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## Numerical Analysis of Sub-Critical Crack Growth in Particulate Ceramic Composites

Z. Majer<sup>a,b,\*</sup>, M. Pletz<sup>c</sup>, C. Krautgasser<sup>d</sup>, L. Náhlík<sup>a,e</sup>, P. Hutař<sup>e</sup>, R. Bermejo<sup>c</sup><sup>a</sup>Brno University of Technology, Faculty of Mechanical Engineering, Technická 2, 616 69 Brno, Czech Republic<sup>b</sup>Institute of Physics of Materials, Žitkova 22, 616 62 Brno, Czech Republic<sup>c</sup>Institut für Struktur- und Funktionskeramik (ISFK), Montanuniversität Leoben, Peter Tunner Strasse 5, A-8700 Leoben, Austria<sup>d</sup>Materials Center Leoben Forschung GmbH, Roseggerstrasse 12, A-8700, Leoben, Austria<sup>e</sup>CEITEC IPM, Institute of Physics of Materials, Žitkova 22, 616 62 Brno, Czech Republic

### Abstract

The strength of glass or ceramic containing materials can be affected by the environment (“stress corrosion”). Under applied stress, crack-like defects may grow (sub-critically) for stress intensity factors,  $K_I$ , below the fracture toughness of the material,  $K_{Ic}$ . The aim of the present work was to develop a two-dimensional finite element model to analyze the subcritical crack growth behavior of ceramic-based particulate composites. The maximum tangential stress criterion (MTS) was used to predict the direction of the crack propagation, in the framework of linear elastic fracture mechanics. The modeled material was a Low Temperature Co-fired Ceramic (LTCC), containing alumina particles embedded in a glass matrix. The experimentally determined SCCG material behavior (i.e.  $v$ - $K_I$  data) was implemented in the numerical model. The effect of the elastic modulus of the particles on the subcritical crack propagation was investigated. The conclusions of this paper can contribute to a better understanding of the subcritical propagation of cracks in particulate composites.

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**Keywords:** Fracture Mechanics; Low Temperature Co-fired Ceramics; Micro-crack Propagation; Finite Element Method; Lifetime Estimation

\* Corresponding author. Tel.: +420-541-142-857; fax: +420-541-142-876.  
E-mail address: majer@fme.vutbr.cz (Z. Majer).

**Nomenclature**

$a$	crack length
$a_i$	initial crack length
$b$	location of particles
$c$	location of crack
$d$	distance between particles
$da$	crack length increment
$dt$	time increment
$n$	subcritical crack growth exponent
$t_i$	time step
$v_0$	critical crack growth velocity
$D$	particle size
$E$	Young's modulus
$K_{I,II}$	stress intensity factors for mode I and II, respectively
$K_{Ic}$	fracture toughness
$\nu$	Poisson's ratio
$\sigma_{appl}$	applied stress
$\dot{v}$	crack growth rate
$\Omega_s$	micro-crack propagation direction
LTTC	Low temperature Co-fired Ceramics
RH	relative humidity
SCCG	subcritical crack growth

**1. Introduction**

Composite materials are nowadays wide spread and can be found practically in every engineering application. The main advantage of composites is that mechanical properties can be tailored for a specific application or use. One of the most important groups is particulate composites, where (hard) particles are homogeneously distributed in a (soft) matrix. Particles embedded in the matrix are often called “fillers” which, in general, aim to reinforce or change the material properties of the matrix. The mechanical behavior of the composite is associated with particles size, matrix properties, volume filler fraction, etc.). Extensive literature can be found to study the effect of the particles on the overall behavior of the composite, see for instance (Demjen et al., 1998, Park et al., 2004, Ahmad et al., 2008). Numerical models have been developed to study the effect of particle shape (Majer et al., 2011), interphases (Majer et al., 2013), as well as loading conditions (Majer, 2013) on the crack propagation in particulate composites.

In some particular cases, the matrix may be used to tailor the properties of the final composite. One example is that of Low-Temperature Co-fired Ceramics (LTCC), used as substrate material for production of multilayer electronic circuits and sensors for medical, automotive and communication devices (Imanaka, 2005). LTCC ceramics consist of ceramic particles (typically alumina) embedded in a glass matrix. LTCC technology was developed around three decades ago as an alternative to overcome conductivity problems with tungsten metallization in alumina substrates used in High Temperature Co-fired Ceramics (Gongora-Rubio et al., 2001, Thelemann et al., 2002). The main characteristic of LTTC is their low sintering temperature of the ceramic particles (cca. 850°C), which is possible due to the use of the glass matrix with a low melting point (Ewsuk, 1990).

The lifetime prediction of LTCCs is associated with the understanding of crack propagation in a given environment. The strength of glass or ceramic containing materials can be degraded by the effect of the environment (“assisted cracking or stress corrosion”). Important factor for the lifetime of glasses and ceramics is related to the subcritical crack growth (SCCG) phenomenon (see for instance Wiederhorn, 1974). Under applied stress, crack-like defects may grow (sub-critically) for stress intensity factors,  $K_I$ , below the fracture toughness of the material,  $K_{Ic}$ . To obtain data for describing crack propagation the direct as well as indirect methods can be used. With direct methods, the crack velocity is measured as a function of the stress intensity factor. On the other hand, with indirect methods a degradation of strength is used to derive the underlying crack propagation parameters.

In this paper, a two-dimensional finite element model was developed to analyze the subcritical crack growth behavior of LTCC composites. The composite was modeled as two-phase continuum considering (ceramic) particles and (glass) matrix. The path of the propagating crack was determined for different initial micro-crack locations (i.e. distance from the neighboring particle). The crack increment,  $da$ , was recalculated in every step according to subcritical crack growth parameters obtained experimentally elsewhere (Bermejo et al., 2013).

## 2. Materials and experiment

The material model used in this investigation is based on a LTCC substrate containing alumina ceramic particles embedded in a glass matrix. Micro-structural features and analysis can be found elsewhere (Bermejo et al., 2011, Bermejo et al., 2013).

In order to analyse the propagation of subcritical cracks in this composite material, a single-power law (also referred as Paris law) is typically assumed to describe the material model in environments with relatively high humidity content. For most ceramics and glasses, the basic crack growth rate  $\nu$  is given by the empirical power-law relation:

$$\nu = \frac{da}{dt} = \nu_0 \cdot \left( \frac{K_I}{K_{Ic}} \right)^n, \quad (1)$$

where  $a$  is crack size,  $t$  is time,  $K_I$  is the applied mode I stress intensity factor,  $K_{Ic}$  is fracture toughness,  $n$  is the SCCG exponent and  $\nu_0$  is a material constant. In the numerical model, the propagation of subcritical cracks in water is described. Corresponding values of subcritical crack growth parameters (i.e.  $n$  and  $\nu_0$ ) were  $31.2 \pm 1.2$  and  $1.04105 \text{ m} \cdot \text{s}^{-1}$  respectively, in water, as taken from (Bermejo et al., 2013). The fracture toughness,  $K_{Ic}$ , of the LTCC material was determined in air using the Single Edge V-Notch Beam method (SEVNB), resulting in  $1.8 \pm 0.1 \text{ MPa} \cdot \text{m}^{1/2}$ .

## 3. Computational model

### 3.1. Numerical model

The load of an existing crack and its growth in the vicinity of particles embedded in a matrix was studied with a two-dimensional finite element model. From the calculated stress intensity values in Mode I ( $K_I$ ) and Mode II ( $K_{II}$ ) the direction  $\Omega_s$  of propagation was estimated by the maximum tangential stress (MTS) criterion, see Fig. 1b. The size of the crack was chosen so that the  $K$  values stay below  $K_{Ic}$  and sub-critical crack growth takes place. With measured curves of sub-critical crack growth, the velocity of crack growth is predicted by Eq. 1. The mean values of  $K_{Ic}$ ,  $\nu_0$  and  $n$  given in section 2 were used in the model.

The direction of crack propagation,  $\Omega_s$ , and the value of crack extension,  $da$ , were calculated in each step of the model. For the next step, the crack was extended in new direction  $\Omega_s$  by the calculated value  $da$ , re-meshed and loaded again. In this way, the growth of the crack can be modeled. This can be done with either a constant time step  $t_i$  (resulting in corresponding crack extensions  $da$ ) or with constant crack increments  $da$  (resulting in corresponding time steps  $t_i$ ). The usage of a constant crack increment has been proven to be more stable and therefore  $da$  of  $0.05 \text{ } \mu\text{m}$  was used.

Figure 1a shows the geometry of the model. The particles were uniformly dispersed in the matrix with a distance between the particle centers  $d$  of  $1.3 \text{ } \mu\text{m}$ . The diameter  $D$  of the round particles was chosen as  $1 \text{ } \mu\text{m}$ . To model a crack with a sufficient size to grow under the used load, there is a distance  $b$  of  $2 \text{ } \mu\text{m}$  between the left edge of the model and the first column of particles. The region modeled has a total height of  $10.775 \text{ } \mu\text{m}$  and width of  $9.2 \text{ } \mu\text{m}$ . The initial crack is modeled at a distance  $c$  of  $0.8 \text{ } \mu\text{m}$  from the center line and with an initial length  $a_i$  of  $1 \text{ } \mu\text{m}$ . Ideal adhesion between matrix and particles was assumed in the model (i.e. no interface delamination was allowed). The nodes on the top and at the bottom of the model were coupled in the  $y$ -direction. A mean stress  $\sigma_{appl}$  of  $500 \text{ MPa}$  was

applied in those regions and held constant throughout the calculation. A node on the right of the model was constrained both in x and y direction, as shown in Fig. 1a.

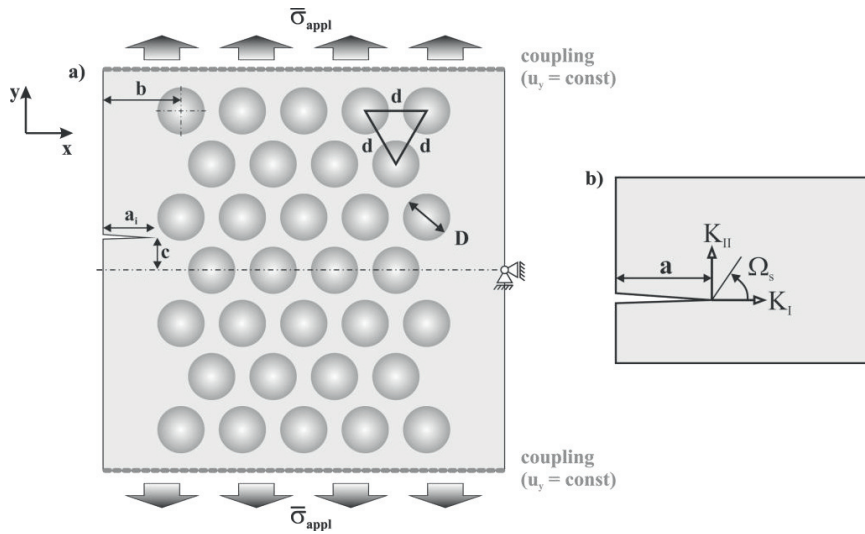


Fig. 1. a) Geometry of the finite element model and used boundary conditions, b) scheme of the crack propagation direction  $\Omega_s$

Only elastic behaviour of the matrix and the particles was considered. The particles are modelled considerably stiffer (Young's modulus of 300 GPa and Poisson's ratio of 0.29) than the glass matrix (Young's modulus of 70 GPa and Poisson's ratio of 0.29), as corresponding values for typical glass and alumina, respectively (Zhang et al., 2009). Plane strain condition and elements with quadratic displacement function (ANSYS type "PLANE183") have been used. The mesh was refined towards the region of the crack tip to give accurate results while still being able to run in reasonable calculation times. Special "crack" finite elements with shifted mid-nodes to capture the stress singularity at the crack tip were used. The model contains approximately 400 000 elements in total and 50 000 elements close to the crack tip.

The values of the stress intensity factors  $K_I$  and  $K_{II}$  were calculated using the standard KCALC procedure implemented in ANSYS. Obtained values  $K_I$  and  $K_{II}$  were used for estimation of the direction of the crack propagation. Residual stresses produced by the sintering of the material are not considered at this stage. Only the crack propagation in the matrix is allowed in the presented model.

### 3.2. Maximum Tangential Stress criterion

The propagation of a micro-crack in the matrix of the particulate composite is influenced by its interaction with particles. To describe the interaction the micro-crack propagation direction has to be known. For the determination of crack propagation direction, numbers of criteria exist in the literature (Chambers et al., 1991, Qian and Fatemi, 1996, Sih, 1991). In this paper the Maximum Tangential Stress criterion has been used (see Erdogan and Sih, 1963 for details). The criterion assumes that the crack propagates in the direction leading to zero  $K_{II}$  values. Determination of crack propagation direction  $\Omega_s$  can then be expressed by the following equation:

$$\Omega_s = \arccos \left( \frac{3K_{II}^2 + K_I \sqrt{K_I^2 + 8K_{II}^2}}{K_I^2 + 9K_{II}^2} \right), \quad (2)$$

where  $\Omega_s$  is micro-crack propagation direction and  $K_I$  and  $K_{II}$  are stress intensity factors for mode I and II, respectively.

#### 4. Results

Fig. 2 shows the stress intensity factor values for mode I ( $K_I$ ) over the crack path. The sub-critical propagation ( $K_{Ic}$  of the matrix is  $1.8 \text{ MPa}\cdot\text{m}^{1/2}$ ) of the crack from an initial crack tip location ( $1 \mu\text{m}$  from the left edge) to the distance of about  $3 \mu\text{m}$  is plotted. Curves for both homogenous material with matrix properties and the particle-reinforced composite are shown. The  $K_I$  values of the loaded homogeneous body are considerably higher than those for the composite. Also, they show a monotonic increase with distance from the edge. The  $K_I$  curve for the composite model features kinks as the crack comes closer to the particles, e.g. close to point 2 in Fig. 2. Crack growth rates become very low there. The paths of the propagating cracks are also shown in Fig. 2. Whereas the crack in the homogeneous material propagates horizontally, the crack in the composite is influenced by presence of rigid particles, i.e. it is drawn towards the particles. Indeed, lower  $K_I$  values in the composite are associated with a slower crack growth, see Eq. (1). The crack growth from the left edge to the distance of  $3 \mu\text{m}$  takes 14 seconds for the homogenous body with matrix material properties and  $2.6 \times 10^7$  seconds (about 300 days) for the composite. Note that only the influence of the different stiffness values of the matrix and particles is incorporated in the presented model. This leads to the crack deflection, reduction of  $K_I$  values and crack growth rate. It can be assumed that the residual stresses produced during production of the composite due to different coefficients of thermal expansion of matrix and particles can significantly influenced the crack propagation rate of sub-critically growing cracks. This will be investigated in further work.

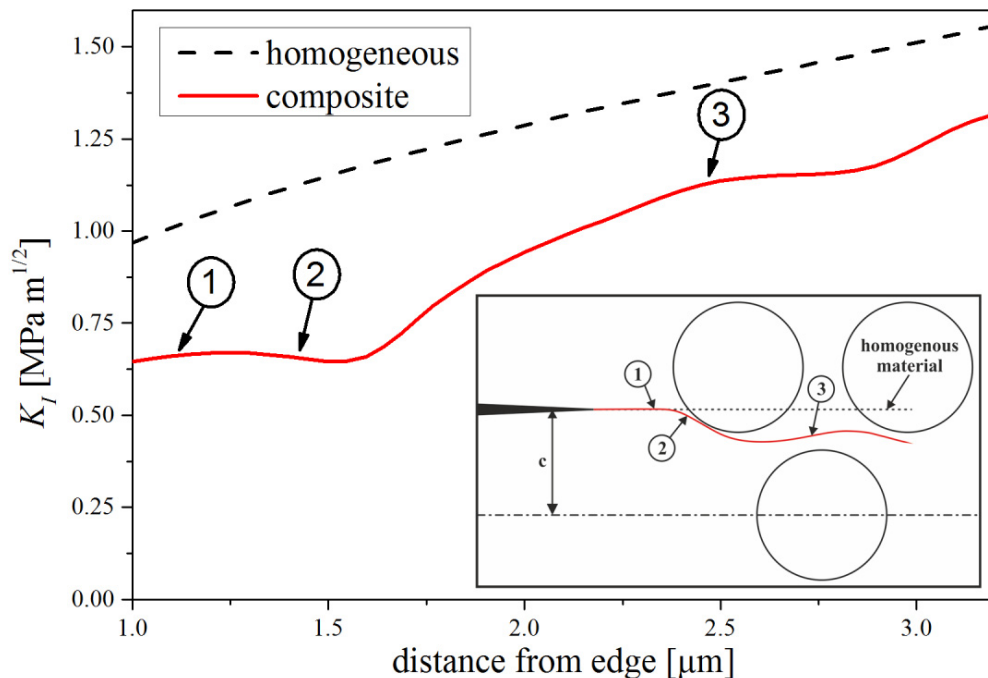


Fig. 2. Calculated values  $K_I$  for two models: homogeneous body with material properties of the matrix (black line) and particulate composite (red line). The initial crack at the distance  $c$  of  $0.8 \mu\text{m}$  from the middle plane is considered. The paths of the cracks for both models are shown in the illustration. Three characteristic points are indicated in both the diagram and the illustration.

#### 5. Conclusions

A simplified finite element model of sub-critical crack growth in particulate composite has been developed. The results show that the stiffness difference of matrix and particles lead to a) a deflection of the growing crack and b) a

considerable reduction of the crack growth rate. It is evident that for a reliable modelling of the sub-critical crack growth in glass-ceramic composites (such as Low Temperature Co-fired Ceramics (LTCCs)) a crack - particle interaction has to be incorporated to the model. Influences of elastic properties of particles and matrix, residual stresses of production, particle fractions and particle shapes on the crack growth rates and crack propagation path can be studied with the model developed in this work. It can be a powerful tool to understand failure phenomena in glass-ceramic composites and to optimize their strength and reliability.

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