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Pseudo Excitation Method and Some Recent Developments

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Abstract

Pseudo Excitation Method (PEM) as a highly efficient and accurate algorithm series of stationary and non-stationary random vibration of structures has been widely used in China. Some well- concerned problems such as the seismic analysis of long-span bridges subjected to differential ground excitations, wind-induced flutter-buffet analysis of slender structures based on 3D finite element modeling have been solved and applied in some important projects taking place in China. Previously, bridges with the main span longer than 150 m were excluded in the 《Specifications of Earthquake Resistant Design for Highway Engineering 》(1989). Since 2008, this official document was replaced by 《Guidelines for Seismic Design of Highway Bridges》, in which PEM is recommended as a basic tool for seismic analysis of long-span bridges. In the design of high speed trains, random vibration can not be effectively analyzed by the traditional approach. The PEM has recently been used instead and some important progresses have been obtained.

Keywords: Random vibration; Pseudo-excitation method; Long-span bridge; High speed train

1. Introduction

In the design of long-span bridges, the spatial effects of earthquakes, including the wave passage effect, the incoherence effect, and the local site effect, must be taken into account（Kiureghian and Neuenhofer 1992）. The random vibration method can fully account for the statistical nature as well as the spatial effects of earthquakes, and so has been widely regarded as a very promising method. Unfortunately the very low computational efficiency has become the bottle-neck of its practical use.

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In the past 30 years, a great number of civil engineering projects, e.g. dams and long-span bridges, have been carried out in China, many of them are located in earthquake regions. Over the past 20 years, a very efficient method, known as the pseudo-excitation method (PEM), to cope with the above computational difficulty have been developed. This method can easily compute the 3D random seismic responses of long-span bridges using finite element models with up to thousands of degrees of freedom on a small personal computer, in which the seismic spatial effect is accounted for accurately. This method is now being applied and developed in China by a great number of scholars. The recently published official document "Guidelines for Seismic Design of Highway Bridges" JTG/T B01-01-2008 recommends the PEM as a basic tool for seismic analysis of long-span bridges.

The method is also introduced by a whole chapter in the "Vibration and Shock Handbook" published by CRC Press (US) in 2005. Owing to its extensive applications in the Chinese civil engineering industry, the PEM has become an important part of the random vibration courses in some Chinese universities and colleges.

2. Pseudo excitation method for structural random vibration analysis

2.1 Structures subjected to stationary random excitations

Consider a linear system subjected to a zero-mean stationary random excitation with a given PSD $S_{xx}(\omega)$. Suppose that for two arbitrarily selected responses $y(t)$ and $z(t)$, the auto-PSD $S_{yy}(\omega)$ and cross-PSD $S_{yz}(\omega)$ are desired. If $H_y(\omega)$ and $H_z(\omega)$ are the corresponding frequency response functions, and if $x(t)$ is replaced by a sinusoidal excitation

$$\tilde{x} = \sqrt{S_{xx}(\omega)}\exp(i\omega t) \tag{1}$$

the responses of $y(t)$ and $z(t)$ would be $\tilde{y} = \sqrt{S_{xx}(\omega)}H_y(\omega)\exp(i\omega t)$ and $\tilde{z} = \sqrt{S_{xx}(\omega)}H_z(\omega)\exp(i\omega t)$. It can be readily verified that (Lin, Zhang and Li 1994)

$$\begin{aligned}\tilde{y}^*\tilde{y} &= \sqrt{S_{xx}(\omega)}H_y^*(\omega)\exp(-i\omega t)\cdot\sqrt{S_{xx}(\omega)}H_y(\omega)\exp(i\omega t)\\ &= \left|H_y(\omega)\right|^2 S_{xx}(\omega) = S_{yy}(\omega)\end{aligned} \tag{2}$$

$$\begin{aligned}\tilde{y}^*\tilde{z} &= \sqrt{S_{xx}(\omega)}H_y^*(\omega)\exp(-i\omega t)\cdot\sqrt{S_{xx}(\omega)}H_z(\omega)\exp(i\omega t)\\ &= H_y^*(\omega)S_{xx}(\omega)H_z(\omega) = S_{yz}(\omega)\end{aligned} \tag{3}$$

If $\boldsymbol{y}(t)$ and $\boldsymbol{z}(t)$ are two arbitrarily selected random response vectors of the structure, and $\tilde{\boldsymbol{y}} = \boldsymbol{a}_y \exp(i\omega t)$ and $\tilde{\boldsymbol{z}} = \boldsymbol{a}_z \exp(i\omega t)$ are the corresponding harmonic response vectors due to the pseudo excitation (1), it can also be proved that the PSD matrices of $\boldsymbol{y}(t)$ and $\boldsymbol{z}(t)$ are

$$\boldsymbol{S}_{yy}(\omega) = \tilde{\boldsymbol{y}}^*\tilde{\boldsymbol{y}}^T = \boldsymbol{a}_y^*\boldsymbol{a}_y^T \tag{4}$$

$$\boldsymbol{S}_{yz}(\omega) = \tilde{\boldsymbol{y}}^*\tilde{\boldsymbol{z}}^T = \boldsymbol{a}_y^*\boldsymbol{a}_z^T \tag{5}$$

This means that the auto- and cross-PSD functions of two arbitrarily selected random responses can be computed using the corresponding pseudo harmonic responses.

Consider a linear structure be subjected to a number of stationary random excitations, which are denoted as an *m* dimensional stationary random process vector $\boldsymbol{x}(t)$ with known PSD matrix $\boldsymbol{S}_{xx}(\omega)$. It is a Hermitian matrix and so it can be decomposed, e.g. by using its eigenpairs $\boldsymbol{\psi}_j$ and d_j $(j = 1,2,...,r)$, into

$$\boldsymbol{S}_{xx}(\omega) = \sum_{j=1}^{r} d_j \boldsymbol{\psi}_j^* \boldsymbol{\psi}_j^T \quad (r \le m) \tag{6}$$

in which r is the rank of $\boldsymbol{S}_{xx}(\omega)$. Next, constitute r pseudo harmonic excitations

$$\tilde{\boldsymbol{x}}_j(t) = \sqrt{d_j}\boldsymbol{\psi}_j \exp(i\omega t) \quad (j = 1, 2, \cdots, r) \tag{7}$$

By applying each of these pseudo harmonic excitations, two arbitrarily selected response vectors $\boldsymbol{y}_j(t)$ and $\boldsymbol{z}_j(t)$ of the structure, which can be displacements, internal forces or other linear responses, may be easily obtained and expressed as

$$\tilde{\boldsymbol{y}}_j(t) = \boldsymbol{a}_{yj}(\omega)\exp(i\omega t) \tag{8}$$

$$\tilde{\boldsymbol{z}}_j(t) = \boldsymbol{a}_{zj}(\omega)\exp(i\omega t) \tag{9}$$

The corresponding PSD matrices can be computed by means of the following formulas (Lin et al. 1994)

$$\boldsymbol{S}_{yy}(\omega) = \sum_{j=1}^{r} \tilde{\boldsymbol{y}}_j^*(t)\tilde{\boldsymbol{y}}_j^T(t) = \sum_{j=1}^{r} \boldsymbol{a}_{yj}^*(\omega)\boldsymbol{a}_{yj}^T(\omega) \tag{10}$$

$$\boldsymbol{S}_{yz}(\omega) = \sum_{j=1}^{r} \tilde{\boldsymbol{y}}_j^*(t)\tilde{\boldsymbol{z}}_j^T(t) = \sum_{j=1}^{r} \boldsymbol{a}_{yj}^*(\omega)\boldsymbol{a}_{zj}^T(\omega) \tag{11}$$

The way used to decompose $\boldsymbol{S}_{xx}(\omega)$ into the form of eqn (105) is not unique. In fact, the Cholesky scheme is perhaps the most efficient and convenient way to do it, i.e. $\boldsymbol{S}_{xx}(\omega)$ is decomposed into

$$\boldsymbol{S}_{xx}(\omega) = \boldsymbol{L}^* \boldsymbol{D} \boldsymbol{L}^T = \sum_{j=1}^{r} d_j \boldsymbol{l}_j^* \boldsymbol{l}_j^T \quad (r \le m) \tag{12}$$

2.2 Structures subjected to non-stationary random excitations

Consider a linear system subjected to an evolutionary random excitation

$$f(t) = g(t)x(t) \tag{13}$$

in which $g(t)$ is a slowly varying modulation function, while $x(t)$ is a zero-mean stationary random process with auto-PSD $S_{xx}(\omega)$. The deterministic functions $g(t)$ and $S_{xx}(\omega)$ are both assumed to be given.

In order to compute the PSD functions of various linear responses due to the action of $f(t)$, the pseudo excitation has the form

$$\tilde{f}(\omega, t) = g(t)\sqrt{S_{xx}(\omega)}\exp(i\omega t) \tag{14}$$

Suppose that $\boldsymbol{y}(t)$ and $\boldsymbol{z}(t)$ are two arbitrarily selected response vectors and $\tilde{\boldsymbol{y}}(\omega,t)$ and $\tilde{\boldsymbol{z}}(\omega,t)$ are the corresponding transient responses due to the pseudo excitation $\tilde{f}(\omega,t)$ with the structure initially at rest. It has been proved that (Lin et al. 1994, 2005)

$$\boldsymbol{S}_{yy}(\omega,t)=\tilde{\boldsymbol{y}}^{*}(\omega,t)\tilde{\boldsymbol{y}}^{T}(\omega,t) \tag{15}$$

$$\boldsymbol{S}_{yz}(\omega,t)=\tilde{\boldsymbol{y}}^{*}(\omega,t)\tilde{\boldsymbol{z}}^{T}(\omega,t) \tag{16}$$

For cases with fully coherent excitations, partially coherent excitations, and non-uniformly modulated evolutionary random excitations, the corresponding pseudo-excitation algorithms are very similar to those for the stationary random excitation cases (Lin et al. 1994, 2005) .

3. Some applications of PEM in China

Seismic Analysis of No.2 Nanjing Yangtze River Bridge at Nancha (Figure 1). It is a five-span dual-tower cable stayed bridge with the total length of 1238m. Three hundred modes were used for mode-superposition when using PEM (Fan , Wang and Chen, 2001).

Seismic analysis of Great Sutong Bridge (Figure 2). The total length of this bridge is 8206 m, its main span is 1088m, being the longest cross-sea bridge in the world (Zhao, Tang and Xu ,2007) .

Seismic analysis of Xiao-Wan arch dam (Figure 3). It has a height of 292m, is the highest arch dam in the world (Chen, 2008)

Seismic Analysis of Xiluodu Hydro-power Station (Figure 4). It takes the form of double curvature arch dam, with the height of 278m. This is the third largest hydro-power station in the world. The seismic analysis was performed by researchers at Tsinghua University using PEM(Wu, Jin and Wu, 2005)

Wind-induced flutter-buffet analysis of Hong Kong Tsing-Ma suspension bridge(Figure 5). The main span of this bridge is 1377m The full 3D FEM-based whole bridge flutter-buffet analysis was completed in 1998 using the PEM with an FE model of 1010 elements, 769 nodes and 2254 DOFs. (Xu,Sun and Ko, 1998)

Ride-comfort analysis of Shanghai maglev trains(Figure 6) . In order to facilitate the riding quality analysis of maglev trains subjected to multi-point coherent excitations, a new analysis method was proposed based on pseudo-excitation algorithm by Zhou Jin-song, Li Da-guang, Shen Gang, etc. at Institute of Railway and Urban Mass Transit, Tongji University (Zhou, Li, etc, 2008)

Coupled train-bridge random vibration analysis of high-speed trains(Figure 7). Train-Bridge coupled random vibration caused by the rail surface roughness has received special attention in the development process of high-speed trains. The PEM is playing an important role for the development of Chinese high-speed trains (Zhang,Lin and Zhang,2010)

Ice-induced random vibration analysis of platforms in the Bohai gulf (Figure 8). Jacket platforms in the Bohai Gulf are often victims of random ice induced vibrations. The PEM has recently been used in the investigation of ice-resistant jacket platforms (Liu, Li, Oberlies and Yue, 2009)

Figure 1: No.2 Nanjing Yangtze River Bridge

Figure 2: Great Sutong Bridge

Figure3. Xiluodu arch dam

Figure4. Xiaowan arch dam

Figure 5. Hong Kong Tsing-Ma bridge

Figure 6: Shanghai maglev train

Figure 7: China high-speed train

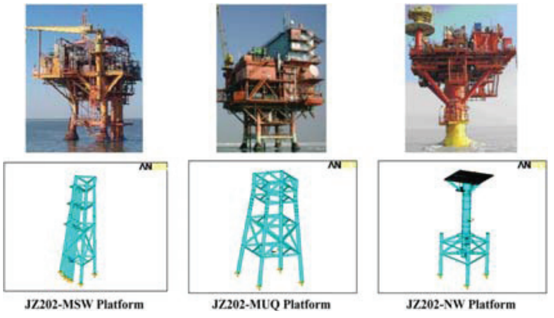

Figure 8: Ice resistant platforms in bohai gulf

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