State Estimation for a Small Scale Flybar-less Helicopter

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Abstract

Unmanned helicopters have been used extensively in the last few decades as a research platform for different applications. Flybarless, single-rotor helicopters are famous for their increased agility and high maneuverability, which makes them a suitable platform for many challenging applications. This paper is concerned with the problem of attitude and flapping angles estimation of a flybar-less small scale single rotor helicopter. A nonlinear model for the Maxi Joker 3 helicopter is used to simulate test data. A Kalman filter is designed and implemented to estimate both the attitude and the flapping angles of the helicopter. Results are shown at the end of the paper to validate the performance of the proposed approach.

Keywords: UAV; Small Scale Flybar-less Helicopter; Attitude Estimation; Kalman Filter; Flapping Angles Estimation.

1. Introduction

Major research attention is aimed at the development of autonomous, small-scale helicopters. Helicopters are considered a viable candidate for many different applications such as surveillance missions, power line inspection, wildlife monitoring, and many military applications. Recently, researchers have focused on the design of various navigation and autonomous control systems to control vehicles in different missions [1]. In order for any control law to work, an accurate knowledge of the different states of the vehicle is needed. In this paper, the Maxi Joker 3 helicopter is utilized (see Figure 1).
During the past few years, a large number of studies addressed the problem of formulating the dynamic model of different small-scale helicopters. In [2-4], a complete six-degrees of freedom dynamic model, with the flapping angles, was introduced for a small-scale helicopter. In [5], the generation of forces and moments from the main and tail rotor was derived. In [6], the system identification of the Joker 3 helicopter was obtained.

Precise state estimation is essential for controlling the helicopter autonomously. Nevertheless, it is hard to obtain accurate values for different helicopter states because of the large drifts, possible measurement bias, and immense noise of the onboard sensors [7, 8]. These sensors are commonly used in VTOL UAV because of their small weight, small size, and small power consumption. By fusing the measurements of different sensors, an accurate estimate can be obtained [9-13].

To estimate the vehicle’s attitude, a number of approaches have been explored. For instance, the strapdown method, which is based on integrating the angular rates to get the Euler angles, was introduced in [9, 10]. Alternatively, the vehicle’s attitude can be estimated using the bi-vector method. In this method, the attitude can be estimated by obtaining the acceleration and the magnetic field readings from the accelerometer and the magnetometer sensors. Moreover, the global positioning system (GPS) and the image sensors were used for vehicles’ attitude estimation in [11].

The strapdown method has the disadvantage of generating an accumulative error during angular rate integration. This error keeps increasing over time because of the MEMS sensors’ offsets. Therefore, for long-term maneuvers, an incorrect estimation will result. On the other hand, the bi-vector method has a different drawback. In this method, the vibration generated from the UAV rotors will affect the accelerometer’s readings, which will lead to an inaccurate estimate of the attitude. A tri-axial magnetometer was used in [12] to enhance the heading angle estimation accuracy by improving the system’s overall state observability. However, the paper uses a kinematic system model rather than incorporating the dynamics of the vehicle. In [13], a helicopter pitch and roll angle estimation technique is proposed that uses a single antenna GPS receiver and gyroscope measurements. The attitude is estimated by determining the thrust vector. The thrust vector is obtained by estimating the helicopter’s acceleration using a Kalman filter.

The helicopter’s flapping angles, while essential to characterizing the vehicle’s dynamic model, cannot be measured directly using any sensor. It is not possible to place a sensor on the main rotor of the helicopter to measure its orientation. Therefore, an estimation algorithm needs to be used to estimate the angles. In this paper, a Kalman filter will be presented to estimate both the attitude and the flapping angles of the helicopter.
2. Helicopter model

The helicopter platform is naturally unstable with a significant amount of cross coupling and high order of states, which makes it challenging to model its dynamics and to control it. This section will discuss different frames which have been used to describe the model. A six degrees of freedom model and a linearized helicopter model are also introduced.

2.1. Reference frames

To describe the position and the orientation of the helicopter, two major different frames are used. The body frame and the inertial frame. The body frame (BF) has its origin in the Centre of Gravity (CG) of the helicopter. Based on the right-hand rule, the orientation of the BF was introduced and it is denoted as $\mathbf{x}^b, \mathbf{y}^b, \mathbf{z}^b$.

On the other hand, the Earth frame (EF) is an inertial frame symbolized by the position and the translational motion of the helicopter are described using the (EF). The EF is positioned on the earth's surface at a fixed point. Figure 2 illustrates the body frame and the earth frame.

![Diagram showing Body Frame and Earth Frame](image)

2.2 Rotation matrix

The forces and the moments in the (BF) are transferred to the (EF) using a rotation matrix. $(R^b_E)$ matrix is used to transfer equations from (EF) to (BF) and vice versa. The Euler angles $[\phi, \theta, \psi]$ represent the rotation around x-axis then y-axis and at last z-axis.

Consequent to the principal of orthonormality transposing principle $(R^b_E)$ is the resulting transformation matrix:

$$R^E_b = \begin{bmatrix} c(\theta) s(\psi) & c(\psi) s(\theta) s(\phi) - s(\psi) c(\phi) & s(\psi) s(\theta) c(\phi) + c(\psi) s(\phi) \\ c(\theta) s(\psi) & s(\theta) s(\phi) + c(\psi) c(\phi) & s(\psi) c(\theta) c(\phi) - s(\phi) s(\psi) s(\theta) \\ -s(\theta) & c(\theta) s(\phi) & c(\theta) c(\phi) \end{bmatrix}$$

(1)

2.3 Six degrees of freedom model (6 DOF)

The position and attitude of a rigid body at any instant in time in 3-D space are defined using 6 DOF model. Three equations to define the position of the rigid body and three to define its attitude [1]. Those
equations are written in the body frame. (2, 3, and 4) are the rotational dynamics equations and (5, 6, 7) describe the translational dynamics.

\[
\begin{align*}
\dot{p} &= \frac{L}{J_x} - \frac{J_z - J_y}{J_x} q r \\
\dot{q} &= \frac{M}{J_y} - \frac{J_x - J_z}{J_y} p r \\
\dot{\phi} &= \frac{M}{J_z} - \frac{J_x - J_y}{J_z} p r \\
\dot{u} &= rv - qw + \frac{F_{xb}}{m} \\
\dot{v} &= -ru + pw + \frac{F_{yb}}{m} \\
\dot{w} &= qu - pv + \frac{F_{zb}}{m}
\end{align*}
\]  

(\(p, q, r\)) are the body rates around \((x^b, y^b, z^b)\) axes respectively, \(m\) is mass for the helicopter. The moments of inertia are \((J_x, J_y, J_z)\). (\(u, v, w\)) are the velocities in the body frame along \((X, Y, Z)\) axes respectively. \((F_{xb}, F_{yb}, F_{zb})\) are the body forces along \((X, Y, Z)\) axes respectively. \((L, M, N)\) are the body moments about \((X, Y, Z)\) axes.

The attitude of the helicopter with respect to the earth frame is described using the Euler angles which are computed based on the 6 DOF model, with the formulae that define the Euler angles are shown in equations (8, 9, and 10).

\[
\begin{align*}
\theta &= q \cdot \cos(\phi) - r \cdot \sin(\phi) \\
\phi &= p + (q \cdot \sin(\phi) + r \cdot \cos(\phi)) \cdot \tan(\theta) \\
\psi &= (p \cdot \sin(\phi) + r \cdot \cos(\phi)) / \cos(\theta)
\end{align*}
\]

2.4 Flapping Dynamics

The main rotor moves when (Elevator and Aileron) servo motors are active [2]. To describe this movement two angles \((a, b)\) are introduced. \((a)\) angle describes how much the tip path plane pitched with respect to the \(Y\)-axis in body frame and the \((b)\) angle describes how much the main rotor plane rolled with respect to the \(X\)-axis in body frame. The dynamics of the tip path plane are described in equations (11, 12).

\[
\begin{align*}
\dot{a} &= -T_a a + T_a P_a \\
\dot{b} &= -T_b b + T_b P_b
\end{align*}
\]

In these equations, \(T_a, T_b\) are the time constants of the main rotor’s movement. \(P_a, P_b\) are the commanding angles from the elevator and aileron servo motors.

2.5 Linearization of Joker 3 helicopter model

To design a linear estimator, a linear system only can be estimated. Therefore, the linearization process is highly important to design the linear estimators. The linearization was calculated near to the
hovering point (attitude angles, attitude rates and body velocities are all the time equal zero). The state-
space format of the linearized model is given by equation (13).

\[
\dot{X} = AX + BU \\
Y = CX + DU
\] (13)

Where \(X\) is the state vector, \(U\) is the input vector, \(A, B, C, D\) are the state-space matrices and \(Y\) is the output vector.

\[
X = [\phi, \theta, \varphi, p, q, r, a, b]^T
\] (14)

The input vector is:

\[
U = [P_a, P_b, P_m, P_l]^T
\] (15)

The \(A\) matrix is computed to be:

\[
A = 
\begin{bmatrix}
0 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -Q_e/J_z & A(4,8) \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & Q_e/J_y \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -T_a \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -T_b \\
\end{bmatrix}
\] (16)

where

\[
A(5,7) = (K_\beta + \pi * K_m * h_{mr})/36)/J_y \\
A(4,8) = (K_\beta + \pi * K_m * h_{mr})/36)/J_x
\]

and the \(B\) matrix is calculated to be:

\[
B = 
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & (k_\ast l_\ast)/J_z \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & (k_\ast l_\ast)/J_x \\
T_a & 0 & 0 & 0 \\
0 & T_a & 0 & 0 \\
\end{bmatrix}
\] (17)

A system identification process for Joker-3 was made in [6] to identify the helicopter’s parameters. According to the datasheet of the Joker 3 helicopter and by identifying some further parameters, the set of
parameters for the Joker 3 helicopter was obtained and used to construct the dynamic model of the helicopter.

The D matrix is a zero matrix, its dimensions are (2×3). The C matrix is a scalar diagonal matrix and its dimensions are (8×8).

3. Kalman filter design

In this section, the linear state estimator for such a linearized dynamic model as described previously will be designed. Kalman filter is a linear state estimator which uses the linear dynamic model described in section II. The Kalman filter requires knowledge of the control inputs to the system and the data from the measurements to estimate the states.

3.1 Algorithm design

The process of designing Kalman filter has few stages which need to be followed in order to get an accurate estimation of the states. The first stage based on the data which is acquired from the linearized model of maxi joker 3 helicopter as described previously. The data acquired at sampling time equal (0.002 s). The state transition matrix is obtained by:

$$F_k = \begin{bmatrix}
0 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -36.6307 & 433.2424 \\
0 & 0 & 0 & 0 & 0 & 0 & 169.0266 & 14.2912 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -10.0000 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -10.0000 \\
\end{bmatrix} \quad (18)$$

The control inputs model is given by:

$$B_k = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.2210 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.3516 \\
10.0000 & 0 & 0 & 0 & 0 \\
0 & 10.0000 & 0 & 0 & 0 \\
\end{bmatrix} \quad (19)$$

$R_k$ is the measurement noise covariance matrix, and it is a diagonal matrix which describes the accuracy of measurements. The value of each diagonal element is the same and computed to be equal to (0.2236). The process noise covariance matrix $Q_k$ is a diagonal matrix, and the main diagonal elements represent the variances of the sensors, and each variance is assumed to be equal (0.0020).

The second stage is the initialization stage, in which an initial value for the state estimate and the state covariance is assigned. In the prediction stage, the priori state estimate and its covariance are formulated. To keep updating the estimate, the estimate covariance, and the measurements, the fourth stage (the updating stage) is needed.
4. Validation results

This section shows the procedure of implementing the state estimation algorithm. Simulated data was acquired based on a mathematical linear dynamic model for the maxi-joker 3 helicopter platform. Attitude angles and body rates are thus modelled, adding white noise to the measurements. Figure 4 shows the simulation structure. Experimental validation was also performed. The results will be shown in an extended publication of this paper.

![Fig. 3. The structure of the experiment.](image)

The results below show the estimated states compared with the measured states which were fused from the modelled sensors.

4.1 Attitude Estimation

The results in figures (4-9) show the estimation of the roll, pitch, and heading angles. Figure 4, shows the actual, measured, and estimated roll angle. It can be seen from the figure that the proposed method accurately estimates the roll angle. Figure 6 shows a zoomed section of figure 5. It is clear from the figure that the estimated roll angle matches the actual helicopter roll angle. Figures 4 and 5 show the roll angle estimation, the estimator shows a great behavior in terms of rejecting the noise and matching the actual roll angle.

As can be seen, figures 6 and 7 show the pitch angle estimation, the estimator shows a great estimation performance in terms of rejecting the noise and matching the actual pitch angle. The heading estimation is quite precise as well, by looking to the figures (8 and 9), the estimated heading is following the actual heading with no delay, and small amount of offset less than (0.1) degree.

![Fig. 4. Estimating the roll angle.](image)

![Fig. 5. Estimating the roll angle (zoomed)].(image)
Fig. 6. Estimating the pitch angle.

Fig. 7. Estimating the pitch angle (zoomed).

Fig. 8. Estimating the heading angle.

Fig. 9. Estimating the heading angle (zoomed).

Fig. 10. Estimating the roll angular rate.

Fig. 11. Estimating the roll angular rate (zoomed).
4.2 Body rates Estimation

The results in figures 10-15 show the estimation of the body rates (roll angular rate, pitch angular rate and the heading angular rate).

As shown, figures 10 and 11 demonstrate the angular roll rate estimation. The behavior of the estimator shows a great estimation performance in terms of rejecting the noise and following the actual roll angular rate. Obviously, The angular pitch rate estimation is relatively accurate, too, as can be seen from figures 9
and 10. The estimated angular pitch rate is matching the actual rate. The figures 11 and 12 prove the angular rate heading estimation, the performance of the estimator shows an excellent estimation process in terms of filtering the measured heading angular rate, and following the actual heading angular rate.

4.3 Flapping Angles Estimation

The results in figures (16,17) show the estimation of the flapping angles (the longitudinal angle and the latitudinal angle). Even though, there is no sensor can measure the flapping angles of the helicopter, the Kalman filter has succeeded in filtering the longitudinal and the latitudinal angles.

5. Conclusion

In this paper, we have applied and implemented a state estimation algorithm using a linear Kalman filter, an attitude, body rates, and flapping angles estimation for a small scale flybar-less helicopter has been achieved. The results showed a great improvement on the estimation of attitude angles and the body rates as well. Also the Kalman filter succeeded in estimating the values of the flapping angles and the body rates as well.

6. References