Reliability analysis of aero-engine blades considering nonlinear strength degeneration

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Abstract To comprehensively consider the effects of strength degeneration and failure correlation, an improved stress–strength interference (SSI) model is proposed to analyze the reliability of aero-engine blades with the fatigue failure mode. Two types of TC4 alloy experiments are conducted for the study on the damage accumulation law. All the parameters in the nonlinear damage model are obtained by the tension–compression fatigue tests, and the accuracy of the nonlinear damage model is verified by the damage tests. The strength degeneration model is put forward on the basis of the Chaboche nonlinear damage theory and the Griffith fracture criterion, and determined by measuring the fatigue toughness during the tests. From the comparison of two kinds of degeneration models based on the Miner’s linear law and the nonlinear damage model respectively, the nonlinear model has a significant advantage on prediction accuracy especially in the later period of life. A time-dependent SSI reliability model is established. By computing the stress distribution using the finite element (FE) technique, the reliability of a single blade during the whole service life is obtained. Considering the failure correlation of components, a modified reliability model of aero-engine blades with common cause failure (CCF) is presented. It shows a closer and more reasonable process with the actual working condition. The improved reliability model is illustrated to be applied to aero-engine blades well, and the approach purposed in this paper is suitable for any actual machinery component of aero-engine rotor systems.

1. Introduction

Aero-engine blades of a low pressure rotor are subjected to highly hostile working conditions. All the improvements in performance, life, and lightweight which manufacturers are interested in are based on not losing reliability. Therefore, it is so worthwhile to accurately estimate reliability of blades after their design is improved. The blades in this paper are made of TC4 (Ti-6Al-4V) alloy. This alloy is widely used in manufacturing blades and disks in the aviation field due to its excellent properties such as high specific strength, heat resistance, and corrosion resistance. For TC4 alloy that is used in blade manufacture, strength degeneration begins slowly. With the cycles increasing, the residual strength decreases rapidly during later stages of fatigue when failures always happen. From microscopic view, the influences caused by dislocations,
slips, and holes generated during the initial stage are too small. However, during the later period, the initial defects grow quickly under cyclic loading. With the propagation of cracks, the effective bearing area decreases rapidly and a failure happens at last.

There are two parts in the study on the strength degeneration of blades. One is the relationship between damage and loading cycles, and the other is the correlation between residual strength and damage.

From the structural fatigue point of view, strength degeneration is proportional to the rate of damage accumulation. The more accurately the damage accumulation process is described, the more precise strength degeneration can be obtained. We can see that it is necessary to consider which damage accumulation law should be used in the study on the regularity of strength degeneration. Among the structural fatigue damage analysis methods, the Palmgren-Miner linear damage model (LDM) is the most popular one. Nevertheless, damage is actually a load-dependent variable, and the contribution of stress below fatigue limit cannot be ignored in the damage accumulation. Based on that, Chaboche and Lesne formulated a nonlinear continuum damage model (CDM) which took the loads below fatigue limit into account.

There are many methods to solve the reliability problem, such as the Petri net method, the Monte-Carlo method, the neural network method, the Bayesian method, and the stress-strength interference (SSI) method. Fatigue failure is the most typical and common failure modes of aero-engine blades. To analyze the fatigue reliability, the SSI method is more suitable. Freudenthal and Gumbel proposed the famous SSI model which is the basis of mechanical structure reliability analysis. The core of the model is regarding stress and strength as random variables. On the premise of already obtaining the probability density functions (PDFs) of stress and strength, reliability is the probability that strength is higher than stress. Stress and strength here are generalized. Stress can be load, temperature, corrosion, etc. Strength can be fatigue strength, heat resistance, corrosion resistance, etc. The SSI method requires information on probability distributions of structure stress and strength. It has been approved that the distribution of material strength generally follows a normal distribution, which can be obtained from tests. Structural stress is related to several variables, such as structure geometry, material properties, working conditions, and external loads. Although stress is difficult to be expressed in mathematical formulas, it can be calculated by using the finite element (FE) method.

Huang and An introduced the application of the SSI method in mechanical structural reliability analysis. However, the traditional SSI method is just suitable for the condition that stress and strength are independent, but the fact is that strength is related to stress. Under continuous loading, the component strength degenerates. As the result of strength degeneration, the reduction of reliability structure is unenviable. Therefore, the assumption of strength independence may lead to error in reliability computation and overestimation of results. To accurately estimate structure reliability, the influence of strength degeneration must be taken into consideration.

With the background mentioned above, this paper develops an improved relationship between residual strength and loading cycles based on a nonlinear damage theory. Two types of fatigue experiments were conducted to obtain the parameters of the residual strength model and validate the nonlinear damage model. An improved SSI model was established to analyze aero-engine blade-disk system reliability. A significant improvement in the reliability calculation was achieved as a result of the introduction of the strength degeneration in the reliability model. An important advantage of the nonlinear residual strength model and the improved SSI model compared to classical approaches is their applicability to any actual component through the FE technique.

2. Nonlinear damage model

According to characteristics of fatigue damage, Chaboche and Lesne proposed a model to describe the component performance degeneration:

\[
\delta D = f(D, \sigma)\delta n
\]

where \(D\) is damage, \(\sigma\) the stress, \(\delta D\) the damage increment, and \(\delta n\) the loading cycle increment.

With regard to the uniaxial fatigue, Chaboche and Lesne presented the following equation on the basis of Eq. (1):

\[
\delta D = \left[1 - (1 - D)^{\beta+1}\right] \left[\frac{\sigma_a}{M_0(1 - b\sigma_m)(1 - D)}\right]^\beta \delta n
\]

where \(\beta\), \(M_0\) and \(b\) are the material parameters, \(\sigma_m\) the mean stress, \(\sigma_a\) the stress amplitude, and \(x\) a parameter related to loadings and damage. Chaboche and Lesne gave the expression of \(x\):

\[
x = 1 - H\left(\frac{\sigma_a - \sigma_l}{\sigma_h - \sigma_a}\right)
\]

where \(\sigma_l < \sigma < \sigma_h\).

From the point of view of metal damage, it is an irreversible process that initial cracks propagate and fail at last. When \(D\) is equal to 0, there is no damage; and when \(D\) is equal to 1, failure happens. When \(D\) is below 1, loading cycles \(n\) is less than fatigue life \(N_t\). The equation of \(n\) and \(D\) is obtained by integrating Eq. (2):

\[
n = \frac{1}{1 - x} \left[1 + \beta\left(\frac{M_0(1 - b\sigma_m)}{\sigma_a}\right)^\beta \left[1 - (1 - D)^{\beta+1}\right]^{1-x}\right]
\]

The model above has been proved in some early work. Then the damage equation is derived as follows:

\[
D = 1 - \left\{1 - \left[\frac{\sigma}{M_0} \left(1 + \beta\right)\left(\frac{\sigma_a}{1 - b\sigma_m}\right)^\beta\right]^{\frac{1 - H(x)}{n}}\right\}^{\frac{1}{1+\beta}}
\]

where \(\sigma = (\sigma_l - \sigma_u)/(\sigma_h - \sigma_u)\), \(\sigma_u\) is fatigue limit under symmetric cyclic loading, and the parameters in Eq. (5) can be obtained from fatigue tests.

3. Experiments

Two types of fatigue tests were conducted, one was tension-compression fatigue tests, and the other was damage tests. Specimens used in both types of tests were the same, and the
geometry of the specimens is shown in Fig. 1. The test section of the specimens was approximately 8 mm in diameter and 44 mm in length. The heat treatments were as follows: annealed at 720 °C, insulated for 1 h, and air cooled. The static stretching test results show that the tensile limit is 1005 MPa, the yield limit 975 MPa, the elongation 16%, and the reduction of area 46%.

The tension–compression fatigue tests were conducted in order to obtain the fatigue limit and the parameters in Eq. (5). The tests were designed on the basis of GB/T 3075—2008, and performed at stress ratio $R$ of 0.1. A sinusoidal loading at the cyclic frequencies of 80–250 Hz was applied.

By using an up and down method, the experimental results of un-notched rods were obtained and presented in Table 1.

The damage tests were conducted to study the relationship between $D_n$ (the damage after $n$ loading cycles) and $n$. According to Ref.16, the damage degree could be measured by the variable of the internal energy of the alloy. Therefore, the damage $D$ was represented by using the variation of toughness degradation which was changing during the fatigue progress. Gu and An17 proposed the equation below to describe the relationship between damage and material toughness:

$$D_n = 1 - \frac{U_n}{U}$$  \hspace{1cm} (6)

where $U$ is toughness of the non-damaged material, and $U_n$ the material toughness after $n$ loading cycles.

Based on Eq. (6), when the damage degree is under a certain fatigue condition, we just need to determine the material tension curve and compare it with the original one. This method is easy to be realized in engineering, and precision can be guaranteed. Meanwhile, because this damage measurement could operate well without measuring the cyclic hysteresis energy and its cumulative value, it could be applied in both high and low cycle fatigue.

The damage tests were also designed according to GB/T 3075—2008. Both the experimental equipment and condition were the same as in the tension–compression tests. The damage tests under $\sigma_n = 700$ MPa were carried out with recycle ratio $n/N_f$ as measurement. Damage results were recorded by recycle ratio from 0.1 to 0.9 respectively.

In order to verify the accuracy of the damage model proposed in this paper, an FE model of the rod specimen built by MSC. Patran (see Fig. 2) was used to calculate the fatigue life and damage. The stress distribution is shown in Fig. 3.

From the results of stress distribution, it can be seen clearly that the center of test section bears the highest stress in the whole loading cycles. As a result, the damage here is the largest in the rod in every single cycle, and so would be the damage accumulation during the whole life. Then, by making use of the CDM proposed in this paper, the damage calculation was taken by recycle ratio from 0.1 to 1.0.

Fig. 4 is the comparison of damage prediction results between CDM and tests. From the comparison, we can see that the damage accumulation ratio is low in most of the time from

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Results of tension–compression tests ($R = 0.1$).</th>
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<tbody>
<tr>
<td>Fatigue limit/MPa</td>
<td>$\beta$</td>
</tr>
<tr>
<td>660.57</td>
<td>1.312</td>
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</tbody>
</table>
the early to middle stage. When the recycle ratio is above 0.7, the damage accumulates more quickly. Particularly when the ratio is close to 1.0, the damage curve becomes almost a vertical line. This phenomenon matches the real fatigue process of TC4.

Due to the high propagation speed, the failure happens too fast to control, so the error of the damage results during this period is the largest between test and computation. Although the mean error is 2.65%, it still shows a great accuracy of the model.

4. Strength degeneration and residual strength

4.1. Linear strength degeneration law

Usually, damage is defined as a dimensionless quantity. When a material is untapped, the damage is zero; when the material fails, the damage is one. It is an irreversible process that the damage of blades grows from zero to one. In this process, the residual strength decreases with the damage accumulation, and both the remaining life and reliability reduce at the same time. The residual strength is a function of load and loading cycles. It is an equivalent variable which reflects the inside damage of structures or materials in micro-scale. Generally, it is hard to obtain the residual strength from fatigue tests, but the regularity of the strength degeneration is able to be presented by damage. Ref. 18 established the relationship between residual strength and damage:

\[ r(n) = r_0[1 - D(n)]^c \]  

where \( r(n) \) is the residual strength of the material under \( n \) loading cycles, \( r_0 \) the initial strength, and \( c \) the material property.

Some earlier researchers used linear damage law, such as the Miner’s law, to describe the progress of material strength degeneration, which was easier and cost less computational work. Ref. 18 put forward a degeneration equation as

\[ r(n) = r_0 \left(1 - \frac{n}{N_f}\right)^a \]  

where \( a \) is equal to \( r_0 \).

Then the relationship between residual strength and loading cycles could be established. By substituting Eq. (11) into Eq. (5), the residual strength expression is

\[ \frac{r(n)}{r_0} = U \left( 1 - \frac{nH(\sigma)(1 + \beta)}{M_0^b} \left( \frac{\sigma_s}{1 - b\sigma_m} \right)^{\beta} \right)^{-\frac{b}{1+\beta}} \]  

From Eq. (10) we can see that the residual strength is a function of load and loading cycles. It would bring a big error if we assume that load and strength are independent.

The strength degeneration curves in 700 MPa by using the linear and nonlinear models respectively are plotted in Fig. 5. It can be seen that the strength curve of the nonlinear strength degeneration model is proportional to the damage accumulation curve, and the strength degeneration accelerates when \( n \) is close to \( N_f \). The curve of the linear model is a line during the whole service life which means the degeneration is a uniform process.

From the microscopic point of view, damage accumulation is a relatively slow process at low loading levels, and the strength reduction is slow too. The accumulation speed would accelerate in the last period of the life. Then cracks appear and propagate in a short time till failure happens, and the residual strength decreases rapidly at the same time.

In other words, the residual strength would stay at a relatively high level until crack initiation. Comparing with the linear model, the nonlinear curve has an obvious advantage in prediction accuracy.

5. Reliability analysis of blade-disk system

5.1. Stress–strength interference model

The stress–strength analysis is a tool widely used in reliability engineering. According to the SSI model, the definition of reliability is the probability that stress is lower than strength. Stress and strength here are both generalized. Stress can be any factor causing failures, such as mechanical load, temperature, humidity, and corrosion. Strength can be any resistance, such as fatigue strength, heat resistance, water logging resistance, and corrosion resistance.

The traditional stress–strength model is shown as follows:

\[ R = P(t > s) = \int_{-\infty}^{\infty} f_1(s) \int_{s}^{\infty} f_i(r) dr ds \]  

Fig. 5 Comparison between the two degeneration models.
where \( f_s(r) \) is the PDF of the component strength, and \( f_s(s) \) the PDF of the stress. Eq. (13) is the general reliability function under single failure mode. The equation is based on the assumption that load and strength are independent, and strength degeneration and loading cycles are not taken into consideration.

\[
R(t) = \exp \left\{ - \int_{0}^{t} \lambda e^{-\sum_{s=0}^{\infty} \frac{x_s^2}{2}} \int_{-\infty}^{x_s} \int_{-\infty}^{\infty} f_s(s) f_s(r, s, i) ds dr \right\}
\]

where \( f_s(r, s, i) \) is the PDF of the strength under \( i \) cycles.

According Eq. (12), the expectation of the residual strength is

\[
r(n) = \sigma \left\{ \frac{U}{U_0} \int_{-\infty}^{\infty} \left( 1 - \frac{M_r^0 (1 - b \sigma_0)^{h(\sigma)}}{n H(\sigma) (1 + \beta) f^o(\sigma)} \right)^{-(1+\beta)} \delta\sigma \right\}^{\frac{1}{\beta}}
\]

Here the number of loading cycles is regarded as a poisson process with intensity \( \lambda \), and the reliability during 0 to \( t + \Delta t \) is

\[
R(t + \Delta t) = R(t) \left\{ \sum_{x=0}^{\infty} P(n = x) \left[ 1 - \lambda \Delta t + \lambda \Delta t P(r(x) > x) \right] \right\}
\]

\[
= R(t) \left\{ \sum_{x=0}^{\infty} P(n = x) \left[ 1 - \lambda \Delta t + \lambda \Delta t F_s(r(n)) \right] \right\}
\]

Eq. (16) can be simplified as

\[
R(t + \Delta t) = R(t) \left\{ 1 - \lambda \Delta t \sum_{x=0}^{\infty} \frac{\lambda^x e^{-\lambda}}{x!} [1 - F_s(r(n))] \right\}
\]

When \( \Delta t \) is sufficiently small, Eq. (17) can be transformed into a differential equation about \( R(t) \):

\[
\frac{dR(t)}{R(t) dt} = \frac{R(t + \Delta t) - R(t)}{R(t) \Delta t} = -\lambda e^{-\sum_{x=0}^{\infty} \frac{\lambda^x}{x!} (1 - F_s(r(n)))}
\]

By solving Eq. (18), the reliability is shown as follows:

\[
R(t) = \exp \left\{ - \int_{0}^{t} \lambda e^{-\sum_{s=0}^{\infty} \frac{x_s^2}{2}} \int_{-\infty}^{x_s} \int_{-\infty}^{\infty} f_s(s) f_s(r, s, i) ds dr \right\}
\]

5.2. Reliability of blades

The height of the blade is 605 cm, and the connection of the blade and disk is a fir tree dovetail joint. Compared with the actual blade, the solid model in this study was established with two simplifications to decrease the computation cost:

1. The disk was simplified as a board and the joint as an interface.
2. In order to make meshes uniform in the height direction, the elements in the tip were tetrahedral and others were hexahedral.

Finally, the blade model (Fig. 6(a)) was meshed into 2167 elements.

To calculate the reliability of a blade, a working cycle should be defined to characterize the damage and residual strength. A complete mission of a civil aircraft (including takeoff, climbing, cruising, approaching, and landing) can be seen as the engine speed rises from zero to the cruising speed and then decreases to zero again. To simplify the calculation, the engine speed curve was taken as a symmetrical curve of the cruise condition, and the specific varying process (takeoff-climbing-cruise) was studied here which could represent the several changing working conditions. Within this paper, according to the actual process from takeoff to cruise of a civil aircraft, the duration was set to 10 min. In this duration, the
rotor speed average accelerated from 0 to 4000 r/min. The calculation step was set to 0.2 s. Within 600 s, the load in 3000 moments was calculated to construct a load-time history. The damage and residual strength were calculated in cycles, while the reliability was usually indicated in hours. From the definition of a computation cycle, 1 cycle could be transformed into 1/6 h.

The initial strength of every element was assumed to be the same, so the element under the maximum load in the whole cycle would fail first. For this reason, we used the maximum stress to be the equivalent stress. Fig. 6(b) shows the distribution of the stress on the blade, and we can see that the inlet flow near the root bears the maximum stress (226 MPa).

The time-dependent reliability $R_{\text{blade}}(t)$ was computed and the result is shown in Fig. 7.

We can see that the reliability is above 0.99 from 0 to 4000 h, and decreases slowly in these stages. When the time is during 4000–7000 h, the reliability decreases more rapidly than during the first 4000 h, but is still above 0.9. The reliability decreases fast to 0.1 in the last stages of service life.

It can be seen clearly that the tendency of the reliability curve is related with the strength degeneration of the component. When the residual strength of the component is at a high level, the reliability reduces slowly. Once the component is in the rapid failure stage, the reliability reduces fast with the strength degeneration.

5.3. Common cause failure (CCF) analysis

All the blades mounted on one disk rotate together at the same speed when an aero-engine is working, so the loads are also the same. The two main kinds of loads on the blades are air force and centrifugal force, which are the most important reason that causes fracture failure. In order to propose the reliability model of blades in considering the failure correlation, some assumptions are put forward as follows:

1. The life probability distribution is the same when blades fail independently.
2. There is more than one failure process, and the different ones are independent.
3. The failure rate depends on the number of failed blades.

The blades compose a typical parallel system shown in Fig. 8. Assuming the initial strength of blades is the same, a parallel system reliability model which takes CCF into consideration is established.

To analyze the reliability of the blades, the reliability of the parallel system composed of 53 components was computed. For a parallel system, when there is the same load, the reliabilities with and without CCF, respectively, are

$$R_{\text{par}}(t) = \frac{1}{k} \left( R_i(r) \right)^k$$

$$R_{\text{par}}(t) = 1 - \left( F_i(r) \right)^k$$

where $k$ is the number of components.

Fig. 9 is the comparison between the calculation results of the reliability model with and without CCF.

From Fig. 9, it can be observed that the reliability calculated with CCF is lower than the one without CCF. The conclusion is that the reliability of a parallel blade system with the consideration of CCF is lower than that with the assumption of failure independence.

6. Conclusions

1. Comparing with the classical linear method, the nonlinear strength degeneration model proposed in this paper has significant advantages in both accuracy and maneuverability of residual strength prediction.
2. The prediction results show that the reliability of a single blade is above 0.99 from 0 to 4000 h, and still above 0.9 from 4000 to 7000 h.
3. The reliability with CCF is lower than that without CCF.
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