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The static and seismic bearing capacity factor N_γ for footings adjacent to slopes

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Abstract

Foundations resting adjacent to the crest of a slope exhibit a smaller bearing capacity respect to the case of horizontal ground. In this paper the static and seismic values of the bearing capacity factor $N_{\gamma g}$ for shallow strip foundations adjacent to slopes were evaluated using the method of characteristics, extended to the seismic case by means of the pseudo-static approach. $N_{\gamma g}$ was evaluated for different values of the slope angle and, under seismic conditions, accounting only for the effect of horizontal and vertical inertia forces arising in the foundation soil. The results, for both smooth and rough foundations, are presented and checked against those obtained through finite element analyses.

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Keywords: Shallow foundations; bearing capacity; method of characteristics; foundations adjacent to slopes

1. Introduction

The static and seismic behaviour of shallow foundations is a complex soil-structure interaction problem which can be studied through different approaches. Soil-foundation-superstructure interaction analyses are widely carried out to investigate the foundation response far from an ultimate limit state, under both static and seismic loading (e.g. [1-4]). Conversely, the safety against an ultimate limit state, is generally evaluated referring to a soil-foundation system. This paper focuses on this topic for which Cascone and Casablanca [5] recently provided a comprehensive review.

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In routine analyses the bearing capacity of shallow foundations is evaluated using the formula proposed by Terzaghi [6] for a strip footing resting on a homogeneous dry soil subjected to a vertical and uniformly distributed load:

$$q_{\text{ult}} = c \cdot N_{\text{c}} + q \cdot N_{\text{q}} + \frac{1}{2} \cdot \gamma \cdot B \cdot N_{\gamma} \tag{1}$$

In Eqn. 1 q_{ult} represents the ultimate load that the soil can sustain under the assumption of rigid plastic behaviour and the other symbols have the usual meanings.

Structures, such as buildings located in hilly regions, bridge piers, abutments and retaining walls (e.g. [7-8]), are frequently built on shallow foundations resting near the edge of slopes. In these cases, the geometry of the ground surface leads to a smaller bearing capacity in comparison with the case of foundations resting on horizontal ground. This reduction is more remarkable under seismic loading conditions when the effects of inertia forces in the soil involved in the plastic mechanism and the effects of inertia forces transmitted onto the foundation by the superstructure arise. Several solutions dealing with the problem of seismic bearing capacity of shallow foundations resting on sloping ground are available in the literature (e.g. [9-12]). In all the studies the earthquake effect is introduced through the pseudo-static approach, considering horizontal inertia forces acting on the foundation, as transmitted by the superstructure, and horizontal and vertical inertia forces arising in the soil mass involved in the plastic mechanism.

In this paper the evaluation of static ($N_{\gamma\text{g}}$) and seismic ($N_{\gamma\text{gE}}$) bearing capacity factors were carried out for shallow strip footings adjacent to the edge of a slope of angle β, by using the method of characteristics (*MC*), which was extended to the seismic condition by means of the pseudo-static approach. The differential governing equations are solved numerically, via a finite difference formulation, according to the procedure described by Cascone and Casablanca [5]. For the seismic condition $N_{\gamma\text{gE}}$ was evaluated accounting only for the effect of horizontal and vertical inertia forces arising in the soil, introducing the horizontal (k_{h}) and the vertical (k_{v}) seismic acceleration coefficients. The results are presented and checked against those obtained through finite element analyses.

2. Bearing capacity for static conditions

The values of the static bearing capacity factor $N_{\gamma\text{g}}$ were evaluated for both smooth and rough foundations, assuming a soil friction angle φ in the range 15° ÷ 45° and values of the slope angle $\beta < \varphi$ up to 45°. The values of $N_{\gamma\text{g}}$ are given in Figure 1 a-b in a semi-plot scale; the values N_{γ} of the bearing capacity factor $N_{\gamma\text{g}}$ corresponding to the case of foundation resting on horizontal ground, i.e. $N_{\gamma} = N_{\gamma\text{g}}$ ($\beta = 0$), are also plotted in Figure 1 a-b and coincide with those recently proposed by [5].

It can be observed that, whatever is φ, the influence of the sloping ground on the bearing capacity factor is remarkable for both smooth and rough foundations. As an example (Fig. 1a, Table 1) for $\varphi = 35°$, in the smooth case, it is $N_{\gamma\text{g}} = 11.26$ or 6.64 for β equal to 10° or 20°, respectively; correspondingly, a reduction $\Delta N_{\gamma\text{g}}$ of the bearing capacity factor equal to about 36 % and 62 % can be estimated in comparison with the case of horizontal ground surface ($\beta = 0$) for which it is $N_{\gamma} = 17.57$. Larger reductions, equal to about 44% and 70% can be estimated in the case $\varphi = 45°$, while a smaller influence of the slope angle can be computed in the case $\varphi = 25°$ (about 30% and 58% for β equal to 10° or 20°, respectively). Accordingly, it can be stressed that the larger is φ, the greater is the reduction of the bearing capacity factor due to the sloping ground. As it is shown in Table 1, in the case of rough foundation, the influence of β is quite similar, meaning that, for the static case, the reduction in the bearing capacity factor due to the presence of a sloping ground is not significantly affected by the roughness of the foundation.

For each of the curves plotted in Figure 1 a-b, a limiting value β^* of the slope angle can be also detected, denoting a condition for which the bearing capacity of the soil-foundation system reduces to zero (i.e. $N_{\gamma\text{g}} = 0$). This condition occurs for $\beta^* = \varphi$ since, in this case, the problem at hand degenerates into a slope stability problem. Finally, Figure 2 a-b show the network of the characteristic lines obtained for $\varphi = 30°$ and $\beta = 10$; it can be observed that, as in the case $\beta = 0$ [5], the roughness of the foundations modifies the inclination of the characteristic lines, mainly in the Cauchy zone, leading to a larger and deeper plastic mechanism involving half of the foundation width.

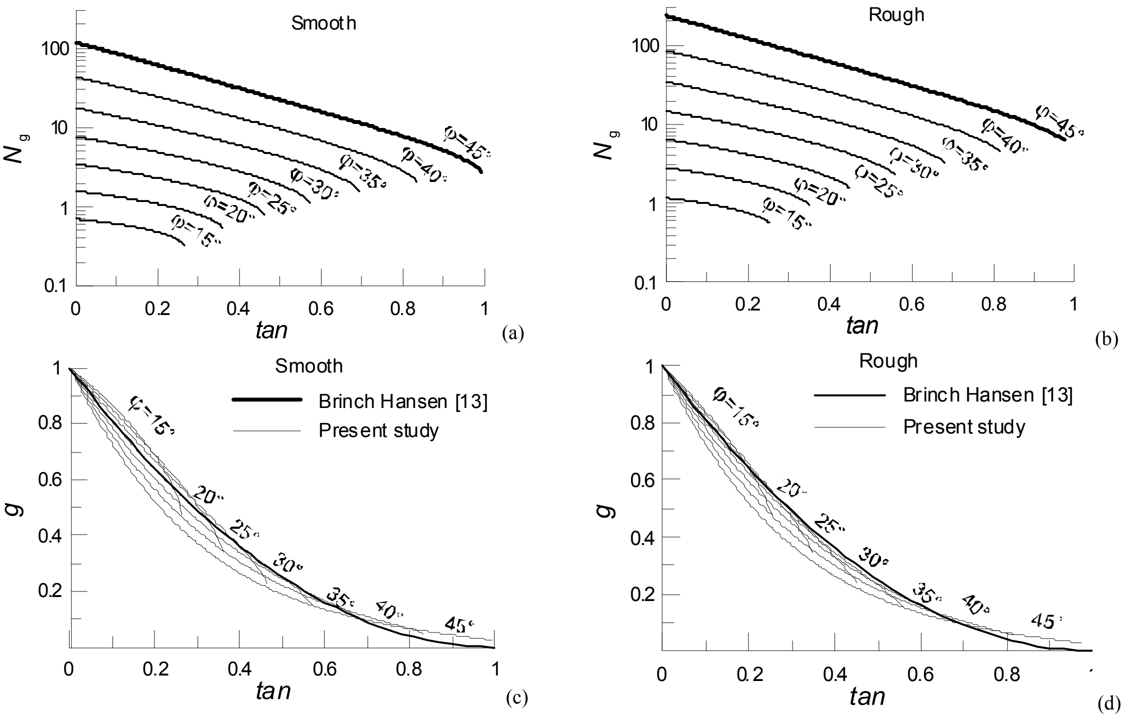


Fig. 1. Static bearing capacity factor (a,b) and corresponding corrective coefficient (c,d) for sloping ground, evaluated for a smooth (a,c) and for a rough (b,d) foundation.

Table 1. Some values of the static bearing capacity factor $N_{\gamma g}$ and of the corresponding corrective coefficient g_γ.

	smooth				rough			
φ (°)	β (°)	$N_{\gamma g}$	$\Delta N_{\gamma g}$ (%)	g_γ	β (°)	$N_{\gamma g}$	$\Delta N_{\gamma g}$ (%)	g_γ
25	0	3.45			0	6.47		
	10	2.43	-29.6	0.70	10	4.42	-31.6	0.68
	20	1.44	-58.3	0.42	20	2.51	-61.2	0.39
35	0	17.57			0	34.43		
	10	11.26	-35.9	0.64	10	21.83	-36.6	0.63
	20	6.64	-62.2	0.38	20	12.65	-63.3	0.37
45	0	117.70			0	235.07		
	10	66.27	-43.7	0.56	10	131.75	-44.0	0.56
	20	35.22	-70.1	0.30	20	69.63	-70.4	0.30

$N_{\gamma g}$ can be conveniently normalized with respect to N_γ, introducing the following corrective coefficient:

$$g_\gamma = N_{\gamma g} / N_\gamma \tag{2}$$

The values of g_γ evaluated for the cases of Figure 1 a-b are plotted in Figure 1 c-d, respectively, and, for the examples previously described, are also listed in Table 1. As already observed, it is evident that the roughness of the foundation does not significantly affects the reduction of $N_{\gamma g}$ due to the sloping ground. Conversely, it should be stressed that, differently from most of existing solutions for the evaluation of g_γ, the results obtained herein point out that this corrective coefficient is contemporarily affected by both β and φ. Thus, solutions giving g_γ as a function of only φ could lead to erroneous predictions of the bearing capacity factor $N_{\gamma g}$. As an example, the solution provided by Brinch-Hansen [13] is plotted in Figure 1 c-d (thick line) for comparison with the solutions proposed in this paper.

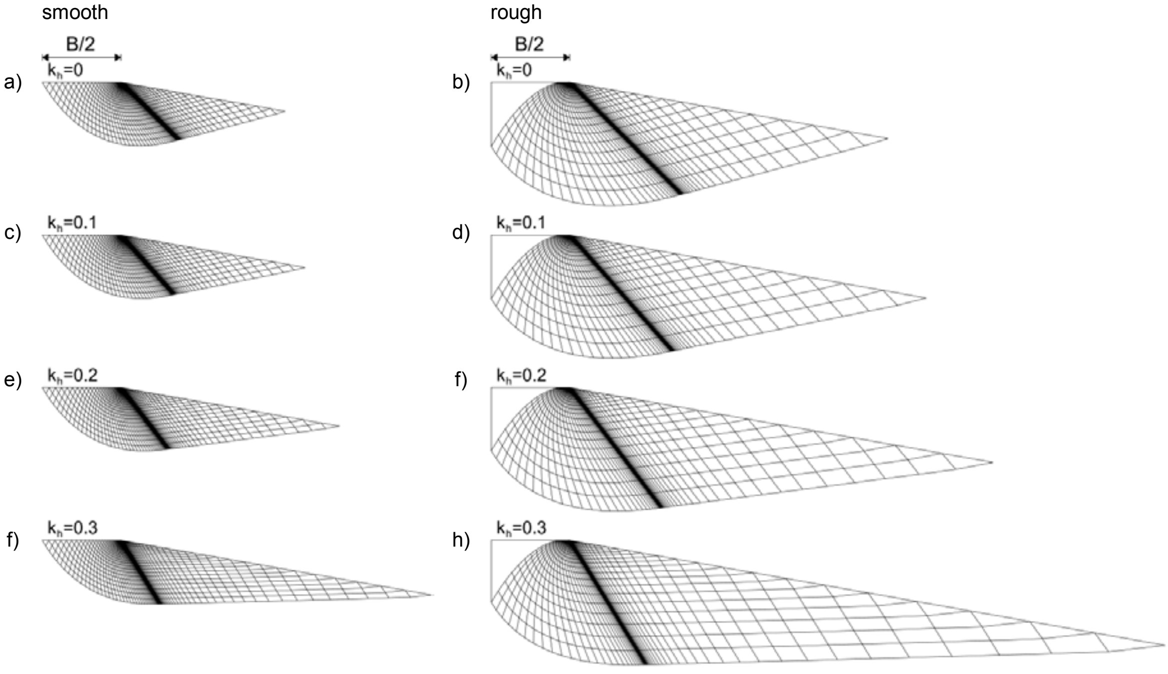


Fig. 2. Characteristic lines network for the $N_{\gamma g}$ problem evaluated for $\varphi = 30°$, β=10° and $k_v = 0$ for a smooth (a) and a rough (b) foundations.

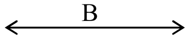
In order to check the values of the proposed bearing capacity factor, a comparison was performed with the solution of the $N_{\gamma g}$ problem obtained through *FE* plane-strain analyses, carried out using the commercial code Plaxis [14]. In these analyses the soil behaves as an elasto-perfectly plastic material, obeys to the Mohr-Coulomb failure criterion, and the flow rule is associated. Vertical boundaries were restrained in the horizontal direction, while displacements of the bottom of the mesh were restrained both horizontally and vertically. For the case $\varphi = 30°$, the result of the comparison is presented in Figure 3 a-b in terms of superposition between the characteristic line network obtained with the *MC* and the contour of total incremental displacements computed through the *FE* analyses. For this latter analysis only a portion of the FE domain is shown in Fig. 3). The comparison is satisfactory since, for both the smooth and the rough case, the stress and the displacement fields are practically coincident.

3. Bearing capacity for seismic conditions

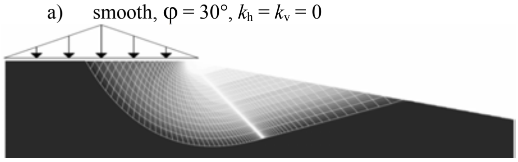
The results obtained for the seismic condition, analyzed via the pseudo-static approach, are described in terms of the seismic bearing capacity factor $N_{\gamma gE}$ and in terms of the corresponding corrective coefficient $g_{\gamma E}$ defined normalizing $N_{\gamma gE}$ with respect to the corresponding static value N_γ computed for an horizontal ground surface:

$$g_{\gamma E} = N_{\gamma gE} / N_\gamma \qquad (3)$$

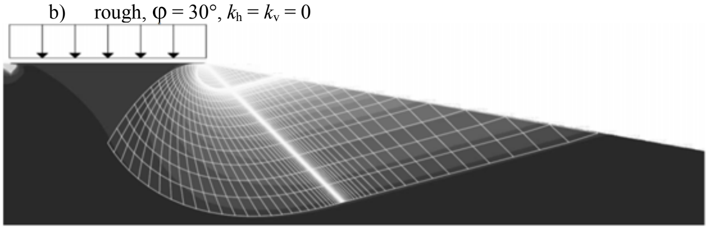
Some of the results, for both smooth and rough foundations, are shown in Figure 4 for the case $\varphi = 30°$, k_h up to 0.6 (with $\Omega = k_v/k_h = 0$ and $\pm$ 0.5) and β equal to 10° and 20°; for comparison, the values of $N_{\gamma gE}$ and $g_{\gamma E}$ computed for the case of foundation on horizontal ground ($\beta = 0$) are also presented. For the same cases with $k_v = 0$, Figure 2 c-h show the characteristic lines network. For both smooth and rough foundations, the effect of the soil inertia on the bearing capacity factor appears to be relevant whatever the slope angle β is (Fig. 4 a-b); specifically, regardless k_v, as k_h increases, $N_{\gamma gE}$ drastically reduces and it vanishes when k_h reaches a limit value $k_{h,lim}$ representing a generalized fluidification condition [12] for the case of sloping ground; according to [5], for $\beta = 0$ it is $k_{h,lim} = \tan\varphi/(1+\Omega\cdot\tan\varphi)$.



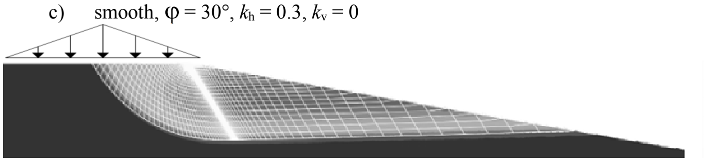
a) smooth, $\varphi = 30°$, $k_h = k_v = 0$



b) rough, $\varphi = 30°$, $k_h = k_v = 0$



c) smooth, $\varphi = 30°$, $k_h = 0.3$, $k_v = 0$



d) rough, $\varphi = 30°$, $k_h = 0.3$, $k_v = 0$

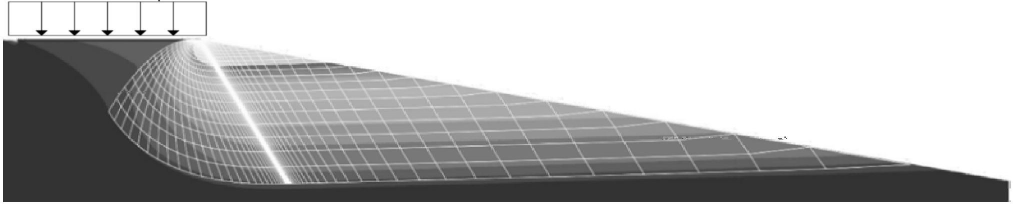


Fig. 3. Comparison between the characteristic lines network and the total incremental displacement contours obtained in the $N_{\gamma g}$ problem of a smooth (a,c) and of a rough (b,d) foundation, with $\varphi = 30°$, through static (a,b) and pseudo-static (c,d) *FE* analyses.

For a given value of k_h, the condition $k_v > 0$ (i.e. soil inertia forces directed upward) always leads to the largest reduction in the seismic bearing capacity factor $N_{\gamma gE}$, regardless the value of the slope angle β. Similar conclusions can be drawn for the corrective coefficient $g_{\gamma E}$ (Fig. 4 c, d). As an example, for a smooth foundation with a slope angle $\beta = 10°$ and $k_h = 0.15$, the bearing capacity factor reduces from its static value $N_{\gamma g} = 5.15$ to about 3.94 (with a reduction $\Delta N_{\gamma\beta E}$ of about 23%) and, since it is $N_\gamma = 7.65$, it is $g_{\gamma E} \approx 0.5$; if $\beta = 20°$ it is $N_{\gamma gE} = 1.67$, $\Delta N_{\gamma\beta E} \approx 46\%$ and $g_{\gamma E} \approx 0.22$.

In the case of a rough foundation ($N_\gamma = 14.71$) it is $N_{\gamma g} = 9.75$ and $N_{\gamma gE} = 7.63$ for $\beta = 10°$, while for $\beta = 20°$ it is $N_{\gamma g} = 5.72$ and $N_{\gamma gE} = 3.19$; all these values are significantly different from those obtained in the smooth case, however, it is again $g_{\gamma E} \approx 0.5$ for $\beta = 10°$ and $g_{\gamma E} \approx 0.22$ for $\beta = 20°$. Thus, also for the seismic loading conditions, the influence of the roughness of the foundation on the bearing capacity factor for the case of sloping ground is quite negligible. As in the static case, the seismic bearing capacity factor $N_{\gamma gE}$ and the corresponding corrective coefficient $g_{\gamma E}$ obtained with the proposed approach, were compared with those computed from the results of *FE* pseudo-static plane-strain analyses. For the cases previously discussed ($\varphi = 30°$, k_h up to 0.6, $\Omega = 0$ and $\pm$ 0.5, β equal 10° and 20°) the comparison is presented in Figure 4 a-b where the *FE* analyses results (empty circlets) are superimposed to the *MC* analyses result; with the exception of few cases, the comparison is satisfactory for any of the adopted values of the slope angle and of the seismic coefficients. The comparison with the *FE* analyses results is presented also in terms of characteristic lines network and contour of total incremental displacements (Fig. 3 c-d); again, it can be observed a good agreement between the results obtained with the two method of analysis since the stress and the displacement fields are practically coincident for both the smooth and the rough case. Finally, from the characteristic lines networks shown in Figure 2 c-h it can be observed that, for both smooth (Fig. 2 c, e, g) and rough (Fig. 2 d, f, h) foundations, as the inertia forces arising in the foundation soil increase, the characteristic lines are appreciably modified in the Cauchy domain, the Reimann zone degenerates into a line and larger plastic volumes are involved in the failure mechanism, even if its depth remain quite constant.

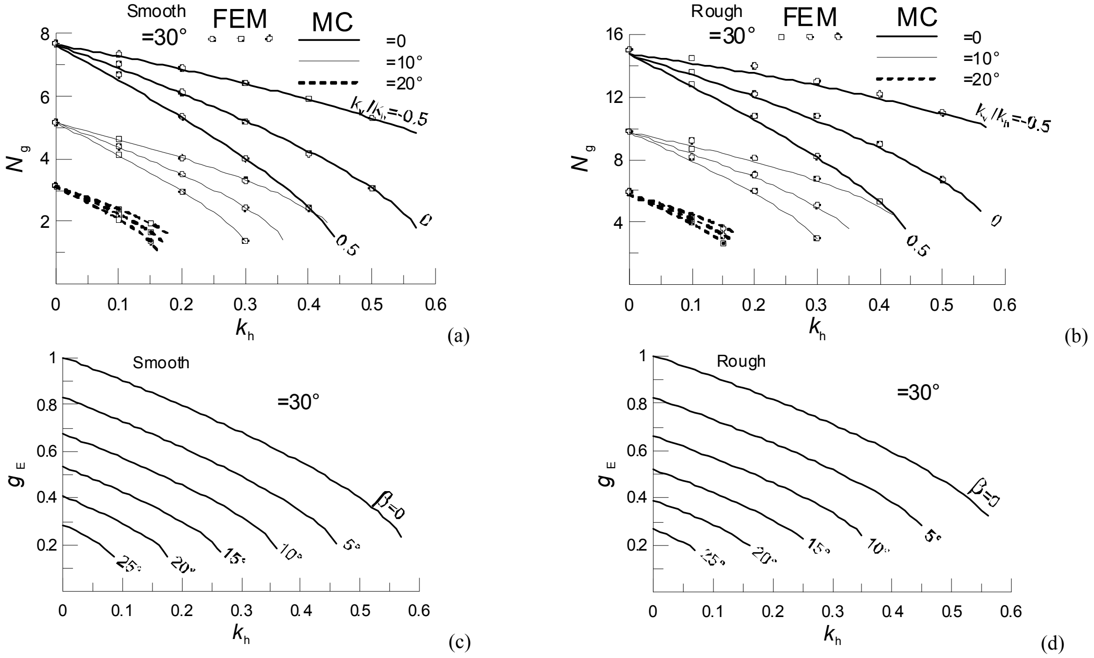


Fig. 4. Seismic bearing capacity factor (a,b) and corrective coefficient (c,d) for sloping ground evaluated for a smooth (a,c) and for a rough (b,d) foundation with φ = 30° and comparison with *FE* analysis results.

References

[1] G. Abate, C. Caruso, M.R. Massimino, M. Maugeri. Evaluation of shallow foundation settlements by an elasto-plastic kinematic-isotropic hardening numerical model for granular soil. Geomechanics and Geoengineering, 3(1) (2008) 27-40.

[2] J.P. Stewart, G.L. Fenves, R.B. Seed. Seismic soil-structure interaction in buildings. I: analytical methods. J. Geotechnical and Geoenvironmental Engineering, 125(1) (1999) 26-37.

[3] G. Abate, M.R. Massimino, M. Maugeri. Finite element modeling of a shaking table test to evaluate the dynamic behaviour of a soil-foundation system. AIP Conference Proceedings. V. 1020, PART 1 (2008) 569-576.

[4] G. Biondi, M.R. Massimino, M. Maugeri. Experimental study in the shaking table of the input motion characteristics in the dynamic SSI of a SDOF model. Bulletin of Earthquake Engineering, 13(6) (2015) 1835-1869.

[5] E. Cascone, O. Casablanca. Static and seismic bearing capacity of shallow strip footings. Soil Dynamics and Earthquake Engineering, 84 (2016) 204-223.

[6] K. Terzaghi K. Theoretical soil mechanics. John Wiley & Sons Inc., New York (1943).

[7] K.L. Fishman., Jr R. Richards, D. Yao. Inclination factors for seismic bearing capacity. J Geotech. Geoenviron. Eng. ASCE, 129(9) (2003)861–865.

[8] G. Biondi, E. Cascone, M. Maugeri M. Displacement versus pseudo-static evaluation of the seismic performance of sliding retaining walls. Bulletin of Earthquake Engineering, 12(3) (2014) 1239-1267.

[9] K. Sarma. Seismic bearing capacity of shallow strip footings adjacent to a slope. 2nd Int. Conf. Earthquake Geotechnical Engineering, Lisbon (1996) 176-184.

[10] D. Choudhury, K.S. Subba Rao. Seismic bearing capacity of shallow strip footings embedded in slope. Int. J. of Geomech., 6 (2006) 176-184.

[11] F. Castelli F., E. Motta. Bearing capacity of strip footings near slopes. Geotechnical and Geological Engineering, 28(29) (2010) 187-198.

[12] R. Richards Jr, D.G. Elms, M. Budhu. Dynamic fluidization of soils. J. Geotech. Eng., ASCE,116(5) (1990)740–59.