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Super- and CP-symmetric QCD in higher dimensions

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Abstract

An extremely precise global symmetry is necessary in the Peccei–Quinn solution to the strong CP problem. Such symmetry arises when colored chiral fermions are localized in an internal space. We present a supersymmetric model that incorporates the above mechanism. Extra colored chiral multiplets around the supersymmetry-breaking scale are a generic prediction of the supersymetric model.

1. Introduction

The Peccei–Quinn mechanism [1] is a promising solution to the strong CP problem, yet it requires an extremely precise global symmetry. The explicit breaking of the symmetry gives rise to an extra scalar potential of the axion. The energy scale relevant to the mechanism has to be 10^{10}–10^{12} GeV, while the extra potential has to be 10^{-10} times smaller than the potential generated by the QCD dynamics. What is the origin of such a precise global symmetry?

Suppose that the $SU(3)_C$ gauge field propagates in extra space dimensions, and that some of $SU(3)_C$-charged chiral fermions are localized in the internal space. Then, an approximate chiral symmetry naturally arises when the chiral fermions are separated sufficiently in the internal space. Such a symmetry might well be stable even against possible quantum gravitational corrections. Our previous papers [2] identified it with the Peccei–Quinn symmetry.

Supersymmetry is a leading candidate to be discovered through experiments in no distant future. As such, it is tempting to extend the higher-dimensional QCD, which naturally realizes the strong CP invariance in the nonsupersymmetric case mentioned above, to a supersymmetric setup. In this Letter, we present an explicit model, and show that colored chiral multiplets are around the supersymmetry-breaking scale in addition to the particles of the minimal supersymmetric standard model (MSSM).

2. Accidental axial symmetry

In this section, we provide a basic supersymmetric setup for the accidental axial (Peccei–Quinn) symmetry.

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2.1. Bulk color gauge theory

Let us consider four-dimensional Minkowski spacetime M_4 along with one-dimensional extra space S^1, whose coordinate y extends from $-l$ to l (that is, two points at $y = l$ and $y = -l$ are identified). The $\mathrm{SU}(3)_C$ vector field is assumed to propagate on the whole spacetime $M_4 \times S^1$. Thus, the $\mathrm{SU}(3)_C$ vector field belongs to a vector multiplet Ψ of the minimal supersymmetry (SUSY) in five dimensions. One vector multiplet Ψ consists of one vector multiplet V and one chiral multiplet Φ of $\mathcal{N} = 1$ SUSY in four-dimensional spacetime.

The minimal SUSY in five-dimensional spacetime corresponds to $\mathcal{N} = 2$ SUSY in four dimensions. Thus, description of gauge theories based on four-dimensional $\mathcal{N} = 2$ SUSY is useful in considering SUSY theories in five dimensions. The action of a U(1) vector multiplet $\Psi = (V, \Phi)$ of $\mathcal{N} = 2$ in four dimensions is given in terms of a holomorphic function $\mathcal{F}(\Psi)$:

$$K = \frac{1}{4\pi} \,\mathrm{Im}\left(\frac{\partial \mathcal{F}(\Phi)}{\partial \Phi} \Phi^\dagger\right),$$
$$W = \frac{-i}{16\pi} \left(\frac{\partial^2 \mathcal{F}(\Phi)}{\partial \Phi^2}\right) \mathcal{W}^\alpha \mathcal{W}_\alpha, \tag{1}$$

where K and W denote the Kähler potential and superpotential, respectively.

In five-dimensional spacetime, the imaginary part of the scalar component in Φ is the 4th polarization of the five-dimensional vector field. The superfields $(V(x), \Phi(x))$ are promoted to fields in five dimensions $(V(x, y), \Phi(x, y))$. The Kähler potential and superpotential are now integrated over $d^4x\, dy\, d^2\theta\, d^2\bar{\theta}$ and $d^4x\, dy\, d^2\theta$, respectively. The coordinate y is integrated along S^1. Let us consider a prepotential that is an at-most-cubic polynomial

$$\mathcal{F} = i \frac{1}{2} \frac{4\pi}{g^2} \Psi^2 + i h \Psi^3. \tag{2}$$

The quadratic term provides the gauge kinetic term, and the cubic term contains the Chern–Simons term

$$-\int \frac{3}{\sqrt{2}8\pi} h A_4 F_{\mu\nu} F_{\kappa\lambda} \epsilon^{\mu\nu\kappa\lambda}\, d^4x\, dy, \tag{3}$$

where A_4 denotes the 4th polarization of the gauge field and $F_{\mu\nu}$ the field strength in four dimensions. The Lorentz symmetry and gauge covariance can be restored by modifying the action [3,4]. The precise form of the Chern–Simons term in five dimensions in the case of nonabelian gauge group is found in Ref. [5] (with supergravity). We now turn to the $\mathrm{SU}(3)_C$ gauge group.

Kaluza–Klein reduction to the four-dimensional spacetime, however, yields an unwanted $\mathcal{N} = 1$ chiral multiplet in the $\mathrm{SU}(3)_C$-**adj.** representation in the low energy spectrum, and the $\mathcal{N} = 2$ SUSY is left unbroken. Hence, we consider an S^1/Z_2 orbifold instead of S^1. The multiplets $V(x, y)$ and $\Phi(x, y)$ of $\mathcal{N} = 1$ SUSY are now under a constraint

$$V(x, y) = V(x, -y), \qquad \Phi(x, y) = -\Phi(x, -y). \tag{4}$$

Then, we have only an $\mathrm{SU}(3)_C$ vector multiplet of the $\mathcal{N} = 1$ SUSY without the chiral multiplet Φ after the Kaluza–Klein reduction. In order to define the theory consistently, the action in $M_4 \times S^1$ should be invariant under the Z_2 transformation. The invariance is achieved as long as (i) the coefficient of the quadratic term $1/g^2$ is even under the Z_2 transformation, and (ii) that of the cubic term h is odd. We take $1/g^2$ to be y-independent and $h(y)$ as

$$h(y) = c \frac{y}{|y|}, \tag{5}$$

where c is a constant that plays the same role as the corresponding one in the non-SUSY models [2] (see below).[1]

2.2. Boundary extra quarks

There are two fixed points in the S^1/Z_2 orbifold: $y = 0$ and $y = l$. Let us put $\mathrm{SU}(3)_C$-charged chiral multiplets as extra quarks on the fixed-point boundaries: chiral multiplets Q in the $\mathrm{SU}(3)_C$-**3** representation at $y = 0$ and the same number of chiral multiplets $\bar{Q}$ in the $\mathrm{SU}(3)_C$-**3*** representation at $y = l$. The action of the extra-quark multiplets contains

$$\int_{y=0} d^4x\, d^2\theta\, d^2\bar{\theta}\; Q^* e^V Q + \int_{y=l} d^4x\, d^2\theta\, d^2\bar{\theta}\; \bar{Q} e^{-V} \bar{Q}^*. \tag{6}$$

[1] In Appendix A, the parameter $h(y)$ is interpreted as a vacuum expectation value of a background field, where a possible origin of the kink configuration is also discussed.

These chiral multiplets by themselves give rise to a triangle anomaly of $\mathrm{SU}(3)_C$ at each fixed point. However, the anomaly at $y = 0$ is the same as that at $y = l$ with the opposite sign. Thus, the anomaly can be canceled through its flow between $y = 0$ and $y = l$ implemented by the Chern–Simons interaction (3), provided the constant c is adequately chosen.[2]

When the extra dimension is sufficiently large, interactions involving both Q and $\bar{Q}$ are highly suppressed. Let the cutoff scale (such as the grand unification or Planck scale) of the model be given by M. Then the effects of particles with masses of order M may generically induce such terms as

$$e^{-Ml} M Q\bar{Q} \tag{7}$$

in the effective superpotential. We assume that $Ml \gtrsim 10^2$ to suppress the effects of such terms. Then there is an accidental axial symmetry

$$Q \to e^{i\xi} Q, \qquad \bar{Q} \to e^{i\xi} \bar{Q}, \tag{8}$$

which is to be identified with the Peccei–Quinn symmetry. Although interactions comprised of only Q's or $\bar{Q}$'s are not expected to be suppressed by e^{-Ml}, they are higher-dimensional operators, and are irrelevant to the axion potential, as we explicitly see in the next section.

The internal dimension is moderately large: we take $Ml \simeq 10^2$–10^4. It follows that $M \simeq (10^{-1}$–$10^{-2})M_{\mathrm{pl}}$, and $l^{-1} \simeq (10^{-3}$–$10^{-6})M_{\mathrm{pl}}$, where $M_{\mathrm{pl}} \simeq 2.4 \times 10^{18}$ denotes the Planck scale. The setup described in this section reduces to a four-dimensional $\mathcal{N} = 1$ $\mathrm{SU}(3)_C$ gauge theory with an accidental axial symmetry below the Kaluza–Klein scale.

3. The model

The superpotential virtually contains no terms involving both Q and $\bar{Q}$, and hence the extra quarks are not forced to develop a chiral condensation. Thus, another vector multiplet is introduced in the five-dimensional bulk, so that the chiral condensation is formed dynamically, and that a composite axion is obtained through the spontaneous chiral symmetry breaking.

For definiteness, let us introduce an $\mathrm{SU}(5)_H$ gauge theory[3] in addition to the usual color $\mathrm{SU}(3)_C$. The extra quarks[4] at $y = 0$ consists of chiral superfields $Q = ({Q^i}_\alpha, {Q^4}_\alpha)$ in the $(\mathbf{3} + \mathbf{1}) \times \mathbf{5}^*$ representation under $\mathrm{SU}(3)_C \times \mathrm{SU}(5)_H$, and their conjugates $\bar{Q} = ({\bar{Q}_i}^{\alpha}, {\bar{Q}_4}^{\alpha})$ are at $y = l$,[5] where $i = 1, 2, 3$ and $\alpha = 1, \ldots, 5$.

The $\mathrm{SU}(5)_H$ gauge theory has four flavors of extra quarks. Thus, an effective superpotential is generated due to the $\mathrm{SU}(5)_H$ interaction [6]:

$$W_{\mathrm{eff}} = \frac{\Lambda_H^{11}}{\det Q\bar{Q}}, \tag{9}$$

where Λ_H denotes the dynamical scale of the $\mathrm{SU}(5)_H$ gauge theory. The run-away potential from Eq. (9) is stabilized by supersymmetry-breaking effects such as $V = m^2|Q|^2$ in the scalar potential. We assume gravity-mediated supersymmetry breaking in this article for simplicity. The Peccei–Quinn scale F_{PQ} is of the order of

$$\sqrt{\langle Q\bar{Q}\rangle} \sim \left(\frac{\Lambda_H^{11}}{m}\right)^{1/10}. \tag{10}$$

The spectrum below the Peccei–Quinn scale F_{PQ} consists of chiral multiplets in the $\mathrm{SU}(3)_C$-($\mathbf{adj}. + \mathbf{3} + \mathbf{3}^*$) representations and two singlets, in addition to the particle contents of the MSSM. All the fermions and real scalars in the extra multiplets acquire masses of the order of the supersymmetry-breaking scale m. Pseudo-scalars in the $\mathrm{SU}(3)_C$-charged chiral multiplets receive radiative corrections at the one-loop level, and have masses at least of the order of $\sqrt{\alpha_{\mathrm{QCD}}}\, m$. They are pseudo-Nambu–Goldstone bosons and cannot remain exactly massless, because the $\mathrm{SU}(3)_C$ interaction explicitly breaks the corresponding chiral global symmetry. One of the singlet pseudo-scalars also has a mass of the order of m due to the mixed anomaly with $\mathrm{SU}(5)_H$ (and an explicit breaking of an R symmetry). Only one pseudo-scalar field remains massless below the supersymmetry-breaking scale, and that field plays the role of the QCD axion.

[2] See Ref. [2] for numerical details.

[3] This corresponds to $\mathrm{SU}(3)_H$ in Ref. [2].

[4] The usual quark and lepton chiral superfields are also assumed to be localized at $y = 0$.

[5] The color singlets ${Q^4}_\alpha$ and ${\bar{Q}_4}^{\alpha}$ are introduced so that the QCD axion is obtained just in the same way as in Ref. [2].

Although there exists an accidental Peccei–Quinn symmetry as advocated above, higher-dimensional operators such as

$$\int_{y=0} d^4x\, d^2\theta\, d^2\bar{\theta}\, \frac{z_0}{M^4}(Q)^3 Q D^2 Q + \int_{y=l} d^4x\, d^2\theta\, d^2\bar{\theta}\, \frac{z_l}{M^4}(\bar{Q})^3 \bar{Q} D^2 \bar{Q} \tag{11}$$

break it explicitly, where z_0 and z_l are dimensionless coupling constants of order one, D denotes the covariant superderivative, and the implicit gauge indices are contracted so that the terms are gauge-invariant.[6]

Let us make a conservative estimate of the QCD axion effective potential [7] induced by the explicit breaking operators localized at the fixed points. On dimensional grounds, the dominant contributions to the axion potential turn out to be of order

$$\frac{m^3}{F_{\rm PQ}^{20}} \Lambda_H^{11} F_{\rm PQ}^8 \frac{z_0^*}{M^4} \frac{z_l^*}{M^4} F_{\rm PQ}^{10} \sim m^4 \left(\frac{F_{\rm PQ}}{M}\right)^8, \tag{12}$$

where the spurion charges of Λ_H^{11} for the selection rules are apparent from Eq. (9) and the supersymmetry-breaking scale factor m^3 originates from the superderivatives and superspace integrals. These corrections should not be too large to make the Peccei–Quinn mechanism ineffective:

$$m^4 \left(\frac{F_{\rm PQ}}{M}\right)^8 = \left[10^{-24}\ {\rm GeV}^4\right] \times \left(\frac{m}{10^4\ {\rm GeV}}\right)^4 \left(\frac{F_{\rm PQ}}{M} \times 10^5\right)^8 \lesssim 10^{-10} \frac{m_u m_d}{m_u + m_d} \Lambda_{\chi {\rm SB}}^3, \tag{13}$$

where m_u and m_d denote the masses of the up and down quarks, and $\Lambda_{\chi{\rm SB}}$ is the energy scale of the QCD chiral symmetry breaking.

[6] The operators in (11) are not necessarily gauge-invariant and are absent, when the extra quarks have non-trivial $\mathrm{U}(1)_Y$ charges. The extra potential of the axion due to the explicit-breaking operators can be suppressed more if it is the case.

4. Cosmological issue

Owing to the dynamical condensation $\langle Q\bar{Q}\rangle \sim F_{\rm PQ}^2$, there exist colored particles in the pseudo-Nambu–Goldstone multiplets in the low-energy spectrum, as mentioned above. These particles, if they were to live too long, would constitute dark matter and lead to a cosmological difficulty [8].

We allow[7] such gauge-invariant terms as

$$\int_{y=0} d^4x\, d^2\theta\, d^2\bar{\theta}\, \frac{z}{M} Q^* Q \bar{d}, \tag{14}$$

where z is a dimensionless coupling constant of order one, $\bar{d}$ denotes the down-quark chiral superfield at $y = 0$ (see footnote 4), and the implicit gauge indices are contracted. In the presence of this operator, all the colored particles have sufficiently short lifetime.

We note that the presence of the above terms does not alter the conclusion in the previous section. In fact, the operator (14) does not contribute to the extra potential of the axion, as seen in the same analysis as that at the end of Section 3.

5. Flow of gauge coupling constants

The colored extra multiplets in the low-energy spectrum has impacts not only in cosmology but also in the renormalization-group flow of gauge coupling constants. In particular, the $\mathrm{SU}(3)_C$ coupling becomes asymptotically non-free, and the model described in Section 3 no longer serves as a good description at very short distance scale, since the QCD interaction becomes non-perturbative. A typical renormalization-group flow of the gauge-coupling constants is shown in Fig. 1.[8] It shows that the Kaluza–Klein scale can be as high as 10^{15} GeV, and the cutoff scale as high as 10^{16} GeV, for $m \simeq 10^{3.5}$ GeV. Thus, some of the extra QCD-charged particles can be within the reach of LHC, while sufficient suppression $F_{\rm PQ}/M \lesssim 10^{-5}$ is obtained in Eq. (13), even if the terms Eq. (11) are not forbidden by the $\mathrm{U}(1)_Y$ charge.

[7] The hypercharge should be assigned to the extra-quarks appropriately. We do not go into the arguments on the gravitational anomaly cancellation in this Letter.

[8] The running of the QCD coupling in the high-energy regime is insensitive to the Peccei–Quinn scale at the one-loop level.

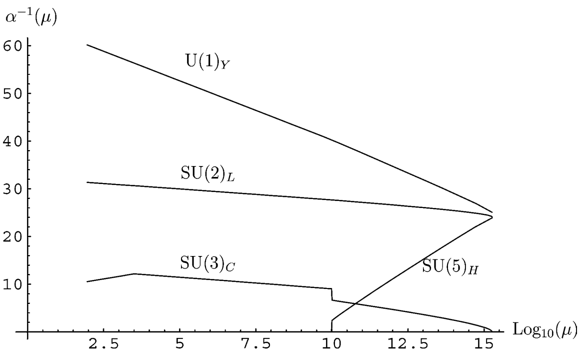

Fig. 1. Renormalization-group flow of the gauge coupling constants of the model. The gauge-coupling constants of the MSSM at 91 GeV contain threshold corrections from a typical spectrum of the MSSM particles. The averaged mass of the extra particles, some of which are charged under $\mathrm{SU}(3)_C$ and $\mathrm{U}(1)_Y$, are set to be $10^{3.5}$ GeV. The Peccei–Quinn scale F_{PQ} is set to 10^{10} GeV. The renormalization group is based on a $\mathrm{U}(1)_Y$ charge assignment under which ${Q^i}_\alpha$ and ${\bar{Q}_i}^\alpha$ are neutral, for definiteness. Two-loop effects have been taken into account.

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Appendix A. Kink configuration and supersymmetry

It is shown in this appendix that the kink configuration of the coefficient of the Chern–Simons interaction in Eqs. (3) and (5) can be understood as a VEV of a background field. The origin of the kink solution is also explained.

In six-dimensional spacetime, the interaction

$$S_{\mathrm{CS}} = -\int G^{(1)} \wedge I^{(5)}, \tag{A.1}$$

is consistent with the minimal SUSY, where $G^{(1)}$ is 1-form field strength of a scalar field $C^{(0)}$ in a linear hypermultiplet, and $I^{(5)}$ a 5-form that satisfies

$$\mathrm{tr}\, F^3 = dI^{(5)}; \tag{A.2}$$

F is the field strength of a Yang–Mills field. This interaction is known in the context of string theories as a part of the Wess–Zumino interaction

$$\int C^{(p+1)} - G \wedge I_{(0)} \tag{A.3}$$

on Dp-brane world volumes, where $C^{(p+1)}$ is the Ramond–Ramond $(p+1)$-form potential, G a collection of Ramond–Ramond field strengths, and $I_{(0)}$ a differential form that is related with the anomaly polynomial I through $I = dI_{(0)}$ [9].

When there is a magnetic source of the field $C^{(0)}$, its Bianchi identity is given by

$$dG^{(1)} = \sum_i N_i \delta^{(2)}(\mathbf{y} - \mathbf{y}_i), \tag{A.4}$$

where $\delta^{(2)}$ denotes a 2-form supported only on a point $\mathbf{y}_i$ (of a magnetic source), and N_i the magnetic charge located there.

The interaction (A.1) implements the inflow of anomaly: let us introduce a 4-form $I^{(4)}$ which satisfies

$$\delta_\epsilon I^{(5)} = dI^{(4)}, \tag{A.5}$$

where δ_ϵ denotes the gauge variation. Then the variation of the action (A.1) is given by

$$\delta_\epsilon S_{\mathrm{CS}} = -\int G^{(1)} \wedge dI^{(4)} = \sum_i N_i \int_{\mathbf{y}=\mathbf{y}_i} I^{(4)}, \tag{A.6}$$

and hence the triangle anomaly flows into a singularity by the amount proportional to the charge localized there.

Let us assume that the internal space of the two extra dimensions is T^2/Z_2. One can consider a torus which is long in one direction, and short in the other. Then, one has an effective description in five dimensions, by performing Kaluza–Klein reduction in the short direction. This five-dimensional description is what we need in the main text.

Let us suppose that $\mathbf{y} = (y, z) = (0, 0)$ singularity has a unit magnetic charge of $C^{(0)}$, and the $(y, z) = (l, 0)$ singularity has the opposite charge. Then, the 1-form $G^{(1)} = dC^{(0)}$ has a positive rotation and a negative one, respectively, around those singularities. The Hodge dual of the 1-form $G^{(1)}$ is given by a

5-form

$$*\left(G^{(1)}\right) = \tilde{G} \wedge \epsilon_{\mu\nu\kappa\lambda}\, dx^{\mu}\, dx^{\nu}\, dx^{\kappa}\, dx^{\lambda}, \tag{A.7}$$

with $\tilde{G}$ a 1-form on T^2/Z_2 that has a positive divergence and a negative one at the singularities. Then, the interaction (A.1) is rewritten as

$$S_{\mathrm{CS}} = -\int G^{(1)} \wedge I^{(5)} = -\int d^6x \left\langle *\left(G^{(1)}\right), I^{(5)} \right\rangle$$
$$\supset -\int d^5x \left(I^{(5)}\right)^{\mu\nu\kappa\lambda 4} \epsilon_{\mu\nu\kappa\lambda} \tilde{G}_4. \tag{A.8}$$

In the limit where the second extra dimension is small, $\tilde{G}_4$ is constant along $y \in (-l, 0)$ as well as along $y \in (0, l)$. Thus, the kink configuration $h(y)$ is given by $\tilde{G}_4$.

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