Periodic pricing and replenishment policy for continuously decaying inventory with multivariate demand ♠

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Abstract
This paper deals with the joint decisions on pricing and replenishment schedule for a periodic review inventory system in which a replenishment order may be placed at the beginning of some or all of the periods. We consider a single product which is subject to continuous decay and a demand which is a function of price and time, without backlogging over a finite planning horizon. The proposed scheme may adjust periodically the selling price upward or downward that makes the pricing policy more responsive to structure changes in supply or demand. The problem is formulated as a dynamic programming model and solved by numerical search techniques. An extensive numerical study is conducted to attend qualitative insights into the structures of the proposed policy and its sensitivity with respect to major parameters. The numerical result shows that the solution generated by the periodic policy outperforms that by the fixed pricing policy in maximizing discount profit.
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Keywords: Periodic inventory system; Dynamic pricing; Deterioration; Time-value

1. Introduction
This paper deals with the problem of joint decisions on pricing and lot-sizing for a deteriorating item over multi-period planning horizon. More specifically, we consider a single product that is subject to continuous decay, a multivariate demand function of price and time, and a periodic review inventory system in which the selling price is allowed to adjust arbitrarily, upward or downward, in response to changes in market demand over product lifecycle.

One of the earliest works that address pricing-ordering joint decision-making was due to Whitin [1], who extended the basic EOQ model for the staple goods and the newsvendor (or stylized) model for the style merchandise by considering the selling price in addition to the order quantity as the decision variables. Mills [2]
extended the stylized model by specifying mean demand as a function of the price under severe cost assumptions, i.e., no relevant inventory holding or shortage costs being explicitly considered. Zabel [3] introduced some generalizations of the cost parameters in the model that relaxes Mills’ restrictive assumptions. Lau and Lau [4] revisited the stylized model under general stochastic demands with alternative objective functions of maximizing expected profit or maximizing the probability of attaining a given target profit level. The models [1–4] being reviewed determine only a single price and order quantity in single period setting.

The treatment of dynamic planning combined with pricing and inventory joint decisions was initially attempted by Thomas [5] under the assumption of deterministic demands, and then undertaken by Thomas [6], Zabel [7], and Kunreuther and Schrage [8] who considered uncertain demands. A comprehensive survey of related literature is given in [9,10]. One of the recent developments in the area is toward dealing with the dynamic pricing and inventory problems [11–17]. Gilbert [18] took another direction on exploring the multi-product problem in multi-period setting with constrained production capacity. To make the problem tractable, he assumed the selling price for each product in the market is unadjustable over planning horizon. Urban and Baker [19] investigated the impact of multivariate demand functions on pricing–inventory decisions and the corresponding profit.

The aforementioned literature, however, does not take product deterioration into account. Deterioration or decay defined by Raafat [20] is the process that prevents an item from being used for its intended original use such as spoilage of foodstuffs, physical depletion due to pilferage, evaporation of liquids, decay of radioactive substances, degradation of high-tech products, and loss of potency of photographic films and pharmaceutical drugs. In the hospital industry, for example, shrinkage is one of the major contributors to the variance of inventory record, in which the physical balance is usually below the computer balance [21]. A survey conducted by the National Accounting Association reveals that the unreported scrap is the major cause of inventory losses regardless of types of industries or products [22]. These phenomena of deterioration are prevalent and should not be disregarded.

This paper considers the effect of deterioration as a function of the on-hand level of inventory. Literature reviews of this research stream are provided in [20,23,24]. However, most of the references therein and the recent work being published in this journal [25–29] do not incorporate pricing or any other marketing related decision. Some exceptions include the models proposed by Abad [30–32], Cohen [33], Papachristos and Skouri [34], Rajan et al. [35], Wee [36], and Wee and Law [37]. The models of Cohen and Rajan et al. jointly determine the optimal replenishment cycle and price for inventory that is subject to continuous decay along product lifetime under standard EOQ cost assumptions. Abad [30] extended the model of Rajan et al. by allowing shortages that can be partially backlogged at the end of the cycle. The subsequent models [31,32,34,36,37] are variants of [30,35] that deal with the problem under a single period setting and considering additional factors such as quantity discount, partially backlogged, or economic production quantity. Instead, we present a dynamic version of the pricing–ordering decision model over multi-period planning horizon, in which the deterioration rate of the on-hand inventory follows a continuous function of product’s lifetime and the demand is a multivariate function of price and time. In addition, we take time-value of money into account, i.e., the net present value approach. The reason of adopting NPV is due to its practical use in business planning and financial decision making [38,39].

The remainder of this paper is organized as follows. Section 2 outlines the problem and summarizes necessary assumptions and notation. The base model for a static, single period lot-sizing and pricing problem is developed in Section 3. Section 4 extends the base model into a multi-period formulation using a dynamic programming approach that determines the optimal replenishment schedule and associated pricing decisions over the finite planning horizon. Section 5 conducts an in-depth comparative study between the solutions generated by the periodic pricing and that by the fixed pricing. In this section, we also investigate the dynamic behavior of the pricing trajectory and perform sensitivity analysis with respect to major parameters. Concluding remarks and suggestions for future research are given in Section 6.

2. Assumptions and notations

We consider a single item whose inventory and selling price are reviewed periodically at time $t$, $t = 0, 1, 2, \ldots, H$, where $H$ is the planning horizon. At the beginning of each period, a joint decision is made
regarding the lot-size of a new replenishment (if any) and its associated selling price. The problem is equivalent to determining the optimal sequence of times \( z_{i-1} \), \( i = 1, 2, \ldots, n \), at which a new replenishment is issued, the selling price \( p \) is reset, and the lot-size is specified simultaneously so that the discount profit stream over \([0, H]\) is maximized. Since demand is a function of price, the replenishment lot-size accordingly depends on the price.

Based on the setting mentioned above, it is worth noting that \( n \leq H \) (the number of replenishments is less than or equal to the number of planning periods), \( z_0 = 0 \) (the first replenishment is scheduled at the beginning of the planning horizon), \( z_n = H \) (the stopping time epoch of the last replenishment/selling period \([z_{i-1}, z_i]\) is the end of the planning horizon), \( z_{i-1} \) is integer (due to the periodic review policy), and \( z_{i-1} \in [0, H) \). For brevity, we assume the replenishment is instantaneous, no shortages are allowed, and consequently each new replenishment at \( z_{i-1} \) is for the selling period over \([z_{i-1}, z_i]\). In the dynamic system, no inventory is held at the beginning and at the end of the time horizon. If the initial inventory level is positive in the system, no action will be taken until the depletion of inventory.

The demand function considered in this paper satisfies the following mild and realistic assumptions: \( D(p, t) \geq 0 \) and is continuous for \( p \geq 0, t \geq 0 \), \( D(p, t) \) decreases in \( p \), \( D(0, t) < +\infty \) and \( \lim_{p \to \infty} D(p, t) = 0 \) for \( t \geq 0 \), and \( D(p, t) > 0 \) for \( p \in [0, p_{\max}) \), \( t \geq 0 \) (see [35] for further discussions). There is a one-to-one correspondence between prices and demand rates, leading to a one-to-one correspondence between pricing and lot-sizing decisions in a given replenishment period, say \([z_{i-1}, z_i]\).

The following notations are defined and will be used throughout the paper:

\[ \pi(z_{i-1}, p, z_i) \] The present worth of the profit generated from the replenishment/selling period \([z_{i-1}, z_i]\) when the selling price is \( p \) in the market.

\[ \Pi_z \] The cumulated value of present worth of the profit generated over period \([0, z_i]\), noting that \( \Pi_z = \sum_{j=1}^{n} \pi(z_{j-1}, p, z_j) \).

\[ \theta(t) \] The deteriorating coefficient of product over lifetime \( \tau(t) \), noting that \( \tau(t) = t - z_{i-1} \), for \( t \in [z_{i-1}, z_i], i = 1, 2, \ldots, n \).

\[ I(p, t) \] The inventory level at time \( t \) when the selling price is \( p \).

\[ R(p, t) \] The inventory level at time \( t \) when the product lifetime is \( \tau(t) \) and the selling price is \( p \).

\[ A \] The fixed ordering cost per lot at time zero.

\[ c \] The purchasing or production cost per unit at time zero.

\[ h \] The holding cost per unit of time at time zero.

### 3. The base model

We first derive a generic base model for determining the optimal lot-size and price joint decisions over an arbitrary selling period \([z_{i-1}, z_i]\) in Section 3.1. In Section 3.2, the concavity property of the model is analyzed based on a linear demand function. The model is further extended into a multi-period formulation in Section 4 where the optimal sequence of replenishments \( z_{i-1} \) and associated price \( p^* \) and lot-size \( Q^* \) for \( i = 1, 2, \ldots, n \), are determined using a dynamic programming approach.

#### 3.1. The model formulation

Taking into account the demand requirement and the effect of deterioration loss over period \([z_{i-1}, z_i]\), the change of inventory level can be represented by the differential equation:

\[
\frac{\partial}{\partial t} I(p, t) = -I(p, t)\theta(\tau(t)) - D(p, t), \quad \text{for } t \in [z_{i-1}, z_i].
\]

Multiplying \( e^{\int_{z_{i-1}}^{t} \theta(u) \, du} \) on both sides of the equation, integrating by part, applying the assumption that the inventory level at the end of the cycle is zero, and taking some simple algebraic operations, the inventory level at time \( t \) can be simplified to:


\[ I(p, t) = e^{-\int_{-1}^{0} (\theta(u)) \, du} \int_{t}^{\infty} D(p, u) e^{\int_{-1}^{0} (\theta(u)) \, du} \, du = \int_{t}^{\infty} D(p, u) e^{\int_{-1}^{0} (\theta(u)) \, du} \, du. \]  

(1)

Derivation of Eq. (1) can refer to Rajan et al. [35] for details. Using Eq. (1), the replenished quantity is the inventory level at the beginning of the replenishment:

\[ Q(z_{t-1}, p, z_t) = I(p, z_{t-1}) = \int_{z_{t-1}}^{\infty} D(p, u) e^{\int_{-1}^{0} (\theta(u)) \, du} \, du. \]  

(2)

The present worth of profit generated from the replenishment/selling period \([z_{t-1}, z_t]\) is the revenue minus the relevant inventory cost which includes the ordering cost, the purchase cost, and the holding cost:

\[ \pi(z_{t-1}, p, z_t) = \int_{z_{t-1}}^{\infty} e^{-R_t} pD(p, t) \, dt - \left( A e^{-R_{z_{t-1}}} + c e^{-R_{z_{t-1}}} I(p, z_{t-1}) + h \int_{z_{t-1}}^{\infty} e^{-R_t} I(p, t) \, dt \right). \]

Again, using Eq. (1), manipulating algebraic operations, and rearranging terms, the above equation can be re-expressed as follows:

\[ \pi(z_{t-1}, p, z_t) = \int_{z_{t-1}}^{\infty} \left( e^{-R_t} p - C(t) \right) D(p, t) \, dt - A e^{-R_{z_{t-1}}}, \]  

(3)

where

\[ C(t) = c e^{-R_{z_{t-1}}} e^{-\int_{-1}^{0} (\theta(u)) \, du} + h \int_{z_{t-1}}^{\infty} e^{-R_t} e^{\int_{-1}^{0} (\theta(u)) \, du} \, dt. \]  

(4)

The value of \( C(t) \) represents the present worth of unit cost due to inventory purchase and holding expenses over \([z_{t-1}, t]\), and \( (e^{-R_t} p - C(t)) \) is the present worth of contribution margin per unit at time \( t \). For given \( z_{t-1} \) and \( z_t(0 \leq z_{t-1} < z_t \leq H) \), the optimal price that maximizes the present worth of total profit over period \([z_{t-1}, z_t]\) can be obtained by differentiating Eq. (3) with respect to \( p \) and setting the results equal to zero:

\[ \frac{\partial}{\partial p} \pi(z_{t-1}, p, z_t) = \int_{z_{t-1}}^{\infty} \left( e^{-R_t} D(p, t) + (e^{-R_t} p - C(t)) D_p(p, t) \right) \, dt = 0. \]  

(5)

Let \( p^* \) be the solution of Eq. (5) which represents the optimal price over period \([z_{t-1}, z_t]\). Substituting \( p \) in Eq. (2) with \( p^* \) yields the associated optimal lot-size \( Q^*(z_{t-1}, p^*, z_t) \) at time epoch \( z_{t-1} \).

### 3.2. The linear demand case

It is not difficult to solve the problem under a wide variety of demand functions, but then we can no longer characterize the optimal property of the model. Therefore, we focus on analyzing the base model for the case of linear demand function: \( D(p, t) = a_t - b_t p \), where \( a_t > 0, b_t > 0 \), for all \( t \in [0, H] \).

Let \( a_t = a e^{\lambda t} \) and \( b_t = b e^{\lambda t} \), the demand function becomes

\[ D(p, t) = a e^{\lambda t} - b e^{\lambda t} p = (a - b e^{(\lambda_2 - \lambda_1) t}) p e^{\lambda t} = (a - b e^{\lambda t}) p e^{\lambda t}. \]  

(6)

Coefficient \( \lambda_1 \) of Eq. (6) represents the trend of market demand; a positive value of \( \lambda_1 \) represents time-increasing demand and negative is time-decreasing. On the other hand, coefficient \( \lambda = \lambda_2 - \lambda_1 \) exhibits the changing pattern, direction and magnitude, of the price-sensitivity of demand over the planning horizon.

Choosing this particular form is due to its generality and because it is commonly used in the literature. Other forms of demand models have been proposed in pricing/inventory research such as Smith and Achabal [17] and Urban and Baker [19]. Lau and Lau [40] provided in-depth discussions on the effects of applying different demand curve functions on pricing/inventory decisions in single tiered as well as multi-tiered supply chains.

To show the uniqueness of the solution we shall demonstrate that the discount profit function in Eq. (3) is concave, i.e., its second order sufficient condition is satisfied by

\[ \frac{\partial^2}{\partial p^2} \pi(z_{t-1}, p, z_t) = \int_{z_{t-1}}^{\infty} (2 e^{-R_t} D_p(p, t) + (e^{-R_t} p - C(t)) D_{pp}(p, t)) \, dt < 0. \]  

(7)
Proposition 1. The discount profit function \( \pi(z_{i-1}, p, z_i) \) is concave in \( p \).

Proof. Substituting the demand function with \( (a - be^{\alpha t})e^{\alpha t} \) in Eq. (7) and simplifying terms yields

\[
\frac{\partial^2}{\partial p^2} \pi(z_{i-1}, p, z_i) = -2b \int_{z_{i-1}}^{z_i} e^{(\lambda_2 - R)t} \, dt = -2b \frac{e^{(\lambda_2 - R)z_i} - e^{(\lambda_2 - R)z_{i-1}}}{(\lambda_2 - R)}.
\]

(8)

Since \( b > 0 \) and \( \frac{e^{(\lambda_2 - R)z_i} - e^{(\lambda_2 - R)z_{i-1}}}{(\lambda_2 - R)} > 0 \) for \( z_i > z_{i-1} \), regardless of the value of \( (\lambda_2 - R) \), Eq. (8) is strictly negative. \( \square \)

4. The multi-period model

The base model developed in Section 3 is static and single-period formulation that is based on a given period over \( z_{i-1} \) and \( z_i \). Determination of the optimal replenishment schedule \( z_{i-1}^*, i = 1, 2, \ldots, n \), and the integer subscript \( n \), can be accomplished by extending the base model into a multi-period formulation using dynamic programming. Based on the principle of optimality for DP, given the current state \( z_{i-1} \), the two sub-problems over period \([0, z_{i-1}]\) and period \([z_{i-1}, z_i]\) are independent. The recursive relationship can be constructed as follows:

\[
\Pi_{z_i} = \max_{z_{i-1}} \{ \Pi_{z_{i-1}} + \pi(z_{i-1}, p^*, z_i) : 0 \leq z_{i-1} < z_i \leq H \},
\]

(9)

with boundary condition \( \Pi_0 = 0 \). In the model above, what is optimal for \( \Pi_{z_{i-1}} \) and \( \pi(z_{i-1}, p^*, z_i) \) is optimal for \( \Pi_{z_i} \). The recursive procedure works in a forward fashion to determine the maximal present worth of the total profit over \([0, z_i]\). On the last stage of the procedure, \( \Pi_{z_i} = \Pi_H \) is found that is the maximal discount profit over planning horizon \([0, H]\). The optimal sequence of replenishments \( z_{i-1}^* \), and the associated price \( p^* \) and lot-size \( Q^* \) can be determined by tracking backward from time \( H \) to time 0. Fig. 1 outlines the solution procedure of Model (9), in which \( T_{z_i}, p_{z_{i-1}}^*, Q_{d}^*(z_{i-1}, p^*, z_i) \) are defined below:

- \( T_{z_i} \) the starting point of the last replenishment cycle from time zero to time \( z_i \) (i.e. \( [T_{z_i}, z_i]\)),
- \( T_{z_i} = 0, 1, 2, \ldots, z_i - 1 \) and \( z_i = 1, 2, \ldots, H \),
- \( p_{z_{i-1}}^* \) the optimal price in cycle \([z_{i-1}, z_i]\), and
- \( Q_{d}^*(z_{i-1}, p^*, z_i) \) the optimal deteriorating quantity in cycle \([z_{i-1}, z_i]\).

The notations mentioned above are used to record the optimal replenishment cycles along the planning horizon as well as the optimal price and replenishment quantity within each cycle. As illustrated in Fig. 1, the algorithm initializes the dynamic procedure in step 1. Step 2 works recursively in a forward fashion to determine the maximal present worth of the total profit \( TP_{z_0} \) over the time horizon. Step 3 finally determines the optimal order cycles \([T_{z_i}, z_i]\) as well as the optimal price and replenishment quantity: \( p_{z_{i-1}}^* \) and \( Q^*(z_{i-1}, p^*, z_i) \) by tracking backward from time \( H \) to time 0.

5. Numerical study

Our model is valid for general deteriorating and demand functions. To remain focus, we use only constant deteriorating rate \( \theta(t) = x \) and the linear demand function: \( a_t - b_t \). The basic settings in the study are as follows: the number of periods \( H = 12 \), cost parameters \( A = 30, c = 1 \), and \( h = 0.03 \), time-discounting \( R = 0.02 \), deteriorating rate \( x = 0.15 \), and demand parameters \( a = 200, b = 75 \), and \( \lambda_1 \) and \( \lambda_2 \) are served as the key factors for the experimental design.

The proposed dynamic programming models were implemented on a personal computer with a Pentium CPU at 1.8 GHz under Windows XP operating system using Mathematica version 4.1. The computational time, on average, to solve the models was less than 2 seconds. Several numerical studies were conducted to attend qualitative insights into the structures of the proposed policy and its sensitivity with respect to major parameters. We focused in particular on investigating the solution property as well as the benefit of the periodic pricing over the fixed pricing policy in settings with time-decreasing demand \( (\lambda_1 = \lambda_2 = -0.075 \) and \( \lambda = 0 \)) and time-increasing demand \( (\lambda_1 = \lambda_2 = 0.075 \) and \( \lambda = 0 \)). The fixed pricing policy is conventionally determined
Algorithm DP {
    Step 0: Input parameters: \( A, c, h, H, R, \alpha, a, b, \lambda_1, \lambda_2 \).
    Step 1: Let \( \Pi_0 = 0 \) and \( z_{i-1} = 0 \).
        For \( z_i = 1 \) to \( H \) {
            Solve equation (5) to obtain \( p^* \);
            Solve equation (3) to obtain \( \pi^*(z_{i-1}, p^*, z_i) \);
            Let \( \Pi_{z_i} = \pi^*(z_{i-1}, p^*, z_i), \quad p_{z_i} = p^* \) and \( T_{z_i} = z_{i-1} \).
        } /* end of for-loop */
    Step 2: For \( z_i = 2 \) to \( H \) {
        For \( z_{i-1} = 1 \) to \( z_i - 1 \) {
            Solve equation (5) to obtain \( p^* \);
            Solve equation (3) to obtain \( \pi^*(z_{i-1}, p^*, z_i) \);
            If \( \Pi_{z_i} < \Pi_{z_{i-1}} + \pi^*(z_{i-1}, p^*, z_i) \)
            Then \( \Pi_{z_i} = \Pi_{z_{i-1}} + \pi^*(z_{i-1}, p^*, z_i), \quad p_{z_{i-1:z_i}} = p^* \), and \( T_{z_i} = z_{i-1} \).
        } /* end of for-loop */
        } /* end of for-loop */
    Step 3: Let \( z_{i-1} = H \).
        For \( z_i = H \) to \( 1 \) {
            If \( z_i = z_{i-1} \)
            Then \( z_{i-1} = T_{z_i} \);
            Replenishment cycle = \([T_{z_i}, z_i]\);
            Optimal sales price = \( p_{z_{i-1:z_i}}^* \);
            Replenishment lot size = \( Q^*(z_{i-1}, p^*, z_i) \), using equation (2);
            Cycle deteriorating rate = \( Q^*_d(z_{i-1}, p^*, z_i) \);
            Cycle profit = \( \pi^*(z_{i-1}, p^*, z_i) \);
            Cumulated profit = \( \Pi_{z_i}^* \).
        } /* end of for-loop */
    } /* end of DP algorithm */

Fig. 1. Solution procedure for the dynamic programming model.

in a decentralized, sequential fashion; the marketing department sets the price that maximizes the present worth of the expected gross profit, the market determines the quantity demanded, and the procurement department orders the realized quantity at minimum total cost.

Further, we examined the dynamic behavior of pricing trajectory over planning horizon under settings with an expanded time frame \( H = 24 \). Finally, we explored the impacts of the time-discounting factor \( R \), deteriorating rate \( \alpha \), and cost parameters \( A \) and \( h \) on the solutions.

5.1. Periodic policy vs. fixed policy

The comparison study is based on the basic settings, and let \( \lambda_1 = \lambda_2 = -0.075 \) represent the time-decreasing demand and \( \lambda_1 = \lambda_2 = +0.075 \) represent the time-increasing demand. In both cases, the coefficient of price-
sensitivity remains constant over time, i.e., \( \lambda = \lambda_2 - \lambda_1 = 0 \) such that we can compute a time-invariant price for the fixed price policy. The first scenario simulates the diminishing effect of demand when the product approaches the end of lifecycle and the second is to reflect the upward trend in demand over the growth phase of product lifecycle. The numerical results generated by the two policies are summarized in Table 1 that reports the optimal sequence of replenishments \( z_{i-1} \), the associated price \( p^* \) and lot-size \( Q^* \), and the accumulated discount profit up to the current epoch \( P_{zi} \). In addition, the table also reports the deteriorating quantity \( Q_d \) and the realized demand in the market \( D^* \), noting that the replenished lot-size is the sum of quantity demanded in the market and the loss due to decay: \( Q^* = D^* + Q_d \). In the experiment, the constant price generated from the fixed pricing policy is \( p^{**} = 1.833 \).

For the case of time-decreasing demand, the two policies generate equal number of replenishments \((n = 5)\) and identical sequence of replenishment schedule \((z_{i-1} = 0, 2, 4, 6, 9)\). Notably, the periodic policy generates higher selling price (1.942 and 2.003 vs. 1.833) and consequently produces less quantity demanded in the market (415.91 vs. 494.53), smaller replenishment lot-size (497.0 vs. 592.1), and less deteriorating loss (81.18 vs. 97.46). Furthermore, the proposed policy outperforms the conventional policy in maximizing discount profit by 8.2% increments (127.4 vs. 117.7). As for the case with time-increasing demand, the periodic policy generates less number of replenishments \((7 vs. 8)\), higher price (1.948 and 1.887 vs. 1.833), smaller lot-size (1228.4 vs. 1375.7), and more profits (496.4 vs. 482.5). Interestingly, the periodic pricing trajectory over the planning horizon is downward for the time-increasing demand and upward for the time-decreasing demand.

### 5.2. Dynamic behavior of price trajectory

This study is to investigate the impact of key factors such as \( \lambda_1 \) and \( \lambda \) on the dynamic behavior of price trajectory generated by the periodic policy. For better observation, a longer planning horizon \( H = 24 \) was applied. Coefficients \( \lambda_1 \) and \( \lambda \) represent the trend of market magnitude (increasing or decreasing) and price-sensitivity (increasing or decreasing) of demand, respectively. Fig. 2 graphically shows the price trajectories under various settings of market magnitude and price-sensitivity. In the cases of constant price-sensitivity, the price changes more often for higher values of \( \lambda_1 (= \pm 0.05) \), while the price stays constant for lower values of \( \lambda_1 (= \pm 0.01) \). In the cases of time-varying price-sensitivity \((\lambda \neq 0)\), the price is adjusted downward for the increasing price-sensitivity along time, and upward for the time-decreasing sensitivity.

<table>
<thead>
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<th>( \lambda_1 = \lambda_2 = +0.075 )</th>
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### Table 1

Numerical results

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<th>Fixed policy</th>
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Fig. 2. Price trajectories over the planning horizon.

Fig. 3. Sensitivity analysis.
5.3. Sensitivity analysis

Using the basic settings, we analyzed the sensitivity of the solutions generated by the periodic policy with respect to major parameters. Fig. 3 graphically illustrates the impact of deteriorating rate $\alpha$, time-discounting coefficient $R$, and cost parameters $A$ and $h$ on the profits generated by the policy. It is easy to observe that the profit is quite sensitive to parameters $A$ and $\alpha$, but less sensitive to parameters $R$ and $h$.

6. Conclusions

As opposed to the conventional fixed pricing policy, the proposed policy in this paper is more flexible in response to market demand by changing price upward or downward periodically. We have presented the necessary and sufficient conditions to the maximization problem based on a linear demand function, formulated the problem as a dynamic programming model, and described the solution procedure. An extensive numerical study has been conducted to attend qualitative insights into the structures of the proposed policy, investigate the behavior of price trajectory, and examine its sensitivity with respect to major parameters such as deteriorating rate, time-value factor, and cost parameters. The numerical results have shown that the solution generated by the periodic policy outperforms that by the fixed policy in maximizing discount profit.

Concerning with practical implementation of the proposed policy, frequent price-adjustments are required that can be costly and hence restricts its applicability. Fortunately, with the advance in information and internet technology, the enterprises are capable of implementing the periodic review inventory system and dynamic pricing in a cost-effective fashion. A natural extension of this research is to consider more complicated and practical demand functions such as random demand in the model. Another direction of this research is to develop a prototype of an advanced planning system with an ERP system [41] that integrates the management science techniques into commercial software for collaborative and robust planning.

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References

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