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# Sensitivity analysis of vertically loaded pile reliability

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**Abstract**

The purpose of this paper is to examine the influence of geotechnical uncertainties on the reliability of vertically loaded pile foundations and the use of this information in decision-making support, especially when gathering the information necessary for reliability analyses. Two case studies of single pile foundations were selected, and each uncertainty source was investigated to identify which are the most important and influential in the evaluation of vertical pile resistance under axial loading. Reliability sensitivity analyses were conducted using FORM (the first-order reliability method) and MCS (Monte Carlo simulations). The characterisation of uncertainties is not an easy task in geotechnical engineering. The aim of the analyses described in this paper is to optimise resources and investments in the investigation of the variables in pile reliability. The physical uncertainties of actions, the inherent variability of soil and model error were assessed by experimental in situ standard penetration tests (SPT) or from information available in the literature. For the cases studied, the sensitivity analysis results show that, in spite of the high variability of the soils involved, model error also plays a very important role in geotechnical pile reliability and was considerably more important than soil variability in both case studies. From a comparison of the two reliability methods (FORM and MCS), it was concluded that FORM is applicable in simple cases and as a first approach because it is an approximate method and sometimes does not have the capability to incorporate every detail of the problem, namely a specific probability density function or more specific limit conditions.


*Keywords:* Deep foundation; Pile design; Probability of failure; Reliability theory; Spatial variability; Standard penetration test; Uncertainties; Vertical bearing capacity

## 1. Introduction

Pile foundations are often used for important structures, and thus, reliability evaluation is an important aspect of the design of such structures. Unlike the approach to reliability evaluation used in structural engineering, the traditional procedure used in geotechnical design addresses uncertainties through high global or partial safety factors, mostly based on past experience. This approach to addressing uncertainties does not provide a rational basis for understanding their influence on design. For this reason, and because of regulation codes (JCSS, 2001; CEN, 2002a;

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Watabe et al., 2009; Honjo et al., 2010b), and social concerns (such as sustainability), geotechnical engineers need to improve their ability to deal with uncertainties and probabilities to help with decision-making.

Reliability methods have become increasingly important as decision support tools in civil engineering and in geotechnical applications, especially over the past two decades (Einstein, 2001; Honjo et al., 2002; Paikowsky, 2004; Honjo et al., 2005; Yang, 2006; Cherubini and Vessia, 2007; Fenton and Griffiths, 2007; Phoon, 2008; Juang et al., 2009; Honjo et al. 2010a; Huang et al., 2010; Wang, 2011). Reliability analyses are conducted for the purpose of determining the probability of reaching a behavioural limit and involve introducing estimates of geometric, material and actions variability into the design process. The main benefit of reliability analysis is that it provides quantitative information about the parameters that most significantly influence the behaviour under study. This makes risk control, the determination of the potential causes of adverse effects on the structure, possible.

The design of pile foundations still involves many limitations and uncertainties, particularly when there is not enough investment in soil characterisation and pile load tests. In addition to the uncertainties associated with soil characterisation (pile design based on insufficient data and using theoretical approaches that do not characterise the model error well), physical, statistical, spatial and human uncertainties exist. However, because it is technically and economically impossible to produce designs of pile foundations in the most unfavourable of cases, it is the engineer's goal to minimise the risk and limit it to an acceptable level in the most economical manner possible.

First developed for other areas of engineering design, reliability theory needs to be adapted to the needs and objectives of geotechnical engineering. This requires consideration of spatial correlations and attention to the influence that the number of samples analysed has on the quantification of the standard deviations and means of geotechnical parameters. Although the extent to which this can be accomplished depends on the engineer's knowledge and the project's budget for investigation, geotechnical engineering definitely benefits from the consideration of reliability in design (Christian, 2004; Najjar and Gilbert, 2009).

The primary purpose of this paper is to demonstrate the application of reliability methods to two distinct case studies of vertical single pile foundations under axial loading. This paper also presents a simple and practical approach to performing reliability-based design (RBD) in geotechnical problems and obtaining valuable information from it. For that purpose, sensitivity analyses were conducted to study the influence of each uncertainty type. In addition, two well-known RBD methods, the first-order reliability method (FORM) and Monte Carlo simulations (MCS) were applied to the case studies for comparison.

Another purpose of this paper is to demonstrate the advantages of employing RBD in the decision-making process for pile foundation design. The decision-making related to the economic and research investments required for gathering the information necessary to characterise the uncertainties associated with important random variables, in both pile design and its reliability, is facilitated by this type of balanced reliability analysis. Therefore, this work makes a significant contribution to the application of RBD to pile design. This type of approach is important not only for decision-making but also for identifying the direction in which geotechnical design research should proceed (Honjo, 2011).

## 2. Reliability approach

### 2.1. Reliability levels

A construction project can be evaluated by different methods, the level of accuracy of each one depends on the way that uncertainties are considered in the design (Madsen et al., 1986; Nowak and Collins, 2000; Zhang and Chu, 2009a, b). Very briefly, these levels are classified as follows:

- Level zero: deterministic methods, in which the random variables (RVs) are taken as deterministic and uncertainties are taken into account by a global safety factor (SF) based on past experience.
- Level I: semi-probabilistic methods, in which deterministic formulas are applied to representative values of RVs multiplied by partial SFs. The characteristic values are calculated based on statistical information, while the partial SFs are based on level II or level III reliability methods, defined subsequently.
- Level II: approximate (simplified hypothesis) probabilistic methods, in which RVs are characterised by their distribution and statistical parameters (mean and standard deviation (SD) or coefficient of variation (COV = SD/mean)). The probabilistic evaluation of safety is then achieved using approximate numerical techniques.
- Level III: full probabilistic (simulation) methods, based on techniques that take into account all of the probabilistic characteristics of the RVs.
- Level IV: risk analysis, in which all of the probabilistic characteristics and the consequences of failure are taken into account. The risk (consequences multiplied by the probability of failure) is then used as a measure of the reliability. This allows for the comparison of solutions on an economic basis, taking into account uncertainty, costs and benefits.

Levels zero and I (one) are traditional approaches to design, while levels II (two) and III (three) are approaches commonly used for the evaluation of the probability of failure. Within reliability analysis, the most popular methods are the first-order reliability method and Monte Carlo simulations, which correspond to level II and level III, respectively. MCS is widely used because of its higher level of accuracy and because it is the most straightforward method for reliability analysis, while FORM is very traditional and has been used since the first studies of structural reliability were conducted. These are the two methods applied in this paper for reliability and sensitivity analyses. A full description of these reliability methods can be found in Manohar and Gupta (2005).

### 2.2. General methodology

The following procedure is used for both FORM- and MCS-based reliability analysis:

1. Definition of the significant failure modes and formulation of their functions ($g(X_i)$)

   Generally, the performance function is defined as written in Eq. (1):

   $$M = R - E = g(X_i) \tag{1}$$

   where $M$ is the safety margin, $R$ denotes the resistance, $E$ denotes the action, $g$ is the performance function, and $X_i$ are the random variables;
2. Identification of the random and deterministic variables;
3. Description and characterisation of the RVs, namely the statistical parameters – the mean, SD, COV and distribution types (probability density function, PDF) – as well as identification of the dependencies among them (by their covariance matrix);
4. Selection of the target reliability index ($\beta_T$) or probability of failure ($pf$), which have the relationship shown in Eq. (4) and Fig. 2.

### 2.3. FORM methodology

Level II RBD using FORM is based on successive linear approximations to a nonlinear performance function (Fig. 1), with statistically dependent and/or non-normally distributed RVs. Following the previously described general reliability analysis procedure (steps 1 to 4), reliability analysis using FORM is accomplished as follows:

i. Transforming all RVs into standard normalised RVs $\rightarrow Z \sim N(0,1)$;

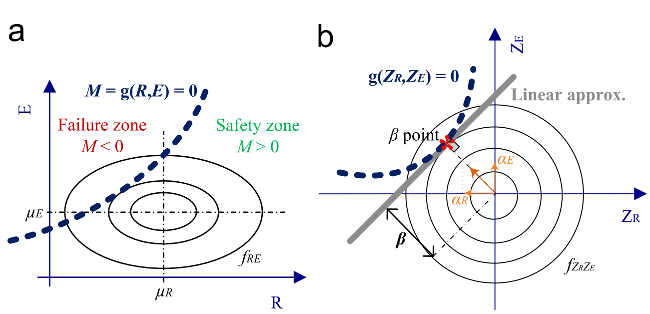


Fig. 1. FORM: transformation of the variables, definition of the design point ($Z^*$), reliability index ($\beta$) and sensitivity factors ($\alpha$). (a) Graphical representation of the problem and (b) Normalised space and linear approximation.

ii. Rewriting the performance function with normalised RVs $\rightarrow g(Z_i)$;
iii. Selecting the design point $\rightarrow Z^*$, that is, the one closest to the origin in the normalised space; and
iv. Evaluating the reliability index ($\beta$) as the distance between the origin and the design point $Z^*$. This method includes sensitivity factors ($\alpha$) that are determined as shown in Fig. 1b (for the case of two RVs, $E$ and $R$).

Sensitivity factors help to evaluate the influence of each RV; therefore, the necessity or importance of each of the basic RVs of the problem is characterised by its sensitivity factor. A positive $\alpha$ indicates that an increase in the corresponding RV means an increase in safety, while a negative $\alpha$ indicates the opposite. Evaluation of the sensitivity factors makes it possible to reduce the number of RVs taken into account without compromising the accuracy of the reliability calculation. Even though there may be a great number of possible RVs, only the variability of the most important and influential ones warrant consideration (Baecher and Christian, 2003). These calculations were performed using software that executes an iterative procedure based on the FORM process (Henriques et al., 1999).

### 2.4. MCS methodology

Simulation methods are level III RBD methods. They can be applied to RVs with non-normal distributions and complex performance characteristics (e.g., requiring nonlinear functions or finite element methods). The application of simulation methods makes use of all of the statistical information pertaining to the RVs, such as the mean, SD or COV and PDF.

Ordinary MCSs were conducted in this study. The authors believe that a MCS is the easiest and most simple approach to level III RBD because it does not require deep mathematical and statistical knowledge to understand its application to engineering. MCS is a robust method often used as reference for validation of other reliability methods. Even though many methods have been proposed for reducing the number of calculations required for MCS (variance reduction techniques), the application of such methods here was unnecessary because when dealing with simple performance functions, by using prediction formulas such as Eqs. (5) and (6), the computational effort required is not extensive (Wang, 2011).

Therefore, following the general steps (1–4) previously described, reliability analysis using MCS is accomplished as follows:

i. Based on the desired reliability ($\beta_T$), select the number of simulations $\rightarrow n$;
ii. Generate $n$ values for each RV based on the variability information (mean, SD or COV and PDF) by applying existing correlations;

iii. Calculate the value of the performance function for each generation; and
iv. Determine the probability of failure as the sum of the simulations that fail ($g(X_i)<0$) divided by the total number of simulations $n$, Eq.(2);

$$pf = \frac{1}{n} \cdot \sum_{1}^{n} I, \quad I = \begin{cases} 1 & if \quad g(X_i) \le 0, \quad failure \\ 0 & if \quad g(X_i) > 0, \quad safety \end{cases} \tag{2}$$

where $pf$ is the probability of failure, $n$ is the number of simulations, $I$ is the failure indicator and $g(X_i)$ is the performance function, where $X_i$ represents the RVs.

The number of simulations ($n$) must be chosen carefully, and its stability should always be studied by repeating the set of $n$ simulations and analysing the fluctuation of the final result. For the case studies considered in this paper, stability was achieved for 100,000 and 150,000 simulations for case studies 1 and 2, respectively. The results were considered stable for probabilities that were approximately $10^{-3}$. All MCS calculations were implemented using a routine in the software *R*, a free programming language and environment for statistical and graphical computation (R Development Core Team, 2009).

### 2.5. Uncertainties definition and characterisation

In civil engineering, uncertainties are normally divided into the following groups:

- Physical uncertainties are associated with the inherently uncertain nature of materials and components, their geometry and the variability and simultaneity of different actions/loads, among other things. These uncertainties are generally not known at first, but can be estimated through observations or past experience and can be addressed using a large database or quality control.
- Modelling uncertainties arise from the theoretical approaches used to model the behaviour of materials and the simplifications associated with these approaches. Modelling uncertainties can be addressed using a coefficient that represents the ratio between the real and predicted response.
- Statistical uncertainties include the uncertainty associated with the finite size of and variations in the samples used to estimate the relevant statistical parameters. This type of uncertainty is impossible to reduce or eliminate.
- Human error is due not only to natural variation in the execution of multiple tasks but also to intervention and error in the processes of documentation, design, communication, construction and use of the structure. Knowledge of these uncertainties is limited. Nevertheless, it is clear that the potential for human error increases the uncertainty of the strength of a man-made structure. An adequate margin of safety against human error is important because this type of uncertainty is not considered in RBD, which lacks a mechanism to account for it (Simpson, 2011).

In geotechnical engineering, when data from the specific site in study are not available or are insufficient to estimate the variability of the RVs, uncertainty can be characterised by the COV observed at other sites (assumed to be similar). Some geotechnical and soil uncertainties have been researched and discussed in studies by Kamien (1997), Baecher and Christian (2003) and Watabe et al. (2009). In addition, Kulhawy and Mayne (1990) and Phoon and Kulhawy (1999a, b) conducted a literature review of the COV of inherent variability, the scale of fluctuation, and the COV of measurement error. Typical values of the COV for soil properties and in situ test results have been compiled and reported by Phoon et al. (1995), Jones et al. (2002), and more recently by Phoon (2008).

As mentioned previously, geotechnical engineers also need to address spatial variability. Many geotechnical RVs vary continuously over space or time and are referred to as random fields (autocorrelation between variables). Normally, values of a parameter measured at locations at considerable distances apart from one another are independent, but if the value of a parameter is measured, the uncertainty in the value at a nearby point becomes less uncertain because it is highly correlated to the value of the first point (Vanmarcke, 1977). Based on this theory, it is possible to reduce the SD of a soil parameter (by taking a local average). To do this, it is only necessary to determine the autocorrelation distance of that parameter (more details are given in Honjo and Setiawan, 2007).

Statistical estimation error also influences the SD of a parameter. The variance function, in both the vertical and horizontal directions, is based on the relative position of the pile and the location where the parameter was measured. The SD of that parameter is calculated as shown in Eq. (3).

$$\sigma_{final} = \sqrt{(\sigma_{corr})^2 + (\sigma_{stat})^2} \tag{3}$$

where $\sigma_{final}$ is the final SD reduced based on statistical estimation error and spatial variability, $\sigma_{corr}$ is the SD reduced based on spatial variability, and $\sigma_{stat}$ is the SD reduced based on statistical estimation error (depending on the number of sampling points).

### 2.6. Target reliability index

The target reliability index depends on many factors, such as the type of structure (its function, occupancy and design working life), the social tolerance for non-compliance (failure, rupture, etc.) and the average number of victims in the case of structural failure. The determination of the target reliability index can be based on previous similar construction projects that met predefined requirements or on recommendations in design codes.

Eurocodes and the International Standard Organisation recommend reliability index ranges for ultimate limit state design of 3.3–4.3 and 1.3–4.3, respectively (CEN, 2002a; ISO 2394, 1998). These values correspond to probabilities of failure of approximately $10^{-1}$ and $10^{-5}$ (Eq. (4)), respectively, and depend primarily on the limit state considered, the failure consequences and the relative costs of safety measures.

$$pf = \Phi(-\beta) = 1 - \Phi(\beta) \tag{4}$$

where $\Phi$ is the normal cumulative density function with mean 0 and variance 1. The relationship of the reliability index ($\beta$) to the probability of failure ($pf$) is shown in Fig. 2.

## 3. Sensitivity analysis for vertically loaded piles

### 3.1. Performance function

Throughout the world, the SPT (standard penetration test) is the most common method of soil investigation and often is the only available source of information for pile design (Yamamoto and Karkee, 2004; Shariatmadari et al., 2008; Lutenegger, 2009; Zhang and Chu, 2009a, b; Kusakabe and Kobayashi, 2010; Dung et al., 2011). In fact, since the early years of modern foundation engineering, the $N$ value obtained from the SPT has been used extensively in design, especially for predicting the bearing capacity of piles (Terzaghi and Peck, 1948; Meyerhof, 1976; Shioi and Fukui, 1982; Robert, 1997). Thus, many of the major specifications pertaining to piles have adopted pile bearing capacity estimation formulas based on the $N$ value obtained from the SPT (AASHTO, 2007; CFEM, 2006; CEN, 2007b; JRA, 2001).

Because the SPT is simple, widely used and familiar in numerous countries, it was selected for the evaluation of the vertical bearing capacity and reliability analyses described in this paper. Consequently, the basic formula for the performance function, Eq. (1), was transformed into Eq. (5) using an empirical method for the evaluation of the vertical bearing capacity for the purposes of this pile foundation study.

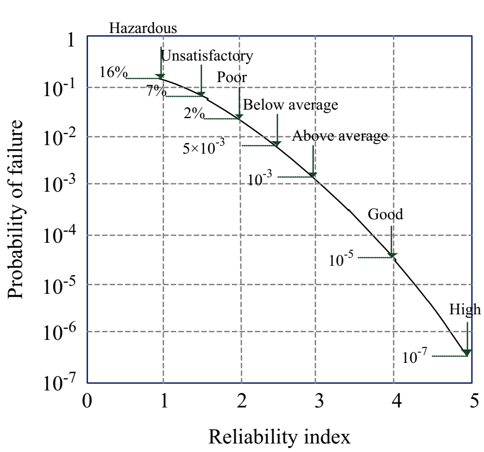


Fig. 2. Probability of failure versus reliability index, a classification proposed by US Army Corps of Engineers in 1997, adapted from Phoon (2008).

$$M = (R_{toe} + R_{side}) - (G + Q) = \left(\delta_t \times Q_{toe} + \delta_f \times F_{side}\right) - \left(\delta_G \times G_k + \delta_Q \times Q_k\right) \tag{5}$$

where $M$ is the safety margin of the vertical pile under axial loading, $R_{toe}$ is the toe resistance of the pile, $R_{side}$ is the side resistance of the pile, $G$ is the permanent action, $Q$ is the variable action, $\delta$ are the factors that take into account the relevant uncertainties ($\delta_t$ for the model error uncertainty of the toe resistance, $\delta_f$ for the model error uncertainty of the side resistance, $\delta_G$ for the permanent actions uncertainties and $\delta_Q$ for the variable actions uncertainties), $Q_{toe}$ is the predicted toe resistance, $F_{side}$ is the predicted side resistance, $G_k$ is the characteristic value of permanent action and $Q_k$ is the characteristic value of variable action.

The failure zone is defined by the conditions $M<0$ or $g(X_i)<0$. All uncertainties were considered as independent, including toe and side resistances, due to the lack of information available about them for the empirical method used. The consideration of any correlation between variables is taken into account in the RV simulation/generation step, maintaining the formulation of the performance function.

### 3.2. Evaluation of vertical bearing capacity

The vertical bearing capacity (resistance) of the pile is evaluated based on the empirical method recommended in the Specifications for Highway Bridges in Japan (JRA, 2001). Eq. (6) presents the formulation of this method, based on the classic type of bearing capacity calculation formulas, using uncorrected $N$ values and empirical factors (for more detail about this method, refer to Honjo et al., 2002). This is just one of the empirical methods available for the evaluation of vertical bearing capacity (Shariatmadari et al., 2008; Viana da Fonseca and Santos, 2008), but most of the available methods have not provided any information about the error associated with the predictions. This method was chosen because it includes a characterisation of the model error for both toe and side predictions (Okahara et al., 1991).

$$\begin{aligned} R_u &= Q_{toe} + F_{side} = q_{toe} \cdot A + U \cdot \sum (L_i \cdot f_i) \\ q_{toe} &= 100 \cdot N(<3000) \\ f_{sand} &= 5 \cdot N(<200) \\ f_{clay} &= 10 \cdot N(<150) \end{aligned} \tag{6}$$

where $R_u$ is the limit vertical bearing capacity predicted for the pile (kN), $Q_{toe}$ is the toe resistance, $F_{side}$ is the side resistance, $q_{toe}$ is the limit bearing capacity of the pile toe for a unit area (kN/m$^2$), $A$ is the section area of the pile toe (m$^2$), $U$ is the perimeter of the pile (m), $L_i$ is the thickness of the soil layer $i$ along the pile (m), $f_i$ is the maximum unit side resistance for layer $i$ (kN/m$^2$), and $N$ is the parameter from the SPT.

### 3.3. Evaluation of actions

Evaluation of the actions requires knowledge of the project and design documentation for the pile. When no information is provided, the permanent and variable loads can be estimated based on the prediction of the vertical bearing capacity and on applying partial SFs proposed by design codes. For the case studies considered in this paper, the partial SFs from Eurocodes were applied (CEN, 2002b; CEN, 2007a, b). The permanent and variable loads were considered equal in magnitude, and the predicted bearing capacity (Eq. (6)) was compared with the load test results (a static load test for case study 1 and a dynamic load test for case study 2). The values of the load tests were only used for assessment of these predictions and were not considered in any of the reliability calculations.

### 3.4. Uncertainties

In the reliability analysis of the case study pile foundations, using the performance function shown in Eq. (5), a total of six types of uncertainty from three distinct sources were considered:

- The modelling uncertainty (or model error) in the evaluation of the pile bearing capacity (resistance) by an empirical method, both toe and side components;
- The inherent soil variability characterised by the variation in the N value from the SPT or other soil tests, both toe and side components; and
- The physical uncertainties of actions, permanent and variable.

Pile dimensions, such as length and diameter, were considered to be deterministic because their uncertainties have very low importance, especially with engineers' control on site. Furthermore, human error was excluded from the analyses, for the reasons previously discussed.

### 3.5. Procedure for sensitivity analysis

The objective of the sensitivity analyses is to evaluate the relative influence of the uncertainty associated with each RV on the final result (i.e., the probability of failure). In the words of Christian (2004), "we can reduce uncertainty by obtaining more information, especially when the search for more information is guided by a rational understanding of the nature of uncertainty and its impact on our decision". These analyses are similar to those of a parametric study where the impact on both the performance of the pile and its reliability is assessed by analysing different lengths and different combinations of the uncertainties (considering and not considering a specific uncertainty; see Table 1).

Table 1
Combinations of uncertainties studied for sensitivity analysis.

| Uncertainty | Combination | | | | | |
|---|---|---|---|---|---|---|
| | 1.1 | 1.2 | 2 | 3 | 4 | 5 |
| Model | ✓ | ✓ | – | ✓ | (✓) | (✓) |
| Toe | ✓ | ✓ | – | ✓ | ✓ | – |
| Side | ✓ | ✓ | – | ✓ | – | ✓ |
| Soil | ✓ | ✓* | ✓ | – | (✓) | (✓) |
| $N_{SPT}$,toe | ✓ | ✓* | ✓ | – | ✓ | – |
| $N_{SPT}$,side | ✓ | ✓* | ✓ | – | – | ✓ |
| Actions | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Permanent | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Variable | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |

✓—the uncertainty was considered,
(✓)—the uncertainty was considered partially.
*Note that in this calculation the reduction of variance based on autocorrelation (spatial variability) was ignored.

## 4. Application examples

### 4.1. Description of case study 1

Case study 1 pertains to an experimental site in the north of Portugal. The Faculty of Engineering of the University of Porto (FEUP) developed this experimental site with 14 piles for a prediction event in 2004 (International Site Characterisation—ISC'2 conference; more information is provided in Viana da Fonseca and Santos, 2008). Residual soil from granite, a very common type of soil in the northwestern part of Portugal (Fig. 3), is found at this site. The site is characterised geologically by an upper layer of heterogeneous residual (saprolitic) granite soil of varying thickness, overlying a relatively weathered granite in contact with high-grade metamorphic rocks. Bedrock is found at a depth of approximately 20 m, and the ground water line (GWL) is found at a depth of approximately 10 m. An extensive in situ and laboratory investigation was conducted, but because the SPT is one of the most commonly used in situ tests for geotechnical design and soil characterisation, SPT results were used for the calculations performed. The pile considered is a reinforced concrete bored pile (id: E9) that is 6 m in length and 0.6 m in diameter. The ultimate capacity of the pile, measured under static loading to failure, was 1350 kN.

### 4.2. Description of case study 2

Case study 2 pertains to a pile from a railway bridge that is 2.7 km long and located in the south of Portugal. The soil for each pile foundation of this bridge is different, along the riverbed and on the riverside. The pile under study is installed in the riverbed. The soil around it consists of an upper layer of mud (soft and dark

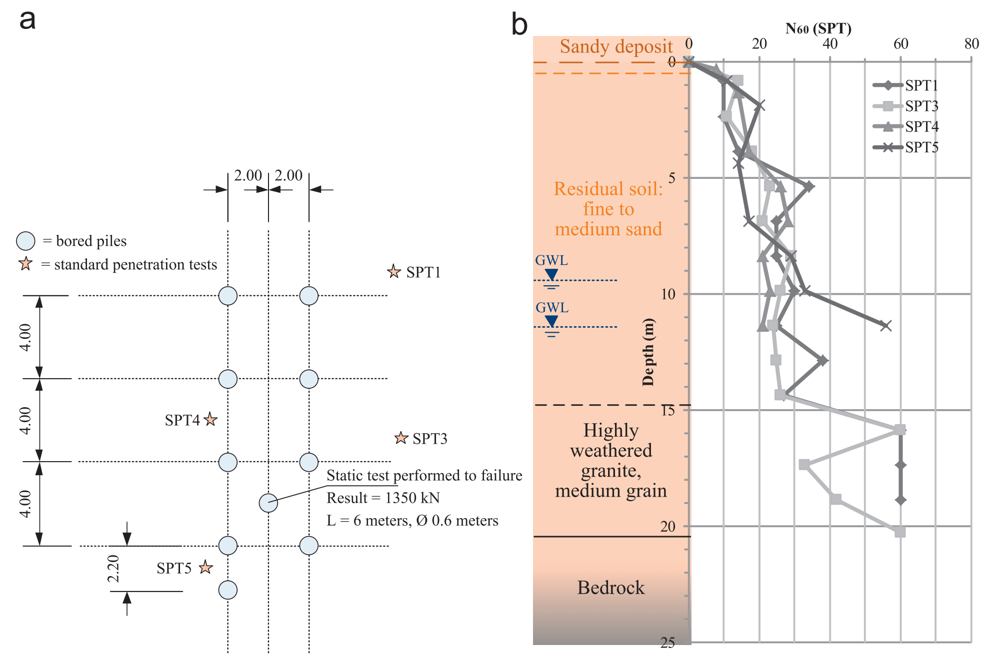


Fig. 3. Case study 1, adapted from Viana da Fonseca and Santos (2008). (a) Layout of the experimental site and (b) Geological and SPT profiles.

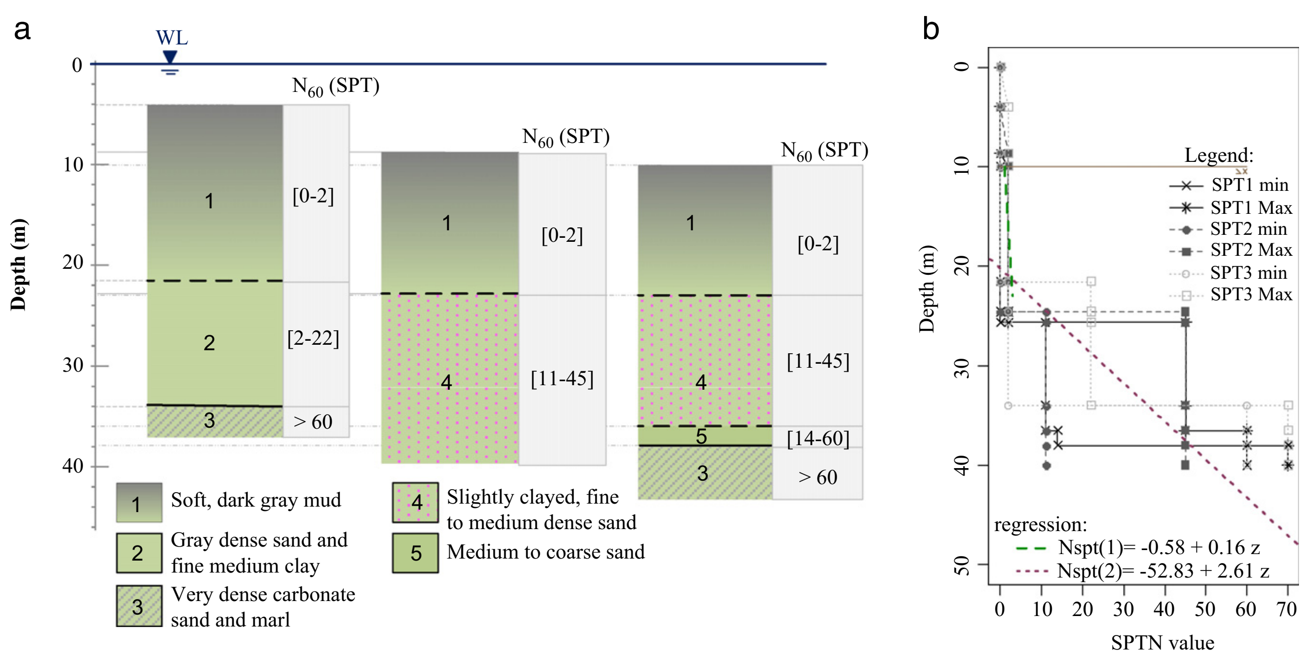


Fig. 4. Case study 2. (a) Geological and SPT profiles and (b) SPT trends.

grey) over a layer of dense to medium-dense slightly clayey sand, a layer of medium to coarse sand and, finally, at a depth of approximately 35–40 m, a layer of very dense carbonate sand and marl. The results from the soil investigation by SPT and the geological profile are depicted in Fig. 4. The pile considered is an open-ended steel pipe pile (id: PPR1-B) with a length of 43.5 m (33.5 m of which is embedded), a diameter of 1.12 m and a wall thickness of 12.4 mm. A dynamic load test was performed to determine the pile's bearing capacity, indicating an ultimate capacity of approximately 4000 kN.

### 4.3. Uncertainty values

Uncertainties for actions were gathered from the documents of JCSS (2001) and Holicky et al. (2007), while for the model error, the study by Okahara et al. (1991) was used. Table 1 shows the combinations that were studied for the sensitivity analysis of the uncertainties, while Table 2

Table 2
Uncertainty values for both case study 1 and case study 2.

| | Modelling uncertainty ($\delta$) | | Soil variability ($N_{SPT}$) | | Actions' uncertainties ($\delta$) | |
|---|---|---|---|---|---|---|
| | Toe | Side | Toe | Side | Permanent | Variable |
| Case study 1[a] | | | | | | |
| Mean value | 1.12 | 1.07 | 10.26+1.91z | | 1.0 | 0.6 |
| Standard deviation | 0.706 | 0.492 | 4.6[b] | 4.6[c] | 0.10 | 0.21 |
| Distribution | Lognormal | Lognormal | Normal | Normal | Normal | Gumbel |
| Case study 2[d] | | | | | | |
| Mean value | 1.12 | 1.07 | (1)—0.58+0.16z<br>(2)—52.83+2.61z | | 1.0 | 0.6 |
| Standard deviation | 0.706 | 0.492 | (1) 5.1[b]<br>(2) 20.4[b] | 5.1[c]<br>20.4[c] | 0.10 | 0.21 |
| Distribution | Lognormal | Lognormal | Normal | Normal | Normal | Gumbel |

(1) Branch from [0–22] m
(2) Branch from [22–60] m
[a] $G_k = Q_k = 463$ kN.
[b] Reduced by taking into account the influence zone on the pile toe (3 × diameter) by averaging over the thickness.
[c] Reduced by taking into account the length of the pile by averaging over the thickness.
[d] $G_k = Q_k = 3074$ kN.

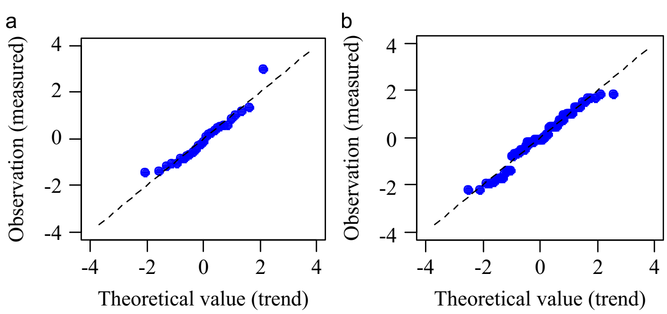


Fig. 5. Q–Q plots of the residuals of $N_{SPT}$. (a) Case study 1 and (b) Case study 1.

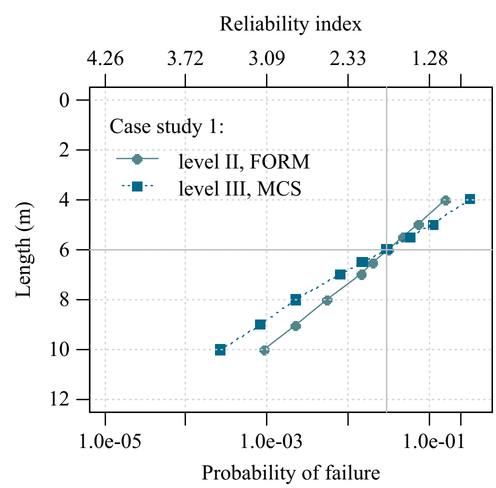


Fig. 6. Results of the reliability analyses using FORM and MCS for case study 1.

shows the values of the uncertainties considered for the two cases under study.

Spatial variability was considered as follows in the variance of the test parameter ($N_{SPT}$):

- First, the trend was defined, and the mean and SD were obtained;
- Then, the residuals were calculated (the difference between the trend and the actual values);
- The residuals were analysed by plotting the histogram and Q-Q plot (a graphical method to compare the distribution of a set of residuals with a normal distribution); and
- The autocorrelation graph of the test was plotted to determine the autocorrelation distance.

The SPT trends are shown in Table 2, and for case study 2, the trends are also depicted in Fig. 4. The Q–Q plots are shown in Fig. 5 and display a good approximation of the residuals ($N_{SPT,trend}$–$N_{SPT,measured}$) to a normal distribution. Furthermore, the SD of the $N_{SPT}$ value was calculated based on autocorrelation as explained previously, incorporating the different components of soil variability. The statistical estimation error was not considered because local averaging was taken into account, and the autocorrelation distance was assumed to be equal to 1 m because of the lack of data points for its evaluation (a value of 1 m is suggested in the literature on the basis of past experience, Phoon and Kulhawy, 1999a, b).

### 4.4. Results for case study 1

The previously described methodologies were applied to case study 1. All uncertainties were considered, and the results of FORM and MCS are compared. The pile length was varied between 4 and 10 m (Fig. 6). Some differences can be observed in the results. We concluded that the FORM results are acceptable when compared with those

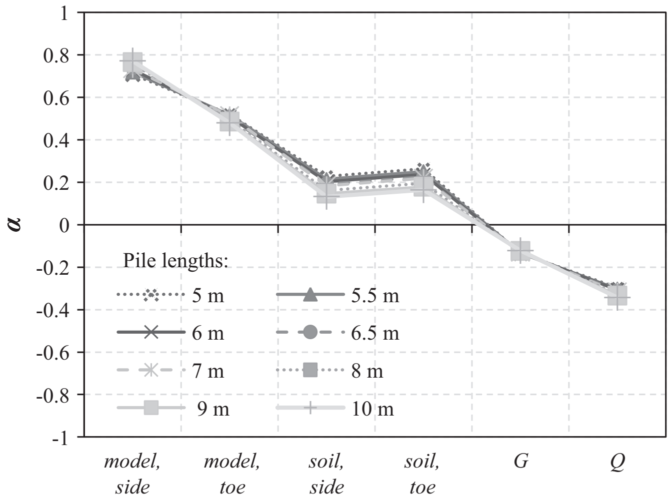


Fig. 7. Values of the FORM sensitivity factors for case study 1.

obtained using MCS. Furthermore, the 6 m pile length has a reliability index that does not meet that recommended by the codes ($\beta=1.86$ with FORM and $\beta=1.88$ with MCS).

The α values (sensitivity factors) from FORM are shown in Fig. 7, and the results from combinations of these uncertainties are depicted in Fig. 8a for FORM and Fig. 8b for MCS.

Fig. 7 shows which RVs have values on the safe side (positive α) and which RVs are the most influential (higher α values). Fig. 8 shows the combinations of results that deviate most from the comb.1.1, meaning that the corresponding RV is the most influential factor in this case study. Fig. 9 demonstrates the relative influence of model error, soil variability, and toe and side components obtained from sensitivity analyses with FORM and MCS. In comparing Fig. 8a and b, the FORM results exhibit greater variation than the MCS results; nevertheless, Fig. 9 shows that FORM and MCS results are consistent.

### 4.5. Results for case study 2

The same procedure was repeated for case study 2. First, all uncertainties were considered, and the pile length was varied between 30 and 50 m. The FORM and MCS results were then compared (Fig. 10). For this case study, the limitations of FORM were exhibited with respect to the performance functions, in particular. The high values of the pile bearing capacity (resistance) that should be controlled by the limit conditions imposed by the empirical method used could not be included in these calculations.

In Fig. 10, the value of the length of the pile (43.5 m, 33.5 m of which was embedded) and the corresponding reliability are noted. The actual pile installed achieved the reliability index recommended by the codes ($\beta=3.2$ for MCS).

Fig. 11 presents the results of the sensitivity analysis for MCS only because FORM was not considered applicable and because its α values showed very high variability. Nevertheless, and although the results of *pf* from FORM deviate considerably from those of MCS, the relative influence of the model error, soil variability, and toe and side components obtained show consistency between the two methods (Fig. 12).

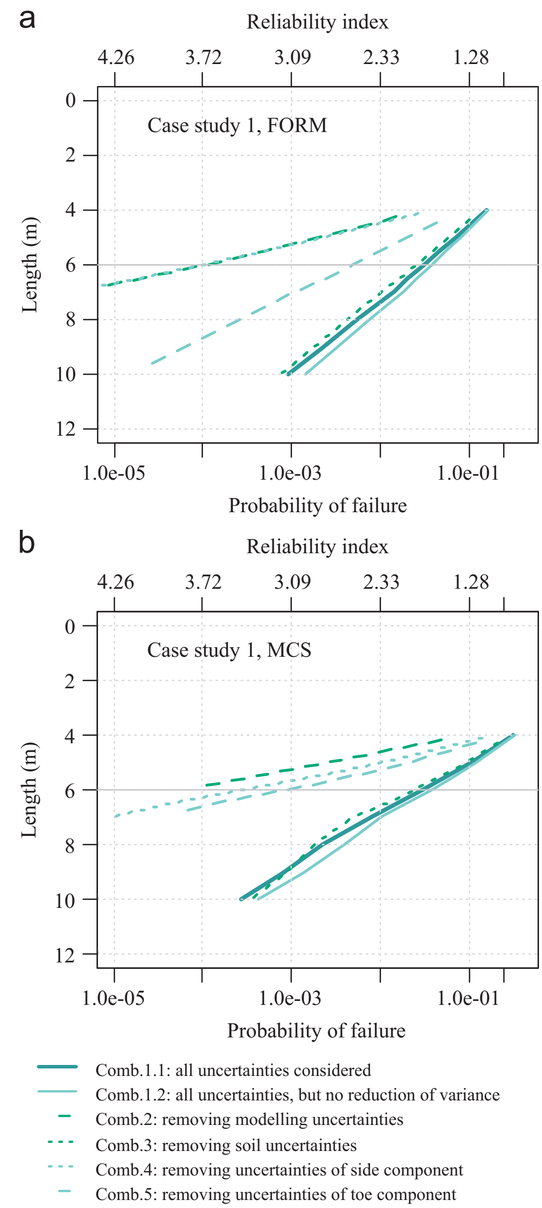


Fig. 8. Results of the reliability sensitivity analyses for case study 1. (a) Sensitivity analysis using FORM and (b) sensitivity analysis using MCS.

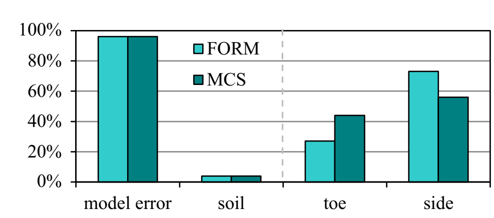


Fig. 9. Comparison of the relative influence of the uncertainties (sensitivity analysis results) for case study 1.

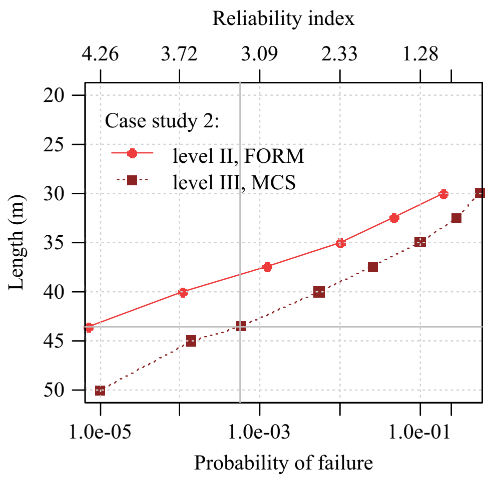


Fig. 10. Results of the reliability analyses using FORM and MCS for case study 2.

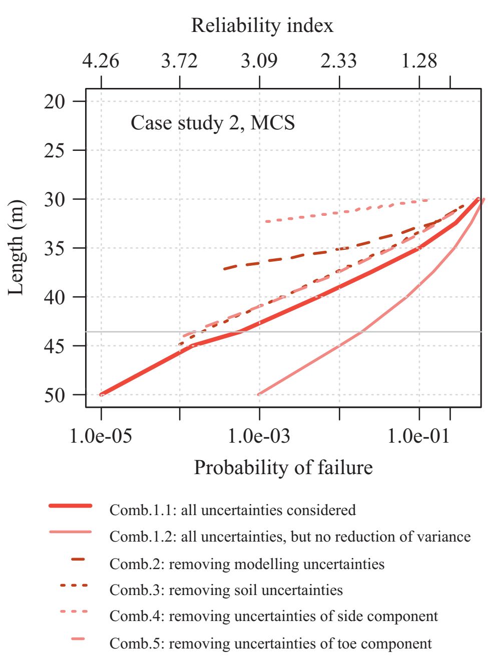


Fig. 11. Results of the reliability sensitivity analyses using MCS for case study 2.

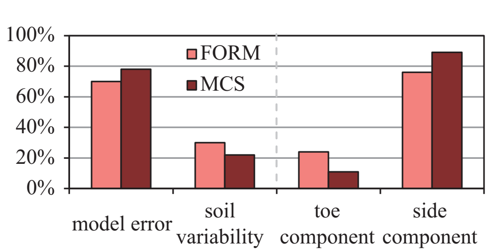


Fig. 12. Comparison of the relative influence of the uncertainties (sensitivity analysis results) for case study 2.

### 4.6. Discussion of the results

For case study 1, the following points can be highlighted:

- FORM yields an acceptable approximation in comparison with MCS results;
- The reliability obtained for the actual length of the pile (6 m) is lower than that recommended;
- For the sensitivity analysis using MCS, in comparing various combinations, the following was observed:
  - comb.1.1 versus comb.2 and comb.3 (modelling uncertainty and soil variability): one can conclude that, without doubt for this case, the model error is the most important uncertainty. Even though soil variability is always described as being very important, that was not true in this case, as evidenced by the comb.3 results being very similar to the comb.1.1 results.
  - comb.1.1 versus comb.4 and comb.5 (toe and side component uncertainties): the results of comb.4 are similar to comb.5. This comparison yields information on the influence of the toe and side components, and for this case, the toe and side components have a ratio of approximately 2 in value. However, the uncertainty associated with the toe component is greater, yielding an influence similar to that of the final result.
- The sensitivity analysis results were very similar for both FORM and MCS. It was very clear that the model error was the most important uncertainty in this case study;
- Furthermore, the FORM sensitivity factors agree with the sensitivity analysis: the model error has a higher $\alpha$ than other uncertainties. The actions uncertainties are not within the scope of this paper, but they are also believed to have a considerable influence on the results.

For case study 2, the following points can be highlighted:

- FORM was considered not applicable because of its limitations;
- The reliability obtained for the actual length of the pile (43.5 m, 33.5 m of which are embedded) meets the recommendations in the codes,
- For the sensitivity analysis using MCS, in comparing various combinations, the following was observed:
  - comb.1.1 versus comb.2 and comb.3 (modelling uncertainty and soil variability): the same kind of behaviour as is seen in case study 1 was observed. The model error has a large influence on the results, and soil variability (comb.3) has almost no influence.
  - comb.1.1 versus comb.4 and comb.5 (toe and side component uncertainties): the importance of the side component is demonstrated: this pile has a greater embedded length, which results in a much bigger influence of the side component uncertainty, as expected.

In conclusion, model error uncertainty had the largest influence on the reliability of the pile in both cases studied.

Moreover, in both of these cases, an approximately linear relationship was observed between the probability of failure (on a log scale) and the length of the pile (on a linear scale).

Furthermore, spatial autocorrelation allowed for the reduction of the SD of the N value from the SPT. It is obvious that spatial autocorrelation will lead to a more reliable result, and as one can see from Figs. 8 and 11 (comb.1.2), when this reduction is not considered, the result is more conservative, but not correct, especially in terms of economy.

FORM was only successfully applied to case study 1. FORM cannot incorporate limit conditions that the empirical method requires for calculation of pile bearing capacity (resistance from Eq. (6)) demands. These calculations were successfully conducted in case study 1 because the limits were not necessary (due to low resistance), but in case study 2 (with a higher magnitude of resistance), the method did not provide realistic results (the resistances predicted were too high), leading to a possible problem of convergence.

## 5. Conclusions

This paper describes the application of reliability-based methodologies and sensitivity analyses applied to two distinct case studies of vertically loaded single piles (a bored pile and a steel pipe pile). This work is intended to contribute to preventing the loss of intuitive understanding when applying these tools to design problems, which is an important issue in geotechnical engineering. This work is also intended as an aid to pile design decision-makers in assessing the uncertainties associated with the random variables that most influence both the probability of failure and pile behaviour. Characterisation of uncertainty in geotechnical problems is a difficult task, and the values recommended in the literature often cannot be applied to a particular case under study due to high soil variability.

Considering the physical uncertainties of actions, the inherent soil variability characterised through SPT results and the bearing capacity (resistance) model error for the reliability studies presented, the following summary statements can be made:

- The application of two reliability methods (FORM and MCS), repeated for different pile lengths and different combinations of the uncertainties, reveals that FORM was only successfully applied to case study 1 (the bored pile). This suggests that FORM can be used as a first approach to reliability analysis. However, for more complex analyses, such as those conducted for case study 2 (a steel pipe pile), MCS should be used for the assessment of the probability of failure.
- The reliability indexes obtained for the actual pile lengths were $\beta=1.88$ (case study 1, 6 m) and $\beta=3.2$ (case study 2, 43.5 m). While the reliability index in case study 2 satisfied the standard recommendations, the reliability index in case study 1 did not. This can be explained by the fact that case study 1 is an experimental field case study in which failure is obviously of minor consequence.

The results of the sensitivity analyses for both case studies, using both FORM and MCS, confirm that not considering spatial correlation is conservative but technically incorrect. In addition, the results of these two case studies show that soil uncertainties were not as important as was expected. However, model uncertainties contributed greatly to the probability of failure for both cases studied. Meanwhile, the contribution of toe and side uncertainties depends greatly on the type of pile and the ratio between these two resistances.

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