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Dark energy from quantum wave function collapse of dark matter

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ABSTRACT

Dynamical wave function collapse models entail the continuous liberation of a specified rate of energy arising from the interaction of a fluctuating scalar field with the matter wave function. We consider the wave function collapse process for the constituents of dark matter in our universe. Beginning from a particular early era of the universe chosen from physical considerations, the rate of the associated energy liberation is integrated to yield the requisite magnitude of dark energy around the era of galaxy formation. Further, the equation of state for the liberated energy approaches $w \to -1$ asymptotically, providing a mechanism to generate the present acceleration of the universe.

1. Introduction

More than a century of development of physical theory since the advent of quantum mechanics and relativity has led to profound advancements of our understanding of the microcosm as well as the macrocosm. Yet certain deep mysteries have emerged in the study of particular aspects like the quantum measurement problem [1] of the former, and the mechanism for the presently accelerating universe [2] of the latter arena. Satisfactory resolution of these two fundamental challenges encountered by modern physics may call for close introspection of any possible domain of overlap between the solutions that have been offered separately for either of them.

The linearity and unitary evolution of quantum theory give rise to the entanglement of elementary particles having widespread and fascinating applications [3]. Basic quantum theory predicts the persistence of quantum entanglement even for macrosystems [4]. However, in practice, it is difficult to realize the quantum entanglement of macroscopic entities over large distance and time scales. The emergence of classicality observed in the real world is hard to understand in terms of any simple limiting behaviour of quantum theory. A key issue not explained by quantum theory is *how* a definite outcome occurs as the result of an individual measurement on a quantum system [1]. Over the years several approaches have been suggested to tackle this problem, such as environment induced decoherence [5], the quantum state diffusion picture [6], the consistent histories approach [7], the Bohmian ontological interpretation [8], and the dynamical models of wave function collapse [9–13].

In dynamical collapse models wave function collapse is regarded as a real physical process describing the measurement dynamics without discontinuity or added interpretations. In the 'Spontaneous localization' models [9] the unitary Schrödinger evolution is modified by stochastic nonlinear terms that affect the dynamics on time scales relevant to typical macrophysical situations. The emergence of classicality and the occurrence of single outcomes in measurements is achieved by the interaction of the fluctuating modes of a scalar field with the relevant wave functions at a rate proportional to the number or the mass [10] of the particles involved. The mass-dependent collapse rate is somewhat similar to the spirit of gravity induced state vector reduction [11]. Recently, relativistic generalizations of collapse models have been made [13], displaying the conservation of the energy exchanged between the scalar field and the collapsing matter. In essence, the dynamical collapse models are able to achieve the quantum to classical transition within standard quantum theory with just the additional postulated role played by a fluctuating scalar field. It is thus desirable that the existence of such an energy liberating scalar field be motivated by some other physical considerations.

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In the cosmological arena one or more scalar fields have been invoked to account for the observed features of the universe since its very early stages. The inflationary paradigm based on the dominant scalar field energy is widely accepted as an essential extension of the standard big-bang cosmological model [14]. Scalar fields play central roles in unified particle physics (electroweak and grand-unification) models and string- and brane-theory models as well, and much of the physics of the early universe is inspired by these models [14].

Observations of high redshift Type Ia Supernovae (SN Ia) [2] led to the conclusion that our universe is presently undergoing a phase of accelerated expansion. This behaviour of the present universe is possible through the presence of a dominant "dark" energy component. Apart from SN Ia observations, indirect evidences from CMB anisotropy and large-scale structure studies show that the dark energy constitutes about 70% of the total energy density of the universe at present [15], is smoothly distributed in space, and has large negative pressure. Several possibilities, such as the existence of a cosmological constant, for dark-energy candidates have been proposed (see Ref. [16] for reviews). The idea that a scalar field rolling along the slope of its potential (quintessence [17] or *k*-essence [18] models) provides the required amount of dark energy has gained some popularity. A generic feature of such models is the "ad-hoc" construction of the scalar field potentials to ensure compatibility with observational constraints [16]. The problem of "cosmic coincidence" [16], as to why the scalar field energy density starts dominating just before the present era, remains.

The motivation for this Letter is to look for a possible connection between these two independently well-founded proposals involving scalar fields in separate domains, namely dynamical wave function collapse advocated for the emergence of classicality of the quantum world, and the mechanism for the scalar field driven present acceleration of the universe, respectively. Our purpose here is to gain additional insights on these problems in a scenario where the cosmic scalar field causes dynamical wave function collapse of the constituents of dark matter in the universe.

2. The scheme

We begin by considering the wave function collapse process in spontaneous localization models [9,10,12,13] which is triggered for a wave function involving one or more particle(s) when the scale of their superposition in position space exceeds the value given by a parameter a_*. The associated rate of energy liberation in mass-dependent dynamical collapse models [9,10,13] for a system with mass M is given by

$$\frac{dE}{dt} = \frac{3\hbar^2 M}{4m_0^2 a_*^2 T_*} \equiv \beta, \tag{1}$$

where $m_0 = 10^{-24}$ g, $a_* = 10^{-5}$ cm and $T_* = 10^{16}$ s are the parameters of the collapse models [9,10,12,13]. These values are chosen such that any superposition of the wave functions of microparticles like the proton is kept intact over the relevant distance and time scales, and are consistent with the results of all laboratory experiments performed so far [12]. Let us consider a situation where there is a uniform distribution of mass in a region of volume V. The rate of energy liberation Eq. (1) can be written as

$$\frac{d}{dt}(\rho V) = \beta, \tag{2}$$

where ρ is the density of the energy gained by the region V. If the fluctuating scalar field ϕ that drives the collapse has an energy density ρ_ϕ, at any given time a part of it is pumped into the region V with an instantaneous rate $\dot{\rho}_\phi$ such that the rate of energy loss by the field in the volume V at this particular instance of time is given by $\dot{\rho}_\phi V$. Conservation of energy [13] between the scalar field and the collapsing matter in the region V dictates that

$$\dot{\rho}_\phi V = -\beta \tag{3}$$

using Eq. (1).

We now apply the above arguments to the dynamical collapse of the dark matter constituents of our universe, driven by the interaction with the fluctuating modes of a cosmological scalar field. We have in mind the typical scenario of the early universe where matter is distributed uniformly in our expanding Robertson–Walker (RW) Hubble volume with scale factor R. The expansion of the universe aids the collapse process since the physical separation between two comoving and entangled wave packets (the physical length scale of the superpositions in position space) increases with time. The rate of wave function collapse should be higher at earlier times since the corresponding rate of expansion is also higher. These considerations were used to evaluate the effect of the energy liberation due to the dynamical collapse of baryonic wave functions [19]. In the present analysis we focus on the collapse of the dark matter since its contribution to the total energy density of the universe exceeds that of baryonic matter by more than one order of magnitude [15]. Replacing ρ by ρ_m (where ρ_m is the energy density of dark matter), in Eq. (2), one obtains

$$\dot{\rho}_m + 3\rho_m \frac{\dot{R}}{R} = \frac{\beta}{R^3}. \tag{4}$$

It is apparent that due to the expansion of the universe all of the energy supplied by the scalar field (r.h.s. of Eq. (4)) does not contribute towards increasing the matter energy.

The activation of the process of dynamical collapse of the constituents of dark matter require the following criteria to hold. The physical size of the superposed wave functions of the dark matter particles in position space should be comparable to or greater than the parameter a_*. Secondly, the scalar field should possess a finite energy density to drive the collapse. The integration of Eq. (4) over a time period during which dynamical collapse of matter in the expanding RW background is effective with the rate given by Eq. (4) (say, from t_i to $t_{\rm end}$) gives the total energy density of dark matter at the time $t_{\rm end}$, i.e.,

$$\int_{t_i}^{t_{\rm end}} \dot{\rho}_m \, dt = (\rho_m)_{\rm end} + (\rho_m)_{\rm extra} \tag{5}$$

where $(\rho_m)_{\rm end}$ is the standard amount of the dark matter energy density at time $t_{\rm end}$ (that can be obtained from the initial density at time t_i by the usual scaling due to expansion), and $(\rho_m)_{\rm extra}$ is the excess contribution coming from the dynamical collapse process.

In the standard model of particle physics, mass of particles is created by the electroweak symmetry breaking. Further, there may not be any matter (even of exotic types) in the universe before the electroweak phase transition, since the matter–antimatter asymmetry in the universe may itself be created at that epoch [14]. Any particle created earlier is very quickly annihilated by its antiparticle. Since we are considering a mass-dependent wave function collapse scheme in the present analysis, it is justifiable to put a lower limit on the time scale on which collapse is activated in the early universe to be of the order of $t_{\rm EW}$, the epoch of the electroweak phase transition in the early universe. Moreover, if supersymmetry is unbroken, a finite vacuum energy for any field is not possible (any contribution from a bosonic field gets cancelled by its fermionic superpartner). Therefore, the vacuum energy of the scalar field does not exist and thus could not be responsible for driving collapse before supersymmetry breaking in the early universe. Since, in several plausible models of high energy physics, supersymmetry is broken just prior to the epoch of the electroweak phase transition [14], this further motivates our choice of $t_i = t_{EW}$.

On the other hand, the rate of energy gain in Eq. (4) is not valid beyond the time when most of the dark matter constituents have decoupled from the background RW expansion due to clustering in the process of structure formation in the universe. Thus we set $t_{end} = t_{galaxy}$ (where t_{galaxy} is the time scale of galaxy formation [14]). Note that the process of wave function collapse does not of course end at this time, but since the dark matter constituents are not homogeneously distributed any longer, the rate given by Eq. (4) ceases to be valid beyond this epoch. Thus, setting $t_i = t_{EW}$, and $t_{end} = t_{galaxy}$, and upon performing the integral in Eq. (5) (from t_{EW} to t_{EQ} first using $R \propto t^{1/2}$ in the radiation-dominated era, and then, with matching boundary conditions, from t_{EQ} to t_{galaxy} using $R \propto t^{2/3}$ in the matter-dominated era), we obtain

$$\rho_m \simeq (\rho_m)_{galaxy} + \frac{\beta t_{EQ}}{R^3_{galaxy}} \tag{6}$$

where $(\rho_m)_{galaxy}$ is the observed density of dark matter at the era of galaxy formation, and t_{EQ} is the epoch of matter–radiation equality. Note that the expression for β given in Eq. (1) contains M that is the total mass of all dark matter in the universe. Making use of the fact that dark matter contributes to about 50% of the critical density around the era of galaxy formation, we set $M/R^3_{galaxy} = 0.5\rho_c$ in Eq. (6). Substituting the standard value [14] of $t_{EQ} = 10^{11}$ s, one gets

$$\rho_m \approx 0.5\rho_c + \mathcal{O}\big(10^{-22}\big)\rho_c, \tag{7}$$

where ρ_c is the critical energy density at t_{galaxy}. One sees that the increase of the matter energy density by the dynamical collapse process is negligible, which retains its standard value (of the order of $0.5\rho_c$) at this era.

At this stage it may be noted that through our Eqs. (4)–(7), we have calculated the net effect of the wave function collapse process on the dark matter energy density at the epoch of galaxy formation in the universe. We have seen that the dark matter energy does not increase in any non-negligible way. However, conservation of the energy liberated by the scalar field aids in the expansion of the universe, as we show now. In order to compute the total energy liberated by the scalar field ϕ from the era t_{EW} to the era t_{galaxy}, we now integrate its instantaneous rate of energy liberation (the scalar field pumps in energy at this rate throughout the above span of time, regardless the state of the individual constituents of dark matter particles) obtained from Eq. (3), i.e.,

$$\dot{\rho}_\phi = -\frac{\beta}{R^3} \tag{8}$$

during the time from $t_i = t_{EW}$ to $t_{end} = t_{galaxy}$. Again using the relations $R \propto t^{1/2}$, and $R \propto t^{2/3}$, in the radiation and matter-dominated eras, respectively, and using Eqs. (1) and (3), we obtain the total magnitude of the energy liberated by the field ϕ till the era of galaxy formation to be

$$(\rho_*)_{galaxy} \equiv \int_{t_{EW}}^{t_{galaxy}} (-\dot{\rho}_\phi\, dt) \simeq \frac{2\beta t_{EQ}}{R^3_{galaxy}} \left(\frac{t_{galaxy}}{t_{EQ}}\right)^2 \left(\frac{t_{EQ}}{t_{EW}}\right)^{1/2}. \tag{9}$$

Putting in the values of $t_{galaxy} = 10^{16}$ s, and $t_{EW} = 10^{-10}$ s [14], one obtains

$$(\rho_*)_{galaxy} \approx (\rho_m)_{galaxy}, \tag{10}$$

where the right-hand side denotes the standard amount of dark matter (obtained from usual scaling due to expansion of the universe) at this epoch. Therefore, the excess energy ρ_* forms a significant part ($\sim 50\%$) of the total energy density at about the era of galaxy formation. (Note that galaxy formation is a continuous process, but we have set $t_{end} = t_{galaxy} \approx 10^{16}$ s implying a cut-off beyond which more than half of the total matter density in the universe is gravitationally clustered, and hence the rate of energy gain given by Eq. (4), as also the rate of energy liberation given by Eq. (8) would no longer be valid.) This energy liberated by the scalar field does not, of course, add to the matter energy, as we have seen from Eq. (7), but nonetheless does contribute to the expansion of the universe.

If the RW expansion takes place in the standard adiabatic manner [14], the total energy density ρ_T of all the constituents and the pressure p must satisfy the relation

$$\frac{d}{dt}\big(\rho_T R^3\big) = -p\frac{d}{dt}\big(R^3\big). \tag{11}$$

Around the time of galaxy formation in the matter-dominated era, $\rho_T \simeq \rho_m + \rho_*$ (assuming that any remnant energy residing in the scalar field is negligible compared to ρ_m and ρ_*) and $p_m \approx 0$ since the energy liberated through wave function collapse is unable to substantially increase the kinetic energy or temperature of the dark matter constituents (similar to the wave function collapse of ordinary matter as verified by the results of laboratory experiments [12]). Using Eqs. (4) and (8) in Eq. (11) one obtains,

$$p = p_* = -\rho_* - \frac{2\beta}{3R^2\dot{R}}. \tag{12}$$

Since the second term in the Eq. (12) falls off as $1/t$, it follows that the $(w = p/\rho)$ parameter approaches $(w = -1)$ asymptotically. The equation of state for the "dark" energy (DE) around the era of galaxy formation (t_{galaxy}) is hence given by

$$p = p_* = -\rho_* - \frac{\beta t_{galaxy}}{R^3_{galaxy}} \simeq -\rho_* - \mathcal{O}\big(10^{-17}\big)\rho_*, \tag{13}$$

resembling closely the equation of state for the cosmological constant [14]. Till the time Eq. (8) is approximately valid, the DE density increases as $\rho_* \sim -1/t$ in the matter-dominated era. Hence, the liberated dark energy ρ_* with Eq. (13) as its equation of state can generate the accelerated expansion of the universe once it exceeds the dark matter density around the era of galaxy formation.

3. Observational implications

Beyond t_{galaxy}, the matter energy density ρ_m falls off as $1/t^2$, whereas the dynamical collapse process for the matter continues adding to the "dark" energy (DE), albeit with a different rate from that given in Eq. (8). The computation of such a rate would need to take into account the back-reaction [20] of structure formation on the Robertson–Walker metric, which is beyond the scope of our present analysis. Assuming a ΛCDM model of the universe, the time t_{galaxy} ($= 10^{16}$ s) corresponds to a red-shift $z_{galaxy} \approx 13$. The equation governing the evolution of the DE density $\rho_*(z)$ (from Eq. (8)) is

$$\frac{d\rho_*}{dz} = -\frac{\beta(1+z)^2}{H(z)}, \tag{14}$$

where $H(z)$, the Hubble parameter at red-shift z is

$$H(z) = H_0\sqrt{\Omega_m(0)(1+z)^3 + \frac{\rho_*(z)}{\rho_c(0)}}, \tag{15}$$

$H_0, \Omega_m(0)$ and $\rho_c(0)$ being the Hubble parameter, the non-relativistic matter density fraction and the critical density at the current time ($z = 0$). A numerical integration of Eq. (14) starting from $z = z_{\text{galaxy}}$ when $\rho_*(z)$ was half of the critical density at z_{galaxy} to $z = 0$ does not lead to the requisite amount of DE ($0.73\rho_c(0)$) at the present time. This just shows that a naive extrapolation of the rate equation (Eq. (8)) beyond t_{galaxy} ($= 10^{16}$ s) is not useful as the dark matter particles will decouple from the FRW expansion.

Recent additions to SN Ia data sets obtained from various projects like the Hubble Space Telescope (HST) ($z \geqslant 1$) [21], the Supernova Legacy Survey (SNLS) ($z \leqslant 1$) [22] and the ESSENCE SN Ia survey ($z \leqslant 0.7$) [23] place strong constraints on the DE equation-of-state index $w(z)$, its variation with red-shift and also the epoch of transition from the matter-dominated deceleration phase to the negative-pressure DE-dominated acceleration phase. Let us now obtain some order-of-magnitude estimates of the variation of $w(z)$ ($= p_*(z)/\rho_*(z)$) with redshift and also the red-shift of transition to the accelerating phase determined by the cross-over of the deceleration parameter $q(z)$ from positive to negative values. To this end, we first model the DE density evolution by a linear law as follows. Let $\Omega_*(z)$ be the ratio of the DE density at any z with respect to the current value of the critical density. Then,

$$\Omega_*(z) \equiv \frac{\rho_*(z)}{\rho_c(0)} = A_0 + A_1(1+z), \tag{16}$$

where the constants A_0 and A_1 are determined by the two conditions: $\Omega_*(0) = 0.73$ and the equality of the dark-matter and DE at z_{galaxy}, i.e.

$$\Omega_*(z_{\text{galaxy}}) = \frac{1}{2}\left[\Omega_m(1+z)^3 + \Omega_*(z)\right]_{z=z_{\text{galaxy}}}. \tag{17}$$

Using Eq. (16) in the DE equation-of-state (easily obtained from Eq. (12) we find that $\frac{dw}{dz}$ is $\mathcal{O}(10^{-17})$ for all z upto z_{galaxy} which is in conformity with (HST) observations which rule out rapidly evolving DE at early times [21]. The epoch of transition from deceleration to acceleration z_T is defined by

$$q(z_T) \equiv \frac{1}{2} + \frac{4\pi}{H^2(z_T)}p_*(z_T) = 0, \tag{18}$$

which, using Eq. (16), comes out to be $z_T \approx 19$. It is interesting that the above prediction of our model from this crude estimate is approximately within an order of magnitude of the ΛCDM value of $z_T = 0.73$ and observational estimates from joint analysis of SN Ia and CMB data [24]: $z_T = 0.39 \pm 0.03$ for the best-fit of the DE models considered and $z_T = 0.57 \pm 0.07$ on assuming ΛCDM priors on the $\Omega_m(0)$ and H_0 at the present epoch. As stated earlier, more refined calculations incorporating the effects of back-reaction [20] of structure formation on the associated energy liberation rate are required to obtain accurately the evolution of $\Omega_*(z)$ at low red-shifts in order to confront our model with present observations.

4. Conclusions

To summarize, the above calculations show that dynamical wave function collapse [9,10,13] when applied to the constituents of dark matter in the universe, offers a possibility for the generation of dark energy responsible for the present acceleration of the universe [2]. A unified framework for dark energy and dark matter has been presented since in this approach the former is generated through the interaction of the latter with a cosmic scalar field. This scheme, though resembling in spirit some other unified approaches to dark matter and dark energy [25], is formally quite distinct from them. Also, unlike in many other models of dark energy involving the scalar field [16–18], construction of complicated potentials is avoided, and the requisite magnitude of dark energy with equation of state ($w = -1$) emerges at about the era of galaxy formation. This energy which could reside either as a kinetic or a potential energy component of the scalar field, is liberated in a scheme of quantum mechanical disentanglement of the constituents of dark matter. The scheme presented here combines some essential features of two hitherto distinct solutions offered respectively for the quantum measurement problem [1] and the dark energy problem [2].

Of course, more detailed calculations are needed to develop our model further. For this purpose it should be particularly useful to evaluate the energy liberated after the era of galaxy formation up to the present time. Such calculations will have to take into account the complex technical and conceptual aspects [26] of the interface of quantum coherence and gravitational collapse. Further, the full dynamics of the scalar field including its possible interactions with matter has to be considered in the setting of the expanding universe in order to make our analysis more comprehensive. Recently, the scheme of dynamical localization through a quantized scalar field has been developed [27] which promises to have rich consequences on gravitational physics and the physics of the early universe. It might be worthwhile to further develop our idea in the context of such a scheme. The application of quantum entanglement in the dark energy problem has also been considered in other different contexts [28].

Finally, we wish to emphasize that the transfer of energy between the "environment" and the "system" is a generic feature of the quantum decoherence paradigm [29]. Though it may be coincidental for our present simplistic calculation to yield the requisite magnitude of dark energy, it is relevant to note that the mathematical structure of the collapse models [9,10,13] has striking similarities [30] (in spite of interpretational differences) with that of other decoherence schemes such as the quantum state demolition approach [6], and the consistent histories approach [7]. Of course, detailed microscopic modelling of the "system-environment" interaction processes in the cosmological background are needed in the context of the various models [6,7,13] to confront the idea of decoherence induced dark energy with observational data [15,16]. Our present analysis aims to show that quantum wave function collapse may indeed play a role in the emergence of the accelerating phase of the universe.

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