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Late time cosmic acceleration from natural infrared cutoff

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ABSTRACT

In this paper, inspired by the ultraviolet deformation of the Friedmann–Lemaître–Robertson–Walker geometry in loop quantum cosmology, we formulate an infrared-modified cosmological model. We obtain the associated deformed Friedmann and Raychaudhuri equations and we show that the late time cosmic acceleration can be addressed by the infrared corrections. As a particular example, we applied the setup to the case of matter dominated universe. This model has the same number of parameters as ΛCDM, but a dynamical dark energy generates in the matter dominated era at the late time. According to our model, as the universe expands, the energy density of the cold dark matter dilutes and when the Hubble parameter approaches to its minimum, the infrared effects dominate such that the effective equation of state parameter smoothly changes from $w_{\rm eff}=0$ to $w_{\rm eff}=-2$. Interestingly and nontrivially, the unstable de Sitter phase with $w_{\rm eff}=-1$ is corresponding to $\Omega_m=\Omega_d=0.5$ and the universe crosses the phantom divide from the quintessence phase with $w_{\rm eff}>-1$ and $\Omega_m>\Omega_d$ to the phantom phase with $w_{\rm eff}<-1$ and $\Omega_m<\Omega_d$ which shows that the model is observationally viable. The results show that the universe finally ends up in a big rip singularity for a finite time proportional to the inverse of the minimum of the Hubble parameter. Moreover, we consider the dynamical stability of the model and we show that the universe starts from the matter dominated era at the past attractor with $w_{\rm eff}=0$ and ends up in a future attractor at the big rip with $w_{\rm eff}=-2$.

1. Introduction

Cosmological observations indicate that the Universe accelerates positively at the small redshifts [1] which leads to the so-called dark energy problem [2,3]. In the standard ΛCDM model, cosmological constant dominates at the late time and derives cosmic speed-up. But, the models in favor of cosmological constant clue to the cosmological constant problem due to the possible identification of the cosmological constant with the vacuum energy of the quantum fields [2,4]. Furthermore, increasing evidences from the cosmological data reveal that the energy density corresponds to the dark energy evolves very slowly in time and the associated equation of state parameter lies in a narrow strip around $w=-1$ [1]. Thus, cosmological constant with sharp value $w=-1$ for the equation of state parameter is an appropriate candidate in the first order of approximation [5]. In order to explain the dynamical nature of the dark energy, the quintessence scenarios with $w>-1$ and phantom models with $w<-1$ are proposed. In this respect, one usually interested in models which support the transition from the quintessence era to the phantom phase. These scenarios are usually based on two postulates: i) assuming general relativity is applicable even on cosmological scales and then considering some sort of unusual matter component(s) costing violation of some energy conditions, ii) deformation of general relativity at the cosmological scales. For the first case the matter source is usually given by a scalar field [6–8] and for the latter case, there are many candidates such as the extra dimensions models, $f(R)$ theories [9,10] and recently proposed massive gravity models [11].

From the theoretical point of view, de Sitter spacetime is a maximally symmetric space and its constant curvature is completely determined by the cosmological constant. Apart from the very small variation of cosmological constant with time, it can be interpreted as a fundamental constant of nature much similar to the speed of light and Planck constant. It therefore provides a universal infrared (IR) cutoff (corresponding to the large length scale $\sim 10^{-56}$ cm^{-2}) for the universe. For instance, existence of cosmological constant as an IR cutoff is essential for the quantization of scalar field in de Sitter spacetime. More precisely, it provides a minimum scale for the momenta of modes through the uncertainty principle and removes the IR divergences in this setup [12]. In this respect the uncertainty principle will be modified in curved spacetimes in order to respect the existence of cosmological constant as

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a universal IR cutoff [13]. On the other hand, existence of a minimal length scale is suggested by any quantum theory of gravity such as loop quantum gravity [14] and string theory [15]. It is also shown that the uncertainty principle is modified in the presence of a minimal length scale [16]. Thus, the uncertainty principle gets modifications in IR and ultraviolet (UV) regimes in order to respect the existence of cosmological constant and minimal length scale respectively [17]. Taking these universal IR and UV cutoffs into account, the quantum field theories turn out to be renormalizable [18]. Therefore, natural IR and UV cutoffs would be emerged in the context of ultimate quantum gravity theory. While the existence of a universal IR cutoff is supported by the standard general relativity framework through the de Sitter spacetime,[1] there is not any explanation for the UV cutoff (minimal length scale) in this setup. On the other hand, a minimal length scale as a UV cutoff emerges in loop quantum gravity framework [19] but there is not a well-defined explanation for taking a cosmological constant into account in this setup (see however Refs. [20] where some attempts have done in this direction). In this paper, we follow the UV deformation of the Friedmann–Lemaître–Robertson–Walker (FLRW) universe in loop quantum cosmology and we construct the corresponding IR-deformed case. We show that the late time cosmic acceleration arises in this setup which is significantly different from the ΛCDM model such that the universe crosses the phantom divide from $w_{\text{eff}} > -1$ to $w_{\text{eff}} < -1$.

2. FLRW universe

The spatial part of the spatially flat FLRW universe is 3-manifold M with the Euclidean isometry group and $\mathbf{R}^3$ topology. One then can fix a constant orthonormal triad e^a_i and a co-triad ω^i_a compatible with a flat fiducial metric ${}^oq_{ab}$ on M. The corresponding gravitational phase space consists of pairs (A^i_a, E^a_i) on M, where A^i_a is a $SU(2)$ connection and E^a_i is its canonically conjugate field [21]. Because of the symmetries of the 3-manifold M, all the information of the phase space variables (A^i_a, E^a_i) are summarized in two variables (β, V) which satisfy canonical Poisson algebra

$$\{\beta, V\} = \frac{\kappa\gamma}{2}\,, \tag{1}$$

on two-dimensional phase space Γ, where $\kappa = 8\pi G$ (we work in unit $c = 1$, where c is the speed of light in vacuum) and $\gamma \approx 0.2375$ is the Barbero–Immirizi parameter which is fixed by the black hole entropy calculations in loop quantum gravity [22]. These variables are related to the old geometrodynamics variables as

$$\beta = \gamma\,\frac{\dot{a}}{a}, \qquad V = a^3\,. \tag{2}$$

So, V is the comoving volume and its canonically conjugate variable β is (up to a constant) the Hubble parameter.

Considering a perfect fluid as a source for the matter content, the associated energy density consisting of non-relativistic and relativistic matters will be a function of volume as $\rho = \rho(V)$ and the corresponding Hamiltonian function is given by

$$\mathcal{H} = -\frac{3}{\kappa\gamma^2}\beta^2 V + \rho V\,. \tag{3}$$

The Hamiltonian system of the FLRW universe in terms of Ashtekar variables (β, V) is therefore defined by the relations (1) and (3) on two-dimensional phase space Γ: The kinematics is defined by the Poisson bracket (1) and the dynamical evolution is governed by the Hamiltonian (3). It is easy to show that the associated Hamilton's equations together with the Hamiltonian constraint $\mathcal{H} \approx 0$ lead to the standard Friedmann and Raychaudhuri equations.

2.1. UV-deformed phase space

In loop quantum cosmology scenario however this Hamiltonian system gets holonomy corrections at the UV regime. At the semiclassical regime, these UV effects can be taken into account in two equivalent ways on the corresponding UV-deformed phase space Γ_λ. One can work in noncanonical chart on Γ_λ in which the Poisson bracket (1) gets UV modification while the Hamiltonian function (3) retains its standard functional form [26]. Equivalently, one can also work in canonical chart on Γ_λ such that the form of Poisson bracket (1) remains unchanged and the Hamiltonian function (3) gets modified functional form [23]. These two different representations are related to each other through the Darboux transformation and lead to the same Friedmann and Raychaudhuri equations [24,25]. In this paper we work in the first picture in which the Poisson bracket gets UV modification as [26]

$$\{\beta, V\} = \frac{\kappa\gamma}{2}\sqrt{1-\lambda^2\beta^2}, \tag{4}$$

where λ is the UV deformation parameter which is preferably of the order of the Planck length $\lambda \sim l_{\text{Pl}}$. Clearly, β gets maximum value as $\beta < \lambda^{-1}$ in this setup. More precisely, the space of β is compactified to a circle S^1 with radius λ^{-1} [23]. The UV-deformed Poisson algebra (4) implies the following modified Hamilton's equations

$$\dot{V} = \{V, \mathcal{H}\} = \frac{\kappa\gamma}{2}\sqrt{1-\lambda^2\beta^2}\,\frac{\partial\mathcal{H}}{\partial\beta}\,, \tag{5}$$

$$\dot{\beta} = \{\beta, \mathcal{H}\} = -\frac{\kappa\gamma}{2}\sqrt{1-\lambda^2\beta^2}\,\frac{\partial\mathcal{H}}{\partial V}\,. \tag{6}$$

The above equations correctly reduce to the standard Hamilton's equations in the limit of $\lambda \to 0$. Substituting (3) into (5) gives

$$\dot{V} = \frac{3V\beta}{\gamma}\sqrt{1-\lambda^2\beta^2}.$$

Using Hamiltonian constraint $\mathcal{H} \approx 0$ and after some manipulations, it is straightforward to obtain the following UV-deformed Friedmann equation

$$H^2 = \frac{\kappa}{3}\rho\left(1 - \frac{\rho}{\rho_{\max}}\right), \tag{7}$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter and we have also defined $\rho_{\max} = \frac{3}{\kappa\gamma^2\lambda^2}$. The energy density and the Hubble parameter get maximum bounds $\rho \le \rho_{\max}$ and $H < H_{\max} = \left(\frac{\kappa\rho_{\max}}{12}\right)^{\frac{1}{2}}$ in this setup and the big bang singularity problem resolves such that the initial singularity in standard model of cosmology replaces with a bounce [27]. Existence of the minimal length scale as a UV cutoff for the system thus naturally leads to the spacetime singularity resolution in cosmological setup. In this paper, we are interested to study the effect of the existence of an IR cutoff on the late time cosmic acceleration in order to address the dark energy problem. As we will show in the next subsection, taking an IR cutoff into account leads to the self-accelerating universe at the late time.

[1] Note that the anti-de Sitter spacetime with negative cosmological constant is also an appropriate candidate from the theoretical point of view. But it rejects by cosmological observations.

2.2. IR-deformed phase space

In the absence of a fundamental theory at the IR regime, we would like to construct an IR-deformed Hamiltonian system following the way by which the UV cutoff is taken into account in the Hamiltonian system of the FLRW universe in loop quantum cosmology scenario. In loop quantum cosmology scenario, the space of β is compactified to a circle S^1 which leads to the deformed Friedmann equation (7). For the case of the IR-deformed Hamiltonian system, we should explore the modification of the space of V. At the first glance, it seems that we should consider the modified S^1 geometry for the space of V. But, let us more elaborate on this point. Indeed, the UV and IR cutoffs are universal and therefore the space of the associated variable should be maximally symmetric [28]. From the global point of view, for the case of one-dimensional space (with which we are interested in this paper) there are only two possibilities: a circle S^1 with compact $SO(2)$ symmetry and a hyperbolic space H^1 with open $SO(1,1)$ symmetry. For the UV-deformed phase space, the unique representation of holonomy-flux algebra fixes the space of β to have $SO(2)$ symmetry which local coordinatization (4) [21]. In the absence of any fundamental theory for the IR sector, we consider both of the possible symmetries $SO(2)$ and $SO(1,1)$ for the space of V. Furthermore, inspired by the loop quantum cosmology scenario, we work with Ashtekar variables and also consider the local coordinatization as same as (4). We therefore lead to two possible IR-deformed Poisson algebras

$$\{\beta, V\}_{\pm} = \frac{\kappa\gamma}{2}\sqrt{1 \pm \alpha^2 V^2}, \tag{8}$$

where α is the IR deformation parameter with dimension of inverse of volume. Before applying the above IR-deformed Poisson algebras to the cosmological setup, let us more elaborate on the differences between these two IR-deformed models. For the case of minus sign, clearly there is a maximum value $V_{\max} = \alpha^{-1}$ for the volume of the universe such that $V \in [0, \alpha^{-1})$ while $V \in [0, \infty)$ for the case of plus sign. At the quantum level, the model with minus sign should be defined on a lattice [25,29] while the model with plus sign leads to the generalized uncertainty relation. Following the way suggested in Ref. [30], it is straightforward to show that the associated uncertainty relation implies a minimum uncertainty in measurement of the corresponding conjugate variable β (see also Refs. [17,31,32]).

Taking the IR-deformed Poisson algebras (8) into account, the corresponding IR-deformed Hamilton's equations are given by

$$\dot{V}_{\pm} = \{V, \mathcal{H}\}_{\pm} = \frac{\kappa\gamma}{2}\sqrt{1 \pm \alpha^2 V^2}\,\frac{\partial \mathcal{H}}{\partial \beta}, \tag{9}$$

$$\dot{\beta}_{\pm} = \{\beta, \mathcal{H}\}_{\pm} = -\frac{\kappa\gamma}{2}\sqrt{1 \pm \alpha^2 V^2}\,\frac{\partial \mathcal{H}}{\partial V}, \tag{10}$$

where a dot denotes derivative with respect to the cosmic time t. In the limit $\alpha \to 0$, the Poisson algebra (8) and IR-deformed Hamilton's equations (9) and (10) are reduced to their standard counterparts. Indeed, depending on the value of the deformation parameter α, the system continue to follow the standard non-deformed classical trajectories and deviations start to dominate for the sufficiently large scales: $V \sim \alpha^{-1}$. Substituting the Hamiltonian function (3) into the relation (9) and then using the Hamiltonian constraint $\mathcal{H} \approx 0$, it is straightforward to show that the IR-modified Friedmann equation will be

$$H_{\pm}^2 = \frac{\kappa}{3}\rho\left(1 \pm \alpha^2 V^2\right). \tag{11}$$

Differentiating the above relation with respect to time t and then using the energy conservation relation

$$\dot{\rho} + 3H_{\pm}(\rho + p) = 0, \tag{12}$$

which is not modified in this setup (since just the geometric parts are modified), one can easily obtain the following IR-modified Raychaudhuri equation

$$\dot{H}_{\pm} = -\frac{\kappa}{2}\left[\rho\left(1 \mp \alpha^2 V^2\right) + p\left(1 \pm \alpha^2 V^2\right)\right]. \tag{13}$$

The IR effects would become significant at large volume limit at the late time. We also expect that they would naturally address the late time cosmic acceleration. In order to understand the late time behaviors of the above IR-deformed models, we obtain the associated effective equation of state parameters. Defining effective energy densities and pressures as $\rho_{\rm eff}^{\pm} = \rho(1 \pm \alpha^2 V^2)$ and $p_{\rm eff}^{\pm} = p \mp (2\rho - p)\alpha^2 V^2$ through the relations (11) and (13), the effective equation of state parameters $w_{\rm eff}^{\pm} = p_{\rm eff}^{\pm}/\rho_{\rm eff}^{\pm}$ can be easily obtained as

$$w_{\rm eff}^{\pm} = w \mp \frac{2\alpha^2 V^2}{1 \pm \alpha^2 V^2}, \tag{14}$$

where $w = p/\rho$ is the standard equation of state parameter. The above relation shows that $w_{\rm eff}^{-} \in [w, \infty)$ since $V \in [0, \alpha^{-1})$. This general result show that the minus sign in (8) cannot generate the late time cosmic acceleration. Invoking the cosmological observations which indicate that the universe accelerate at late time [1], we therefore abandon the minus sign. For the case of plus sign, however, we have $w_{\rm eff}^{+} \in (w-2, w]$ when the volume changes as $V \in [0, \infty)$. This is an interesting result since it shows that the plus sign in (8) can potentially address the dark energy problem [2,3]. In the next section we show that the late time cosmic acceleration naturally arises even in the cold dark matter dominated universe.

3. Dark energy from natural IR cutoff

We are interested in the late time cosmic evolution where the radiation component is negligible. Also, we would like to address the dark energy problem in the presented setup. Therefore, we consider the cold dark matter (CDM) dominated universe without cosmological constant. In this respect, our model has the same number of parameters as the standard ΛCDM model such that the effects of cosmological constant will replace with IR parameter α.

Considering the energy density $\rho_m = \rho_{0_m} a_0^3 a^{-3}$ for CDM in (11) and (13) (for the plus sign since we have shown that the minus sign cannot produce acceleration), the IR-deformed Friedmann and Raychaudhuri equations for the CDM dominated universe in this setup are given by

$$H^2 = \frac{\kappa}{3}\rho_m\left(1 + \frac{\rho_{\min}^2}{\rho_m^2}\right), \tag{15}$$

$$\dot{H} = -\frac{\kappa}{2}\rho_m\left(1 - \frac{\rho_{\min}^2}{\rho_m^2}\right), \tag{16}$$

where we have defined the minimum energy density

$$\rho_{\min} = \alpha \rho_{0_m} a_0^3. \tag{17}$$

The above minimum energy density is defined in the sense that at $\rho_m = \rho_{\min}$, the effective energy density

$$\rho_{\rm eff} = \rho_m + \frac{\rho_{\min}^2}{\rho_m}, \tag{18}$$

has the minimum $\rho_{\rm eff} = 2\rho_{\min}$. The existence of this minimum value for the effective energy density implies a minimum value

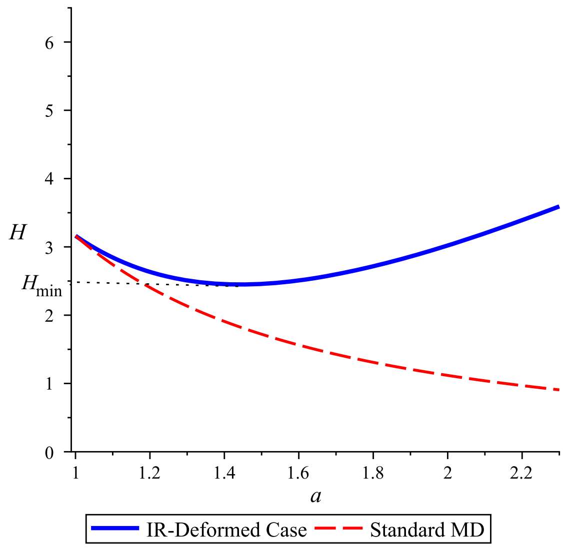

Fig. 1. The Hubble parameter versus the scale factor (as a clock) is plotted. While the Hubble parameter decreases with decreasing rate in the standard matter dominated era with $w = 0$ (the red dashed line), it decreases with increasing rate in our model until it approaches to its minimum (19) which is corresponding to an unstable de Sitter phase with $w = -1$ (the blue solid line). After crossing the phantom divide ($w < -1$) it starts to increase with increasing rate and finally it diverges at the big rip with $w = -2$ through the finite time (23). The figure is plotted for $\kappa = 3$ and $\rho_{\rm min} = 1/3$.

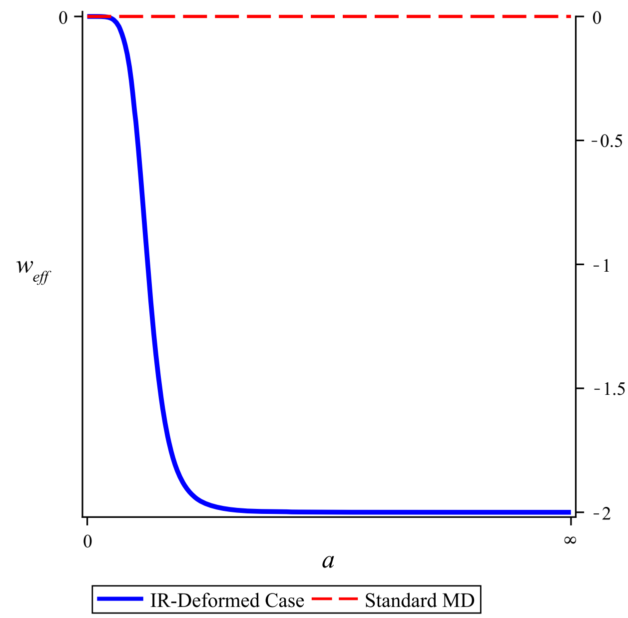

Fig. 2. The effective equation of state parameter versus the scale factor is plotted. It is clear that it is bounded as $0 \leq w_{\rm eff} \leq 2$ in this setup. The unstable point $w_{\rm eff} = 0$ is corresponding to the matter dominated era. As the universe expands, the IR effects starts to dominate and after crossing an unstable de Sitter phase with $w_{\rm eff} = -1$, it enters in a phantom phase with $w_{\rm eff} < -1$. Finally, the universe ends up in a big rip at the finite time (23) with $w_{\rm eff} = -2$. The figure is plotted for $\rho_{\rm min} = 1$.

for the Hubble parameter through the relation (15) (see also the Fig. 1) which is given by

$$H_{\rm min} = \sqrt{\frac{2\kappa\rho_{\rm min}}{3}}. \tag{19}$$

Before scrutinizing the cosmological implication of the model, it is interesting to note that the IR-deformed Friedmann equation (15) can be also deduced from the $f(R)$ theories for the particular case of $f(R) = R + \alpha R^{-1}$ [10]. Furthermore, it can be realized from the Cardassian models of dark energy which is investigated in the context of braneworld scenario [8]. In Cardassian model, the deformed Friedmann equation in matter dominated era is given by $H^2 = A\rho_m + B\rho_m^n$ with $A = \kappa/3$ and B and n are two free parameters of the model. Our model can be realized as a special case by the relevant identification $B = \frac{\kappa}{3}\rho_{\rm min}^2$ and $n = -1$. Note that our model has also one parameter less than the Cardassian models.

Solving (18) for $\rho_m = \rho_m(\rho)$ and then substituting for the effective pressure $p_{\rm eff} = p_m + p_d = p_d$ gives

$$p_{\rm eff} = -\rho_{\rm eff} \pm \rho_{\rm eff}\sqrt{1 - 4\Big(\frac{\rho_{\rm min}}{\rho_{\rm eff}}\Big)^2}. \tag{20}$$

The effective equation of state parameter is then given by

$$w_{\rm eff} = \frac{p_{\rm eff}}{\rho_{\rm eff}} = -1 \pm \sqrt{1 - 4\Big(\frac{\rho_{\rm min}}{\rho_{\rm eff}}\Big)^2}. \tag{21}$$

From the minimum bound that are arisen for the effective energy density as $\rho_{\rm eff} = 2\rho_{\rm min}$, it is clear that the equation of state parameter (21) is also bounded as $-2 \leq w_{\rm eff} \leq 0$ in complete agreement with our previous general treatment though the relation (14) (see Fig. 2). This range for $w_{\rm eff}$ shows that the model can produce the acceleration phase.

To study the fate of the universe, using the IR-deformed Friedmann equation (15) in the conservation relation for the effective energy density

$$\dot{\rho_{\rm eff}} + 3H(\rho_{\rm eff} + p_{\rm eff}) = 0\,, \tag{22}$$

and then substituting for the pressure from the relation (20) with minus sign (corresponding to the phantom phase) gives

$$t_{\rm rip} - t_\Lambda = \frac{1}{\sqrt{3\kappa}}\int_{2\rho_{\rm min}}^{\infty}\frac{d\rho_{\rm eff}}{\sqrt{\rho_{\rm eff}(\rho_{\rm eff}^2 - 4\rho_{\rm min}^2)}} = \sigma\, H_{\rm min}^{-1}\,, \tag{23}$$

where $\sigma = \frac{2\sqrt{\pi}}{3}\frac{\Gamma[5/4]}{\Gamma[3/4]} \approx 0.874$ and also we have used (19). Clearly, the integration in (23) is performed from the lower bound $\rho_{\rm eff} = 2\rho_{\rm min}$ corresponding to the unstable de Sitter phase with $w_{\rm eff} = -1$ (through the relation (21)) to the final state with $\rho_{\rm eff} \to \infty$ and $w_{\rm eff} = -2$. Thus, t_Λ denotes the time at which the system is in unstable de Sitter phase with effective cosmological constant $\Lambda_{\rm IR} = 2\kappa\rho_{\rm min} = 2\kappa\alpha\rho_{0_m}a_0^3$ and $t_{\rm rip}$ corresponds to a big rip since the effective energy density and Hubble parameter diverge at finite time (23) [33]. To be more precise, we should obtain the scale factor at the time of big rip. Substituting pressure from the relation (20) with minus sign into the relation (22) and integrating gives the following integral for the scale factor at the big rip

$$a = \exp\left[\frac{1}{3}\int_{2\rho_{\rm min}}^{\infty}\frac{d\rho_{\rm eff}}{\sqrt{\rho_{\rm eff}^2 - 4\rho_{\rm min}^2}}\right] \longrightarrow \infty\,, \tag{24}$$

which shows that it diverges for infinite energy density at the big rip. From (20) it is clear that the pressure also diverges at the time (23) and therefore the universe finally ends up in a big rip with $\rho_{\rm eff}, |p_{\rm eff}|, a \to \infty$ and $w_{\rm eff} = -2$ at the finite time (23). A big rip is the common fate of the phantom dominated universe [34]. At sufficiently high energy regime $\rho_{\rm eff} \gg \rho_{\rm min}$, the IR effects are negligible and relation (18) gives $\rho_{\rm eff} \approx \rho_m$ which is corresponding to the standard matter dominated era and therefore the standard

Friedmann and Raychaudhuri equations for the matter dominated era can be recovered in this regime.

Note that fixing the IR deformation parameter of the model α (or equivalently fixing $\rho_{\rm min}$), which replaces the cosmological constant in comparison with the ΛCDM, immediately fixes all the observable parameters such as the equation of state parameter $w_{\rm eff}$ and density parameters Ω_m and Ω_d. Thus, fitting the density parameter Ω_d with the observational data immediately gives a fixed value for the effective equation of state parameter $w_{\rm eff} = w_{\rm eff}(\Omega_d)$ which shows the naturalness and predictiveness of the model. Interestingly, the cases $w_{\rm eff} > -1$ and $w_{\rm eff} < -1$ are corresponding to $\Omega_m > \Omega_d$ and $\Omega_m < \Omega_d$ respectively which shows the model is observationally viable. While the model qualitatively is relevant, in contrast to the dynamical dark energy models, it may not fit the observational data in a very precise manner. But, note that the dynamical dark energy models such as the model based on the scalar fields [35] have at least one parameter more that our model and ΛCDM. We could therefore consider another model with more adjustable parameters which fits the observational data in a more precise manner. For instance, adding even a massless scalar field to the matter content can produce cosmological constant like term at late time in this setup. We are going to study such a setup in a new research program [36]. Moreover, similar to the theories which deals with the geometric deformation of the Einstein's equations at the late time such as $f(R)$ theories [10,9] and recently proposed massive gravity models [11], our setup can produce an accelerating universe and crossing the phantom divide without violating any energy condition.

4. Autonomous system and dynamical stability

In this section we consider the dynamical stability of the self-accelerating CDM dominated universe that is presented in the previous section. We define the energy density and pressure of the dark energy component as

$$\rho_d = \frac{\rho_{\rm min}^2}{\rho_m}, \tag{25}$$

$$p_d = -2\rho_d = -2\frac{\rho_{\rm min}^2}{\rho_m}. \tag{26}$$

Using energy conservation relation $\dot{\rho}_m + 3H\rho_m = 0$ for the pressureless CDM, it is easy to show that the energy density (25) and pressure (26) satisfy the following conservation relation

$$\dot{\rho}_d + 3H(\rho_d + p_d) = \dot{\rho}_d - 3H\rho_d = 0\,. \tag{27}$$

The IR-deformed Friedmann equation (15) then rewrites as

$$H^2 = \frac{\kappa}{3}(\rho_m + \rho_d). \tag{28}$$

From the above relation and the energy conservation relation (27), one could easily find the IR-deformed Raychaudhuri equation

$$\dot{H} + H^2 = -\frac{\kappa}{6}(\rho_m - 5\rho_d)\,. \tag{29}$$

As it is clear from the above relation, the energy density of the dark energy ρ_d which purely originates from the IR effects, appears with the minus sign which show how it generates cosmic acceleration at the late time.

In order to consider the dynamical stability of the model, we work with the well-known dimensionless density parameters

$$\Omega_m = \frac{\kappa\rho_m}{3H^2}, \qquad \Omega_d = \frac{\kappa\rho_d}{3H^2}, \tag{30}$$

in terms of which the IR-deformed Friedmann equation (28) becomes

$$\Omega_m + \Omega_d = 1\,. \tag{31}$$

From the definition (25) and using the IR-deformed Friedmann equation (15) together with the definition of the effective energy density (18) it is easy to show that

$$\Omega_m\Omega_d = \frac{H_{\rm min}^4}{H^4} = \frac{\rho_{\rm min}^2}{\rho_m^2}\,, \tag{32}$$

where we have also used the relations (17) and (19). This relation is useful to study the qualitative behavior of the model.

Taking the time derivative of the dark energy density parameter as $\dot{\Omega}_d = \Omega_d\left(\frac{\dot{\rho}_d}{\rho_d} - 2\frac{\dot{H}}{H}\right)$, and then substituting from the relations (27), (28) and (29) gives

$$\frac{d\Omega_d}{d\tau} = 6\Omega_d(1-\Omega_d)\,, \tag{33}$$

where $\tau = \ln a$ and we have also eliminated the CDM density parameter Ω_m by means of the constraint equation (28). As it is clear from (33), the space of states is the one-dimensional segment $\Omega_d \in [0,1]$ and there are two critical points $\Omega_d = 0$ and $\Omega_d = 1$ in this model. The critical point $\Omega_d = 0$ clearly corresponds to the matter dominated era with $\Omega_m = 1$ and the point $\Omega_d = 1$ is corresponding to the dark energy dominated era with $\Omega_m = 0$. In order to consider the stability of the model, we should consider the linear perturbation of the equation (33) around these critical points. Considering the small perturbations $\Omega_d = \delta_1 \to 0$ and $\Omega_d = 1 - \delta_2 \to 1$ in relation (33) immediately leads to the following differential equations [37]

$$\frac{d\delta_1}{d\tau} = 6\delta_1, \qquad \frac{d\delta_2}{d\tau} = -6\delta_2, \tag{34}$$

which have the following solutions

$$\delta_1 = c_1 e^{6\tau}, \qquad \delta_2 = c_2 e^{-6\tau}, \tag{35}$$

where c_1 and c_2 are constants of integrations. The point $\Omega_d = 0$ will be a past attractor since the initially small perturbation δ_1 increases exponentially with time and then taking the system away from the matter dominated era $\Omega_m = 1$. The perturbation δ_2, however, decreases exponentially with the time which shows that the point $\Omega_d = 1$ is a stable equilibrium point. This is a future attractor solution which is corresponding to a big rip at the time (23) with $w = -2$. While the total energy density behaves as $\rho \simeq \rho_m$ at the past attractor point $\Omega_d = 0$ ($\Omega_m = 1$ matter dominated era) and the scale factor behaves as $a \propto t^{2/3}$, it behaves as $\rho \propto \rho_m^{-1} \propto a^3$ at the future attractor point $\Omega_d = 1$ ($\Omega_m = 0$ dark energy dominated era) and therefore the scale factor would behave as $a \propto t^{-2/3}$.

5. Summary and conclusions

Cosmological observations show that the universe accelerates at small redshifts and the cosmological constant derives the desired acceleration in standard ΛCDM cosmology. In the context of general relativity, the cosmological constant can be interpreted as a universal IR cutoff. Quantum gravity candidates such as loop quantum gravity and string theory also suggest the existence of a minimum length scale of the order of the Planck length. The ultimate quantum theory of gravity then should contain universal IR and UV cutoffs. While the standard general relativity accommodates the existence of IR cutoff through the de Sitter spacetime with positive cosmological constant, it cannot support the existence of a UV cutoff. On the other hand, loop quantum cosmology scenario suggests the existence of a minimum length scale as UV cutoff for the system under consideration, but it does not support the existence of any IR cutoff. In this paper, following the UV deformation of the FLRW gravitational phase space in loop quantum cosmology, we have formulated deformed phase space which supports the existence of an IR cutoff. We obtained the associated IR-deformed Friedmann and Raychaudhuri equations and we showed that the IR corrections derives the late time cosmic acceleration. The model has the same number of parameters as ΛCDM such that the IR effects replace the effects of cosmological constant. But, the dynamics of the universe in our model is very different from the standard ΛCDM cosmology. For instance the Hubble parameter and energy density turned out to be bounded from the below in this model. As a particular example, we applied the setup to the simple case of CDM dominated universe. We divide the cosmic evolution in this model into the following three phases:

- *Quintessence phase* ($-1 < w_{\rm eff} \leq 0$): The universe starts from the standard matter dominated era with equation of state parameter $w_{\rm eff} \approx w_m = 0$ for $\rho_{\rm eff} \gg \rho_{\rm min}$ when the IR effects are negligible. In this phase, as the universe expands and the total energy density (18) dilutes, the IR effects become more and more appreciable up to $\rho_{\rm eff} \sim \rho_{\rm min}$. The effective equation of state parameter, which is given by the plus sign of the relation (21) in this regime, then smoothly decreases from $w_{\rm eff} = 0$ in matter dominated era to the negative values $w < 0$ and the universe starts to accelerate when it crosses over the value $w_{\rm eff} = -\frac{1}{3}$. From the relations (21) and (32), it is clear that the accelerating phase with $-1 < w_{\rm eff} < -\frac{1}{3}$ is corresponding to $\Omega_m > \Omega_d$.
- *Unstable de Sitter phase* ($w_{\rm eff} = -1$); *Transition from* $w_{\rm eff} > -1$ *to* $w_{\rm eff} < -1$: The effective equation of state parameter (21) then approaches to the value $w_{\rm eff} = -1$ when the energy density of CDM approaches to the critical value $\rho_m = \rho_{\rm min}$. At this momentum the Hubble parameter approaches to its minimum $H = H_{\rm min}$ that is corresponding to an unstable de Sitter phase with effective cosmological constant $\Lambda_{\rm IR} = 3H_{\rm min}^2 = 2\kappa\alpha\rho_{0_m} a_0^3$. From (21) and (32), one can easily see that the model includes a transition from the quintessence era with $w > -1$ and $\Omega_m > \Omega_d$ to the phantom phase with $w_{\rm eff} < -1$ and $\Omega_m < \Omega_d$ when crossing over the unstable de Sitter phase with $w_{\rm eff} = -1$ and $\Omega_m = \Omega_d$. This result makes the model observationally viable.
- *Phantom phase* ($-2 \leq w_{\rm eff} < -1$): After the universe enters into a phantom era with $w_{\rm eff} < -1$ and $\Omega_m < \Omega_d$, the effective equation of state parameter decreases until approaches to its asymptotic value $w_{\rm eff} = -2$ where the universe ends up in a big rip singularity at the finite time (23).

We have also considered the dynamical stability of the model which shows that, in this model, the universe starts at a past attractor in matter dominated era ($w_{\rm eff} = 0$) and after crossing an unstable point, corresponds to a de Sitter phase ($w_{\rm eff} = -1$), it approaches to a future attractor ($w_{\rm eff} = -2$) which is corresponding to the big rip singularity.

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