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Scattering on D3-branes

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Abstract

In a direct open string approach we analyze scattering of massless states on a stack of D3-branes. First we construct vertex operators on the D-branes. The $4+6$ splitting for the fermionic part is made possible by inserting appropriately defined projection operators. With the vertex operators constructed we compute various tree amplitudes. The results are then compared with the corresponding field theory computations of the $\mathcal{N}=4$ SYM with α'-corrections: agreements are found. We comment on applications to AdS/CFT.

1. Introduction

At the heart of the AdS/CFT are the description methods of D-branes. They can be described either as a hypersurface where an open string can end or as a solitonic solution of the closed string theory. In the open string theory description one can use the $D=4$, $\mathcal{N}=4$ SYM theory as a leading order approximation to the full open string description. In particular there has been efforts to compute the anomalous dimensions of some SYM operators.

Although simple and useful the SYM theory does not contain the effects of the massive open string modes since the SYM is a leading order approximation: it may be worth studying a higher order in the approximation accommodating the effects of the massive modes. A first step toward this direction has been taken in [1] where the α'-corrected SYM was considered in the regular field theory approach. One loop scalar four point amplitudes were computed and the counter-terms that remove the divergence were examined. Unlike the Abelian case where the effective action can be obtained in a closed form, in the non-Abelian case one must consider string theory four-point, five-point, etc., separately, and deduce the field theory action from the results. It may be useful for that purpose to know the possible forms of the field theory counter-terms in advance, which is one of the motivations of the work [1].

As stated there, the string-based technique and the field theory technique should be mutually guiding. Here we turn to the string world-sheet physics. Since D-branes are stringy objects it ought to, in general, take the full open string theory for their complete description. Therefore how the massive open string modes figure into AdS/CFT (or matrix theory conjectures for that matter) is an interesting and important issue. The possible relevance of the open string in the context of AdS/CFT was discussed e.g. in [2,3].[1] With the comparison with the field theory in mind we study the scattering of massless states. Although the body of a string lives in ten dimensions its end points remain *on* the D3-branes before and after the scattering. (We only consider such scattering.) For the purpose of analyzing such scattering it is necessary to construct the vertex operators in a direct open string approach: the boundary state formulation for example cannot be applied. Below we will construct the vertex operators. They come in two multiplets which we call the "scalar multiplet" and the "gauge multiplet" respectively. As the name suggests they should respectively correspond to the scalar multiplet and the gauge multiplet in the $\mathcal{N}=2$ field theory language. What makes it possible to separate the scalar multiple

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[1] Related discussions may be found in [4,5].

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from the gauge multiplet (or vice versa) is insertion of appropriately defined projection operators in various places. The momenta of the vertex operators will be such that they have non-zero components only along the D3-brane directions. Physically speaking, for the branes whose location is fixed this choice of momenta seems natural. In fact it also follows at an explicit computational level as a consequence of ensuring the closure of the vertex operator algebra under susy transformation. Once they are constructed various tree amplitudes can be easily computed following the standard procedure. We verify that the field theory computations at α'^2-order can be recovered by expanding the corresponding string computations at the same order.

The organization of the Letter is as follows. In the next section we briefly review the boundary conditions of D3-branes in the Green–Schwarz formulation. We then construct two sets of vertex operators, the "scalar multiplet" and the "gauge multiplet". In Section 3 we compute various tree amplitudes using the standard world-sheet techniques and compare the results with the corresponding amplitudes obtained by using the $\mathcal{N}=4$ SYM with the α'-corrections. By computing the tree graphs we are setting the ground for the loop computation, which is more interesting and important for the reasons that we list in the conclusion. There we also comment on future directions and applications of our results to AdS/CFT.

2. Vertex operator construction

In this section we construct the vertex operators in a direct open string framework. We start with a brief review of the light-cone gauge to set the notations. The vertex operators are constructed based on the closure under susy transformations as in the D9-brane case. The additional task, compared with the D9 case, is that now one should carry out the $(4+6)$ splitting. For the bosonic coordinates the splitting is obvious whereas with the fermionic coordinates it is subtle. As we will see below the fermionic splitting is accomplished through insertion of some projection operators. Throughout we mostly follow the conventions of [6].

2.1. Review of light-cone gauge

In the Green–Schwarz formulation, the string action is given by

$$S=-\frac{1}{2\pi}\int d^2\sigma\,\big(\sqrt{-g}g^{\alpha\beta}\Pi_\alpha{}^M\Pi_\beta{}^N\eta_{MN}+2i\epsilon^{\alpha\beta}\partial_\alpha X^M\big(\bar\theta^1\Gamma_M\partial_\beta\theta^1-\bar\theta^2\Gamma_M\partial_\beta\theta^2\big)-2\epsilon^{\alpha\beta}\big(\bar\theta^1\Gamma^M\partial_\alpha\theta^1\big)\big(\bar\theta^2\Gamma_M\partial_\beta\theta^2\big)\big) \tag{1}$$

where $g=|\det g_{\alpha\beta}|$ and $\Pi_\alpha{}^M=\partial_\alpha X^M-i\bar\theta^A\Gamma^M\partial_\alpha\theta^A$. The 32×32 Γ-matrices are such that Γ^M, $M\neq 0$, is real and symmetric and Γ^0 is real and antisymmetric.

Consider a D3-brane extended along the (X^1,X^2,X^3)-directions. We locate it at the origin of the transverse dimensions, i.e., $X^m=0$ at $\sigma=0,\pi$. The boundary conditions for the bosonic coordinates are such that we impose the Neumann conditions for the world volume coordinates, X^μ, and Dirichlet for the transverse ones, $X^4,\dots,X^9$:

$$\partial_\tau X^m=0,\quad \sigma=0,\pi, \tag{2}$$

$$\partial_\sigma X^\mu=0,\quad \sigma=0,\pi. \tag{3}$$

For the fermionic coordinates it is necessary to impose a constraint,

$$\theta^2=\Gamma_{4,\dots,9}\theta^1,\qquad \sigma=0,\pi \tag{4}$$

which in turn implies the usual half supersymmetry breaking condition. After the standard light-cone gauge fixing procedure

$$\Gamma^+\theta^{1,2}=0 \tag{5}$$

one has the following action,

$$S=-\frac{1}{2}\int\big(T\partial_\alpha X^i\partial^\alpha X^i-\frac{i}{\pi}\bar S^a\rho^\alpha\partial_\alpha S^a\big)=-\frac{1}{2\pi}\int\big(\partial_\alpha X^i\partial^\alpha X^i-i\bar S^a\rho^\alpha\partial_\alpha S^a\big) \tag{6}$$

where $S\equiv\sqrt{p^+}\theta$. The mode expansion of the bosonic coordinates is

$$X^\mu(\sigma,\tau)=x^\mu+l^2p^\mu\tau+il\sum_{n\neq 0}\frac{1}{n}\alpha_n^\mu e^{-in\tau}\cos n\sigma,\qquad X^m(\sigma,\tau)=R^m+\frac{1}{\pi}\Delta X^m\sigma+l\sum_{n\neq 0}\frac{1}{n}\alpha_n^m e^{-in\tau}\sin n\sigma \tag{7}$$

where R^m, ΔX^m are the parameters that are associated with the locations of the branes. We locate the branes at the origin of the transverse 6-plane. For an open string with both ends on the D3-branes the transverse coordinates become simpler

$$X^m(\sigma,\tau)=l\sum_{n\neq 0}\frac{1}{n}\alpha_n^m e^{-in\tau}\sin n\sigma. \tag{8}$$

The mode expansion of the fermionic coordinates is

$$S^{1a} = \sum_{-\infty}^{\infty} s_n^a e^{-in(\tau-\sigma)}, \qquad S^{2a} = \sum_{-\infty}^{\infty} \gamma_{4,\dots,9} s_n^a e^{-in(\tau+\sigma)} \tag{9}$$

where $\gamma_{4,\dots,9}$ is an 8×8 matrix. Note that $\gamma^2,\dots,\gamma^8$ are real and antisymmetric matrices while γ^9 is an identity matrix. This mode expansion yields

$$\{S^{Aa}(\sigma), S^{Bb}(\sigma')\} = 2\pi\delta^{ab}\delta^{AB}\delta(\sigma-\sigma'). \tag{10}$$

2.2. Supercharges

Since we will construct the vertex operators mostly based on their susy transformations we first obtain the expressions for the supercharges. Care is needed with the boundary conditions/terms. There are two sets of susy transformations. The first set is

$$\delta\theta^1 = \frac{1}{\sqrt{2}}\eta^a, \qquad \delta\theta^2 = \frac{1}{\sqrt{2}}\eta^a, \qquad \delta X^i = 0 \tag{11}$$

which yields

$$Q^a = \sqrt{2p^+}s_0^a. \tag{12}$$

It has the same form as the D9-brane case. The second set of the susy transformation has the same form as the D9-brane case as well, but it is in terms of modified susy parameters, ε:

$$\delta S = -\frac{1}{\sqrt{2p^+}}\rho\cdot\partial X^i\gamma^i\varepsilon, \qquad \delta X^i = -\frac{i}{\sqrt{2p^+}}\bar{\varepsilon}\gamma^i S. \tag{13}$$

To determine ε, we examine the boundary terms that result from taking the variation on the bosonic term,

$$\sim \partial_\sigma X^i\left(\epsilon_2\gamma^i S_1 - \epsilon_1\gamma^i S_2\right). \tag{14}$$

The $(i=u)$-terms drop due to Neumann boundary condition. Substitution of (4) into the above equation leads to the susy parameters, $\varepsilon = \binom{\epsilon}{-\gamma_{4,\dots,9}\epsilon}$. The supercharges for this transformation are

$$Q^{\dot a} = \frac{1}{\sqrt{p^+}}\left(\sum_n \alpha_n^i\left[(\gamma^i)^T\gamma_{4,\dots,9}\right]s_{-n}^a\right) \tag{15}$$

where $\alpha_0^i = p^i = (p^u, 0)$. It is a column vector. The supercharges Q^a, $Q^{\dot a}$ satisfies the following algebra:

$$\{Q^a, Q^b\} = 2p^+\delta^{ab}, \qquad \{Q^a, Q^{\dot b}\} = -\sqrt{2}p^u\gamma^u\gamma_{4,\dots,9}, \qquad \{Q^{\dot a}, Q^{\dot b}\} = 2H\delta^{\dot a\dot b} \tag{16}$$

where $H = \frac{1}{2p^+}(p^u p^u + 2[\sum_{n=1}^{\infty}\alpha_{-m}^i a_m^i + m s_{-m}^a s_m^a])$. Since $\gamma_{4,\dots,9}$ appears frequently it is convenient to define

$$\gamma \equiv \gamma_{4,\dots,9} = \gamma_{4,\dots,8} \tag{17}$$

where the second equality holds since γ^9 is an 8×8 identity matrix. γ satisfies

$$\gamma^T = -\gamma, \qquad [\gamma, \gamma^m] = 0 = \{\gamma, \gamma^u\}, \qquad \gamma^2 = -1. \tag{18}$$

2.3. Vertex operator

With the supercharges available we are ready to construct the vertex operators by requiring closure under susy.[2] We do that in $k^+ = 0$ frame as in the D9-brane case. It turns out that they come in two pairs: we call them a vector multiplet and a scalar multiplet. With the various gauges and constraints that we have imposed they should correspond the $\mathcal{N}=2$ field theory scalar multiplet and the gauge multiplet. Each pair satisfies the modified susy transformation relations given in (24) and (30) below, which are analogous to the corresponding D9-brane relations

$$\begin{aligned}
&[\eta^a Q^a, V_F(u,k)] \approx V_B(\tilde{\zeta},k), \qquad [\eta^a Q^a, V_B(\zeta,k)] \approx V_F(\tilde{u},k),\\
&[\epsilon^{\dot a} Q^{\dot a}, V_F(u,k)] \approx V_B(\tilde{\tilde{\zeta}},k), \qquad [\epsilon^{\dot a} Q^{\dot a}, V_B(\zeta,k)] \approx V_F(\tilde{\tilde{u}},k).
\end{aligned} \tag{19}$$

[2] The Lorentz transformation can be utilized as well [6]. The discussion of Lorentz invariance in the current case goes parallel to the D9 case. In particular one can show that $[J^{i-}, J^{j-}] = 0$ requires the theory in the critical dimension.

The wave function u satisfies

$$k^+ u^a + k^i \gamma^i_{a\dot{a}} u^{\dot{a}} = 0, \qquad k^- u^{\dot{a}} + k^i \gamma^i_{\dot{a}a} u^a = 0. \tag{20}$$

The $\approx$ means that the equalities are up to total τ-derivative terms. The closure of each multiplet is made possible by inserting the following projection operators,

$$E_+ = \frac{1}{2}(1 + i\gamma), \qquad E_- = \frac{1}{2}(1 - i\gamma). \tag{21}$$

In particular they appear in the fermionic parts of the vertex operators bringing the $(4+6)$ splitting. As a natural trial we choose momenta such that they have non-zero components only along the D3-brane directions. From a physical standpoint this choice seems inevitable for the branes whose location is fixed. In fact we will see that it follows as a consequence of the vertex operator algebra under susy generators. Let us use the convention that μ, ν are the brane direction with $u, v = 2, 3$ and m, n are the transverse directions. With $k^m = 0$ the transverse polarization condition becomes $k^u \zeta^u = 0$.

Defining $k^i = (k^u, 0)$, $\zeta^i = (\zeta^u, 0)$ the vector multiplet is

$$V_{Bg}(\zeta, k) = \left(\zeta^u B^u_g - \zeta^- B^+_g\right) e^{ik\cdot X}, \qquad V_{Fg}(u, k) = \left(u^a E_- F^a_g + u^{\dot{a}} E_+ F^{\dot{a}}_g\right) e^{ik\cdot X} \tag{22}$$

where

$$\begin{aligned}
B^+_g &= p^+, \\
B^u_g &= \left(\dot{X}^u - R^{uj}_g k^j\right), \\
F^{\dot{a}}_g &= \frac{1}{\sqrt{2p^+}}\left[\left((\gamma^u)^T \dot{X}^u S_1\right)^{\dot{a}} - \left((\gamma^m)^T X^{m\prime} S_1\right)^{\dot{a}} + \frac{1}{3}{:}\left((\gamma^i)^T S_1\right)^{\dot{a}} R^{ij}_g{:}k^j\right], \\
F^a_g &= \sqrt{\frac{p^+}{2}} S^a_1
\end{aligned} \tag{23}$$

where $R^{ij}_g = \frac{1}{4} S_1 \gamma^{ij} S_1$. They satisfy the modified vertex operator algebra,

$$\begin{aligned}
&\left[\eta^a E_+ Q^a, V_{Fg}(u,k)\right] \approx V_{Bg}(\tilde{\zeta}, k), \qquad \left[\eta^a E_+ Q^a, V_{Bg}(\zeta,k)\right] \approx V_{Fg}(\tilde{u}, k), \\
&\left[\epsilon^{\dot{a}} E_- Q^{\dot{a}}, V_{Fg}(u,k)\right] \approx V_{Bg}(\tilde{\tilde{\zeta}}, k), \qquad \left[\epsilon^{\dot{a}} E_- Q^{\dot{a}}, V_{Bg}(\zeta,k)\right] \approx V_{Fg}(\tilde{\tilde{u}}, k).
\end{aligned} \tag{24}$$

The wave function u satisfies

$$k^+ u^a + k^u \gamma^u_{a\dot{a}} u^{\dot{a}} = 0, \qquad k^- u^{\dot{a}} + k^u \gamma^u_{\dot{a}a} u^a = 0. \tag{25}$$

How the projection operators bring the closure can be seen e.g. in the computation of

$$\left[\eta^a E_+ Q^a, V_{Fg}(u,k)\right] \approx V_{Bg}(\tilde{\zeta}, k). \tag{26}$$

One of the commutators yields

$$\left[\eta E_+ \sqrt{2p^+} s_0, (u E_+)^{\dot{a}} F^{\dot{a}}\right] = \eta E_+ \gamma^u E_- u \dot{X}^u + \eta E_+ \gamma^m E_- u X'^m = \eta E_+ \gamma^u u \dot{X}^u \tag{27}$$

where in the second equality the second term has dropped due to the presence of the projection operators. Therefore even though there is X'^m in $F^{\dot{a}}$, one produces the correct form of B^u_g.

For the scalar multiplet, we define $k^i = (k^u, 0)$, $\xi^i = (0, \xi^m)$:

$$V_{Bs}(\xi, k) = \xi \cdot B_s e^{ik\cdot X} = \left(\xi^m B^m_s\right) e^{ik\cdot X}, \qquad V_{Fs}(w, k) = w F_s e^{ik\cdot X} = \left(w^a E_- F^a_s + w^{\dot{a}} E_+ F^{\dot{a}}_s\right) e^{ik\cdot X} \tag{28}$$

where

$$\begin{aligned}
B^m_s &= \left(X'^m + R^{mj}_s k^j\right), \\
F^{\dot{a}}_s &= \frac{1}{\sqrt{2p^+}}\left[\left((\gamma^u)^T \dot{X}^u S_1\right)^{\dot{a}} - \left((\gamma^m)^T X^{m\prime} S_1\right)^{\dot{a}} - \frac{1}{3}{:}\left((\gamma^i)^T S_1\right)^{\dot{a}} R^{ij}_s{:}k^j\right], \\
F^a_s &= -\sqrt{\frac{p^+}{2}} S^a_1
\end{aligned} \tag{29}$$

where $R^{ij}_s = \frac{1}{4} S_1 \gamma^{ij} S_1 = R^{ij}_g$. Note that compared with the fermionic term in B^u_g the corresponding term in B^m_s has an opposite sign. (This can be checked by applying on X'^m a Lorentz transformation that takes a state whose only non-zero momentum is k^- to a state that has $k^+ = 0$ with other components non-zero.) It triggers a few sign differences in the subsequent formulas. They satisfy the modified vertex operator algebra,

$$\begin{aligned}&\left[\eta^a E_- Q^a, V_{Fs}(w,k)\right] \approx V_{Bs}(\tilde{\xi},k), \qquad \left[\eta^a E_- Q^a, V_{Bs}(\xi,k)\right] \approx V_{Fs}(\tilde{w},k),\\&\left[\epsilon^{\dot a} E_+ Q^{\dot a}, V_{Fs}(w,k)\right] \approx V_{Bs}(\tilde{\tilde{\xi}},k), \qquad \left[\epsilon^{\dot a} E_+ Q^{\dot a}, V_{Bs}(\xi,k)\right] \approx V_{Fs}(\tilde{\tilde{w}},k).\end{aligned} \tag{30}$$

The wave function w satisfies

$$k^+ w^a + k^u \gamma^u_{a\dot a} w^{\dot a} = 0, \qquad k^- w^{\dot a} + k^u \gamma^u_{\dot a a} w^a = 0. \tag{31}$$

One can see that $k^m = 0$ is required to ensure for example

$$\left[\epsilon^{\dot a} E_+ Q^{\dot a}, V_{Fs}(w,k)\right] \approx V_{Bs}(\tilde{\tilde{\xi}},k). \tag{32}$$

3. Tree level scattering

The respective closure of the scalar multiplet and the vector multiplet is already a strong indication that the construction is correct. We substantiate the claim by computing various tree amplitudes with the vertex operators just constructed. For the vector vertex operator the computations essentially the same as the corresponding computations in the D9-branes. The results are then expanded at α'^2-order and compared with the corresponding computations in the $\mathcal{N}=4$ SYM with the α'-corrections. Agreements are found between the two computations.

3.1. String computation

Consider the vector three point tree graph. Only the cyclically inequivalent permutations are added. The computation is precisely analogous to the D9-brane case yielding

$$\begin{aligned}A(VVV) &= g\,\mathrm{tr}\big(\lambda^a\lambda^b\lambda^c\big)\big\langle\zeta^1,k^1\big|V_g\big(\zeta^2,k^2\big)\big|\zeta^3,k^3\big\rangle + \big((1,a)\leftrightarrow(3,c)\big)\\&= g\,\mathrm{tr}\big(\lambda^a\lambda^b\lambda^c\big)\big(\zeta^1\cdot k^2\zeta^2\cdot\zeta^3 + \zeta^2\cdot k^3\zeta^3\cdot\zeta^1 + \zeta^3\cdot k^1\zeta^1\cdot\zeta^2\big) + \big((1,a)\leftrightarrow(3,c)\big)\\&= 2g\,\mathrm{tr}\big(\lambda^a\lambda^b\lambda^c\big)\big(\zeta^1\cdot k^2\zeta^2\cdot\zeta^3 + \zeta^2\cdot k^3\zeta^3\cdot\zeta^1 + \zeta^3\cdot k^1\zeta^1\cdot\zeta^2\big)\\&= 2igNf^{abc}\big(\zeta^1\cdot k^2\zeta^2\cdot\zeta^3 + \zeta^2\cdot k^3\zeta^3\cdot\zeta^1 + \zeta^3\cdot k^1\zeta^1\cdot\zeta^2\big)\end{aligned} \tag{33}$$

where in the fourth equality we have adopted a normalization $\mathrm{Tr}\,\lambda^a\lambda^b = 2\delta^{ab}$. There is no three point scalar vertex in the $\mathcal{N}=4$ SYM with α'-corrections. The string scalar three point graph indeed produces a vanishing result:

$$\begin{aligned}A(\phi\phi\phi) &= g\,\mathrm{tr}\big(\lambda^a\lambda^b\lambda^c\big)\big\langle\xi^1,k^1\big|V_s\big(\xi^2,k^2\big)\big|\xi^3,k^3\big\rangle\\&= g\,\mathrm{tr}\big(\lambda^a\lambda^b\lambda^c\big)\big\langle\xi^1,k^1\big|\xi_2^m\big(X'm + R^{mv}k_2^v\big)e^{ik_2\cdot X}\big|\xi^3,k^3\big\rangle\\&= g\,\mathrm{tr}\big(\lambda^a\lambda^b\lambda^c\big)\delta(k_1+k_2+k_3)\big\langle\xi^1\big|\xi_2^m R^{mv}k_2^v e^{ik_2\cdot X}\big|\xi^3\big\rangle\\&= g\,\mathrm{tr}\big(\lambda^a\lambda^b\lambda^c\big)\delta(k_1+k_2+k_3)\xi_2^m k_2^v\big(\xi_1^m\xi_3^v - \xi_3^m\xi_1^v\big) = 0.\end{aligned} \tag{34}$$

In the third equality we have used the fact that X'^m does not have a zero mode. Proceeding as in the vector case one gets the fourth equality which is zero since $\xi^v = 0$ for the scalar state. Similarly the vector–vector–scalar vertex can be shown to vanish which is consistent with the field theory. The last example of three point function that does not involve an external fermionic state is the vector–scalar–scalar vertex,

$$\begin{aligned}A(V\phi\phi) &= g\,\mathrm{tr}\big(\lambda^a\lambda^b\lambda^c\big)\big\langle\xi^1,k^1\big|V_g\big(\zeta^2,k^2\big)\big|\xi^3,k^3\big\rangle + \big((1,a)\leftrightarrow(3,c)\big)\\&= g\,\mathrm{tr}\big(\lambda^a\lambda^b\lambda^c\big)\zeta^2\cdot\big(k^3-k^1\big)\xi^1\cdot\xi^3.\end{aligned} \tag{35}$$

Our final example of three point amplitude is $A(\psi\psi A_\mu)$,

$$\begin{aligned}A(\psi\psi A_\mu) &= g\,\mathrm{tr}\big(\lambda^a\lambda^b\lambda^c\big)\big\langle u_1,k^1\big|V_f\big(u_2,k^2\big)\big|\zeta^3,k^3\big\rangle + \big((1,a)\leftrightarrow(2,b)\big)\\&= g\,\mathrm{tr}\big(\lambda^a\lambda^b\lambda^c\big)u_1\gamma^\mu E_- u_2\zeta_3^\mu\end{aligned} \tag{36}$$

where γ^μ for example is an eight by eight matrix. The index μ has appeared as a result of covariantizing the index v.

We turn to the four point amplitudes. For the four vector amplitude, one gets

$$\begin{aligned}A(VV\Delta VV) &= \frac{g^2}{2}\,\mathrm{tr}\big(\lambda^a\lambda^b\lambda^c\lambda^d\big)\big\langle\zeta^1,k^1\big|V_g\big(\zeta^2,k^2\big)\Delta V_g\big(\zeta^3,k^3\big)\big|\zeta^4,k^4\big\rangle\\&= \frac{g^2}{2}\,\mathrm{tr}\big(\lambda^a\lambda^b\lambda^c\lambda^d\big)\big\langle\zeta^1\big|(1+t/2)\zeta^2\cdot\zeta^3 B(1-s/2,-1-t/2)\end{aligned}$$

$$+\big[-\zeta^2\cdot k^1\zeta^3\cdot k^4 - R_0^{uv}\big(\zeta_2^u k_2^v\zeta^3\cdot k^4 - \zeta_3^u k_3^v\zeta^2\cdot k^1\big) + R_0^{uv}R_0^{u'v'}\zeta_2^u k_2^v\zeta_3^{u'}k_3^{v'}\big]B(-s/2, 1-t/2)$$
$$+\big[\zeta^2\cdot k^3\zeta^3\cdot k^4 + \zeta^2\cdot k^1\zeta^3\cdot k^2 + \zeta^2\cdot k^3\zeta^3\cdot k^2 + R_0^{uv}\big(\zeta_2^u k_2^v\zeta^3\cdot k^2 - \zeta_3^u k_3^v\zeta^2\cdot k^3 - k_2^u k_3^v\zeta^2\cdot\zeta^3$$
$$- \zeta_2^u\zeta_3^v k^2\cdot k^3 + k_2^u\zeta_3^v\zeta^2\cdot k^3 + \zeta_2^u k_3^v\zeta^3\cdot k^2\big)\big]B(1-s/2, -t/2)\big|\zeta^4\big\rangle \tag{37}$$

where

$$s = -(k_1+k_2)^2, \qquad t = -(k_2+k_3)^2, \qquad u = -(k_1+k_3)^2. \tag{38}$$

This is valid up to the cyclically inequivalent permutations which will be added below. Compared with the D9-brane case there are some sign flips which are due to different conventions from [6]. They do not persist in the final form of the amplitude given below. Using the following identities

$$\begin{aligned}
&\big\langle\zeta^1\big|\zeta^4\big\rangle = \zeta^1\cdot\zeta^4,\\
&\big\langle\zeta^1\big|R_0^{uv}\big|\zeta^4\big\rangle = -\zeta_4^u\zeta_1^v + \zeta_1^u\zeta_4^v,\\
&\big\langle\zeta^1\big|R_0^{uv}R_0^{u'v'}\big|\zeta^4\big\rangle = \zeta_1^v\zeta_4^{u'}\delta^{uv'} - \zeta_1^u\zeta_4^{u'}\delta^{vv'} - \zeta_1^v\zeta_4^{v'}\delta^{uu'} + \zeta_1^u\zeta_4^{v'}\delta^{vu'}
\end{aligned} \tag{39}$$

one can derive

$$A(VV\Delta VV) = -\frac{g^2}{2}\frac{\Gamma(-s/2)\Gamma(-t/2)}{\Gamma(1-s/2-t/2)}K \tag{40}$$

where

$$\begin{aligned}
K = {}& -\frac{1}{4}(st\zeta_1\cdot\zeta_3\zeta_2\cdot\zeta_4 + su\zeta_2\cdot\zeta_3\zeta_1\cdot\zeta_4 + tu\zeta_1\cdot\zeta_2\zeta_3\cdot\zeta_4)\\
&+\frac{1}{2}s(\zeta_1\cdot k_4\zeta_3\cdot k_2\zeta_2\cdot\zeta_4 + \zeta_2\cdot k_3\zeta_4\cdot k_1\zeta_1\cdot\zeta_3 + \zeta_1\cdot k_3\zeta_4\cdot k_2\zeta_2\cdot\zeta_3 + \zeta_2\cdot k_4\zeta_3\cdot k_1\zeta_1\cdot\zeta_4)\\
&+\frac{1}{2}t(\zeta_2\cdot k_1\zeta_4\cdot k_3\zeta_3\cdot\zeta_1 + \zeta_3\cdot k_4\zeta_1\cdot k_2\zeta_2\cdot\zeta_4 + \zeta_2\cdot k_4\zeta_1\cdot k_3\zeta_3\cdot\zeta_4 + \zeta_3\cdot k_1\zeta_4\cdot k_2\zeta_2\cdot\zeta_1)\\
&+\frac{1}{2}u(\zeta_1\cdot k_2\zeta_4\cdot k_3\zeta_3\cdot\zeta_2 + \zeta_3\cdot k_4\zeta_2\cdot k_1\zeta_1\cdot\zeta_4 + \zeta_1\cdot k_4\zeta_2\cdot k_3\zeta_3\cdot\zeta_4 + \zeta_3\cdot k_2\zeta_4\cdot k_1\zeta_1\cdot\zeta_2).
\end{aligned} \tag{41}$$

It has precisely the same form as the D9-brane case [6]. For a small α'-expansion note that

$$\frac{\Gamma(-s/2)\Gamma(-t/2)}{\Gamma(1-s/2-t/2)} = \frac{4}{st} - \frac{\pi^2}{6} + \cdots. \tag{42}$$

The leading terms in the small α'-expansion are

$$\begin{aligned}
&\frac{4}{st}\big[\mathrm{Tr}\big(\lambda^a\lambda^b\lambda^c\lambda^d\big) + \mathrm{Tr}\big(\lambda^a\lambda^d\lambda^c\lambda^b\big)\big]K + \frac{4}{ut}\big[\mathrm{Tr}\big(\lambda^a\lambda^c\lambda^b\lambda^d\big) + \mathrm{Tr}\big(\lambda^a\lambda^d\lambda^b\lambda^c\big)\big]K\\
&\quad+\frac{4}{su}\big[\mathrm{Tr}\big(\lambda^a\lambda^b\lambda^d\lambda^c\big) + \mathrm{Tr}\big(\lambda^a\lambda^c\lambda^d\lambda^b\big)\big]K.
\end{aligned} \tag{43}$$

The next to leading order terms come at l^4 order:

$$-l^4\frac{g^2}{2}\left(-\frac{\pi^2}{6}\right)K\,\mathrm{Tr}\big(\lambda^a\lambda^b\lambda^c\lambda^d + 5\text{ more terms}\big) = 2\pi^2 g^2\alpha'^2\,\mathrm{STr}\big(\lambda^a\lambda^b\lambda^c\lambda^d\big)K \tag{44}$$

which is the same as the field theory result since $\lambda^a = \sqrt{2}T^a$ where T^a is a generator that is used in the field theory Lagrangian. As an example that does not have a counter-part in the D9 case consider the vector–vector–scalar–scalar amplitude: it turns out to be

$$\begin{aligned}
A(\phi V\Delta V\phi) &= \frac{g^2}{2}\,\mathrm{tr}\big(\lambda^a\lambda^b\lambda^c\lambda^d\big)\big\langle\xi^1, k^1\big|V_g\big(\zeta^2, k^2\big)\Delta V_g\big(\zeta^3, k^3\big)\big|\xi^4, k^4\big\rangle\\
&= \frac{g^2}{2}\,\mathrm{tr}\big(\lambda^a\lambda^b\lambda^c\lambda^d\big)\big\langle\xi^1\big|(1+t/2)\zeta^2\cdot\zeta^3 B(1-s/2, -1-t/2)\\
&\quad+\big[-\zeta^2\cdot k^1\zeta^3\cdot k^4 - R_0^{uv}\big(\zeta_2^u k_2^v\zeta^3\cdot k^4 - \zeta_3^u k_3^v\zeta^2\cdot k^1\big) + R_0^{uv}R_0^{u'v'}\zeta_2^u k_2^v\zeta_3^{u'}k_3^{v'}\big]B(-s/2, 1-t/2)\\
&\quad+\big[\zeta^2\cdot k^3\zeta^3\cdot k^4 + \zeta^2\cdot k^1\zeta^3\cdot k^2 + \zeta^2\cdot k^3\zeta^3\cdot k^2 + R_0^{uv}\big(\zeta_2^u k_2^v\zeta^3\cdot k^2 - \zeta_3^u k_3^v\zeta^2\cdot k^3 - k_2^u k_3^v\zeta^2\cdot\zeta^3\\
&\quad- \zeta_2^u\zeta_3^v k^2\cdot k^3 + k_2^u\zeta_3^v\zeta^2\cdot k^3 + \zeta_2^u k_3^v\zeta^3\cdot k^2\big)\big]B(1-s/2, -t/2)\big|\xi^4\big\rangle.
\end{aligned} \tag{45}$$

After some algebra one can show that the leading term in the α'-expansion is given by

$$Ng^2\big(f^{eab}f^{ecd} + f^{eac}f^{ebd}\big)\zeta_2\cdot\zeta_3\xi_1\cdot\xi_4. \tag{46}$$

The next order term can be computed similarly to the four vector case. It is simpler due to the fact that $\xi\cdot k = 0 = \xi\cdot\zeta$. One gets

$$2\pi^2 g^2\alpha'^2\,\mathrm{STr}\big(\lambda^a\lambda^b\lambda^c\lambda^d\big)\xi_1\cdot\xi_4\left[-\frac{su}{4}\zeta_2\cdot\zeta_3+\frac{s}{2}(\zeta_2\cdot k_4\zeta_3\cdot k_1)+\frac{u}{2}(\zeta_3\cdot k_4\zeta_2\cdot k_1)\right]. \tag{47}$$

Concerning the cyclic symmetry it is not present when there is a mixture of scalar-vertex operators and vector-vertex operators: it is inconsistent with the broken Lorentz symmetry of the D3-brane configuration. Our final example of four point amplitude is four scalar scattering. One gets

$$\frac{g^2}{2}\alpha'^2\,\mathrm{Tr}\big(\lambda^a\lambda^b\lambda^c\lambda^d\big)\frac{\Gamma(-s/2)\Gamma(-t/2)}{\Gamma(1-s/2-t/2)}(su\xi_1\cdot\xi_4\xi_2\cdot\xi_3+tu\xi_1\cdot\xi_2\xi_3\cdot\xi_4+st\xi_2\cdot\xi_4\xi_1\cdot\xi_3). \tag{48}$$

The inequivalent cycling order is to be understood. At α'^2 order it yields

$$-\frac{1}{2}\pi^2\alpha'^2 g_{\mathrm{YM}}^6\,\mathrm{Str}\big(\lambda^a\lambda^b\lambda^c\lambda^d\big)(su\xi_1\cdot\xi_4\xi_2\cdot\xi_3+tu\xi_1\cdot\xi_2\xi_3\cdot\xi_4+st\xi_2\cdot\xi_4\xi_1\cdot\xi_3) \tag{49}$$

after taking the cycling into account.

3.2. Field theory computation

In this section we compute the α'-corrections to various scattering amplitudes in the regular field theory approach. The normalization of the field theory amplitude is such that one should multiply N/g_{YM}^2 to compare with the string theory. Also there could be difference in factors of i which is due to the Wick rotation in some string computations and the lack thereof in the corresponding field theory computations.

For the SYM action we take

$$\begin{aligned}\mathcal{L}_{\mathrm{SYM}}=\bigg[&-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}-\frac{1}{2}\big(\partial_\mu\phi^a_i+f^{abc}A^b_\mu\phi^c_i\big)^2-\frac{1}{2}\bar\psi^a\Gamma^\mu\big(\partial_\mu\psi^a+f^{abc}A^b_\mu\psi^c\big)-\frac{1}{2}f^{abc}\bar\psi^a\Gamma^i\phi^b_i\psi^c\\&-\frac{1}{4}\sum_{i,j}f^{abc}f^{ade}\phi^b_i\phi^c_j\phi^d_i\phi^e_j-\frac{1}{2}\big(\partial_\mu A^\mu_a\big)^2-\frac{1}{2}\partial_\mu\omega^*_a\big(\partial^\mu\omega_a+f^{abc}A^\mu_b\omega_c\big)\bigg].\end{aligned} \tag{50}$$

The comparison of the three point amplitudes that only include bosons is straightforward. For example the vector three point amplitude is given by

$$A(VVV)=2g^4_{\mathrm{YM}}f^{abc}\big(\zeta^1\cdot k^2\zeta^2\cdot\zeta^3+\zeta^2\cdot k^3\zeta^3\cdot\zeta^1+\zeta^3\cdot k^1\zeta^1\cdot\zeta^2\big). \tag{51}$$

The comparison of fermionic amplitudes are less trivial. The reason is that the conventions of the SYM action above are such that the fermionic fields have 32-components (while it has four-dimensional space–time dependence). For example the $(\psi\psi A_\mu)$-computation yields

$$A(\psi\psi A_\mu)=\frac{i}{2}g^4_{\mathrm{YM}}f^{abc}\frac{1}{k_3^2}\frac{1}{\Gamma^0\not k_1}\Gamma^0\Gamma^\mu\frac{1}{\Gamma^0\not k_2}+\big((1,a)\leftrightarrow(2,b)\big). \tag{52}$$

Implementing the reduction procedure one gets

$$A(\psi\psi A_\mu)=g^4_{\mathrm{YM}}f^{abc}\mathcal{U}_1\Gamma^0\Gamma^\mu\mathcal{U}_2\zeta^\mu_3 \tag{53}$$

where $\mathcal{U}_{1,2}$ are 32-component spinors. Being a Mayorana–Weyl spinor, they can be reduced to 16-component spinors, $U_{1,2}$, rendering the above expression

$$A(\psi\psi A_\mu)=-g^4_{\mathrm{YM}}f^{abc}U_1\Gamma^\mu U_2\zeta^\mu_3 \tag{54}$$

where the minus sigh came from the 32×32 gamma matrix, Γ^0. As further dimensional reduction one keeps only the lower half components for U_1 and upper half for U_2,

$$A(\psi\psi A_\mu)=-g^4_{\mathrm{YM}}f^{abc}w_1\gamma^{\mu T}w_2\zeta^\mu_3=g^4_{\mathrm{YM}}f^{abc}w_1\gamma^\mu w_2\zeta^\mu_3 \tag{55}$$

where γ^μ is an 8×8 matrix. Matching with the string theory identifies

$$w_{1,2} = E_- u_{1,2}. \tag{56}$$

Four point amplitudes are in order. In the leading order the vector four point function is

$$\begin{aligned}&-ig_{\rm YM}^6\big(\zeta_1\cdot\zeta_3\zeta_2\cdot\zeta_4 f^{eab}f^{ecd} - \zeta_1\cdot\zeta_4\zeta_2\cdot\zeta_3 f^{eab}f^{ecd} + \zeta_1\cdot\zeta_2\zeta_3\cdot\zeta_4 f^{eac}f^{ebd} - \zeta_1\cdot\zeta_4\zeta_3\cdot\zeta_2 f^{eac}f^{ebd}\\ &\quad + \zeta_1\cdot\zeta_3\zeta_2\cdot\zeta_4 f^{ead}f^{ecb} - \zeta_1\cdot\zeta_2\zeta_3\cdot\zeta_4 f^{ead}f^{ecb}\big).\end{aligned} \tag{57}$$

The next order result is

$$8\pi^2\alpha'^2 i g_{\rm YM}^6 K\,\mathrm{STr}\big(T^aT^bT^cT^d\big) \tag{58}$$

which matches the corresponding string theory computation. In the leading order $\langle A_\mu^a(x_1)\phi_k^b(x_2)\phi_k^c(x_3)A_\sigma^d(x_4)\rangle$ yields

$$ig_{\rm YM}^6\big(f^{eab}f^{ecd} + f^{eac}f^{ebd}\big)\zeta_2\cdot\zeta_3\xi_1\cdot\xi_4. \tag{59}$$

To compare with the string results of the previous section we need the α'-corrections to the SYM. They were obtained in ten dimensions [8–13]. We keep the α'^2-order terms and reduce it to four dimensions. The complete list of the terms at α'^2-order were presented in [1]. Here we quote only the terms that are relevant for the present computations. For the four vector scattering it is essentially the same as the D9 case so we will not repeat here. The vertices for the two scalar and two vector scattering are

$$(2\pi\alpha')^2\,\mathrm{Str}\bigg[-\frac{1}{8}F_{\mu\nu}F^{\mu\nu}D_\rho\phi_k D^\rho\phi^k - \frac{1}{2}D_\nu\phi_i F_{\nu\rho}F^{\rho\sigma}D^\sigma\phi^i\bigg]. \tag{60}$$

It is straightforward to show that they yield

$$4i\pi^2g^2\alpha'^2g_{\rm YM}^6\,\mathrm{STr}\big(T^aT^bT^cT^d\big)\xi_1\cdot\xi_4\bigg[-\frac{su}{2}\zeta_2\cdot\zeta_3 + s(\zeta_2\cdot k_4\zeta_3\cdot k_1) + u(\zeta_3\cdot k_4\zeta_2\cdot k_1)\bigg] \tag{61}$$

which is consistent with the string theory computation. At α'^2-order the relevant vertices for the four scalar amplitude are

$$(2\pi\alpha')^2\,\mathrm{Str}\bigg[-\frac{1}{8}D_\mu\phi_j D^\mu\phi^j D_\nu\phi_k D^\nu\phi^k + \frac{1}{4}D_\nu\phi_i D^\nu\phi^k D_\sigma\phi_k D^\sigma\phi^i\bigg] \tag{62}$$

which yields

$$-2i\pi^2\alpha'^2g_{\rm YM}^6\,\mathrm{Str}\big(T^aT^bT^cT^d\big)(su\xi_1\cdot\xi_4\xi_2\cdot\xi_3 + tu\xi_1\cdot\xi_2\xi_3\cdot\xi_4 + st\xi_2\cdot\xi_4\xi_1\cdot\xi_3). \tag{63}$$

It again agrees with the previous string computation at the same order.

4. Conclusion

In this Letter we computed several tree amplitudes. One obvious future direction is one-loop graphs. With the vertex operators constructed and tested here we are in a good position to tackle the problem. The one loop analysis will be presented elsewhere [7].

There are several reasons for the importance of one loop amplitudes. In the loop computation one expects to face divergence. One will need to come up with a regularization how to handle the divergence in the string theory context. The task will be interesting on its own right. However, what makes it more so is the possibility that one might encounter a non-trivial geometry arising while handling the divergence. (This issue is tied with the question whether/how an open string attached on a D-brane can feel the gravitational effects that are produced by the brane.) In e.g. [14] an explicit map was obtained between the quantum (and non-perturbative) effects and the $\mathrm{AdS}_5\times \mathrm{S}^5$ geometry. There only the pure SYM part was considered. We expect that the massive modes will have their contribution to the picture. The work of [14] was in a regular field theory context. It will be very interesting to see how the geometry arise in the current set-up of the string world sheet analysis. Perhaps could it arise through a Fishler–Susskind type mechanism?

The one loop should also be useful to study the string corrections to the anomalous dimensions of the SYM operators. While $\mathcal{N}=4$, $D=4$ SYM theory is a super-renormalizable theory the presence of the new vertices generates divergence. As well known open superstring yields finite results for various scattering amplitudes. Therefore it is natural to expect that there should be a procedure to obtain finite results from the SYM. The divergence would have to be cancelled by counter-terms. It will be interesting to see how the way that string theory deals with divergence is related to that of the field theory. Once the divergence is removed one will be able to compute the string corrections to the anomalous dimensions.[3] We will report on these issues in the future.

[3] For that matter one may try to compute the anomalous dimensions directly in the world sheet framework without detouring to the field theory.

References

[1] I.Y. Park, arXiv: 0704.2853 [hep-th].
[2] I.Y. Park, Phys. Lett. B 468 (1999) 213, hep-th/9907142.
[3] I.Y. Park, Phys. Rev. D 64 (2001) 081901, hep-th/0106078.
[4] P. Di Vecchia, A. Liccardo, R. Marotta, F. Pezzella, Int. J. Mod. Phys. A 20 (2005) 4699, hep-th/0503156.
[5] H. Kawai, T. Suyama, arXiv: 0706.1163 [hep-th].
[6] M.B. Green, J.H. Schwarz, E. Witten, Superstring Theory, vols. 1, 2, Cambridge Univ. Press, Cambridge, 1987.
[7] J.Y. Park, One loop scattering on D-branes, in preparation.
[8] S.J. Gates Jr., S. Vashakidze, Nucl. Phys. B 291 (1987) 172.
[9] E. Bergshoeff, M. Rakowski, E. Sezgin, Phys. Lett. B 185 (1987) 371.
[10] A.A. Tseytlin, hep-th/9908105.
[11] M. Cederwall, B.E.W. Nilsson, D. Tsimpis, JHEP 0106 (2001) 034, hep-th/0102009;
M. Cederwall, B.E.W. Nilsson, D. Tsimpis, JHEP 0107 (2001) 042, hep-th/0104236.
[12] E.A. Bergshoeff, A. Bilal, M. de Roo, A. Sevrin, JHEP 0107 (2001) 029, hep-th/0105274.
[13] P. Koerber, A. Sevrin, JHEP 0110 (2001) 003, hep-th/0108169.
[14] F. Gonzalez-Rey, B. Kulik, I.Y. Park, M. Rocek, Nucl. Phys. B 544 (1999) 218, hep-th/9810152.