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Models for little rip dark energy

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A B S T R A C T

We examine in more detail specific models which yield a little rip cosmology, i.e., a universe in which the dark energy density increases without bound but the universe never reaches a finite time singularity. We derive the conditions for the little rip in terms of the inertial force in the expanding universe and present two representative models to illustrate in more detail the difference between little rip models and those which are asymptotically de Sitter. We derive conditions on the equation of state parameter of the dark energy to distinguish between the two types of models. We show that coupling between dark matter and dark energy with a little rip equation of state can alter the evolution, changing the little rip into an asymptotic de Sitter expansion. We give conditions on minimally coupled phantom scalar field models and on scalar-tensor models that indicate whether or not they correspond to a little rip expansion. We show that, counterintuitively, despite local instability, a little rip cosmology has an infinite lifetime.

1. Introduction

The current acceleration of the universe is often attributed to dark energy, an unknown fluid with effective equation of state (EoS) parameter w close to -1. The observational data [1] favor ΛCDM with $w = -1$. However, phantom ($w < -1$) or quintessence ($-1/3 > w > -1$) dark energy models are not excluded by observational data [2]. In both cases, it is known that the universe may evolve to a finite time future singularity. Phantom dark energy models can lead to a singularity in which the scale factor and density become infinite at a finite time; such a singularity is called a big rip [3,4], or Type I singularity [5]. For quintessence dark energy, one can have a singularity for which the pressure goes to infinity at a fixed time, but the scale factor and density remain finite; this is called a sudden singularity [6,7], or a Type II singularity [5]. Alternately, the density and pressure can both become infinite with a finite scale factor at a finite time (a Type III singularity), or higher derivatives of the Hubble parameter H can diverge (a Type IV singularity) [5]. The occurrence of a singularity at a finite time in the future may lead to some inconsistencies. Several scenarios to avoid a future singularity have been proposed so far: coupling with dark matter [8], inclusion of quantum effects [9], additional changes in the equation of state [10] or special forms of modified gravity [10].

Recently, a new scenario to avoid a future singularity has been proposed in Ref. [11]. In this scenario, w is less than -1, so that the dark energy density increases with time, but w approaches -1 asymptotically and sufficiently rapidly that a singularity is avoided. This proposed non-singular cosmology was called a "little rip" because it leads to a dissolution of bound structures at some point in the future (similar to the effect of a big rip singularity). It can be realized in terms of a general fluid with a complicated EoS [5,12]. The evolution of the little rip cosmology is close to that of ΛCDM up to the present, and is similarly consistent with the observational data.

The present Letter is devoted to further study of the properties of the little rip cosmology. In the next section, the inertial force interpretation of the little rip is developed, and it becomes clear why a dissolution of bound structures occurs. Coupling of the little rip fluid with dark matter is considered in Section 3. It is shown that as the result of such a coupling an asymptotically de Sitter universe

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can eventually evolve to have a little or big rip. In Section 4, the little rip cosmology is reconstructed in terms of scalar field models. Our results are summarized in Section 5.

2. Inertial force interpretation of the little rip

As the universe expands, the relative acceleration between two points separated by a comoving distance l is given by $l\ddot{a}/a$, where a is the scale factor. An observer a comoving distance l away from a mass m will measure an inertial force on the mass of

$$F_{\mathrm{iner}} = ml\ddot{a}/a = ml\big(\dot{H} + H^2\big). \tag{1}$$

Let us assume the two particles are bound by a constant force F_0. If F_{iner} is positive and greater than F_0, the two particles become unbound. This is the "rip" produced by the accelerating expansion. Note that Eq. (1) shows that a rip always occurs when either H diverges or $\dot{H}$ diverges (assuming $\dot{H} > 0$). The first case corresponds to a "big rip" [13], while if H is finite, but $\dot{H}$ diverges with $\dot{H} > 0$, we have a Type II or "sudden future" singularity [6,7,5], which also leads to a rip. However, as noted in Ref. [11], it is possible for H, and therefore, F_{iner}, to increase without bound and yet not produce a future singularity at a finite time; this is the little rip. Both the big rip and little rip are characterized by $F_{\mathrm{iner}} \to \infty$; the difference is that $F_{\mathrm{iner}} \to \infty$ occurs at a finite time for a big rip and as $t \to \infty$ for the little rip.

An interesting case occurs when H is finite and $\dot{H}$ diverges but is negative. In this case, even though the universe is expanding, all structures are crushed rather than ripped. An example is given by

$$H = H_0 + H_1(t_c - t)^{\alpha}. \tag{2}$$

Here H_0 and H_1 are positive constants and α is a constant with $0 < \alpha < 1$.

By using the FRW equations

$$\frac{3}{\kappa^2}H^2 = \rho, \qquad -\frac{1}{\kappa^2}\big(2\dot{H} + 3H^2\big) = p, \tag{3}$$

we may rewrite (1) in the following form:

$$F_{\mathrm{iner}} = -\frac{ml\kappa^2}{6}(\rho + 3p). \tag{4}$$

Here $\kappa^2 = 8\pi G$ and G is Newton's gravitational constant. Not surprisingly, we see that the inertial force is sourced by the quantity $\rho + 3p$. Then if we consider the general equation of state,

$$p = -\rho + f(\rho), \tag{5}$$

we find

$$F_{\mathrm{iner}} = \frac{ml\kappa^2}{6}\big(2\rho - 3f(\rho)\big). \tag{6}$$

As noted in Ref. [11], when $w \to -1$ but $w < -1$, a rip can occur without a singularity. If we ignore the contribution from matter, the equation of state (EoS) parameter w of the dark energy can be expressed in terms of the Hubble rate H as

$$w = -1 - \frac{2\dot{H}}{3H^2}. \tag{7}$$

Then if $\dot{H} > 0$, we find $w < -1$.

Now consider the following example:

$$H = H_0 e^{\lambda t}. \tag{8}$$

Here H_0 and λ are positive constants. Eq. (8) tells us that there is no curvature singularity for finite t. By using Eq. (7), we find

$$w = -1 - \frac{2\lambda}{3H_0}e^{-\lambda t}, \tag{9}$$

and therefore $w < -1$ and $w \to -1$ when $t \to +\infty$, and w is always less than -1 when $\dot{H}$ is positive. From Eq. (1), we have

$$F_{\mathrm{iner}} = ml\big(\lambda H_0 e^{\lambda t} + H_0^2 e^{2\lambda t}\big), \tag{10}$$

which is positive and unbounded. Thus, F_{iner} becomes arbitrarily large with increasing t, resulting in a little rip.

As another example, consider the model:

$$H = H_0 - H_1 e^{-\lambda t}. \tag{11}$$

Here H_0, H_1, and λ are positive constants and we assume $H_0 > H_1$ and $t > 0$. Since the second term decreases when t increases, the universe goes to asymptotically de Sitter space–time. Then from Eq. (7), we find

$$w = -1 - \frac{2\lambda H_1 e^{-\lambda t}}{3(H_0 - H_1 e^{-\lambda t})^2}. \tag{12}$$

As in the previous example, $w < -1$ and $w \to -1$ when $t \to +\infty$. For H given by Eq. (11), however, the inertial force, given by (1), is

$$F_{\mathrm{iner}} = ml\{\lambda H_1 \mathrm{e}^{-\lambda t} + (H_0 - H_1 \mathrm{e}^{-\lambda t})^2\}, \tag{13}$$

which is positive but bounded and $F_{\mathrm{iner}} \to mlH_0^2$ when $t \to +\infty$. Therefore if we choose H_0, H_1, and λ small enough, we do not obtain a rip. When t becomes large, the scale factor a is given by that of the de Sitter space–time $a \sim a_0 \mathrm{e}^{H_0 t}$, and the energy density ρ has the following form:

$$\rho = \frac{3}{\kappa^2} H^2 \sim \frac{3}{\kappa^2}\left(H_0^2 - 2H_0 H_1 \mathrm{e}^{-\lambda t}\right) \sim \frac{3}{\kappa^2}\left(H_0^2 - 2H_0 H_1 \left(\frac{a}{a_0}\right)^{-\frac{\lambda}{H_0}}\right), \tag{14}$$

which is an increasing function of a and becomes finite as $a \to \infty$.

For $t \to \infty$, Eq. (12) gives the asymptotic behavior of w to be

$$w \sim -1 - \frac{2\lambda H_1 \mathrm{e}^{-\lambda t}}{3H_0^2}, \tag{15}$$

which is identical with (9) if we replace $\lambda H_1/H_0$ with λ.

These results indicate that knowledge of the asymptotic ($t \to \infty$) behavior of $w(t)$ is insufficient to distinguish models with a rip from models which are asymptotically de Sitter. The reason for this becomes clear when we derive the expression for $\rho(t)$ as a function of $w(t)$. The evolution of ρ is given by:

$$\frac{d\rho}{dt} = -3H(\rho + p), \tag{16}$$

which can be expressed as

$$\rho^{-3/2}\frac{d\rho}{dt} = -\sqrt{3}\kappa(1 + w). \tag{17}$$

Integrating between initial and final times t_i and t_f gives:

$$\rho_i^{-1/2} - \rho_f^{-1/2} = -\frac{\sqrt{3}}{2}\int_{t_i}^{t_f}\left[1 + w(t)\right] dt. \tag{18}$$

Evolution leading to a little rip implies that $\rho_f \to \infty$ as $t_f \to \infty$, while asymptotic de Sitter evolution requires $\rho_f \to$ constant as $t_f \to \infty$. However, in either case, the integral on the right-hand side simply approaches a constant as the upper limit goes to infinity. Thus, the asymptotic functional form for $w(t)$ is not a good test of the asymptotic behavior of ρ.

On the other hand, expressing the equation of state parameter as a function of the scale factor a instead of the time t does provide a clearer test of the existence of a future rip. Eq. (16) can be written in terms of the scale factor as

$$\frac{a}{\rho}\frac{d\rho}{da} = -3\left[1 + w(a)\right], \tag{19}$$

from which it follows that

$$\ln\left(\frac{\rho_f}{\rho_i}\right) = -3\int_{a_i}^{a_f}\left[1 + w(a)\right]\frac{da}{a}. \tag{20}$$

Thus, ρ is asymptotically constant if the integral of $(1 + w)/a$ converges at its upper limit, while ρ will increase without bound, leading to a rip, when the integral diverges. Then if $1 + w(a)$ behaves as an inverse power of a, as in $1 + w(a) \sim a^{-\epsilon}$ with arbitrary positive constant ϵ when $a \to \infty$, the integration on the right-hand side of (20) is finite when $a_f \to \infty$, and therefore a rip does not occur. If $1 + w(a)$ vanishes more slowly than any power of a when $a \to \infty$, e.g., $1 + w(a) \sim 1/\ln a$, the integration on the right-hand side of (20) diverges when $a_f \to \infty$, and therefore a rip is generated.

We now consider what kind of perfect fluid realizes the evolution of H in Eqs. (8) or (11). The FRW equations give

$$\rho = \frac{3}{\kappa^2}H^2, \qquad \rho + p = -\frac{2}{\kappa^2}\dot{H}. \tag{21}$$

Consider first the model given by Eq. (8). By substituting Eq. (8) into Eq. (21) and eliminating t, we obtain:

$$(\rho + p)^2 = \frac{4\lambda^2}{3\kappa^2}\rho. \tag{22}$$

On the other hand, for the case corresponding to Eq. (11), we obtain:

$$\rho = \frac{3H_0^2}{\kappa^2} + \frac{3H_0}{\lambda}(\rho + p) + \frac{3\kappa^2}{4\lambda^2}(\rho + p)^2. \tag{23}$$

3. Coupling with dark matter

In Ref. [8], it was shown that the coupling of zero-pressure dark matter with phantom dark energy could avoid a big rip singularity, and the universe might evolve to asymptotic de Sitter space. Here we investigate the possibility that coupling with the dark matter could avoid a little rip. We consider the equation of state Eq. (22), for which a little rip occurs in the absence of such a coupling. We show that by adding a coupling with dark matter, a little rip can be avoided, and the universe can evolve to de Sitter space.

We now consider the following conservation law [8]

$$\dot{\rho} + 3H(\rho + p) = -Q\rho, \qquad \dot{\rho}_{\mathrm{DM}} + 3H\rho_{\mathrm{DM}} = Q\rho. \tag{24}$$

Here ρ_{DM} is the energy density of the dark matter and Q is a positive constant. The right-hand sides in Eqs. (24) express the decay of the dark energy into dark matter. We assume the equation of state given in Eq. (22), for which a rip could occur. Then the first equation in (24) can be rewritten as

$$\dot{\rho} - \frac{2\lambda\sqrt{3\rho}}{\kappa}H = -Q\rho. \tag{25}$$

Note that $\rho + p < 0$ since we are considering the model $w < -1$.

We now assume the de Sitter solution where H is a constant: $H = H_0 > 0$. If we neglect the contribution from everything other than the dark energy and dark matter, the first FRW equation

$$\frac{3}{\kappa^2}H^2 = \rho + \rho_{\mathrm{DM}}, \tag{26}$$

indicates that $\rho + \rho_{\mathrm{DM}}$ is a constant. Then Eq. (24) becomes

$$0 = 3H_0(\rho + p + \rho_{\mathrm{DM}}). \tag{27}$$

Since $H = H_0 > 0$, we find

$$\rho_{\mathrm{DM}} = -\rho - p. \tag{28}$$

Note that the above equation (28) can be obtained from the conservation law (24) and the first FRW equation (26) without using any equation of state.

Now we assume the equation of state (22). Combining Eqs. (22) and (28), we get

$$\rho = \frac{3\kappa^2}{4\lambda^2}\rho_{\mathrm{DM}}^2. \tag{29}$$

Since $\rho + \rho_{\mathrm{DM}}$ is a constant, Eq. (29) implies that ρ_{DM} and therefore ρ is a constant. Then the second equation in (24) gives

$$\rho_{\mathrm{DM}} = \frac{4H_0\lambda^2}{\kappa^2 Q}, \tag{30}$$

and therefore, from (29), we find

$$\rho = \frac{12H_0^2\lambda^2}{\kappa^2 Q^2}. \tag{31}$$

Then by using the FRW equation (26), we find

$$H_0 = \frac{4\lambda^2}{3Q(1 - \frac{4\lambda^2}{Q^2})}. \tag{32}$$

This requires

$$\frac{\lambda}{Q} < \frac{1}{2}. \tag{33}$$

By using (32), we can rewrite (30) and (31) as

$$\rho_{\mathrm{DM}} = \frac{16\lambda^4}{3\kappa^2 Q^2(1 - \frac{4\lambda^2}{Q^2})}, \qquad \rho = \frac{64\lambda^6}{3\kappa^2 Q^4(1 - \frac{4\lambda^2}{Q^2})^2}. \tag{34}$$

Then we obtain

$$\frac{\rho_{\mathrm{DM}}}{\rho} = \frac{Q^2(1 - \frac{4\lambda^2}{Q^2})}{4\lambda^2}. \tag{35}$$

At the present time, $\rho_{\mathrm{DM}}/\rho \sim 1/3$, and the fact that this ratio is of order unity today is called the coincidence problem. This observed ratio can be obtained in our model when $\lambda^2/Q^2 \sim 3/16$.

De Sitter space can be realized by the a-independent energy density. The energy density of the phantom dark energy increases by the expansion but it decreases by the decay into the dark matter. On the other hand, the energy density of the dark matter decreases by the expansion but it increases by the decay of the dark energy. In the above solution, the decay of the dark energy into the dark matter balances with the expansion of the universe, and the energy densities of both the dark energy and dark matter become constant. This mechanism is essentially identical to one found in [8].

If the solution corresponding to de Sitter space–time is an attractor, the universe becomes asymptotic de Sitter space–time and any rip might be avoided. In order to investigate if the de Sitter space–time is an attractor or not, we consider the perturbation from the de Sitter solution in (32) and (34):

$$H=\frac{4\lambda^2}{3Q(1-\frac{4\lambda^2}{Q^2})}+\delta H,\qquad \rho_{\mathrm{DM}}=\frac{16\lambda^4}{3\kappa^2Q^2(1-\frac{4\lambda^2}{Q^2})}+\delta\rho_{\mathrm{DM}},\qquad \rho=\frac{64\lambda^6}{3\kappa^2Q^4(1-\frac{4\lambda^2}{Q^2})^2}+\delta\rho. \tag{36}$$

Then the first FRW equation (26) gives

$$\frac{8\lambda^2}{\kappa^2Q(1-\frac{4\lambda^2}{Q^2})}\delta H=\delta\rho+\delta_{\mathrm{DM}}. \tag{37}$$

The conservation laws (24) and (25) give

$$\delta\dot\rho=\frac{16\lambda^4}{\kappa^2Q^2(1-\frac{4\lambda^2}{Q^2})}\delta H-\frac{Q}{2}\delta\rho,$$
$$\delta\dot\rho_{\mathrm{DM}}=-\frac{16\lambda^4}{\kappa^2Q^2(1-\frac{4\lambda^2}{Q^2})}\delta H+Q\,\delta\rho-\frac{4\lambda^2}{Q(1-\frac{4\lambda^2}{Q^2})}\delta\rho_{\mathrm{DM}}. \tag{38}$$

By eliminating δH in (38) using (37), we obtain

$$\frac{d}{dt}\begin{pmatrix}\delta\rho\\ \delta\rho_{\mathrm{DM}}\end{pmatrix}=\begin{pmatrix}-\frac{Q}{2}(1-\frac{4\lambda^2}{Q^2}) & \frac{2\lambda^2}{Q}\\ Q(1-\frac{2\lambda^2}{Q^2}) & -\frac{\frac{2\lambda^2}{Q}(3-\frac{4\lambda^2}{Q^2})}{1-\frac{4\lambda^2}{Q^2}}\end{pmatrix}\begin{pmatrix}\delta\rho\\ \delta\rho_{\mathrm{DM}}\end{pmatrix}. \tag{39}$$

In order for the de Sitter solution in (32) and (34) to be stable, all the eigenvalues of the matrix in (39) should be negative, which requires the trace of the matrix to be negative and the determinant to be positive, giving

$$-\frac{Q}{2}\left(1-\frac{4\lambda^2}{Q^2}\right)-\frac{\frac{2\lambda^2}{Q}(3-\frac{4\lambda^2}{Q^2})}{1-\frac{4\lambda^2}{Q^2}}<0,\quad \lambda^2>0. \tag{40}$$

The second condition can be trivially satisfied, and the first condition is also satisfied as long as (33) is satisfied. Therefore the de Sitter solution in (32) and (34) is stable and therefore an attractor. This tells us that the coupling of the dark matter with the dark energy as in (24) eliminates the little rip.

Thus, if the universe where the dark energy dominates is realized, the universe will expand as in (8). If there is an interaction as given in (24), the dark energy decay into dark matter will yield asymptotic de Sitter space–time corresponding to Eq. (32).

4. Scalar field little rip cosmology

4.1. Minimally coupled phantom models

First consider a minimally coupled phantom field ϕ which obeys the equation of motion

$$\ddot\phi+3H\dot\phi-V'(\phi)=0, \tag{41}$$

where the prime denotes the derivative with respect to ϕ. A field evolving according to Eq. (41) rolls uphill in the potential. In what follows, we assume a monotonically increasing potential $V(\phi)$. If this is not the case, then it is possible for the field to become trapped in a local maximum of the potential, resulting in asymptotic de Sitter evolution.

Kujat, Scherrer, and Sen [16] derived the conditions on $V(\phi)$ to avoid a big rip, namely $V'/V\to 0$ as $\phi\to\infty$, and

$$\int\frac{\sqrt{V(\phi)}}{V'(\phi)}\,d\phi\to\infty. \tag{42}$$

When these conditions are satisfied, w approaches -1 sufficiently rapidly that a big rip is avoided.

We now extend this argument to determine the conditions necessary to avoid a little rip. Clearly, we will have $\rho\to$ constant if $V(\phi)$ is bounded from above, so that $V(\phi)\to V_0$ (where V_0 is a constant) as $\phi\to\infty$. We can show that this is also a necessary condition. Suppose that $V(\phi)$ is not bounded from above, so that $V(\phi)\to\infty$ as $\phi\to\infty$. Then the only way for the density of the scalar field to remain bounded is if the field "freezes" at some fixed value ϕ_0. However, this is clearly impossible from Eq. (41), since it would require $\ddot\phi=\dot\phi=0$ while $V'(\phi)\neq 0$. Thus, boundedness of the potential determines the boundary between little rip and asymptotic de Sitter evolution. Phantom scalar field models with bounded potentials have been discussed previously in Ref. [9].

4.2. Scalar-tensor models

Using the formulation in Ref. [14], we now consider what kind of scalar-tensor model, with an action given by

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}R - \frac{1}{2}\omega(\phi)\partial_\mu\phi\partial^\mu\phi - V(\phi)\right\}, \tag{43}$$

can realize the evolution of H given in Eqs. (8) or (11). Here $\omega(\phi)$ and $V(\phi)$ are functions of the scalar field ϕ. Since the corresponding fluid is phantom with $w < -1$, the scalar field must be a ghost with a non-canonical kinetic term. If we consider the model where $\omega(\phi)$ and $V(\phi)$ are given by a single function $f(\phi)$ as follows,

$$\omega(\phi) = -\frac{2}{\kappa^2}f''(\phi), \qquad V(\phi) = \frac{1}{\kappa^2}\left(3f'(\phi)^2 + f''(\phi)\right), \tag{44}$$

the exact solution of the FRW equations has the following form:

$$\phi = t, \qquad H = f'(t). \tag{45}$$

Then for the model given by Eq. (8), we find

$$\omega(\phi) = -\frac{2\lambda H_0}{\kappa^2}e^{\lambda\phi}, \qquad V(\phi) = \frac{1}{\kappa^2}\left(3H_0^2e^{2\lambda\phi} + \lambda H_0 e^{\lambda\phi}\right). \tag{46}$$

Furthermore, if we redefine the scalar field ϕ to φ by

$$\varphi = \frac{2e^{\frac{\lambda}{2}\phi}}{\kappa}\sqrt{\frac{2H_0}{\lambda}}, \tag{47}$$

we find that the action (43) has the following form:

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{3\lambda^2\kappa^2}{64}\varphi^4 - \frac{\lambda^2}{8}\varphi^2\right\}. \tag{48}$$

Note that in the action (48), H_0 does not appear. This is because the shift of t in (8) effectively changes H_0. The parameter A in [11] corresponds to $2\lambda/\sqrt{3}$ in (8) and is bounded as $2.74 \times 10^{-3}\ \mathrm{Gyr}^{-1} \leqslant A \leqslant 9.67 \times 10^{-3}\ \mathrm{Gyr}^{-1}$, or $2.37 \times 10^{-3}\ \mathrm{Gyr}^{-1} \leqslant \lambda \leqslant 8.37 \times 10^{-3}\ \mathrm{Gyr}^{-1}$, by the results of the Supernova Cosmology Project [15]. In [11], it was shown that the model defined by Eq. (8) can give behavior of the distance modulus versus redshift almost identical to that of ΛCDM, so this model can be made consistent with observational data.

As in Ref. [11], we can generalize the behavior of this model to

$$H = H_0e^{Ce^{\lambda t}}. \tag{49}$$

Here H_0, C, and λ are positive constants. Then we find

$$\omega(\phi) = -\frac{2}{\kappa^2}H_0C\lambda e^{Ce^{\lambda\phi}}e^{\lambda\phi}, \qquad V(\phi) = \frac{1}{\kappa^2}\left(3H_0^2e^{2Ce^{\lambda\phi}} + H_0C\lambda e^{Ce^{\lambda\phi}}e^{\lambda\phi}\right). \tag{50}$$

If we redefine the scalar field ϕ to φ by

$$\varphi = \frac{\sqrt{2H_0C\lambda}}{\kappa}\int d\phi\, e^{\frac{C}{2}e^{\lambda\phi}}e^{\frac{\lambda}{2}\phi} = \frac{1}{\kappa}\sqrt{\frac{8H_0C}{\lambda}}\int^{e^{\frac{\lambda}{2}\phi}} dx\, e^{\frac{C}{2}x^2} = \frac{2}{\kappa}\sqrt{\frac{H_0\pi}{\lambda}}\mathrm{Erfi}\left[\sqrt{\frac{C}{2}}e^{\frac{\lambda}{2}\phi}\right], \tag{51}$$

we may obtain the action where the kinetic term of the scalar field φ is $+\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi$. In (51), $\mathrm{Erfi}[x] = \mathrm{Erf}[ix]/i$ with $i^2 = -1$, where $\mathrm{Erf}[x]$ is the error function. In [11], it was shown that the model given by Eq. (49) can also be consistent with the observations.

As in Ref. [11], we can easily find models which show more complicated behavior of H such as

$$H = H_0e^{C_0e^{C_1e^{C_2e^{\lambda t}}}}. \tag{52}$$

On the other hand, in the model given by Eq. (11), we find

$$\omega(\phi) = -\frac{2\lambda H_1}{\kappa^2}e^{-\lambda\phi}, \qquad V(\phi) = \frac{1}{\kappa^2}\left\{3\left(H_0 - H_1e^{-\lambda\phi}\right)^2 + \lambda H_1e^{-\lambda\phi}\right\}, \tag{53}$$

and by the redefinition

$$\varphi = \frac{2e^{-\frac{\lambda}{2}\phi}}{\kappa}\sqrt{\frac{2H_1}{\lambda}}, \tag{54}$$

we find that the action (43) has the following form:

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{\kappa^2}\left[3\left(H_0 - \frac{\lambda\kappa^2}{8}\varphi^2\right)^2 + \frac{\lambda^2\kappa^2}{8}\varphi^2\right]\right\}. \tag{55}$$

In the action given by Eq. (55), H_1 does not appear. This is because the shift of t in (11) effectively changes H_1.

Eq. (47) shows that in the infinite future $t = \phi \to +\infty$, φ also goes to infinity, that is, the scalar field climbs up the potential to infinity. This climbing up the potential makes the Hubble rate grow and generates a rip due to the inertial force (1). On the other hand, Eq. (54) tells us that when $\phi \to +\infty$, φ vanishes. Note that the potential in (55) is a double well potential similar to the potential of the Higgs field, and $\varphi = 0$ corresponds to the local maximum of the potential. Therefore, in the model given by (55), the scalar field climbs up the potential and arrives at the local maximum after an infinite time. The behavior of the scalar field is different from that of the canonical scalar field, which usually rolls down the potential. This phenomenon of how the scalar field climbs up the potential occurs due to the non-canonical kinetic term. For the canonical scalar field φ_c, the field equation has the form of $\nabla_t^2\varphi_c = -V'(\phi)$, but if the sign of the kinetic term is changed, we obtain $\nabla_t^2\varphi_c = V'(\phi)$ for a non-canonical scalar field. That is, the sign of the "force" is effectively changed.

We now investigate the stability of the solution (45) in the model given by Eqs. (43) and (44) by considering the perturbation from the solution (45):

$$\phi = t + \delta\phi(t), \qquad H = f'(t) + \delta h(t). \tag{56}$$

By using the FRW equations

$$\frac{3}{\kappa^2}H^2 = \frac{1}{2}\omega(\phi)\dot{\phi}^2 + V(\phi), \qquad -\frac{1}{\kappa^2}\left(2\dot{H} + 3H^2\right) = \frac{1}{2}\omega(\phi)\dot{\phi}^2 - V(\phi), \tag{57}$$

we find

$$\frac{d}{dt}\begin{pmatrix}\delta h\\ \delta\phi\end{pmatrix} = \begin{pmatrix}-6f'(t) & 6f'(t)f''(t) + f'''(t)\\ -3\frac{f'(t)}{f''(t)} & 3f'(t)\end{pmatrix}\begin{pmatrix}\delta h\\ \delta\phi\end{pmatrix}. \tag{58}$$

In order for the solution (45) to be stable, all the eigenvalues of the matrix in (58) should be negative, which requires the trace of the matrix to be negative and the determinant to be positive, giving

$$-3f'(t) < 0, \qquad 3\frac{f'(t)f'''(t)}{f''(t)} > 0. \tag{59}$$

The first condition is trivially satisfied in the expanding universe since $f'(t) = H > 0$. If the universe is in the phantom phase, where $f''(t) = \dot{H} > 0$, the second condition reduces to $f'''(t) = \ddot{H} > 0$. Then the model corresponding to (11) is unstable but the model corresponding to (8) is stable. There are no local maxima in the potential in (48), so one would expect the field to climb the potential well to infinity, generating a rip. In general, in a model which generates a big or little rip, H goes to infinity, which requires $\ddot{H} > 0$. Therefore in the scalar field model generating a big or little rip, the solution corresponding to the rip is stable, and models that are asymptotically de Sitter can eventually evolve to have a rip.

5. Including matter

In the previous sections, we have neglected the contribution from matter except for the dark matter in Section 3. In this section, we now consider the affect of additional matter components. We assume each component has a constant EoS parameter w^i_{matter}. Then the energy density and pressure contributed by all of these components can be expressed as

$$\rho_{\mathrm{matter}} = \sum_i \rho^i_0 a^{-3(1+w^i_{\mathrm{matter}})}, \qquad p_{\mathrm{matter}} = \sum_i w_i\rho^i_0 a^{-3(1+w^i_{\mathrm{matter}})}. \tag{60}$$

Here the ρ^i_0's are constants. Even including these additional matter components, we can construct the scalar-tensor model realizing the evolution of H by, instead of (44),

$$\begin{aligned}\omega(\phi) &= -\frac{2}{\kappa^2}g''(\phi) - \sum_i \frac{w^i_{\mathrm{matter}} + 1}{2}\rho^i_0 a_0^{-3(1+w^i_{\mathrm{matter}})}\mathrm{e}^{-3(1+w^i_{\mathrm{matter}})g(\phi)},\\ V(\phi) &= \frac{1}{\kappa^2}\left(3g'(\phi)^2 + g''(\phi)\right) + \sum_i \frac{w^i_{\mathrm{matter}} - 1}{2}\rho^i_0 a_0^{-3(1+w^i_{\mathrm{matter}})}\mathrm{e}^{-3(1+w^i_{\mathrm{matter}})g(\phi)}.\end{aligned} \tag{61}$$

Then the solution of the FRW equations (3) is given by

$$\phi = t, \qquad H = g'(t) \qquad \left(a = a_0\mathrm{e}^{g(t)}\right). \tag{62}$$

We may consider the example of (8), which gives

$$a(t) = a_0\mathrm{e}^{\frac{H_0}{\lambda}\mathrm{e}^{\lambda t}}. \tag{63}$$

Then by using the FRW equations (3), we find the EoS parameter w_{DE} corresponding to the dark energy is given by

$$\begin{aligned}w_{\mathrm{DE}} &= \frac{\frac{3}{\kappa^2}H^2 - \rho_{\mathrm{matter}}}{-\frac{1}{\kappa^2}(2\dot{H} + 3H^2) - p_{\mathrm{matter}}}\\ &= \frac{\frac{3}{\kappa^2}H_0^2\mathrm{e}^{2\lambda t} - \sum_i \rho^i_0 a_0^{-3(1+w^i_{\mathrm{matter}})}\mathrm{e}^{-\frac{3(1+w^i_{\mathrm{matter}})H_0}{\lambda}\mathrm{e}^{\lambda t}}}{-\frac{1}{\kappa^2}(2\lambda H_0\mathrm{e}^{\lambda t} + 3H_0^2\mathrm{e}^{2\lambda t}) - \sum_i w^i_{\mathrm{matter}}\rho^i_0 a_0^{-3(1+w^i_{\mathrm{matter}})}\mathrm{e}^{-\frac{3(1+w^i_{\mathrm{matter}})H_0}{\lambda}\mathrm{e}^{\lambda t}}}.\end{aligned} \tag{64}$$

When t becomes large, the contribution from the matter components decreases rapidly and w_{DE} in (64) coincides with w in (9). The density parameter Ω_{DE} of the dark energy is also given by

$$\Omega_{\mathrm{DE}} = \frac{\frac{3}{\kappa^2}H^2 - \rho_{\mathrm{matter}}}{\frac{3}{\kappa^2}H^2} = 1 - \frac{\kappa^2}{3H_0^2}\sum_i \rho_0^i a_0^{-3(1+w^i_{\mathrm{matter}})} \mathrm{e}^{-\frac{3(1+w^i_{\mathrm{matter}})H_0}{\lambda}\mathrm{e}^{\lambda t} - 2\lambda t}, \tag{65}$$

which rapidly goes to unity when t becomes large. It would be interesting to consider the cosmological perturbation in the model including the contribution from these matter components.

Let $t = 0$ represent the present. We now assume the matter consists only of dust with a vanishing EoS parameter. Then we find $\rho_{\mathrm{matter}} = \rho_0 a_0^{-3} \mathrm{e}^{-\frac{3H_0}{\lambda}}$ and $p_{\mathrm{matter}} = 0$. Since $\Omega_{\mathrm{DE}} = 0.74$, we find $\rho_{\mathrm{matter}} = 0.26 \times \frac{3H_0^2}{\kappa^2}$ by using (65). Since H_0 is the Hubble parameter in the present universe, we find $H_0 = 7.24 \times 10^{-2}\ \mathrm{Gyr}^{-1} (\approx 70\ \mathrm{km/\ s\,Mpc})$. Since $2.37 \times 10^{-3}\ \mathrm{Gyr}^{-1} \leqslant \lambda \leqslant 8.37 \times 10^{-3}\ \mathrm{Gyr}^{-1}$ (see below (48)), by using (64), we find $-0.97 < w_{\mathrm{DE}} < -0.72$, which could be consistent with the observed value $w_{\mathrm{DE}} = -0.972^{+0.061}_{-0.060}$.

6. Discussion

Little rip models provide an evolution for the universe intermediate between asymptotic de Sitter expansion and models with a big rip singularity. We have shown that the EoS parameter w as a function of time is a less useful diagnostic of such behavior than is w as a function of the scale factor. As for the case of big rip singularities, a little rip can be avoided if the dark energy is coupled to the dark matter so that energy flows from the dark energy to the dark matter. Minimally coupled phantom scalar field models can lead to viable little rip cosmologies. The models we investigated that yield little rip evolution turned out to be stable against small perturbations, and we found that big rip evolution is also consistent with the conditions for stability. For phantom field models, rip-like behavior is an attractor.

It is interesting that it was recently demonstrated that the little rip cosmology may be realized by a viscous fluid [17]. It turns out that the viscous little rip cosmology can also be stable.

Scalar little rip dark energy represents a natural alternative to the ΛCDM model, which also leads to a non-singular cosmology. It remains to consider the coupling of such a model with matter and to confront its predictions with observations.

It is known [18] that in a local frame with a flat background, a classical field theory with $w < -1$ has a negative kinetic energy term, and the corresponding quantum field theory has a tachyonic instability and a vacuum decay lifetime which appears finite, although possibly greater than the age of the universe. Our result shows that in the presence of a rip, the space–time expansion is so fast that this tachyonic instability does not have time to destabilize the global geometry and shows, interestingly, that the extraordinary conditions of a little rip can lead to an infinite lifetime.

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References

[1] S. Perlmutter, et al., SNCP Collaboration, Astrophys. J. 517 (1999) 565, arXiv:astro-ph/9812133;
A.G. Riess, et al., Supernova Search Team Collaboration, Astron. J. 116 (1998) 1009, arXiv:astro-ph/9805201;
D.N. Spergel, et al., WMAP Collaboration, Astrophys. J. Suppl. 148 (2003) 175, arXiv:astro-ph/0302209;
D.N. Spergel, et al., WMAP Collaboration, Astrophys. J. Suppl. 170 (2007) 377, arXiv:astro-ph/0603449;
E. Komatsu, et al., WMAP Collaboration, Astrophys. J. Suppl. 180 (2009) 330, arXiv:0803.0547 [astro-ph];
E. Komatsu, et al., WMAP Collaboration, Astrophys. J. Suppl. 192 (2011) 18, arXiv:1001.4538 [astro-ph.CO].
[2] M. Li, X.-D. Li, S. Wang, Y. Wang, arXiv:1103.5870 [astro-ph.CO].
[3] P.H. Frampton, T. Takahashi, Phys. Lett. B 557 (2003) 135, arXiv:astro-ph/0211544.
[4] R.R. Caldwell, M. Kamionkowski, N.N. Weinberg, Phys. Rev. Lett. 91 (2003) 071301, arXiv:astro-ph/0302506.
[5] S. Nojiri, S.D. Odintsov, S. Tsujikawa, Phys. Rev. D 71 (2005) 063004, arXiv:hep-th/0501025.
[6] J.D. Barrow, Class. Quant. Grav. 21 (2004) L79, arXiv:gr-qc/0403084.
[7] J.D. Barrow, Class. Quant. Grav. 21 (2004) 5619, arXiv:gr-qc/0409062.
[8] S. Nojiri, S.D. Odintsov, Phys. Lett. B 686 (2010) 44, arXiv:0911.2781 [hep-th].
[9] E. Elizalde, S. Nojiri, S.D. Odintsov, Phys. Rev. D 70 (2004) 043539, arXiv:hep-th/0405034.
[10] K. Bamba, S. Nojiri, S.D. Odintsov, JCAP 0810 (2008) 045, arXiv:0807.2575 [hep-th];
S. Nojiri, S.D. Odintsov, arXiv:1011.0544 [gr-qc].
[11] P.H. Frampton, K.J. Ludwick, R.J. Scherrer, Phys. Rev. D 84 (2011) 063003, arXiv:1106.4996 [astro-ph.CO].
[12] S. Nojiri, S.D. Odintsov, Phys. Rev. D 72 (2005) 023003, arXiv:hep-th/0505215;
H. Stefancic, Phys. Rev. D 71 (2005) 124036, arXiv:astro-ph/0504518.
[13] R.R. Caldwell, Phys. Lett. B 545 (2002) 23, arXiv:astro-ph/9908168.
[14] S. Nojiri, S.D. Odintsov, Gen. Rel. Grav. 38 (2006) 1285, arXiv:hep-th/0506212.
[15] R. Amanullah, C. Lidman, D. Rubin, G. Aldering, P. Astier, K. Barbary, M.S. Burns, A. Conley, et al., arXiv:1004.1711 [astro-ph.CO].
[16] J. Kujat, R.J. Scherrer, A.A. Sen, Phys. Rev. D 74 (2006) 083501, arXiv:astro-ph/0606735.
[17] I. Brevik, E. Elizalde, S. Nojiri, S.D. Odintsov, arXiv:1107.4642 [hep-th].
[18] S.M. Carroll, M. Hoffman, M. Trodden, Phys. Rev. D 68 (2003) 023509, arXiv:astro-ph/0301273;
J.M. Cline, S. Jeon, G.D. Moore, Phys. Rev. D 70 (2004) 043543, arXiv:hep-ph/0311312.