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MHD flow of dusty nanofluid over a stretching surface with volume fraction of dust particles

Sandeep Naramgari, C. Sulochana *

Department of Mathematics, Gulbarga University, Gulbarga 585106, India

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KEYWORDS

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Abstract In this study we analyzed the momentum and heat transfer behavior of MHD nanofluid embedded with conducting dust particles past a stretching surface in the presence of volume fraction of dust particles. The governing equations of the flow and heat transfer are transformed into non-linear ordinary differential equations by using similarity transformation and then solved numerically using Runge–Kutta based shooting technique. The effect of non-dimensional governing parameters on velocity and temperature profiles of the flow are discussed and presented through graphs. Additionally friction factor and the Nusselt number have also been computed. Under some special conditions, numerical results obtained by the present study were compared with the existed studies. The result of the present study proves to be highly satisfactory. The results indicate that an increase in the interaction between the fluid and particle phase enhances the heat transfer rate and reduces the friction factor.

1. Introduction

The fluid flow and heat transfer over a stretching surface have wide range of applications in engineering and its allied areas. Now a days, low thermal conductivity in convectional fluids such as water, ethylene glycol, and oil encountered variety of problems in engineering electronic devices. To overcome this drawback and enhance the thermal conductivity in the convectional fluids, past few decades many researchers concentrated on mixing of nano or micrometer sized particles in the base fluids. Mixing of nano meter sized particles into base fluid is called nanofluid, which helps to enhance the thermal conductivity of the mixture fluid. Mixing of milli or micrometer sized particles (dust particles) in the base fluids also helps to improve the thermal conductivity of the base fluid and it is called dusty fluid. Till now, researchers concentrated on investigating the momentum and heat transfer behavior of either dusty or nanofluids. In this study we are taking initiation to analyze the momentum and heat transfer characteristics of dusty nanofluids, that is the mixture of milli or micro meter sized conducting dust particles into the nanofluid. This also may help to enhance the thermal conductivity of the base fluid for some combination of dusty and nano mixtures.

We have tremendous applications of dusty and nanofluids individually in engineering and sciences [1,2]. Saffman [3]

* Corresponding author.
E-mail addresses: nsreddy.dr@gmail.com (S. Naramgari), math.sulochana@gmail.com (C. Sulochana).
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was the first person who studied the stability of the laminar flow by uniformly distributed dust particles into the base fluid. Magnetohydrodynamic flow of a viscoelastic fluid embedded with dust particles over aligned parallel plates was discussed by Singh and Singh [4]. Unsteady MHD flow of a dusty fluid between two oscillating plates by considering heat source was illustrated by Debnath and Ghosh [5]. Momentum and heat transfer characteristics of MHD flow over a stretching sheet was analyzed by Chakrabarti and Gupta [6].

Datta and Dalal [7] discussed the flow and heat transfer behavior of dusty fluid over a circular pipe. Unsteady convective flow of a dusty fluid over rectangular channel was discussed by Dalal et al. [8]. Bagewadi and Shantharajappa [9] extended the previous work by considering flow over frenet frame. Allan et al. [10] presented a dusty viscous fluid flow through naturally occurred porous medium. Dusty viscous fluid between two infinite parallel plates by considering aligned magneticfield was discussed by Sandeep and Sugunamma [11]. Very recently the researchers [12,13] analyzed the magneticfield, radiation and chemical reaction effects on dusty viscous flow. Makinde and Aziz [14] analyzed the boundary layer flow of a nanofluid over a stretching sheet by considering convective boundary conditions.

Hady et al. [15] studied the radiation effect on viscous nanofluid flow over a nonlinear stretching sheet. Makinde et al. [16] discussed the buoyancy effects on MHD stagnation point flow and heat transfer of a nanofluid past a stretching/shrinking sheet. Nadeem et al. [17] presented a numerical study for MHD boundary layer flow of a Maxwell fluid past a stretching sheet in the presence of nanoparticles. The similar type of study by considering tangent hyperbolic fluid was discussed by Akbar et al. [18]. Many researchers like Oztop and Abu-Nada [19], Mohan Krishna et al. [20], Sandeep et al. [21] discussed the heat transfer characteristics on nanofluids by immersing the high conductivity nano materials in the base fluids and they concluded that the effective thermal conductivity of the fluid increases appreciably and consequently enhances the heat transfer characteristics by suspending the high thermal conductivity of nano materials into the base fluids. MHD stagnation point flow of a nanofluid toward a stretching surface by considering radiation effect was illustrated by Akbar et al. [22]. Hajmohammadi et al. [23] discussed the flow and heat transfer characteristics of Cu and Ag nano particles over permeable surfaces. Rana and Bhargava [24] used finite element and finite difference methods for nonlinear stretching sheet problem. Zaimi et al. [25] extended the work of Rana and Bhargava and they studied heat transfer and steady boundary layer flow of a nanofluid over a stretching/shrinking sheet. Effect of heat generation or absorption on nanofluid flow over a vertical plate was discussed by Ghalambaz and Noghrehabadi [26]. The researchers [27–31] proposed the interesting semi-analytical methods to solve different types of ordinary differential equations. Hajmohammadi et al. [32] presented an analytical solution for two-phase flow between two rotating cylinders filled with power law liquid and a micro layer of gas.

All the references cited above studied either dusty or nanofluid flows through different channels. To the authors' knowledge no studies have been investigated the flow and heat transfer behavior of the dusty nanofluid over stretching sheet by considering volume fraction of the dust particles. In this paper we studied the momentum and heat transfer behavior of MHD nanofluid embedded with conducting dust particles past a stretching surface in the presence of volume fraction of dust particles. The governing equations of the flow are solved numerically by using Runge–Kutta based shooting technique with Matlab package. Effect of governing parameters on velocity, temperature, friction factor and Nusselt number is discussed and presented through graphs. Comparisons of the present study are made with the existed studies.

2. Flow analysis of the problem

Consider two dimensional steady, laminar, incompressible, electrically conducting boundary layer flow of a dusty nanofluid past a stretching sheet. The sheet is along the plane $y = 0$ and the flow being confined to $y > 0$. The flow is generated by the two equal and opposite forces acting along the x-axis and y-axis is normal to it. The sheet is being stretched with the velocity $u_w(x)$ along the x-axis. The flow field is exposed to the influence of external magneticfield strength B_0 along x-axis as displayed in Fig. 1. Here both the fluid and dust particle clouds are supposed to be static at the beginning. The dust particles are assumed as uniform in size and conducting. Spherical shaped nano and dust particles are considered. Number density of dust particles along with volume fraction is taken into account. Here the number density of the dust particles is assumed as constant throughout the flow. It is also assumed that the external electric field due to polarization of charge is negligible. The drag force is taken into account for the fluid and particle interaction.

The boundary layer equations that govern the present flow as per above assumptions are given by (see [33,34]).

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$(1-\phi_p)\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = (1-\phi_p)\nu_{nf}\frac{\partial^2 u}{\partial y^2} + \frac{KN}{\rho_{nf}}(u_p - u) - \frac{\sigma B_0^2}{\rho_{nf}}u - \frac{\nu_{nf}}{k_1}u, \tag{2}$$

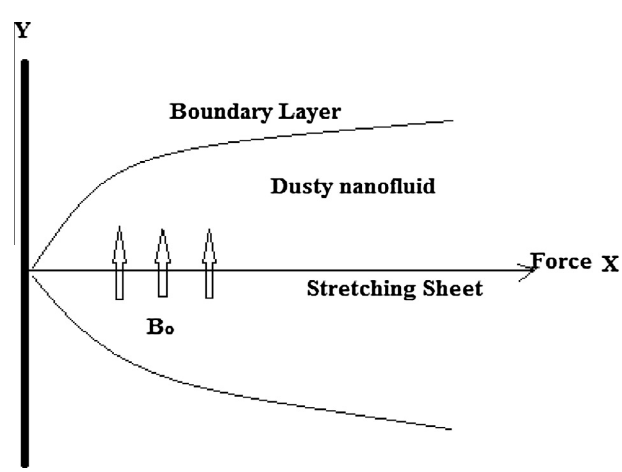

Figure 1 Physical model of the problem.

$$u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} = \frac{K}{m}(u - u_p), \tag{3}$$

$$\frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y} = 0, \tag{4}$$

with the boundary conditions

$$u = u_w(x), v = 0 \text{ at } y = 0,$$
$$u \to 0, u_p \to 0, v_p \to v \text{as } y \to \infty, \tag{5}$$

where (u, v) and (u_p, v_p) are the velocity components of the nanofluid and dust phases in the x and y directions respectively, ϕ_p is the volume fraction of the dust particles (i.e. the volume occupied by the dust particles per unit volume of mixture), v_{nf} is the kinematic viscosity of the nanofluid, K is the stokes resistance, m is the mass of the dust particle, N is the number density of the dust particles, ρ_{nf} is the density of the nanofluid, σ, B_0 are the electrical conductivity, induced magneticfield respectively, k_1 is the permeability of porous medium and $u_w(x) = cx, c > 0$ is the stretching sheet velocity.

For similarity solution, we introduced the following similarity transformation

$$u = cxf'(\eta), v = -v^{1/2}c^{1/2}f(\eta), \eta = v^{-1/2}c^{1/2}y,$$
$$u_p = cxF'(\eta), v_p = -v^{1/2}c^{1/2}F(\eta), \tag{6}$$

Eq. (6) identically satisfies Eqs. (1) and (4). Now Eqs. (2) and (3) become

$$f''' + ff'' - f'^2 + \frac{\alpha\beta}{(1-\phi_p)}(F' - f') - \frac{(M + K_1)}{(1-\phi_p)}f' = 0, \tag{7}$$

$$F'^2 - FF'' + \beta(F' - f') = 0, \tag{8}$$

with the transformed boundary conditions

$$f'(\eta) = 1, f(\eta) = 0 \text{ at } \eta = 0,$$
$$f'(\eta) = 0, F'(\eta) = 0, F(\eta) = f(\eta) \text{ as } \eta \to \infty, \tag{9}$$

where $\alpha = Nm/\rho_{nf}$ is the mass concentration of the dust particles, $\beta = K/cm$ is the fluid particle interaction parameter for the velocity, $M = \sigma B_0^2/c\rho_{nf}$ is the magneticfield parameter, $K_1 = v_{nf}/ck_1$ is the porosity parameter.

3. Heat transfer analysis

The governing boundary layer heat transport equations for dusty nanofluid are

$$(\rho c_p)_{nf}\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k_{nf}\frac{\partial^2 T}{\partial y^2} + \frac{N_1(c_p)_{nf}}{\tau_T}(T_p - T) + \frac{N_1}{\tau_v}(u_p - u)^2, \tag{10}$$

$$N_1 c_m\left(u_p\frac{\partial T_p}{\partial x} + v_p\frac{\partial T_p}{\partial y}\right) = -\frac{N_1(c_p)_{nf}}{\tau_T}(T_p - T), \tag{11}$$

where T and T_p are the temperature of the nanofluid and dust particles respectively, k_{nf} is the effective thermal conductivity of the nanofluid, $(c_p)_{nf}, c_m$ are the specific heat of the nanofluid and dust particles respectively, $N_1 = Nm$ is the density of the particle phase, τ_T is the thermal equilibrium time, τ_v is the relaxation time of the dust particle. For nanofluid constants (see [21]).

We considered temperature boundary conditions in order to solve Eqs. (10) and (11) as

$$T = T_w = T_\infty + A(x/l)^2 \text{ at } y = 0,$$
$$T \to T_\infty, T_p \to T_\infty \text{ as } y \to \infty, \tag{12}$$

where T_w, T_∞ are the temperatures near the wall and far away from the wall respectively and $l = v^{1/2}c^{-1/2} > 0$ is a characteristic length.

We now introduce the following non-dimensional variables to get the similarity solutions of Eqs. (10) and (11)

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \theta_p(\eta) = \frac{T_p - T_\infty}{T_w - T_\infty}, \tag{13}$$

where $T - T_\infty = A(x/l)^2\theta(\eta), A > 0,$

Using (12) and (13) in Eqs. (10) and (11), we get the ordinary differential equations as

$$\theta'' - \Pr(2f'\theta - f\theta') + \Pr\alpha\beta_T(\theta_p - \theta) + \Pr Ec\alpha(F' - f')^2 = 0, \tag{14}$$

$$2F'\theta_p - F\theta_p' + \gamma\beta_T(\theta_p - \theta) = 0, \tag{15}$$

the transformed boundary conditions are

$$\theta(\eta) = 1 \quad \text{at } \eta = 0,$$
$$\theta(\eta) = 0, \theta_p(\eta) = 0 \quad \text{as } \eta \to \infty, \tag{16}$$

where $\Pr = \mu_{nf}(c_p)_{nf}/k_{nf}$ is the Prandtl number, $\beta_T = 1/c\tau_T$ is the fluid interaction parameter, $Ec = cl^2/A(c_p)_{nf}$ is the Eckert Number, $\gamma = (c_p)_{nf}/c_m$ ratio of the specific heat of the nanofluid to dust particles.

For the engineering interest the skin friction coefficient C_f and the local Nusselt number Nu_x are defined as

$$C_f = \tau_w/\rho_{nf}u_w^2, \ \mathrm{Nu}_x = xq_w/k_{nf}(T_w - T_\infty), \tag{17}$$

where the surface shear stress τ_w and the surface heat flux q_w are given by

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0}, \ q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0}, \tag{18}$$

Using non-dimensional variables, we get

$$C_f\mathrm{Re}_x^{1/2} = f''(0), \ \mathrm{Nu}_x\mathrm{Re}_x^{-1/2} = -\theta'(0), \tag{19}$$

where $\mathrm{Re}_x = u_w x/v_{nf}$ is the local Reynolds number.

4. Results and discussion

The coupled ordinary differential Eqs. (7), (8), (14), and (15) with respect to the boundary conditions (9) and (16) are solved numerically using Runge–Kutta based shooting technique. For numerical results we considered $\phi_p = 0.1, M = \beta = 1$, $\beta_T = 0.3, Ec = 2, \Pr = 1, \gamma = 1$, and $K_1 = 0.2$. These values are kept as common in entire study except the varied values as displayed in the respective figures and tables. The results obtained show the influences of the non-dimensional governing parameters, namely magneticfield parameter M, mass concentration of dust particles α, volume fraction of dust particles ϕ_p, fluid particle interaction parameter for velocity β, fluid particle interaction parameter for temperature respectively as β_T

and the porosity parameter K_1 on velocity, temperature profiles along with the friction factor and Nusselt number.

Figs. 2 and 3 depict the influence of magneticfield parameter on the velocity and temperature profiles for both fluid and dust phases. It is evident from the figures that an increase in magneticfield parameter depreciates the velocity profiles and enhances the temperature profiles for both fluid and dust phases. This is due to the fact that a raise in the magneticfield parameter develops the opposite force to the flow, is called Lorentz force. This force has tendency to reduce the velocity boundary layer and enhance the thermal boundary layer thickness. Due to this reason we seen hike in temperature profiles of the flow. The effect of mass concentration of dust particles α on the velocity and temperature profiles for both fluid and dust phases are displayed in Figs. 4 and 5. It is clear from the figures that an enhancement in α declines the velocity and temperature profiles of fluid and particle phases. It is obvious that the increase in mass concentration of dust particles reduces the velocity and temperature boundary layers.

Figs. 6 and 7 illustrate the influence of the fluid particle interaction parameter for velocity β on velocity and temperature profiles for both fluid and dust phases. From Fig. 6 it is interesting to observe that an increase in the value of β enhances the velocity of the dust phase and depreciates the velocity of the fluid phase. This may happen due to the fact that the interaction between the fluid and particle phase is high then the particle phase develops the opposite force to the fluid phase until the particle velocity reaches the fluid velocity. From Fig. 7 it is clear that enhancement in the value of fluid particle interaction parameter for velocity increases the temperature profiles of the fluid and dust phases. This agrees with the general fact that the interaction between the fluid and particle phase is more then it causes to improve the thermal conductivity of the flow. Figs. 8 and 9 represent the effect of

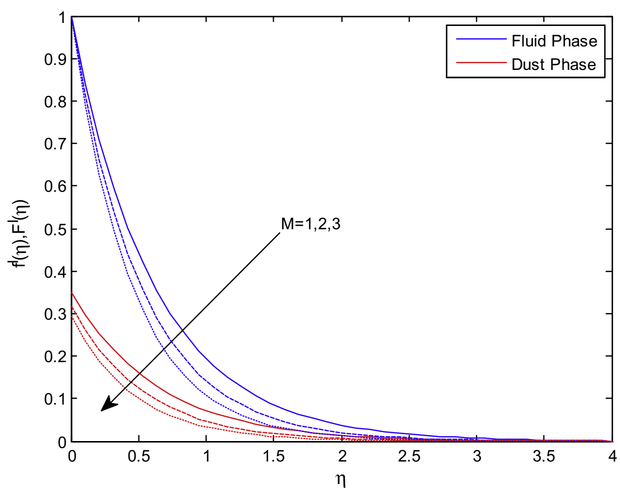

Figure 2 Velocity profiles for various values of magneticfield parameter M.

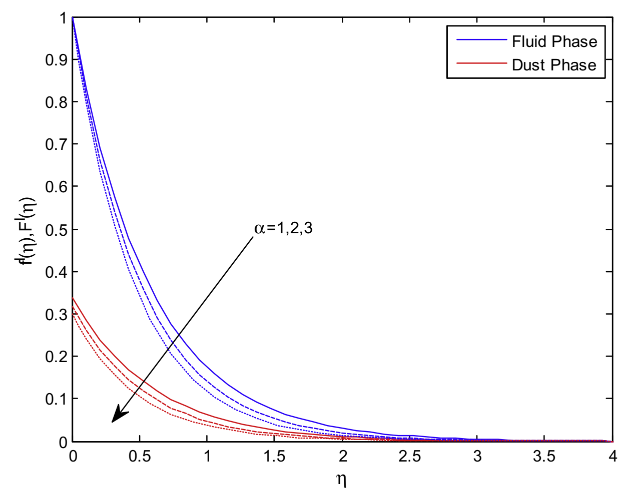

Figure 4 Velocity profiles for various values of mass concentration of dust particles α.

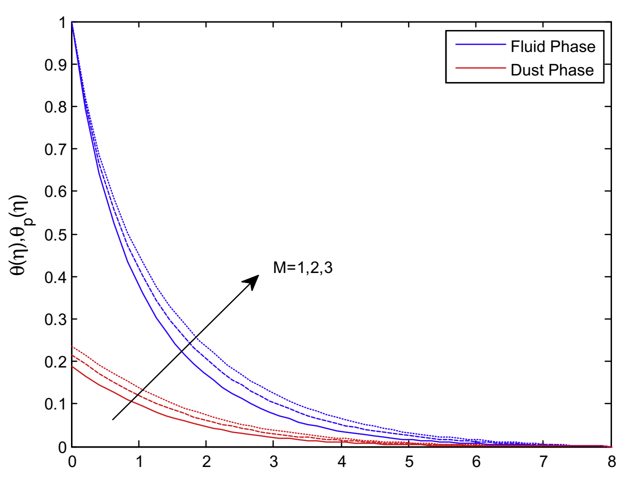

Figure 3 Temperature profiles for various values of magneticfield parameter M.

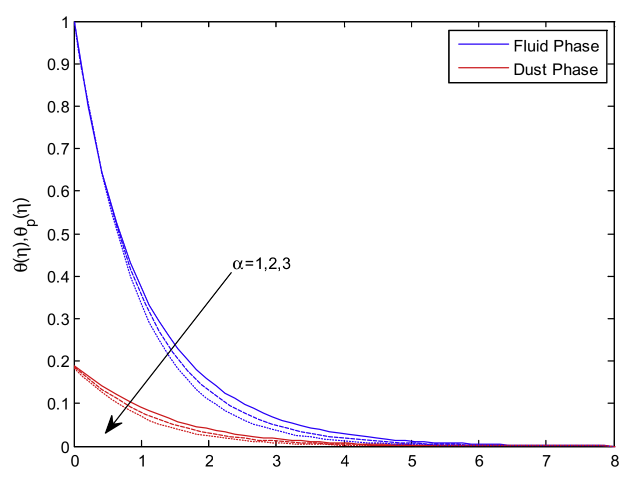

Figure 5 Temperature profiles for various values of mass concentration of dust particles α.

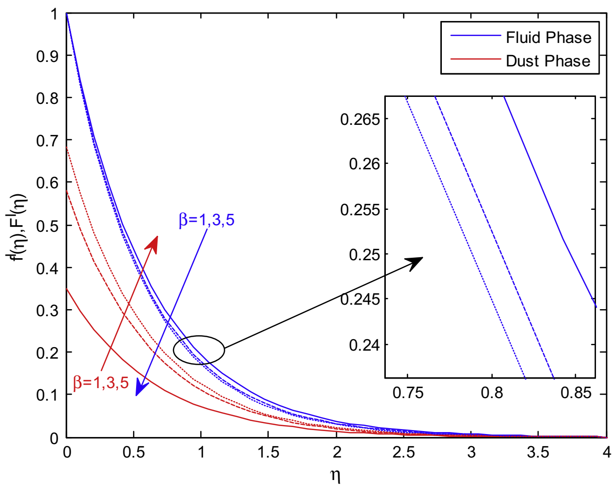

Figure 6 Velocity profiles for various values of fluid particle interaction parameter for velocity β.

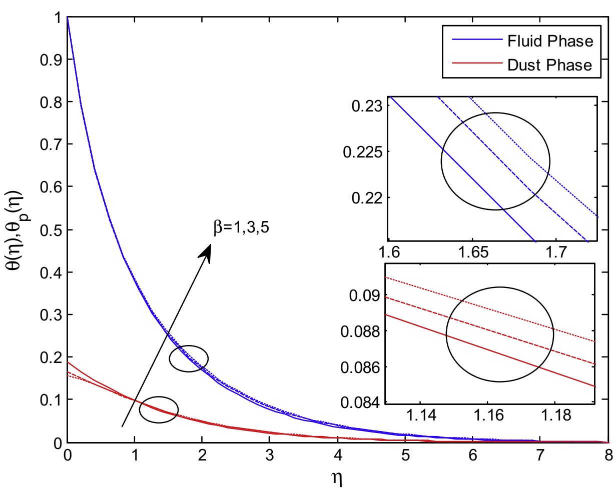

Figure 7 Temperature profiles for various values of fluid particle interaction parameter for velocity β..

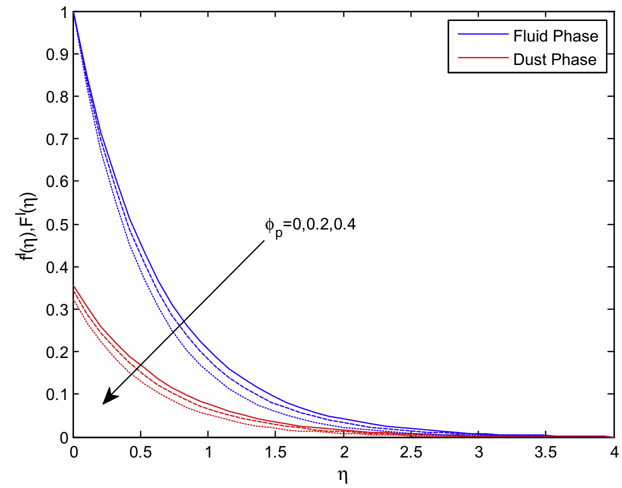

Figure 8 Velocity profiles for various values of volume fraction of dust particles ϕ_p.

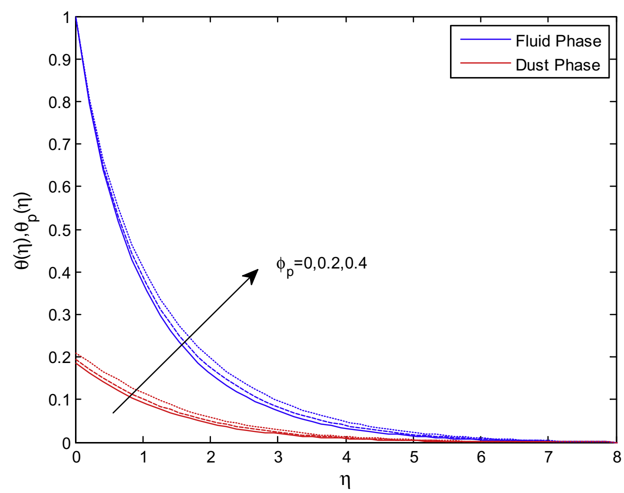

Figure 9 Temperature profiles for various values of volume fraction of dust particles ϕ_p.

volume fraction of the dust particles ϕ_p on the velocity and temperature profiles of fluid and dust phases. It is observed from the figures that a raise in the volume fraction of the dust particles depreciates the velocity profiles and enhances the temperature profiles of both fluid and dust phases. Generally the volume occupied by the dust particles per unit volume of mixture is high, then fluid as well as dust concentration will increase, this causes to decrease the velocity boundary layer thickness. Due to the increased thermal conductivity by the additional dust particles we observed hike temperature profiles of the flow.

Figs. 10 and 11 display the influence of the porosity parameter on velocity and temperature profiles for both fluid and dust phases. It is clear from the figures that the enhancement in the porosity parameter decreases the velocity profiles and enhances the temperature profiles for both fluid and dust phases. The reason behind this is the increase in porosity parameter means widen the holes of porous medium, in this case resistive forces act opposite to the flow and reduce the velocity profiles. From

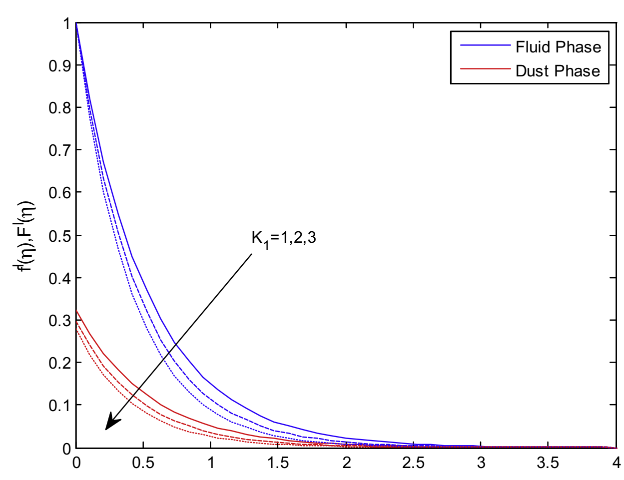

Figure 10 Velocity profiles for various values of porosity parameter K_1.

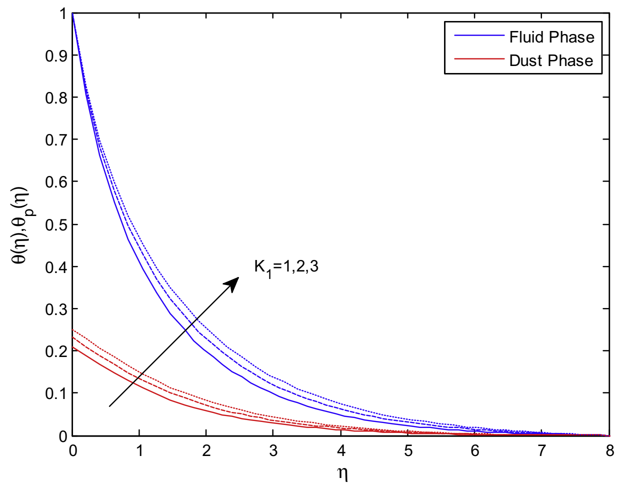

Figure 11 Temperature profiles for various values of porosity parameter K_1.

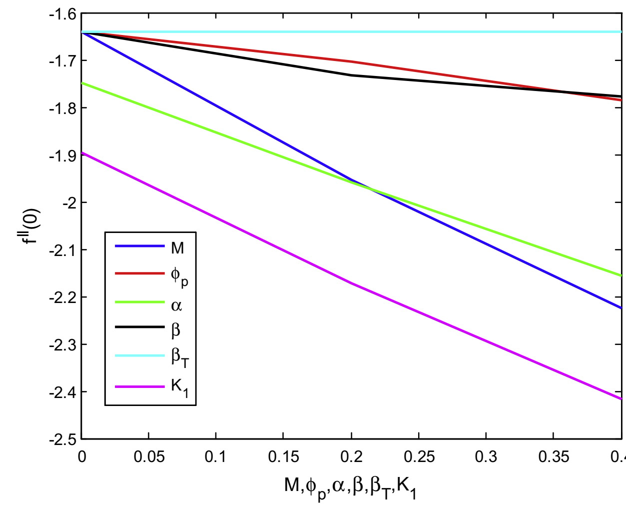

Figure 13 Variation in Skin friction coefficient $f''(0)$.

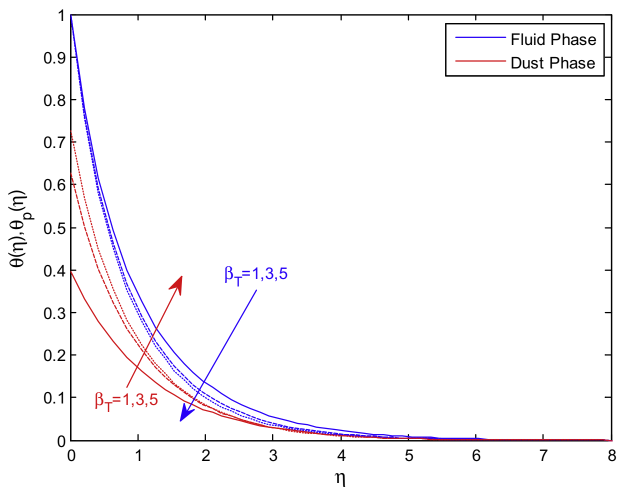

Figure 12 Temperature profiles for various values of fluid particle interaction parameter for temperature β_T.

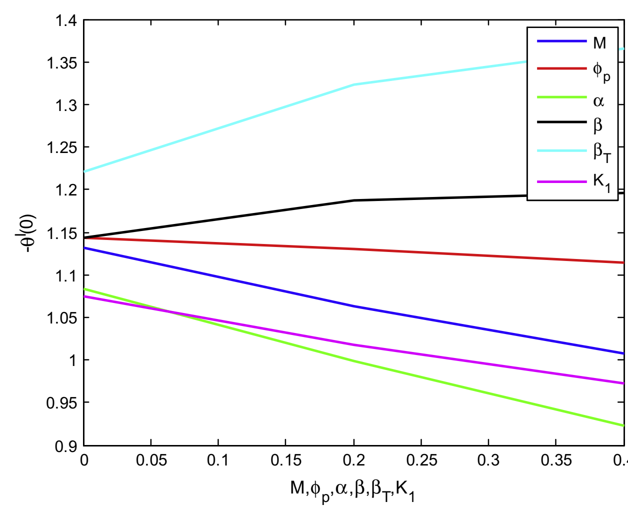

Figure 14 Variation in Nusselt Number $-\theta'(0)$.

Fig. 11 we observed hike in temperature profiles. Physically this means that increase in porosity parameter releases the internal heat to the flow, this helps to enhance the temperature profiles of the fluid and dust phases. Fig. 12 depicts the effect of fluid particle interaction parameter for temperature β_T on the temperature profiles for both fluid and dust phases. It reveals that an increase in β_T declines the temperature profiles in the fluid phase and enhances the temperature profiles of dust phase. This may happen due to the domination of conductive heat transfer in dust phase due to the enhanced interaction between the nano and dust particle phase.

Figs. 13 and 14 illustrate the influence of the non-dimensional governing parameters on the friction factor and rate of heat transfer. From Fig. 13 it is evident that enhancement in the magneticfield parameter, mass concentration of dust particles, porosity parameter, fluid particle interaction parameter for velocity and the volume fraction of the dust particles reduces the friction factor. But a raise in the value of fluid particle interaction parameter enhances the skin friction coefficient. From Fig. 14 it is noticed that the enhancement in the fluid particle interaction for velocity and temperature parameters improves the heat transfer rate. An increase in volume fraction of dust particles showed partial decrement in heat transfer rate. But it is expected that after certain period of time it may take reverse action and shows the enhancement in the Nusselt number. A raise in magneticfield parameter, porosity parameter and the mass concentration of dust particles declines the heat transfer rate. Table 1 shows the influence of various non-dimensional governing parameters on the friction factor and the rate of heat transfer. This phenomenon was already explained with the help of Figs. 13 and 14. Table 2 depicts the comparison of the present results with the existed results of Gireesha et al. [34] and Abel et al. [35]. We noticed an excellent agreement of the present results with the existed studies. This shows the validity of the present results.

Table 1 Variation in $f''(0)$ and $-\theta'(0)$ for different values of non-dimensional governing parameters.

ϕ_p	α	β	β_T	M	K_1	$f''(0)$	$-\theta'(0)$
0	0.5	1	0.3	1	0.2	−1.640069	1.144146
0.2	0.5	1	0.3	1	0.2	−1.704612	1.130511
0.4	0.5	1	0.3	1	0.2	−1.784459	1.114033
0.1	1	1	0.3	1	0.2	−1.749168	1.084180
0.1	2	1	0.3	1	0.2	−1.958297	0.997968
0.1	3	1	0.3	1	0.2	−2.155880	0.922651
0.1	0.5	1	0.3	1	0.2	−1.640069	1.144146
0.1	0.5	3	0.3	1	0.2	−1.732946	1.186799
0.1	0.5	5	0.3	1	0.2	−1.778637	1.195443
0.1	0.5	1	1	1	0.2	−1.639956	1.221163
0.1	0.5	1	3	1	0.2	−1.639956	1.322617
0.1	0.5	1	5	1	0.2	−1.639956	1.366266
0.1	0.5	1	0.3	1	0.2	−1.639956	1.132423
0.1	0.5	1	0.3	2	0.2	−1.954443	1.062415
0.1	0.5	1	0.3	3	0.2	−2.223702	1.008081
0.1	0.5	1	0.3	1	1	−1.895866	1.074931
0.1	0.5	1	0.3	1	2	−2.172591	1.017992
0.1	0.5	1	0.3	1	3	−2.417130	0.972258

Table 2 Comparison of the results for wall temperature gradient $-\theta'(0)$ in the case of $\beta = \beta_T = Ec = \phi_p = \phi = 0$.

Pr	Gireesha et al. [34]	Abel et al. [35]	Present study
0.72	1.0886	1.0885	1.088561
1	1.3333	1.3333	1.333333
10	4.7968	4.7968	4.796817

5. Conclusions

This paper presents a similarity solution for the MHD boundary layer flow and heat transfer behavior of a dusty nanofluid over a stretching sheet. The governing mathematical equations are reduced into ordinary differential equations, which are then solved numerically by using Runge–Kutta based shooting technique with MATLAB package. The effects of some governing parameters on the flow, heat transfer, friction factor and Nusselt number are presented graphically and discussed. The findings of the numerical results are summarized as follows:

- A raise in the volume fraction of the dust particles has tendency to slow down the velocity profiles and improves the temperature profiles of the flow.
- An increase in magneticfield parameter, mass concentration of dust particles and porosity parameters help to gradual decrement in the friction factor.
- Enhancement in the fluid particle interaction parameter for velocity and temperature improves the heat transfer rate and reduces the skin friction coefficient.
- Fluid particle interaction parameter for temperature has tendency to enhance the thermal conductivity of the particle phase.
- Increase in the mass concentration of the dust particles reduces the velocity and thermal boundary layer thicknesses.
- Increase in the mass concentration of the dust particles cases to gradual reduction in the heat transfer rate.

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Dr.N.Sandeep obtained his M.Sc., and Ph.D. degrees from Sri Venkateswara University, Tirupati, and M.Phil degree from Madurai Kamaraj University, Madurai. He published more than 30 papers in reputed scientific journals and presented papers in national and international conferences. At present he is serving as a Post Doctoral Fellow under UGC Dr.D.S. Kothari Fellowship Scheme at Department of Mathematics, Gulbarga University, Gulbarga. His area of interest is Fluid Mechanics.

Prof.C.Sulochana obtained her M.Sc., and M.Phil., degrees from Gulbarga University, Gulbarga, Karnataka, and Ph.D degree from Sri Krishna Devaraya University, Anantapur. She produced 6 Ph.D and 7 M.Phil degrees. She published more than 35 papers in reputed scientific journals and presented papers in national and international conferences. She handled various prestigious positions at Gulbarga University. At present she is serving as a professor at Department of Mathematics, Gulbarga University, Gulbarga. Her area of interest is Fluid Mechanics.