Individual Reinforcement and Social Reinforcement: Analytical Model of Individual Behavior in Social Context

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Abstract

Social cascades can be beneficial as well as harmful to the society itself. In order to understand the tendencies of such social dynamics, we designed an analytical model that considers the preference of individuals and the influence of social choices based on a binary choice model. To navigate the public away from a harmful cascade, it is important for leaders to slowdown the cascading effect as they think of ways to diverge public opinion. From our model we concluded that diverse opinions and the ability for individuals to think are important to dampen cascades to achieve a robust society.

Keywords: logit model; multi-agent model; availability cascade; individual preference; social influence; reinforcement; freedom of speech; diverse opinions; education; robustness

1. Introduction

In our daily lives we often face the dilemma of making a decision for ourselves or conforming to the decision of the social group to which we belong. A group’s decision and action, whether in reality or through social media networks such as Twitter and Facebook, can be effective and productive enough to cause political regime changes, as we have seen in the Arab Spring in recent years [1-2]. On the other hand, group dynamics can also be harmful especially if the members in the group are ignorant allowing the few individuals with an agenda to cause a social information availability cascade [3]. Additionally, blindly following a trend can also be unproductive, a phenomenon often seen in management or educational methodologies. The effectiveness of various schools of thought are difficult to prove, therefore they come and go with time [4-5]. Social media networks and the mob mentality can also be misleading, as several people were wrongfully accused for the Boston Marathon bombings in 2013 [6].
Whether the choice that emerges through the social aggregate being good or bad has been a topic of debate, based on recent studies. The fact that an assembly of ignorant agents making wise choice has been supported by studies dating back to the early 20 century [7]. More recently, there was a popular book providing principles to nourish wise “societies” or smaller groups such as teams [8]. This book was also appreciated in the business community to enhance the success of teams in corporate businesses. On the other hand, a recent study showed how individual’s “correct” choice can be influenced so easily that it can undermine the group arriving to a “wise” conclusion [9].

While some social movements may be for a good cause for the public, others may be harmful and cause economic damages as seen in the examples above [3, 10-12]. Since a social movement is an aggregate of agents acting within the society, it may be important to understand and to manage the individual agents to prevent harmful social cascades that may damage the society itself. In this paper we present a model that considers the individual’s preference reinforcement and social reinforcement on individuals in a binary choice setting and evaluate important qualities based on how the society behaves overall. This model may help us to understand and confirm what is necessary to maintain a healthy and dynamic society.

2. The Mathematical Model

We derive a logit model for the personal preference with two parameters that result from the model, “indecisiveness” and the “amount of change” in the preference at each time step. Indecisiveness reflects the agents’ ambiguity when making a decision. This delays the cascading time because the agent takes its time to make a choice. The amount of change reflects how radical an agent may change per time step. This personal reinforcement is combined with the social reinforcement which is the fraction of the population that made one choice over the entire population. The personal reinforcement and the social reinforcement are processed by a third parameter, the weight factor, which represents the extent of reinforcement by these two terms. Details are explained below.

2.1. Individual reinforcement

Zajonc found that when people are exposed to a stimulus repeatedly, people’s affection toward the stimulus increases [13]. Such behavior can be seen especially in marketing, therefore it is applied to advertising for merchandise [14] or campaigning for a political figure [15]. Whether to choose a particular product or a political figure can be simplified and described as a binary choice model. This dynamics is expressed by using the following logit model based on stochastic utility theory for individual decision making. We elaborate the work done by Namatame and Matsuyama [16] and replace the difference in the utility so that it is represented by a single parameter that is controllable. We express the deterministic utilities of individuals by Ui, where i can take the value A or B.

\[
\mu = \frac{1}{1 + \exp\left(-\frac{(U_A - U_B)}{\lambda}\right)} \tag{1}
\]

Normally, \( \mu \) represents the choice probability of A as a function of the difference of the utility between the choices of A and B. However, considering the payoff matrix (Table 1) for the two choices \( U_A \) and \( U_B \), the payoff for \( U_A \) is \( a \) and the payoff for \( U_B \) is \( 1-a \). The utility of B, \( U_B \), can be expressed as \( 1-U_A \). Then we have \( U_A-U_B=a-(1-a)=2a-1 \). The value of \( U_A-U_B \) varies as \(-1 \leq (U_A-U_B) \leq 1 \). Hence, \( 0 \leq a \leq 1 \).
Table 1. Payoff matrix for utility of binary choices, A and B.

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<th>$U_A$</th>
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<tr>
<td>$U_A$</td>
<td>$a$</td>
<td>0</td>
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<tr>
<td>$U_B$</td>
<td>0</td>
<td>$1-a$</td>
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Therefore we get,

$$\mu = \frac{1}{1 + \exp\left\{-(2a-1)/\lambda\right\}}$$

Parameter $\lambda$ stands for the indecisiveness of making a decision. To represent the psychological factor of increasing or decreasing affection [13-15], we change the value $a$ at every time step with $\Delta a$. The sign of $\Delta a$ is decided stochastically, i.e. we compare $\mu$ with a random number. If $\mu$ is larger (smaller) than the random number, $\Delta a$ is positive (negative). When $a(t+1) = a(t) + \Delta a$, the agent prefers A more and when $a(t+1) = a(t) - \Delta a$, the agent prefers A more. When A is chosen, choice for B can be expressed as $b(t+1) = 1 - (a(t) + \Delta a)$. Therefore at $t+1$, $U_A - U_B = a(t+1) - b(t+1) = 2a + 2\Delta a - 1$. This formulation can also be used to describe the diffusion of innovative technological products in the consumer market, when old technologies get replaced by new technologies [16].

2.2. Social reinforcement

Considering human psychology of preference, as long as people are social creatures, people are influenced by each other’s opinions. Assuming that agents make reasonable decision, very often the majority’s opinions influence the individuals [17]. This is a cause of fads such as soft technologies which are management methodologies based on little or no supportive evidence [18] or baseless social movements that may actually be damaging for the society [3]. In the context of a binary choice A and B, the fraction that a choice A is chosen by a given group can be expressed by the number of population choosing A over the entire population of the group,

$$F = \frac{A(t)}{A(t) + B(t)}.$$  

2.3. Combined together

We assume that an individual has a choice to follow its own preference or to be reinforced by the preference of the community to which the individual belongs. We use the parameter $\alpha$ ($0 < \alpha < 1$) to shift the preference influence from individual reinforcement to social reinforcement. The social reinforcement grows stronger as the individual preference decline. This preference tradeoff parameter is expressed with $\alpha$ on the individual preference term and $1-\alpha$ on the social reinforcement term.

$$\mu(t+1) = \alpha \ast \frac{1}{1 + \exp\left\{-(2\alpha t-1)/\lambda\right\}} + (1-\alpha) \ast \frac{A(t)}{A(t) + B(t)}$$
3. Simulation method

We controlled three parameters, indecisiveness ($\lambda$), amount of change in preference per cycle ($\Delta a$), and social reinforcement on the individuals ($\alpha$). The models are expressed as a graph with the probability of choosing one option in the x-axis. The right most value of the x-axis, $x=1$, is the probability of choosing the choice A and the left most value of the x-axis, $x=0$, is the probability of not choosing A, in other words the probability of choosing B. If the indecisiveness is high, the agents start in the middle of the x-axis, $x=0.5$, where the probability of choosing A or B is 50%. The y-axis is the number of agents choosing the probability on x-axis. In this model, we use 200 agents, however the graphs display only up to 100 agents due to space limitations.

4. Simulation results

First we show how the indecisiveness $\lambda$ and the amount of change in preference per time step $\Delta a$ contribute to the behavior of the agents. Then we present the way the influence of the individual reinforcement and social reinforcement affect the overall dynamics.

4.1. Initial conditions

When the indecisiveness $\lambda$ is low, the preferences of the agents are scattered evenly over the preference probability (Fig 1 (a)). When the indecisiveness is high, the agents tend to start with preferences closer together, leaving the preference population distribution in the form of a normal probability distribution, centered in the middle of the x-axis (Fig 1 (b)).

![Fig. 1. Graphs of initial conditions when (a) the indecisiveness $\lambda$ is low ($\lambda=0.05$, $\Delta a=0.03$, $\alpha=1$) and (b) the indecisiveness is high ($\lambda=0.95$, $\Delta a=0.03$, $\alpha=1$).](image)

4.2. The contribution of the amount of change in preference per time step, $\Delta a$

The amount of change in preference per time step results in agents grouping together in smaller groups with increasing distance as the amount of change becomes larger (Fig 2).

4.3. Individual reinforcement and social reinforcement, $\alpha$

In this section we focus on how the parameter $\alpha$ contributes to the results. The section is largely subdivided into 3 sections, when there is individual reinforcement only ($\alpha=1$), when there is social reinforcement only ($\alpha=0$) and when there is both individual and social reinforcement ($0<\alpha<1$).
4.3.1. Case 1: Individual reinforcement only ($\alpha=1$)

The population divides evenly most of the time when there is a strong individual reinforcement and little or no social reinforcement, when $\alpha$ is equal to 1 or close to it (Fig 3). Fig 3 (d) is the time progression of how the agents’ preference average changes over time.

4.3.2. Case 2: Social reinforcement only ($\alpha=0$)

The other extreme from the individual reinforcement is when there is a strong social reinforcement (Fig 4). Similar to the case of strong individual reinforcement, the result has a high reoccurrence rate, although whether the population tends to one choice or the other is unpredictable. The social choice would depend on the unbalance in the layout of the initial values. Because of such a strong social reinforcement, the population quickly runs to one extreme or the other with little hesitation or irregular dynamics in the intermediate process.

4.3.3. Case 3: Both Individual and social reinforcement ($0<\alpha<1$)

This section shows results when the agents receive influence from both the individual reinforcement and social reinforcement. It is largely separated as the agents dividing and agents cascading to one choice. The behavior may differ for every run because of stochastic reasons as well as from the contributions from other parameters, $\lambda$ and $\Delta \alpha$. 
4.3.3.1. The population divides unevenly and the average is off the center.

The population divides unevenly (Fig 5). This occurs when \( \lambda \) value moves away from 1 and the social reinforcement becomes stronger \((a=0.4)\). Not only does the population divide unevenly, but also the two extremes preferences are attracted to each other because of the social reinforcement. As a result, the center of the two preferences will move toward the preference with more popularity. Consequently, the average is off the center.

4.3.3.2. The population cascades to one extreme.

Below are various timing in how the agents cascade. Some cascade quickly (Fig 6), some cascade suddenly after a gentle initial cascade (Fig 7), and others take their time as they approach the end (Fig 8).

4.3.3.2.1. The population comes together at the center and then moves to one side. On occasions, it drags a tail when approaching one end.

When \( \lambda \) is higher than approximately 0.2, the population tends to bond at the center (Fig 6 (a)). If the personal reinforcement \( a \) is small \((a<0.5)\), the population of the preference probability will move to one extreme quickly (Fig 6 (b), (c), and (e)). If \( a \) is approximately 0.5, the crowd will have individuals that resist being reinforced by the population and therefore the crowd will leave a trail of individuals behind (Fig 6 (d)). These individuals are affected by the population and are eventually absorbed by the majority population (Fig 6 (e)).
4.3.3.2. The population divides, maintains the division and moves to one side, and gradually forms a one sided cascade.

The population divides and stays close to the center of the probability distribution for a while, maintains the division as the population moves to one extreme, and all agents are absorbed to that side (Fig 7). As seen earlier in section 4.2, when the amount of change in the preference $\Delta a$ is high, it tends to make the grouping of agents stronger. This is believed to be how the divided population is maintained in that form for a while. Since the individual preference is low ($\alpha=0.2$) and therefore the social reinforcement is high, the population is eventually absorbed to one end.

4.3.3.2.3. The population wrestles itself to one side.

The individuals within the group tend to stick close together as the crowd overall moves toward one extreme (Fig 8). This occurs when there is a relative tension balance between the individual preference and social preference at approximately $\alpha=0.7$ with respect to other parameters. The change in preference $\Delta a$ is small enough that the individuals do not make radical moves, and the indecisiveness is fairly high ($\lambda>0.4$) so that the group tend to stick together.
Fig. 7. Graphs of time progression of agent preference when the agents’ preference cascades in the end. $\lambda=0.05$, $\Delta a=0.2$, $\alpha=0.2$. (a) At time=0. (b) At time=8. (c) At time=75. (d) At time=270. (e) Graph of preference average vs. time (The time goes up to 250 in this graph).

Fig. 8. Graphs of time progression of agent preference when the agents’ preference cascades in the end. $\lambda=0.8$, $\Delta a=0.03$, $\alpha=0.6$. (a) At time=0. (b) At time=15. (c) At time=50. (d) At time=123. (e) Graph of preference average vs. time.
5. Discussion

The model presented in this paper describes the probability of choices of agents based on personal reinforcement and social reinforcement. According to Zajonc [13], the more people are exposed to a stimulus, the more they are prone to liking it. Such psychological tendency is reinforced if they had selected that stimulus to begin with. Without social reinforcement, agents tend to divide evenly due to their preference (Fig 3). On the other hand, without individual reinforcement, agents choice will be completely swayed by the society (Fig 4). Since people are social beings as well as being individuals, they are influenced by each other’s choices. This will result in the unbalance of population distribution between the two choices (Fig 5). Even though the agents are divided, when social reinforcement is present their preferences are not as extremely separated away as when there was no social reinforcement. The agents of two different preferences are attracted to each other, therefore they settle closer together (Fig 5). This shows that when there is social reinforcement, even if there are two different choices, the agents that choose one preference is also aware and drawn to the other opinion. We observe this to be a natural balancing trend. This may be interpreted as the importance of freedom of speech and encouraging diverse opinions in order to achieve a healthy balanced society to prevent social cascades [3]. When there is too much social reinforcement and suppression of independent thought, the entire society will cascade to a one-sided opinion [19]. As a society, ideological cascading may be damaging in terms of economics, operational safety, and convenience. When the indecisiveness is low and the agents are under strong social reinforcement, the population cascades very quickly as in Fig 6. To increase the thinking time and therefore to make it easier to prevent a cascade, or at least to allow more time to think, it will be necessary to increase the indecisiveness. In terms of society, this means to educate individuals so that each person can think analytically and make their own decisions. Even though the agents cascade, Fig 8 has a higher indecisiveness and it takes longer to complete the cascading process. Allowing more individual reinforcement leaves individuals resisting to be affected by the social dynamics. This is seen in Fig 6 (d) where the population drags a tail of agents behind. Fig 7 also shows a resistance for a cascade where the population maintains two divided preferences at the center for a while, then shifts to one of the extremes while maintaining the divided two preferences, then gradually the population of the minority gets absorbed by the majority.

The amount of change in preference per time step pushes the agents’ probability of choice further away from other preference groups and closer together within its own preference groups. In other words, the amount of change in preference per time steps makes each subgroup of preference more distinct and dissimilar from other preference groups (Fig 2).

In order to hamper a social cascade, our model suggests that individuals are responsible to think, and the community must respect diverse opinions (Fig 6 (d), 7, 8). With the presence of diverse opinions, each preference group will be reinforced by the other preference groups (Fig 5). Our model may be objectively suggesting key principles to achieve and maintain a healthy balanced society.

6. Conclusion

We constructed a combined model that considers both the individual reinforcement of agents and the reinforcement by social trends of the agents. Depending on the degree of parameters, some society preferences divide and others cascade. The trends of cascade can be analyzed further where the cascade occurs instantly or taking a longer time. Social cascades can be seen occasionally in different professions and its damages can be extensive. Cascade taking a longer time may be a key solution to prevent harmful social cascades since time can allow leaders to evaluate the situation and navigate the public to reconsider their opinions based on better understanding and further studies. In order to gain time in the cascading aggregate of the agents, individuals are encouraged to form diverse opinions and to think for themselves. These support the modern democratic values of freedom of speech and individual education to think analytically in order to maintain a robust society. As for
future progression of this research, it is important to consider the network structure of the agents involved in
the preference reinforcement of individuals in the social context. In this current research, all agents are affected
evenly by the social choice in a way that would be a complete graph structure. Since in reality the society has a
“small world” property, we can connect the agents using a small-world network [20]. Such consideration of
networks may give us a better understand of the influence of social media networks.

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