Study on stress wave propagation in fractured rocks with fractal joint surfaces

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Abstract

This paper presents the experimental and theoretical investigation of property of stress wave propagation in jointed rocks by means of SHPB technique and fractal geometry method. Our aim focuses on the influence of the rough joint surface configuration on stress wave propagation. The comparison of behavior of reflection and transmission waves, deformation and energy dissipation of a rough joint surface characterized by its fractal feature with that of a smooth plane joint has been carried out. It has shown that the rough joint surface distinctly affects the stress wave propagation and energy dissipation in the jointed rocks. The rougher the joint surface was, the more permanent deformation occurred and the more attenuation stress wave took place as well. A nonlinear relationship between the normalized energy dissipation ratio

\[ \frac{W_J}{W_I} \]

of the jointed rock and the joint roughness in terms of the fractal dimension has been formulated. It seems that the ratio

\[ \frac{W_J}{W_I} \]

of a roughly jointed rock, however, tends to be the same as that of a smoothly jointed rock if the fractal dimension is less than a critical value

\[ D_c = 2.20 \]

The energy dissipation ratio at the critical point

\[ D_c \]

seem to be a constant, not dependent of rock type but fractal joint configuration.

Keywords: Stress wave propagation; Jointed rocks; Joint surface configuration; Fractal dimension; Roughness

1. Introduction

Blasting is one of the most widely adopted excavation techniques for underground mining, tunneling and civil engineering. Rock blasting results in ground shock and vibration which may cause damage to either the rock bodies or the surrounding structures such as buildings, dams, slopes, bridges and tunnels. Of importance
to ensure the safety of rock bodies and surrounding structures and make it run effectively is to well understand the way that blasting stress wave propagates in rocks and to establish an efficient method to evaluate the deformation and failure of rocks under these stresses. As it well known, the natural rock is a sort of composite separated by the embedded joints, interfaces, cracks, pores and faults. The discontinuous and heterogeneous composition of the rock leads to a complicated mechanical property and make it very difficult to evaluate the wave propagation and attenuation and to quantify its deformation and failure process during stress wave transmission.

In addition to the mechanical properties of intact rocks and the geological factors, the joints connecting the fragmented rocks significantly affect the wave propagation and attenuation. This problem has been investigated theoretically, experimentally and numerically by going through the sound wave, seismic wave or blasting wave effects. A good number of theoretical, empirical and numerical models concerning the geometrical and mechanical properties of the joint (or interface) have been developed to figure out the wave propagation and attenuation effects in jointed rocks (e.g., Ghaboussi et al., 1973; Murty, 1975, 1976; Blair and Spathis, 1982; Bandis et al., 1985; Lemos, 1987; Pyrak-Nolte et al., 1990; Vashisth et al., 1991; Smyshlyaev and Willis, 1994; Roy and Pyrak-Nolte, 1995; Pyrak-Nolte, 1996; Capuani and Willis, 1997; Wu et al., 1998; Chen and Zhao, 1998; Cai and Zhao, 2000; Jing and Hudson, 2002; King, 2002; Stavropoulou et al., 2003; Dineva et al., 2004; Fan et al., 2004; Krasnova et al., 2005; Gulyayev and Ivanchenko, 2006). These relevant studies have shown that the performance of stress wave propagation through the joints or interfaces depends on the stress wave type, amplitude, history, joint configuration, interface property, contact modes and bond characters.

Basically, the joints herein were classified, in the previous studies, as the micro joints and the macro joints. The micro joints refer to those cracks whose length and width are smaller as compared with the wavelength. The scattering phenomenon occurred when the stress wave passed through these interfaces. The scattering of the wave can be analytically solved using the dispersion theory with the assumption of the linear or nonlinear interface contact modes (Murty, 1975; Comninou and Dundurs, 1980; Hudson, 1981; Angle and Achenbach, 1985; Chevalier et al., 1991; Hirose and Achenbach, 1993; Smyshlyaev and Willis, 1994; Capuani and Willis, 1997). The macro joints are those cracks whose length is longer but the width is smaller as compared with the wavelength. For the case of applied stress wave with lower amplitude, the discontinuous deformation as well as the wave transmission and reflection through these cracks can be quantified using the linear models (Schoenberg, 1980, 1983; Pyrak-Nolte et al., 1990; Roy and Pyrak-Nolte, 1995; Pyrak-Nolte, 1996). For the nonlinear deformation caused by the stress waves with higher amplitudes, a couple of nonlinear discontinuous displacement models have been established to characterize the transmission and reflection of waves through the joined surfaces (Capuani and Willis, 1997; Chen and Zhao, 1998; Cai and Zhao, 2000; King, 2002). Mostly, the researches focused on the two-dimensional analyses.

An interesting question arising from the previous attempts is that in most cases the natural joint surface has been assumed to be a two-dimensional smooth plane. The trace, i.e., the intersection of the plane and its vertical section protruding out of the rock formation, is therefore simplified as a one-dimensional straight line. For the normally projected incident stress wave, this plane was treated to be perpendicular to the stress wave direction and no shear deformation along the surface has been taken into account. However, a natural joint surface is a three-dimensional (Indeed the real dimension should be greater than 2.0 and less than 3.0) irregular, i.e., non-smooth surface. Its trace turns to be a one-dimensional (Indeed the real dimension should be greater than 1.0 and less than 2.0) irregular curve instead of a straight line (see Fig. 1). It is hard to distinguish whether the stress wave is vertical or somehow inclined to this irregular surface comprising of different segments with different directions. Even though the joint surface can be pictured as a set of various small straight segments, the conventional elastic wave theories can not directly apply to analyzing the stress wave propagation since the varied directions, uncertain contact mode and incompatible deformation of sub-surfaces make the stress wave transmission and reflection undetermined. The stress at the neighborhood of the joint is in a complex state rather than a uniaxial state where the conventional elastic wave theories apply. This means that the assumption of the smooth plane is a simplification upon which the deformation and property of stress wave propagation of jointed rocks might not be properly characterized. It remains as a difficult problem in the evaluation of the stress wave propagation of fractured rocks.

The aim of our paper is that we expect to quantify the influence of irregular configuration of the natural joint on stress wave propagation in fractured rocks. The three-dimensional features of the rough joint surface
will be considered in this attempt. The property of stress wave across the three-dimensional irregular surface will be characterized in terms of the geometrical parameters of the joint configuration and the strain energy dissipation. To achieve this goal, the fractal theory and the SHPB (Spit Hopkinson Pressure Bar) experimental technique have been employed.

Foundational work concerning the geometrical description of the irregular joint surface has been carried out, to the best of our knowledge, either by means of the statistical parameter methods or by the fractal measurement methods (Mandelbrot, 1982; Johnson, 1985; Brown, 1987; Zhao, 1997; Xie, 1993; Xie et al., 1998, 1999, 2001; Falconer, 2003). The relationship between the static mechanical property of rocks and the rough configuration has been analyzed (Xie, 1993; Kwaśniewski and Wang, 1997; Zhao, 1997; Xie et al., 1997; Kulatilake et al., 1998; Falconer, 2003). However, few studies on the stress wave propagation have taken the effect of the three-dimensional rough joint into account.

Our experimental investigation comprises of two parts. First, we generated some irregular surfaces using the three-point bending fracture tests, and measured the fractality of these surfaces by means of a 3D digital laser scanning profilometer and the fractal measurement method (Xie et al., 1998, 1999; Zhou and Xie, 2003). To exam the effect of a two-dimensional smooth plane for comparison purpose, we split a rock cylinder using an electric cutter to produce a smooth plane. Subsequently, we combined the two fractured parts as an entire thin cylinder jointed by a fractal surface or a smooth plane, respectively. The jointed rock samples were then mounted into the SHPB system to collect the data of the incident wave, transmission and reflection waves through the impact. The relationship between the wave attenuation characters, the energy dissipation and the fractality of rock joints has been analyzed in terms of the different joint configurations.

2. Experimental outline

2.1. Generation of joint surfaces and fractality measurement

To investigate the influences of a rough joint and a smooth joint on wave propagation, we devised and tested the various joint surfaces with different profiles. The specimens were made out of marbles and granites, respectively. The setup for the three-point bending fracture tests and the sample dimensions are briefly illustrated in Fig. 2. Fig. 3 presents the photographs of the typical fractured surfaces of rocks and the 3D laser scanning profilometer. An example of three-dimensional profile of a fractured surface of a marble specimen attained using the laser machine is shown in Fig. 4. The fractality of the joint surface has been measured to identify the three-dimensional configuration by applying the fractal projective covering method (Xie and Wang, 1999; Zhou and Xie, 2003). After scanning and measuring, we incorporated two separate parts of the fractured sample as an entire one jointed either by a rough surface or a smooth plane. A smooth joint
plane was cut out of a rock cylinder. The scanning capacity of the laser machine scopes up to 30 mm with a resolution of 7 μm and a minimum scanning increment of 7.5 μm. Totally, 19 marble and 17 granite specimens have been tested. The average measured compression strength and flexure tensile strength for marbles are 143.39 MPa and 9.72 MPa, respectively, and those values for granites are 194.99 MPa and 15.5 MPa, respectively. The test results have been summarized in Section 3.
2.2. SHPB (Split hopkinson pressure bar) impact tests

The SHPB (Split Hopkinson Pressure Bar) is a common experimental technique nowadays for testing the dynamic performance of solid media. This technique finds its origin in the work of Hopkinson (1914) who used it to measure the pressure-pulse profile using a long thin bar. The practical setup with two long bars and a short specimen between them, widely used today, was introduced and developed by Kolsky (1949). According to the principles of SHPB and one-dimensional elastic wave propagation, the dimension of a sample and its stress state significantly affect the stress wave propagation. To minimize the unexpected effects of inertia, the transversal dispersion and the frictions between ends of the specimens and input and output bars, the authors applied a 12 mm-long jointed sample with a radius of 30 mm, meeting the requirement of \( h = \sqrt{3} v_s a \) and \( h/C_a \), where \( a \) is the radius of a cylinder and \( v_s \) is a dynamic Poisson’s ratio of rock under the impact loads (Davies and Hunter, 1963; Lindholm, 1964; Lifshitz and Leber, 1994). As far as the impact sample preparation is concerned, a 12 mm-long specimen with a joint surface in the middle, no matter a rough one or a smooth one, was cutting down cautiously from a firmly glued specimen. The ends of the sample have been ground very carefully such that the frictionless condition can be satisfied before we installed the sample in between the steel bars. Fig. 5 presents the setup of SHPB test and the pulse measurement method. The incident impact wave was generated and propagated along the longitudinal direction of the cylinder, vertically projected to the joint surface.

Basically, this investigation aims at the effect of the irregular surface configuration on wave transmission. To minimize the side effect of large plastic deformation and additional cracking, we applied an impact striker speed ranging from 6 m/s to 7 m/s such that the plasticity or cracking in the specimen induced by the impact were negligible. It means that no irreversible energy dissipated except for the displacement of joint surface during the wave propagation. To achieve this point, we have firstly proceeded the SHPB tests with intact rocks using different impact speed varying from 4.8 m/s to 8.9 m/s to determine an appropriate speed under which the rock deformed within the scope of elasticity. The impact speed of striker of 6.8 m/s was finally adopted for SHPB test of the jointed rock samples.

The strain gauges were used for recording the stress wave profile and were positioned in the middle of the incident and transmission bars with the same distance to the ends. For the purpose of recording the stress wave continuously and accurately and cleaning up the influence of pulses reflected from the free and contacted ends, the distance between the strain gauges and the ends of the bars remains larger than the length of striker bar (see Fig. 5). The data and wave profiles were acquired automatically by SHPB system.

3. Results and analyses

3.1. Fractal property of joint surface

Figs. 6 and 7 illustrate a few typical three-dimensional profiles of the fractured surfaces and the calculations of the fractal dimensions of rough surfaces using fractal projective covering method for marbles and granites, respectively. At least five samples either for marbles or for granites have been repeatedly tested to make sure the measure-
ment reliable. The diagrams show a very good linear relationship between the covering box count $N$ and the covering scale $d$ in the bi-logarithm coordinates. According to the fractal theory, the fractured surfaces therefore revealed the fractal features and a fractal dimension applies to quantify the irregularity of the joint surface.

However, it should be aware of that the fractal dimension did not appear to be a parameter that sufficiently and completely identifies the rough surface properties (Xie, 1993; Kwaśniewski and Wang, 1997; Xie et al., 1998; Falconer, 2003). It physically specifies the irregularity or the roughness of the three-dimensional surface. The intercept of $y$ coordinate on a $N$–$d$ diagram, for instance, provides another indication showing how much the altitude of each point on the surface vibrates. It may affect the contact property of the joints. Our experimental results show that for marble surfaces the fractal dimension $D$, reflecting the irregularity, varies within the range of 2.2254–2.3836, and the intercept $b$ of $y$ coordinate, reflecting the altitude vibration, falls in the range of 6.1836–7.1211. The characters for the granite surface configuration change within the range of 2.3171–2.3834 and 6.5717–7.4427, respectively. Clearly, the fractal dimension vibrates less than the intercept of $y$ coordinate. It means that the irregularity or the roughness of each fractured surface is fairly close, but the altitude fluctuation relative greatly changes from each other. This measurement gets in accordance with the claim that, for the same category of the rock, the fractal dimension of a fractured surface could be close and its intercept of $y$ coordinate may not be necessarily same, even though the generation method was identical (Kwaśniewski and Wang, 1997; Xie et al., 1998). The variation of the intercept $b$ of a fractured surface, on the other hand, reflects the influence of the material strength and uncertain experimental factors on the surface configuration. The fractality of the fractured surface memorizes the structural changes of the material responding to the external loads or any other actions.

3.2. Property of stress wave propagation

Fig. 8 shows a number of typical curves of stain vs. time of the incident, transmitted and reflected pulses we have collected from the marble and granite samples with the fractal joints, respectively. As a comparison,
Fig. 9 illustrates the relevant curves of the marble and granite samples with the smooth plane joints. To make sure the measurement reliable, at least five or six samples either for marbles or for granites have been tested repeatedly for each type of joints. It is shown that the amplitude \( \varepsilon_T \) of the transmission wave considerably reduced comparing to the original incidence wave \( \varepsilon_I \) no matter what type of the joint, i.e., rough or smooth, was applied. The amplitude \( \varepsilon_T \) of the transmitted pulse attenuated much more when it traveled through a fractal surface than when passed a smooth plane. Meanwhile, the amplitude \( \varepsilon_R \) of the reflected pulse bouncing from the fractal surface is greater than that of wave bouncing from the smooth one. The similar phenomena have been found for both the marble and the granite specimens. In addition, no large separation between the two rough faces of the joint has been observed during the wave transmission.

The experimental observation manifests, however, that a few small permanent deformation (i.e., displacement) did take place by different amount both in the rough surface and in the smooth plane under the stress wave. This permanent deformation brought about stress wave attenuation by different amount depending on the joint type. The overall permanent deformation of a fractal surface is larger than that of a smooth plane with respect to the same incidence wave. This provides with the evidence that the deformation of an irregular joint surface took place not only in the normal direction but also in the other directions. The result shows the way in which the irregular surface affects the deformation and wave propagation through the joint.

On the other hand, from the energy point of view, this permanent deformation leads to a part of irreversible energy that the incidence wave dissipated on the joint during the propagation. The loss of energy of the wave transmission due to the permanent deformation has brought about the stress wave attenuation. In what follows it can be quantitatively explained using the strain energy analysis.
From the SHPB principles (Davies and Hunter, 1963; Lindholm, 1964; Lifshitz and Leber, 1994), considering the length of the specimen is considerably small and no plasticity happened, we therefore suppose that the stress wave propagates along the incidence and the transmission bars without stress attenuation. Denote \( e_I \), \( e_R \) and \( e_T \) the strains of the incident, reflected and transmitted pulses, respectively. From the one-dimensional theory of elastic wave propagation we have (Lindholm, 1964)

\[
u = c_0 \int_0^t \varepsilon \, dt'
\]

where \( \nu \) is the displacement at time \( t \), \( c_0 \) is the longitudinal wave velocity in the impact bars, and \( \varepsilon \) denotes the strain. The displacement \( u_1 \) of the end of the incident bar is the result of both the incident strain pulse \( \varepsilon_I \) and the reflected strain pulse \( \varepsilon_R \) (see Fig. 10). Thus

\[
u_1 = c_0 \int_0^t \varepsilon_I \, dt' - c_0 \int_0^t \varepsilon_R \, dt' = c_0 \int_0^t (\varepsilon_I - \varepsilon_R) \, dt'
\]

Similarly, the displacement \( u_2 \) at the end of the transmission bar can be derived from the transmitted strain pulse \( \varepsilon_T \) as the following
Therefore, the average strain $e_s$ in the specimen can be obtained

$$e_s = \frac{u_1 - u_2}{h_0} = \frac{c_0}{h_0} \int_0^t (e_I - e_R - e_T) \, dt'$$

where the parameter $h_0$ denotes the initial length of the specimen. Since the specimen between the ends of the incidence and transmission bars is small enough, we reasonably suppose the stress across the specimen to be constant. With this assumption, the applied loads at each face of the specimen satisfy (see Fig. 10)

$$E_0 A_0 \varepsilon_I + E_0 A_0 \varepsilon_R - E_0 A_0 \varepsilon_T = 0$$

where indices $E_0$ and $A_0$ denote the modulus of elasticity and the cross-sectional area of the impact bars, respectively. Thus

$$\varepsilon_I + \varepsilon_R - \varepsilon_T = 0$$

Substituting Eq. (6) into Eq. (4) gives

$$u_2 = c_0 \int_0^t \varepsilon_T \, dt'$$

Fig. 8. Typical strain vs. time curves of incident, transmitted and reflected pulses normally directed to fractal joint surfaces. (a) Two marble samples; (b) two granite samples.
Consequently, the average stress, $r_s$, in the sample satisfies

$$r_s A_s = E_0 A_0 e_T$$

Thus

$$\sigma_s = E_0 \left( \frac{A_0}{A_s} \right) e_T$$

where $A_s$ refers to the initial cross-sectional area of the specimen.

Since that the intact parts of the rock were in a state of elasticity and no plasticity occurred, one might apply these formulae to the jointed rocks. The strains $e_I$, $e_R$ and $e_T$ can be obtained from the measured incident, reflected and transmitted pulses shown in Figs. 8 and 9.

From Eqs. (7) and (9), it is not difficult to tell that the overall deformation of the rock specimen with a fractal joint is relatively greater than that of the specimen with a smooth joint as the amplitude $e_R$ of the reflected pulse bouncing from the fractal surface is larger than that of the smooth plane. Meanwhile, the average stress in the rock with the fractal surface turns to be smaller as compared to the sample with the smooth joint because of the more attenuated amplitude $e_T$ of its transmitted pulse.
As far as the energy dissipation is concerned, from above assumptions, the energy of the incident, reflected and transmitted waves denoted by $W_I$, $W_R$ and $W_T$ can be derived as

$$W_I = A_0 \cdot c_0 \cdot \varepsilon_I(t_0)$$

(10)

$$W_R = A_0 \cdot c_0 \cdot \varepsilon_R(t_0)$$

(11)

$$W_T = A_0 \cdot c_0 \cdot \varepsilon_T(t_0)$$

(12)

where $\varepsilon_I(t)$, $\varepsilon_R(t)$ and $\varepsilon_T(t)$ denote the stresses in the pressure bars which can be determined by the relevant strains of the incident, reflected and transmitted pulses shown in Figs. 8 and 9. Furthermore, considering that the longitudinal wave velocity $c_0$ of pressure bars can be written as $C_0 = (E_0/\rho_0)^{1/2}$, where $\rho_0$ is the material density of pressure bars, we rewrite the above formulae.

$$W_I = \frac{A_0}{c_0 \rho_0} \int_0^t \sigma_I'(t') \, dt'$$

(13)

$$W_R = \frac{A_0}{c_0 \rho_0} \int_0^t \sigma_R'(t') \, dt'$$

(14)

$$W_T = \frac{A}{c_0 \rho_0} \int_0^t \sigma_T'(t') \, dt'$$

(15)

Regarding the stress wave propagation as an isothermal process without any heat exchange with exterior, and friction free between the ends of the specimen and pressure bars, we correlate each part of energy in accordance with the first law of thermodynamics

$$W_1 = W_R + W_J + W_T$$

(16)

Thus

$$W_1 = W_1 - W_R - W_T$$

(17)
where $W_J$ represents the overall irreversible energy caused by the permanent deformation of the joint including the normal and tangential components. Substituting $W_I$, $W_R$ and $W_T$ into the Eq. (17) and utilizing Matlab as a tool, one can attain the quantity of $W_J$, the part of dissipated energy during the joint deformation. Tables 1 and 2 calculate the energy $W_J$ dissipated on a fractal joint and a smooth plane of marbles and granites, respectively.

To minimize the incidence wave variation, we set the normalized dimensionless variables $W_R/W_I$, $W_T/W_I$ and $W_J/W_I$, i.e., the ratios of energy of reflected wave, transmitted wave and the dissipated part to the input energy of incident wave, to quantify the energy change induced by the deformed joint. Fig. 11 outlines the relationship between the quantity $W_J/W_I$ and the fractal dimension of the joint surface. The triangular symbol represents the normalized energy $W_J/W_I$ dissipated on the two-dimensional smooth plane. Considering the unexpected disturbance in the SHPB installment, sample alignment and data acquiring, the error data have not been taken into account in Tables 1 and 2, as well as Fig. 11.

Based on the principle of least square method and the regression analysis, the tendency of the energy dissipation ratio $W_J/W_I$ increasing with the fractal dimension $D$ has been plotted in the Fig. 11. It implies that, despite of the data variation, the energy dissipation rate $W_J/W_I$ grow up nonlinearly with an increasing fractal dimension of the surface. It means that the rougher the surface was, the more energy dissipated in the joint. The trajectory of $W_J/W_I$ therefore reveals how the energy dissipated in a joint evolves with the surface irregularity.

Interestingly, the energy dissipation ratio $W_J/W_I$ of a roughly jointed rock tends to be the same as the value of a smoothly jointed rock if the fractal dimension of the joint surface is less than a critical value $D_c = 2.20$.

Table 1
Comparison of energy dissipation with respect to fractal and plane joints of marbles

<table>
<thead>
<tr>
<th>Fractal dimension ($D$)</th>
<th>Incidence wave $W_I$ (J)</th>
<th>Reflection wave $W_R/W_I$ (%)</th>
<th>Transmission wave $W_T/W_I$ (%)</th>
<th>Ratio of energy dissipation $W_J/W_I$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Measured</td>
<td>Average</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.2254</td>
<td>27.0210</td>
<td>4.7696</td>
<td>64.5002</td>
<td>30.7302</td>
</tr>
<tr>
<td>2.3413</td>
<td>24.0959</td>
<td>5.5744</td>
<td>62.9979</td>
<td>31.4727</td>
</tr>
<tr>
<td>2.3673</td>
<td>23.4914</td>
<td>7.2364</td>
<td>62.0423</td>
<td>32.2906</td>
</tr>
<tr>
<td>2.3836</td>
<td>27.0172</td>
<td>3.8638</td>
<td>63.8456</td>
<td>32.6890</td>
</tr>
<tr>
<td>2.3971</td>
<td>24.1472</td>
<td>4.2833</td>
<td>63.0277</td>
<td>32.6890</td>
</tr>
<tr>
<td>Smooth surface $D = 2.0$</td>
<td>24.2917</td>
<td>4.2858</td>
<td>64.3747</td>
<td>31.3395</td>
</tr>
<tr>
<td></td>
<td>23.1371</td>
<td>6.0547</td>
<td>62.3171</td>
<td>31.6282</td>
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<tr>
<td></td>
<td>25.7608</td>
<td>6.8218</td>
<td>63.6614</td>
<td>29.5168</td>
</tr>
</tbody>
</table>

Table 2
Comparison of energy dissipation with respect to fractal and plane joints of granites

<table>
<thead>
<tr>
<th>Fractal dimension ($D$)</th>
<th>Incidence wave $W_I$ (J)</th>
<th>Reflection wave $W_R/W_I$ (%)</th>
<th>Transmission wave $W_T/W_I$ (%)</th>
<th>Ratio of energy dissipation $W_J/W_I$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Measured</td>
<td>Average</td>
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<td></td>
</tr>
<tr>
<td>2.2588</td>
<td>24.8538</td>
<td>3.4116</td>
<td>65.7823</td>
<td>30.8061</td>
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<td>2.2646</td>
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<td>2.3220</td>
<td>23.7879</td>
<td>6.7185</td>
<td>63.1271</td>
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<td>2.3328</td>
<td>23.9182</td>
<td>8.0183</td>
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<td>2.3594</td>
<td>25.1837</td>
<td>3.1965</td>
<td>65.5765</td>
<td>31.2270</td>
</tr>
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<td>2.3683</td>
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<td>6.5958</td>
<td>61.5498</td>
<td>31.8544</td>
</tr>
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<td>Smooth surface $D = 2.0$</td>
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<td>5.2164</td>
<td>63.7221</td>
<td>31.0615</td>
</tr>
<tr>
<td></td>
<td>21.3582</td>
<td>4.7813</td>
<td>65.5547</td>
<td>29.6640</td>
</tr>
<tr>
<td></td>
<td>27.3493</td>
<td>3.8239</td>
<td>64.7753</td>
<td>31.4008</td>
</tr>
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</table>
The normalized quantity \( W_J/W_I \) ascends continuously only when the fractal dimension exceeds the point \( D_c = 2.20 \). The similar tendency has been observed in both marbles and granites. Meanwhile, it shows that the energy dissipation \( W_J/W_I \) of the marble at the critical point \( D_c \) is slightly greater than that of the granite. For the current experimental condition, the values \( W_J/W_I \) for marbles and granites are 30.82 and 30.71, respectively. Disregarding the tiny difference possibly resulted from measurement, the values \( W_J/W_I \) for marble and granite at the point \( D_c \) seem to be identical. It means that the critical energy dissipation ratio is a constant, not dependent of rock type but fractal joint configuration.

For engineering application purpose, applying the regression analysis method, the tendency of the normalized energy rate \( W_J/W_I \) can be formulated in terms of the fractal dimension of the rough joint surface as following

\[
\frac{W_J}{W_I} = \begin{cases} 
(3.63 - 1.19D)D^{3/2}e^D & (D < D_c < 3.0) \\
C_0 & (2.0 \leq D \leq D_c = 2.20)
\end{cases}
\]

where the critical energy dissipation ratio \( C_0 \) is a constant depending upon the joint configuration.

It should be aware of, because of the approach to producing the fractured surfaces, that the fractal dimensions \( D \) of the joint surfaces varied within the range of 2.00–2.40. The deformation of a joint appeared relatively small. However, a natural joint surface could be more rough and the fractal dimension could be larger.
than 2.40. Therefore, the suitability of the analysis to the case of the fractal dimension greater than 2.40 needs to be verified.

Clearly, the current attempt that authors presented above was to explain the mechanism how the roughness or irregularity of the joint surface affect the stress wave transmission. Nevertheless, from the engineering application point of view, for a natural joint surface, unfortunately, it is difficult to directly determine the fractal dimension that we used in the analysis and formulae above. A realistic joint does not allow us to directly unveil and measure the surface as what we have done in Section 2.1. Fortunately, a few theories and techniques that have been developed to picture the surface (Russ, 1994; Xie and Wang, 1999; Develi et al., 2001; Xie et al., 2001; Zhou and Xie, 2003) might be helpful to solve this problem. Both the laboratory test and the mathematical analysis have indicated that the fractal dimension of a three-dimensional rough surface basically correlates with the values of its ‘one-dimensional’ protruding traces. The fractal interpolation and generation strategy can be used to build a three-dimensional surface using the ‘one-dimensional’ information of the trace profiles. The properties of the fractal surface can therefore be estimated, including the fractal dimensions. This could be a way to find out the fractal dimension of an embedded natural joint surface and to apply the method we propose here to estimate its wave transmission in engineering.

4. Summary and conclusions

The SHPB investigation and application of fractal geometry show that a rough joint surface distinctly affected the stress wave transmission in the jointed rocks compared with a smooth plane joint. The irregular surface configuration brought on the permanent deformation not only in the normal direction and also in the other direction of the surface. The rougher the joint surface was, the more permanent deformation occurred. It was the permanent deformation that caused the energy dissipation in the joint and led to the stress wave attenuation. The rougher a joint surface was, the more attenuation that stress wave took place. The energy of the incidence wave has been dissipated by different amount depending on the roughness of joint configuration.

The fractal dimension was employed to identify the roughness of the three-dimensional joint surface. A relationship between the fractal dimension of the joint surface and the dimensionless normalized energy dissipation rate $W_J/W_I$ has been formulated. The ratio of energy dissipation $W_J/W_I$ presents how much the energy of the incident wave has been dissipated in the joint. It is shown that the energy dissipation rate $W_J/W_I$ grows up nonlinearly with an increasing fractal dimension of the surface if a fractal dimension exceeds the critical point $D_c = 2.20$. The energy dissipation ratio $W_J/W_I$ of a rough joint tends to be the same as the value of a smooth joint if the fractal dimension is less than $D_c = 2.20$. Disregarding the tiny experimental difference, the values $W_J/W_I$ for marbles and granites at the point $D_c$ seem to be identical. It implies that the critical energy dissipation ratio turns to be a constant, not dependent of rock type but joint configuration.

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