}}\bar{\zeta}V_{21}^H-\frac{\kappa}{\sqrt{2}}V_{11}^H+\sqrt{\frac{2}{3}}\zeta V_{31}^H-\frac{\rho}{3\sqrt{2}}V_{51}^H\right)U_{61}^H\bigg|^2 F_{11}\big(m^S,Q\big)
\end{aligned}
\tag{130}
$$

$$
\begin{aligned}
K_{F\bar{H}}=\sum_{a=1}^{d(F)}\sum_{a'=2}^{d(H)}\bigg|&\big(i\gamma V_{1a}^F U_{4a'}^H+\gamma U_{3a'}^H V_{2a}^F-\bar{\gamma}V_{2a}^F U_{2a'}^H+\kappa V_{4a}^F U_{4a'}^H+i\kappa V_{2a}^F U_{6a'}^H\big)U_{11}^H\\
&+\left(\frac{4\eta}{\sqrt{3}}V_{2a}^F U_{3a'}^H+2\sqrt{3}i\eta V_{1a}^F U_{4a'}^H-\sqrt{3}\bar{\zeta}V_{4a}^F U_{4a'}^H-\frac{2i\bar{\zeta}}{\sqrt{3}}V_{2a}^F U_{6a'}^H\right.\\
&\left.-\bar{\zeta}V_{2a}^F U_{5a'}^H+\bar{\gamma}V_{2a}^F U_{1a'}^H\right)U_{21}^H\\
&-\left(\frac{4}{\sqrt{3}}\eta V_{2a}^F U_{2a'}^H+\frac{2i\zeta}{\sqrt{3}}V_{2a}^F U_{6a'}^H+\zeta V_{2a}^F U_{5a'}^H+\sqrt{3}\zeta V_{4a}^F U_{4a'}^H+\gamma V_{2a}^F U_{1a'}^H\right)U_{31}^H\\
&+\left(i\zeta V_{1a}^F U_{5a'}^H-\sqrt{3}\zeta V_{1a}^F U_{6a'}^H-\frac{i\rho}{\sqrt{3}}V_{4a}^F U_{6a'}^H-2\sqrt{3}i\eta V_{1a}^F U_{2a'}^H-i\gamma V_{1a}^F U_{1a'}^H\right.\\
&\left.-\kappa V_{4a}^F U_{1a'}^H+\sqrt{3}\bar{\zeta}V_{4a}^F U_{2a'}^H+\sqrt{3}\zeta V_{4a}^F U_{3a'}^H\right)U_{41}^H\\
&+\left(\bar{\zeta}V_{2a}^F U_{2a'}^H-\frac{i\rho}{3}V_{2a}^F U_{6a'}^H-i\zeta V_{1a}^F U_{4a'}^H+\zeta V_{2a}^F U_{3a'}^H\right)U_{51}^H\\
&+\left(\sqrt{\frac{4}{3}}i\zeta V_{2a}^F U_{3a'}^H+\sqrt{\frac{4}{3}}i\bar{\zeta}V_{2a}^F U_{2a'}^H+\sqrt{3}\zeta V_{1a}^F U_{4a'}^H\right.\\
&\left.+\frac{i\rho}{\sqrt{3}}V_{4a}^F U_{4a'}^H+\frac{i\rho}{3}V_{2a}^F U_{5a'}^H-i\kappa V_{2a}^F U_{1a'}^H\right)U_{61}^H\bigg|^2 F_{12}\big(m_a^F,m_{a'}^H,Q\big)
\end{aligned}
$$

$$
\begin{aligned}
&-\sum_{a'=2}^{\mathrm{d(H)}} 2g_{10}^2\big|i\big(V_{1a'}^{H*}U_{11}^H+V_{2a'}^{H*}U_{21}^H\\
&+V_{3a'}^{H*}U_{31}^H+V_{5a'}^{H*}U_{51}^H+V_{6a'}^{H*}U_{61}^H\big)\big|^2F_{12}\big(m_{a'}^H,m_{\lambda_F},Q\big)\\
&-2g_{10}^2\big|i\big(V_{11}^{H*}U_{11}^H+V_{21}^{H*}U_{21}^H+V_{31}^{H*}U_{31}^H+V_{51}^{H*}U_{51}^H+V_{61}^{H*}U_{61}^H\big)\big|^2F_{11}(m_{\lambda_F},Q)
\end{aligned}
\tag{131}
$$

$$
\begin{aligned}
K_{GH}=&\sum_{a=1}^{\mathrm{d}(G)}\sum_{a'=2}^{\mathrm{d}(H)}\Bigg|\Bigg(\gamma V_{2a}^GV_{3a'}^H+\frac{\gamma}{\sqrt{2}}V_{3a}^GV_{3a'}^H\\
&-\sqrt{2}i\bar{\gamma}V_{5a}^GV_{4a'}^H+\bar{\gamma}V_{2a}^GV_{2a'}^H-\frac{\bar{\gamma}}{\sqrt{2}}V_{3a}^GV_{2a'}^H-\kappa V_{1a}^GV_{5a'}^H+\frac{i\kappa}{\sqrt{2}}V_{3a}^GV_{6a'}^H\Bigg)U_{11}^H\\
&+\Bigg(2\sqrt{\frac{2}{3}}\eta V_{3a}^GV_{3a'}^H+\frac{4\eta}{\sqrt{3}}V_{2a}^GV_{3a'}^H-\sqrt{\frac{2}{3}}i\bar{\zeta}V_{3a}^GV_{6a'}^H-\frac{\bar{\zeta}}{\sqrt{2}}V_{3a}^GV_{5a'}^H-\bar{\zeta}iV_{1a}^GV_{6a'}^H\\
&+\bar{\gamma}V_{2a}^GV_{1a'}^H+\frac{\bar{\gamma}}{\sqrt{2}}V_{3a}^GV_{1a'}^H\Bigg)U_{21}^H\\
&+\Bigg(\frac{4\eta}{\sqrt{3}}V_{2a}^GV_{2a'}^H-\frac{4\eta}{\sqrt{6}}V_{3a}^GV_{2a'}^H-2\sqrt{6}\eta iV_{5a}^GV_{4a'}^H-\sqrt{\frac{2}{3}}i\zeta V_{3a}^GV_{6a'}^H\\
&+i\zeta V_{1a}^GV_{6a'}^H-\frac{\zeta}{\sqrt{2}}V_{3a}^GV_{5a'}^H+\gamma V_{2a}^GV_{1a'}^H-\frac{\gamma}{\sqrt{2}}V_{3a}^GV_{1a'}^H\Bigg)U_{31}^H\\
&+\big(2\sqrt{6}i\eta V_{4a}^GV_{2a'}^H-\sqrt{2}i\zeta V_{4a}^GV_{5a'}^H\\
&+\sqrt{6}\zeta V_{4a}^GV_{6a'}^H+\sqrt{2}\gamma iV_{4a}^GV_{1a'}^H+2\sqrt{3}\lambda V_{2a}^GV_{4a'}^H-\sqrt{6}\lambda V_{3a}^GV_{4a'}^H\big)U_{41}^H\\
&+\Bigg(\frac{\zeta}{\sqrt{2}}V_{3a}^GV_{3a'}^H-\kappa V_{1a}^GV_{1a'}^H\\
&-\frac{i\rho}{3\sqrt{2}}V_{3a}^GV_{6a'}^H+\sqrt{2}i\bar{\zeta}V_{5a}^GV_{4a'}^H+\frac{\bar{\zeta}}{\sqrt{2}}V_{3a}^GV_{2a'}^H\Bigg)U_{51}^H\\
&+\Bigg(\sqrt{\frac{2}{3}}i\zeta V_{3a}^GV_{3a'}^H+i\zeta V_{1a}^GV_{3a'}^H+\sqrt{\frac{2}{3}}i\bar{\zeta}V_{3a}^GV_{2a'}^H\\
&-i\bar{\zeta}V_{1a}^GV_{2a'}^H+\frac{2\rho}{3\sqrt{3}}V_{6a'}^HV_{2a}^G+\frac{i\rho}{3\sqrt{2}}V_{5a'}^HV_{3a}^G\\
&-\frac{i\kappa}{\sqrt{2}}V_{1a'}^HV_{3a}^G+\sqrt{6}\bar{\zeta}V_{4a'}^HV_{5a}^G\Bigg)U_{61}^H\Bigg|^2F_{12}\big(m_a^G,m_{a'}^H,Q\big)\\
&-\sum_{a'=2}^{\mathrm{d}(H)}2g_{10}^2\Bigg|\frac{-i}{\sqrt{5}}\big(U_{1a'}^{H*}U_{11}^H+U_{2a'}^{H*}U_{21}^H+U_{3a'}^{H*}U_{31}^H-4U_{4a'}^{H*}U_{41}^H+U_{5a'}^{H*}U_{11}^H\\
&+U_{6a'}^{H*}U_{61}^H\big)\Bigg|^2F_{12}\big(m_{a'}^H,m_{\lambda_G},Q\big)\\
&+\sum_{a=1}^{\mathrm{d}(G)}\Bigg|\Bigg(\gamma V_{2a}^GV_{31}^H+\frac{\gamma}{\sqrt{2}}V_{3a}^GV_{31}^H-\sqrt{2}i\bar{\gamma}V_{5a}^GV_{41}^H
\end{aligned}
$$

$$
\begin{aligned}
&+\bar{\gamma} V_{2a}^G V_{21}^H - \frac{\bar{\gamma}}{\sqrt{2}} V_{3a}^G V_{21}^H - \kappa V_{1a}^G V_{51}^H + \frac{i\kappa}{\sqrt{2}} V_{3a}^G V_{61}^H\bigg) U_{11}^H \\
&+\bigg(2\sqrt{\frac{2}{3}}\eta V_{3a}^G V_{31}^H + \frac{4\eta}{\sqrt{3}} V_{2a}^G V_{31}^H \\
&-\sqrt{\frac{2}{3}} i\bar{\zeta} V_{3a}^G V_{61}^H - \frac{\bar{\zeta}}{\sqrt{2}} V_{3a}^G V_{51}^H - i\bar{\zeta} V_{1a}^G V_{61}^H \\
&+\bar{\gamma} V_{2a}^G V_{11}^H + \frac{\bar{\gamma}}{\sqrt{2}} V_{3a}^G V_{11}^H\bigg) U_{21}^H \\
&+\bigg(i\zeta V_{1a}^G V_{61}^H - \sqrt{\frac{2}{3}} i\zeta V_{3a}^G V_{61}^H - \frac{\zeta}{\sqrt{2}} V_{3a}^G V_{51}^H - 2\sqrt{6} i\eta V_{5a}^G V_{41}^H \\
&+\gamma V_{2a}^G V_{11}^H - \frac{\gamma}{\sqrt{2}} V_{3a}^G V_{11}^H + \frac{4\eta}{\sqrt{3}} V_{2a}^G V_{21}^H - \frac{4\eta}{\sqrt{6}} V_{3a}^G V_{21}^H\bigg) U_{31}^H \\
&+\big(2\sqrt{6} i\eta V_{4a}^G V_{21}^H - \sqrt{2} i\zeta V_{4a}^G V_{51}^H \\
&+\sqrt{6}\zeta V_{4a}^G V_{61}^H + \sqrt{2} i\gamma V_{4a}^G V_{11}^H + 2\sqrt{3}\lambda V_{2a}^G V_{41}^H - \sqrt{6}\lambda V_{3a}^G V_{41}^H\big) U_{41}^H \\
&+\bigg(-\frac{i\rho}{3\sqrt{2}} V_{3a}^G V_{61}^H - \kappa V_{1a}^G V_{1a'}^H + \sqrt{2} i\bar{\zeta} V_{5a}^G V_{41}^H + \frac{\zeta}{\sqrt{2}} V_{3a}^G V_{31}^H + \frac{\bar{\zeta}}{\sqrt{2}} V_{3a}^G V_{21}^H\bigg) U_{51}^H \\
&+\bigg(\sqrt{\frac{2}{3}} i\zeta V_{3a}^G V_{31}^H + i\zeta V_{1a}^G V_{31}^H + \sqrt{\frac{2}{3}} i\bar{\zeta} V_{3a}^G V_{21}^H \\
&- i\bar{\zeta} V_{1a}^G V_{21}^H + \frac{2\rho}{3\sqrt{3}} V_{2a}^G V_{61}^H + \frac{i\rho}{3\sqrt{2}} V_{3a}^G V_{51}^H \\
&-\frac{i\kappa}{\sqrt{2}} V_{3a}^G V_{11}^H + \sqrt{6}\bar{\zeta} V_{5a}^G V_{41}^H\bigg) U_{61}^H\bigg|^2 F_{11}\big(m_a^G, Q\big) \\
&-2g_{10}^2\bigg|\frac{-i}{\sqrt{5}}\big(U_{11}^{H*} U_{11}^H + U_{21}^{H*} U_{21}^H + U_{31}^{H*} U_{31}^H - 4U_{41}^{H*} U_{41}^H + U_{51}^{H*} U_{51}^H \\
&+U_{61}^{H*} U_{61}^H\big)\bigg|^2 F_{11}(m_{\lambda_G}, Q)
\end{aligned}
\tag{132}
$$

Appendix B

Table 2
Solution 1: Values of the NMSGUT–SUGRY–NUHM parameters at M_X derived from an accurate fit to all 18 fermion data and compatible with RG constraints. Unification parameters and spectra of superheavy and superlight fields are also given.

Parameter	Value	Field [SU(3), SU(2), Y]	Masses (units of 10^{16} GeV)
χ_X	0.3988	$A[1,1,4]$	1.68
χ_Z	0.1168	$B[6,2,5/3]$	0.0718
$h_{11}/10^{-6}$	3.4611	$C[8,2,1]$	0.94, 2.41, 5.15
$h_{22}/10^{-4}$	3.0937	$D[3,2,7/3]$	0.08, 3.39, 6.02
h_{33}	0.0230	$E[3,2,1/3]$	0.09, 0.71, 1.85
$f_{11}/10^{-6}$	$0.0038 + 0.2167i$		1.854, 2.65, 5.33
$f_{12}/10^{-6}$	$-1.0760 - 2.0474i$	$F[1,1,2]$	0.29, 0.57
$f_{13}/10^{-5}$	$0.0632 + 0.1223i$		0.57, 3.33
$f_{22}/10^{-5}$	$5.0702 + 3.6293i$	$G[1,1,0]$	0.015, 0.14, 0.50
$f_{23}/10^{-4}$	$-0.3765 + 1.7999i$		0.498, 0.65, 0.68
$f_{33}/10^{-3}$	$-0.9059 + 0.2815i$	$h[1,2,1]$	0.291, 2.32, 3.41
$g_{12}/10^{-4}$	$0.1310 + 0.1177i$		4.89, 23.26
$g_{13}/10^{-5}$	$-8.5199 + 6.9958i$	$I[3,1,10/3]$	0.23
$g_{23}/10^{-4}$	$-3.1937 - 1.2230i$	$J[3,1,4/3]$	0.201, 0.65, 1.21
$\lambda/10^{-2}$	$-3.8826 + 1.0500i$		1.21, 3.83
η	$-0.3134 + 0.1210i$	$K[3,1,8/3]$	1.86, 3.84
ρ	$0.6305 - 0.5268i$	$L[6,1,2/3]$	1.93, 2.56
k	$0.1926 + 0.2311i$	$M[6,1,8/3]$	2.17
ζ	$0.9082 + 0.8524i$	$N[6,1,4/3]$	2.04
$\bar{\zeta}$	$0.2737 + 0.6140i$	$O[1,3,2]$	2.77
$m/10^{16}$ GeV	0.0086	$P[3,3,2/3]$	0.64, 3.56
$m_\Theta/10^{16}$ GeV	$-2.375e^{-i\,\mathrm{Arg}(\lambda)}$	$Q[8,3,0]$	0.181
γ	0.3234	$R[8,1,0]$	0.08, 0.24
$\bar{\gamma}$	-3.6166	$S[1,3,0]$	0.2828
x	$0.78 + 0.58i$	$t[3,1,2/3]$	0.16, 0.45, 0.90, 2.52
$\Delta_X^{tot}, \Delta_X^{GUT}$	0.67, 0.74		4.08, 4.37, 25.68
$\Delta_G^{tot}, \Delta_G^{GUT}$	$-20.46, -23.49$	$U[3,3,4/3]$	0.238
$\Delta\alpha_3^{tot}(M_Z), \Delta\alpha_3^{GUT}(M_Z)$	-0.0126, 0.0020	$V[1,2,3]$	0.187
$\{M^{\nu^c}/10^{12}$ GeV$\}$	0.000648, 0.99, 37.28	$W[6,3,2/3]$	1.95
$\{M_{II}^{\nu}/10^{-10}$ eV$\}$	2.41, 3700.98, 138823.42	$X[3,2,5/3]$	0.063, 2.068, 2.068
M_ν (meV)	1.169109, 7.32, 41.46	$Y[6,2,1/3]$	0.08
$\{\mathrm{Evals}[f]\}/10^{-6}$	0.017143, 26.28, 985.21	$Z[8,1,2]$	0.24
Soft parameters	$m_{\frac{1}{2}} = -152.899$	$m_0 = 11400.993$	$A_0 = -2.0029 \times 10^5$
at M_X	$\mu = 1.5966 \times 10^5$	$B = -1.7371 \times 10^{10}$	$\tan\beta = 51.0000$
	$M_{\bar{H}}^2 = -2.0655 \times 10^{10}$	$M_H^2 = -1.7978 \times 10^{10}$	$R_{\frac{b\tau}{s\mu}} = 0.1998$
$Max(\lvert L_{ABCD}\rvert, \lvert R_{ABCD}\rvert)$	8.1104×10^{-22} GeV^{-1}		
Susy contribution to $\Delta_{X,G,3}$	$\Delta_X^{Susy} = -0.070$	$\Delta_G^{Susy} = 3.04$	$\Delta\alpha_3^{Susy} = -0.015$

Table 3
Solution 1: Fit with $\chi_X = \sqrt{\sum_{i=1}^{17}(O_i - \bar{O}_i)^2/\delta_i^2} = 0.3988$. Target values, at M_X of the fermion Yukawa couplings and mixing parameters, together with the estimated uncertainties, achieved values and pulls. Eigenvalues of the wave function renormalization for fermion and Higgs lines are given with Higgs fractions $\alpha_i, \bar{\alpha}_i$ which control the MSSM fermion Yukawa couplings.

Parameter	Target $= \bar{O}_i$	Uncert. $= \delta_i$	Achieved $= O_i$	Pull $= (O_i - \bar{O}_i)/\delta_i$
$y_u/10^{-6}$	2.062837	0.788004	2.066323	0.004424
$y_c/10^{-3}$	1.005548	0.165915	1.010599	0.030440
y_t	0.369885	0.014795	0.369792	−0.006256
$y_d/10^{-5}$	11.438266	6.668509	12.421488	0.147443
$y_s/10^{-3}$	2.169195	1.023860	2.189195	0.019534
y_b	0.456797	0.237078	0.527664	0.298917
$y_e/10^{-4}$	1.240696	0.186104	1.224753	−0.085665
$y_\mu/10^{-2}$	2.589364	0.388405	2.603313	0.035911
y_τ	0.543441	0.103254	0.532427	−0.106669
$\sin\theta_{12}^q$	0.2210	0.001600	0.2210	−0.0003
$\sin\theta_{13}^q/10^{-4}$	29.1907	5.000000	29.0755	−0.0230
$\sin\theta_{23}^q/10^{-3}$	34.3461	1.300000	34.3574	0.0087
δ^q	60.0212	14.000000	59.7774	−0.0174
$(m_{12}^2)/10^{-5}(\text{eV})^2$	5.2115	0.552419	5.2189	0.0133
$(m_{23}^2)/10^{-3}(\text{eV})^2$	1.6647	0.332930	1.6650	0.0011
$\sin^2\theta_{12}^L$	0.2935	0.058706	0.2926	−0.0152
$\sin^2\theta_{23}^L$	0.4594	0.137809	0.4412	−0.1317
$\sin^2\theta_{13}^L$	0.0250	0.019000	0.0267	0.0892
$(Z_{\bar{u}})$	0.957467	0.957908	0.957908	
$(Z_{\bar{d}})$	0.950892	0.951332	0.951333	
$(Z_{\bar{\nu}})$	0.925116	0.925579	0.925580	
$(Z_{\bar{e}})$	0.944853	0.945306	0.945308	
(Z_Q)	0.968740	0.969189	0.969190	
(Z_L)	0.949564	0.950011	0.950013	
$Z_{\bar{H}}, Z_H$	0.000273	0.001151		
α_1	$0.1609 - 0.0000i$	$\bar{\alpha}_1$	$0.1188 - 0.0000i$	
α_2	$-0.3140 - 0.6026i$	$\bar{\alpha}_2$	$-0.4802 - 0.2961i$	
α_3	$-0.0477 - 0.4786i$	$\bar{\alpha}_3$	$-0.4842 - 0.2469i$	
α_4	$0.3903 - 0.1942i$	$\bar{\alpha}_4$	$0.5795 + 0.0171i$	
α_5	$-0.0449 + 0.0061i$	$\bar{\alpha}_5$	$-0.0415 - 0.1241i$	
α_6	$-0.0071 - 0.2982i$	$\bar{\alpha}_6$	$0.0274 - 0.1349i$	

Table 4
Solution 1: Standard Model fermion masses in GeV at M_Z compared with masses obtained from GUT derived Yukawa couplings run down from M_X^0 to M_Z, before and after threshold corrections. Fit with $\chi_Z = \sqrt{\sum_{i=1}^{9}(m_i^{MSSM} - m_i^{SM})^2/(m_i^{MSSM})^2} = 0.1153$.

Parameter	$SM(M_Z)$	$m^{GUT}(M_Z)$	$m^{MSSM} = (m + \Delta m)^{GUT}(M_Z)$
$m_d/10^{-3}$	2.90000	1.08183	3.01515
$m_s/10^{-3}$	55.00000	19.06631	53.14737
m_b	2.90000	3.17508	3.05602
$m_e/10^{-3}$	0.48657	0.45157	0.45925
m_μ	0.10272	0.09594	0.09902
m_τ	1.74624	1.65725	1.65734
$m_u/10^{-3}$	1.27000	1.10509	1.27687
m_c	0.61900	0.54048	0.62449
m_t	172.50000	145.99987	170.88573

Table 5
Solution 1: Values (in GeV) of soft Susy parameters at M_Z (evolved from the soft SUGRY–NUHM parameters at M_X) determine Susy threshold corrections to fermion Yukawas. Matching of run down fermion Yukawas in the MSSM to the SM parameters determines soft SUGRY parameters at M_X. Note the heavier third sgeneration. $\mu(M_Z)$ and $B(M_Z) = m_A^2 \sin 2\beta/2$ are determined by electroweak symmetry breaking conditions. m_A is the mass of the CP odd scalar in the doublet Higgs. The sign of μ is assumed positive.

Parameter	Value	Parameter	Value
M_1	210.10	$M_{\tilde{\bar{u}}_1}$	14446.81
M_2	569.81	$M_{\tilde{\bar{u}}_2}$	14445.85
M_3	1000.14	$M_{\tilde{\bar{u}}_3}$	24609.79
$M_{\tilde{\bar{l}}_1}$	1761.31	$A_{11}^{0(l)}$	−121907.75
$M_{\tilde{\bar{l}}_2}$	210.71	$A_{22}^{0(l)}$	−121757.58
$M_{\tilde{\bar{l}}_3}$	20777.09	$A_{33}^{0(l)}$	−77289.04
$M_{\tilde{L}_1}$	15308.21	$A_{11}^{0(u)}$	−148456.63
$M_{\tilde{L}_2}$	15258.47	$A_{22}^{0(u)}$	−148455.19
$M_{\tilde{L}_3}$	21320.16	$A_{33}^{0(u)}$	−76985.25
$M_{\tilde{\bar{d}}_1}$	8402.95	$A_{11}^{0(d)}$	−122521.00
$M_{\tilde{\bar{d}}_2}$	8401.45	$A_{22}^{0(d)}$	−122518.53
$M_{\tilde{\bar{d}}_3}$	51842.14	$A_{33}^{0(d)}$	−44046.92
$M_{\tilde{Q}_1}$	11271.93	$\tan\beta$	51.00
$M_{\tilde{Q}_2}$	11270.77	$\mu(M_Z)$	125591.16
$M_{\tilde{Q}_3}$	40274.01	$B(M_Z)$	2.7861×10^9
$M_{\bar{H}}^2$	-1.6336×10^{10}	M_H^2	-1.7391×10^{10}

Table 6
Solution 1: Spectra of supersymmetric partners ignoring generation mixing. Due to large values of μ, B, A_0 the LSP and light chargino are essentially pure bino and wino ($\tilde{W}_\pm$). The light gauginos and light Higgs h^0, are accompanied by a light smuon and sometimes selectron. The rest of the sfermions have multi-TeV masses. The decoupled and mini-split supersymmetry spectrum and large μ, A_0 parameters help avoid problems with FCNC and CCB/UFB instability [48]. Sfermion masses are ordered by generation not magnitude.

Field	Mass (GeV)
$M_{\tilde{G}}$	1000.14
$M_{\chi^\pm}$	569.81, 125591.22
M_{χ^0}	210.10, 569.81, 125591.20, 125591.20
$M_{\tilde{\nu}}$	15308.069, 15258.322, 21320.059
$M_{\tilde{e}}$	1761.89, 15308.29, 211.57, 15258.60, 20674.72, 21419.56
$M_{\tilde{u}}$	11271.80, 14446.76, 11270.63, 14445.80, 24607.51, 40275.87
$M_{\tilde{d}}$	8402.99, 11272.10, 8401.48, 11270.95, 40269.19, 51845.93
M_A	377025.29
$M_{H^\pm}$	377025.30
M_{H^0}	377025.28
M_{h^0}	124.00

Table 7
Solution 1: Spectra of supersymmetric partners calculated including generation mixing effects. Inclusion of such effects changes the spectra only marginally. Due to the large values of μ, B, A_0 the LSP and light chargino are essentially pure bino and wino ($\tilde{W}_\pm$). Note that the ordering of the eigenvalues in this table follows their magnitudes, comparison with the previous table is necessary to identify the sfermions.

Field	Mass (GeV)
$M_{\tilde{G}}$	1000.72
$M_{\chi^\pm}$	570.11, 125537.00
M_{χ^0}	210.22, 570.11, 125536.98, 125536.98
$M_{\tilde{\nu}}$	15257.98, 15307.71, 21350.169
$M_{\tilde{e}}$	242.61, 1765.59, 15258.25, 15307.93, 20733.03, 21453.81
$M_{\tilde{u}}$	11258.18, 11270.54, 14444.57, 14445.53, 24609.90, 40301.29
$M_{\tilde{d}}$	8400.19, 8401.71, 11258.52, 11270.84, 40294.63, 51879.28
M_A	377430.83
$M_{H^\pm}$	377430.84
M_{H^0}	377430.82
M_{h^0}	124.13

Table 8
Solution 2: See Table 2 caption.

Parameter	Value	Field [SU(3), SU(2), Y]	Masses (units of 10^{16} GeV)
χ_X	0.1326	$A[1,1,4]$	1.36
χ_Z	0.0558	$B[6,2,5/3]$	0.0966
$h_{11}/10^{-6}$	3.9601	$C[8,2,1]$	0.93, 5.17, 7.45
$h_{22}/10^{-4}$	3.6120	$D[3,2,7/3]$	0.29, 6.07, 9.09
h_{33}	0.0176	$E[3,2,1/3]$	0.11, 0.75, 2.60
$f_{11}/10^{-6}$	$-0.0130+0.1591i$		2.600, 4.85, 8.23
$f_{12}/10^{-6}$	$-1.0217-1.8123i$	$F[1,1,2]$	0.19, 0.65
$f_{13}/10^{-5}$	$0.0723+0.3387i$		0.65, 4.30
$f_{22}/10^{-5}$	$6.5536+4.3762i$	$G[1,1,0]$	0.025, 0.20, 0.76
$f_{23}/10^{-4}$	$-0.7338+2.3513i$		0.773, 0.77, 0.85
$f_{33}/10^{-3}$	$-1.2731+0.5157i$	$h[1,2,1]$	0.335, 2.67, 5.57
$g_{12}/10^{-4}$	$0.1284+0.1895i$		7.65, 17.54
$g_{13}/10^{-5}$	$-9.5431+2.8232i$	$I[3,1,10/3]$	0.36
$g_{23}/10^{-4}$	$-1.6403-0.6279i$	$J[3,1,4/3]$	0.297, 0.39, 1.44
$\lambda/10^{-2}$	$-4.6906-0.1490i$		1.44, 5.01
η	$-0.2495+0.0683i$	$K[3,1,8/3]$	1.73, 5.14
ρ	$1.1753-0.2967i$	$L[6,1,2/3]$	1.79, 2.60
k	$-0.0175+0.0581i$	$M[6,1,8/3]$	1.95
ζ	$1.2956+0.9514i$	$N[6,1,4/3]$	1.88
$\bar{\zeta}$	$0.2238+0.5885i$	$O[1,3,2]$	3.14
$m/10^{16}$ GeV	0.0104	$P[3,3,2/3]$	0.49, 4.65
$m_\Theta/10^{16}$ GeV	$-2.553e^{-iArg(\lambda)}$	$Q[8,3,0]$	0.309
γ	0.3925	$R[8,1,0]$	0.10, 0.38
$\bar{\gamma}$	-2.4482	$S[1,3,0]$	0.4403
x	$0.85+0.51i$	$t[3,1,2/3]$	0.38, 1.16, 1.67, 3.05
$\Delta_X^{tot}, \Delta_X^{GUT}$	0.80, 0.86		5.18, 5.70, 20.56
$\Delta_G^{tot}, \Delta_G^{GUT}$	$-20.52, -23.43$	$U[3,3,4/3]$	0.382
$\Delta\alpha_3^{tot}(M_Z), \Delta\alpha_3^{GUT}(M_Z)$	$-0.0123, -0.0021$	$V[1,2,3]$	0.261
$\{M^{\nu^c}/10^{12}$ GeV$\}$	0.000244, 2.33, 81.40	$W[6,3,2/3]$	2.50
$\{M_{II}^{\nu}/10^{-10}$ eV$\}$	0.45, 4292.75, 149682.98	$X[3,2,5/3]$	0.088, 2.832, 2.832
M_ν (meV)	1.170731, 7.11, 40.21	$Y[6,2,1/3]$	0.11
$\{\mathrm{Evals}[f]\}/10^{-6}$	0.004259, 40.69, 1418.71	$Z[8,1,2]$	0.38
Soft parameters	$m_{\frac{1}{2}}=0.000$	$m_0=12860.405$	$A_0=-1.9844\times10^5$
at M_X	$\mu=1.7240\times10^5$	$B=-1.4927\times10^{10}$	$\tan\beta=50.0000$
	$M_{\bar{H}}^2=-2.9608\times10^{10}$	$M_H^2=-2.8920\times10^{10}$	$R_{\frac{b\tau}{s\mu}}=5.6405$
$Max(\lvert L_{ABCD}\rvert, \lvert R_{ABCD}\rvert)$	7.7373×10^{-22} GeV^{-1}		
Susy contribution to $\Delta_{X,G,3}$	$\Delta_X^{\mathrm{Susy}}=-0.053$	$\Delta_G^{\mathrm{Susy}}=2.91$	$\Delta\alpha_3^{\mathrm{Susy}}=-0.010$

Table 9
Solution 2: Fit with $\chi_X = 0.1326$. See Table 3 caption.

Parameter	Target $= \bar{O}_i$	Uncert. $= \delta_i$	Achieved $= O_i$	Pull $= (O_i - \bar{O}_i)/\delta_i$
$y_u/10^{-6}$	2.035847	0.777694	2.035834	−0.000017
$y_c/10^{-3}$	0.992361	0.163740	0.994253	0.011560
y_t	0.350010	0.014000	0.350076	0.004715
$y_d/10^{-5}$	10.674802	6.223410	10.374090	−0.048320
$y_s/10^{-3}$	2.024872	0.955740	2.118158	0.097606
y_b	0.340427	0.176682	0.349778	0.052924
$y_e/10^{-4}$	1.121867	0.168280	1.122417	0.003267
$y_\mu/10^{-2}$	2.369435	0.355415	2.364688	−0.013356
y_τ	0.474000	0.090060	0.471211	−0.030967
$\sin\theta^q_{12}$	0.2210	0.001600	0.2210	0.0009
$\sin\theta^q_{13}/10^{-4}$	30.0759	5.000000	30.0765	0.0001
$\sin\theta^q_{23}/10^{-3}$	35.3864	1.300000	35.3924	0.0046
δ^q	60.0215	14.000000	60.0469	0.0018
$(m^2_{12})/10^{-5}\ (\text{eV})^2$	4.9239	0.521931	4.9233	−0.0012
$(m^2_{23})/10^{-3}\ (\text{eV})^2$	1.5660	0.313209	1.5664	0.0011
$\sin^2\theta^L_{12}$	0.2944	0.058878	0.2931	−0.0217
$\sin^2\theta^L_{23}$	0.4652	0.139567	0.4622	−0.0220
$\sin^2\theta^L_{13}$	0.0255	0.019000	0.0260	0.0252
$(Z_{\bar{u}})$	0.972582	0.972763	0.972764	
$(Z_{\bar{d}})$	0.967473	0.967657	0.967659	
$(Z_{\bar{\nu}})$	0.946651	0.946835	0.946838	
$(Z_{\bar{e}})$	0.961973	0.962151	0.962154	
(Z_Q)	0.983138	0.983334	0.983336	
(Z_L)	0.967422	0.967617	0.967619	
$Z_{\bar{H}}, Z_H$	0.000480	0.001284		
α_1	$0.2016 + 0.0000i$	$\bar{\alpha}_1$	$0.1336 - 0.0000i$	
α_2	$-0.4805 - 0.6320i$	$\bar{\alpha}_2$	$-0.5177 - 0.2850i$	
α_3	$0.0105 - 0.3558i$	$\bar{\alpha}_3$	$-0.3597 - 0.2864i$	
α_4	$0.3622 - 0.1474i$	$\bar{\alpha}_4$	$0.4974 + 0.3280i$	
α_5	$-0.0159 - 0.0451i$	$\bar{\alpha}_5$	$0.0535 - 0.2288i$	
α_6	$-0.0007 - 0.2171i$	$\bar{\alpha}_6$	$0.0189 - 0.1050i$	

Table 10
Solution 2: See caption Table 4. Fit with $\chi_Z = 0.0557$.

Parameter	$\text{SM}(M_Z)$	$m^{GUT}(M_Z)$	$m^{MSSM} = (m + \Delta m)^{GUT}(M_Z)$
$m_d/10^{-3}$	2.90000	1.05215	2.80332
$m_s/10^{-3}$	55.00000	21.48237	57.23281
m_b	2.90000	2.77488	2.94586
$m_e/10^{-3}$	0.48657	0.48189	0.48468
m_μ	0.10272	0.10148	0.10207
m_τ	1.74624	1.73337	1.73251
$m_u/10^{-3}$	1.27000	1.09833	1.27302
m_c	0.61900	0.53640	0.62171
m_t	172.50000	146.22372	172.58158

Table 11
Solution 2: See caption Table 5.

Parameter	Value	Parameter	Value
M_1	246.41	$M_{\tilde{\bar{u}}_1}$	12822.53
M_2	590.18	$M_{\tilde{\bar{u}}_2}$	12822.49
M_3	1200.01	$M_{\tilde{\bar{u}}_3}$	48248.96
$M_{\tilde{\bar{l}}_1}$	11957.95	$A_{11}^{0(l)}$	−137311.14
$M_{\tilde{\bar{l}}_2}$	11961.97	$A_{22}^{0(l)}$	−137158.88
$M_{\tilde{\bar{l}}_3}$	38556.41	$A_{33}^{0(l)}$	−93057.53
$M_{\tilde{L}_1}$	15324.76	$A_{11}^{0(u)}$	−147185.28
$M_{\tilde{L}_2}$	15326.33	$A_{22}^{0(u)}$	−147183.69
$M_{\tilde{L}_3}$	30130.38	$A_{33}^{0(u)}$	−81454.63
$M_{\tilde{\bar{d}}_1}$	11245.04	$A_{11}^{0(d)}$	−138168.65
$M_{\tilde{\bar{d}}_2}$	11246.12	$A_{22}^{0(d)}$	−138165.70
$M_{\tilde{\bar{d}}_3}$	49308.99	$A_{33}^{0(d)}$	−76263.17
$M_{\tilde{Q}_1}$	13440.51	$\tan\beta$	50.00
$M_{\tilde{Q}_2}$	13440.94	$\mu(M_Z)$	155715.41
$M_{\tilde{Q}_3}$	48976.61	$B(M_Z)$	4.1869×10^9
$M_{\bar{H}}^2$	-2.5331×10^{10}	M_H^2	-2.5545×10^{10}

Table 12
Solution 2: See caption Table 6.

Field	Mass (GeV)
$M_{\tilde{G}}$	1200.01
$M_{\chi^\pm}$	590.18, 155715.46
M_{χ^0}	246.41, 590.18, 155715.44, 155715.44
$M_{\tilde{\nu}}$	15324.618, 15326.183, 30130.304
$M_{\tilde{e}}$	11958.03, 15324.84, 11961.76, 15326.63, 30125.09, 38560.60
$M_{\tilde{u}}$	12822.48, 13440.40, 12822.42, 13440.85, 48227.49, 48998.14
$M_{\tilde{d}}$	11245.07, 13440.65, 11246.13, 13441.10, 48865.41, 49419.24
M_A	457636.54
$M_{H^\pm}$	457636.55
M_{H^0}	457636.54
M_{h^0}	125.00

Table 13
Solution 2: See caption Table 7.

Field	Mass (GeV)
$M_{\tilde{G}}$	1200.22
$M_{\chi^{\pm}}$	590.28, 155704.39
M_{χ^0}	246.44, 590.28, 155704.37, 155704.37
$M_{\tilde{\nu}}$	15324.61, 15326.19, 30133.937
$M_{\tilde{e}}$	11958.04, 11961.80, 15324.83, 15326.64, 30128.73, 38566.31
$M_{\tilde{u}}$	12822.35, 12822.41, 13440.35, 13459.00, 48229.64, 48995.86
$M_{\tilde{d}}$	11245.00, 11246.06, 13440.60, 13459.25, 48864.06, 49420.94
M_A	457783.97
$M_{H^{\pm}}$	457783.98
M_{H^0}	457783.97
M_{h^0}	125.02

Table 14
$d = 5$ operator mediated proton lifetimes τ_p (yrs), decay rates Γ (yr^{-1}) and Branching ratios in the dominant meson$^+ + \nu$ channels.

Solution	$\tau_p(M^+\nu)$	$\Gamma(p \to \pi^+\nu)$	$\mathrm{BR}(p \to \pi^+\nu_{e,\mu,\tau})$	$\Gamma(p \to K^+\nu)$	$\mathrm{BR}(p \to K^+\nu_{e,\mu,\tau})$
1	9.63×10^{34}	4.32×10^{-37}	$\{1.3 \times 10^{-3}, 0.34, 0.66\}$	9.95×10^{-36}	$\{4.6 \times 10^{-4}, 0.15, 0.85\}$
2	3.52×10^{34}	2.14×10^{-36}	$\{1.7 \times 10^{-3}, 0.18, 0.81\}$	2.62×10^{-35}	$\{1.8 \times 10^{-3}, 0.19, 0.81\}$

Table 15
Unoptimized values for $\mathrm{BR}(b \to s\gamma)$, Δa_μ, $\Delta\rho$, $\epsilon^{CP}_{Leptogenesis}$, δ^{CP}_{PMNS}.

Solution	$\mathrm{BR}(b \to s\gamma)$	Δa_μ	$\Delta\rho$	$\epsilon/10^{-7}$	δ_{PMNS}
1	3.294×10^{-4}	1.06×10^{-9}	6.03×10^{-7}	0.12	6.21°
2	3.289×10^{-4}	1.74×10^{-12}	1.92×10^{-7}	0.01	6.27°

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