

Available online at www.sciencedirect.com**ScienceDirect**

Nuclear Physics B 878 [PM] (2014) 269–294

www.elsevier.com/locate/nuclphysb

F-term stabilization of odd axions in LARGE volume scenario

Xin Gao^{a,b}, Pramod Shukla^{a,*}^a *Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), Föhringer Ring 6, 80805 München, Germany*^b *State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, P.O. Box 2735, Beijing 100190, China*

Received 5 October 2013; accepted 25 November 2013

Available online 28 November 2013

Abstract

In the context of the LARGE volume scenario, stabilization of axionic moduli is revisited. This includes both even and odd axions with their scalar potential being generated by F-term contributions via various tree-level and non-perturbative effects like fluxed E3-brane instantons and fluxed poly-instantons. In all the cases, we estimate the decay constants and masses of the axions involved.

© 2013 The Authors. Published by Elsevier B.V. Open access under [CC BY license](http://creativecommons.org/licenses/by/4.0/).

1. Introduction

From the point of view of constructing (semi-)realistic models in string compactifications, the understanding of moduli stabilization is a very central issue and has been under deep investigation for more than a decade. In order to quantitatively address certain aspects of cosmology and of particle physics, moduli stabilization is a prerequisite, as on the one hand some physical parameters depend on the value of the moduli and on the other hand the existence of such massless scalars is incompatible with observations.

The standard paradigm for string moduli stabilization is described (and better understood) in the framework of Type IIB orientifolds with O7- and O3-planes, and two popular classes of models, namely KKLT [1] and LARGE volume scenarios [2], have been in the market for almost

* Corresponding author.

E-mail addresses: gaoxin@mppmu.mpg.de (X. Gao), shukla@mppmu.mpg.de (P. Shukla).

a decade. In these models, a combination of background three-form fluxes and D3-brane instantons can lead to a potential for the axion–dilaton, the complex structure and the Kähler moduli [3–6]. Usually, the moduli stabilization scheme in these two classes of models is a two-step procedure. In first step, one stabilizes complex structure moduli along with axion–dilaton at the leading order via a tree-level Kähler potential and a perturbative flux-contribution to the superpotential. The Kähler moduli which remains flat (due to a so-called ‘no-scale structure’) at this stage are lifted in a second step by including the subleading non-perturbative corrections to the superpotential W , and the same results in a supersymmetric (KKLT) AdS-minimum [1]. Taking also the leading order perturbative α' -corrections to the Kähler potential into account, a non-supersymmetric AdS-minimum at large overall volume appears. This is the so-called LARGE volume scenario (LVS) [2], which has been exploited in the literature in the context of getting realistic particle physics and realizing inflationary cosmology both (see [7,8] and references therein). Recently, the orientifold even axionic sector was scrutinized [9] leading to the proposal that in the context of the LVS there exists a whole axiverse, which means that the decay constants of the different axions vary over a wide range of values. This is mainly owed to the fact that different axions get different volume suppression factors in their kinetic terms. Further, most of these studies are only focused on the orientifold even sector of axions. A detailed analysis in these directions with orientifold odd axionic sector is missing. However, in some models, moduli stabilization [10], inflationary aspects [11–13] as well as particle pheno model building [14] with the inclusion of involutively odd (1, 1)-cohomology sector have been initiated in the meantime.

These orientifold odd axions also play a crucial role in global model building in string compactification. Here we have in mind the string constraints governing the coexistence of D7-branes, fluxes and instantons, like the chirality problem pointed out in [15] or the Freed–Witten anomalies [16]. The chirality issue comes with the appearance of extra zero modes located at the intersection between the instanton E3-brane and D7-brane supporting the visible sector. This prevents a class of instantons from participating in moduli stabilization. Several approaches have been proposed in building up models which (could) avoid such a problem. One way is, not to support the visible sector on the divisor which gives rise to the non-perturbative superpotential contribution [17–20]. Models which support visible sector with D-branes at singularities have been proposed in [19,20] in which one needs to embed such singularities in a compact Calabi–Yau threefold X with non-zero odd components in cohomology class $H_-^{1,1}(X/\sigma)$. Another way to avoid the chirality issue is to include the gauge flux on the instanton E3-brane supported by the orientifold odd two-cycle [21].

For having the fluxed-instanton contribution, one needs the involutively odd-moduli (b^a, c^a) which arise from the NS–NS field B_2 and R–R field C_2 in Type IIB orientifolds to correct the E3-brane superpotential and remove the extra charged zero modes. These odd axions combine in pure axionic chiral multiplets for which the entire complex boson is made from axions. These new chiral multiplets G^a appear in the effective action, i.e. in the Kähler- and superpotential in a completely different manner than the even moduli so that they must be treated separately. In addition, we also include those axions sitting in the same chiral multiplet as the saxions governing the size of four-cycles having the right topology to support so-called poly-instantons. These are subleading non-perturbative contributions which can be briefly described as instanton corrections to instanton actions. These were introduced in [22], further elaborated on in [23] and have been analyzed recently in the context of the LVS in [24–32]. In order to support the odd moduli in models of (Type IIB) string compactification, a classification of the involutions on the toric Calabi–Yau

threefolds with $h^{1,1} \leq 4$ which may result non-zero odd component of $(1, 1)$ -cohomology class is also studied in [33].

In general, axions play an important role in physics beyond the Standard Model and, via axion driven inflation, might provide a bridge between particle physics with cosmology. On the particle physics side, axions were introduced to solve the strong CP problem of QCD [34–39]. However, due to the shift symmetry of the axions, a non-trivial subleading potential is generated at the non-perturbative level, making them ideal inflaton candidates as well [37,38,40–43].¹ Since in most of the axionic model building purposes (e.g. in axionic inflation), the axionic decay constants are required to be very high, a treatment in a UV-complete theoretical framework such as string theory is desirable and has indeed been pursued. axionic fields are already ubiquitous in superstring theories in ten-dimensions and via compactification generically lead to the order of $10\text{--}10^2$ axionic fields in four dimensions [46,47]. These axions often appear in a chiral supermultiplet, where they are combined with a scalar field, a so-called saxion, which describes the deformation of the underlying compact geometry. These geometric moduli can for instance be the Kähler or complex structure moduli of Calabi–Yau manifolds. For a string model to be realistic, all moduli have to be stabilized as they lead to unobserved fifth force and interfere with the standard big bang cosmology, in particular big bang nucleosynthesis [46–48].

In this article, our main focus is to revisit the F -term moduli stabilization in LARGE volume scenarios. The idea is to include the involutively odd axions and instanton flux effects to generate F -term contributions depending on odd axions. Addressing more involved issues, for example, the ones mentioned in the aforementioned paragraph need more concreteness in the setup, and are beyond the scope of this article. To be specific, we limit ourselves to a toy-model setup, and without supporting a concrete MSSM-like visible sector on D7-brane wrapping a holomorphic divisor, we assume that concrete setups with desired divisor intersections and allowed fluxes consistent with tadpole/anomaly cancellations can be constructed. Apart from moduli stabilization, as a by-product of our investigations regarding the simple estimates of volume scalings in axion decay constants and masses, we explore the possible mass hierarchy among the various moduli.

The article is organized as follows. In Section 2, we start with a brief review of a generic Type IIB orientifold framework, and following [21], we collect the relevant ingredients on fluxed D3-brane instanton contributions to the superpotential. In Section 3, we discuss the moduli stabilization in an extended LARGE volume setup with the inclusion of a single involutively odd axion and fluxed-instanton effects. In Section 4, we extend the analysis for a fluxed poly-instanton setup. For all the cases, we also present some estimates for the decay constants and masses of various even/odd axions. Section 5 presents the overall conclusions followed by Appendices A–C of the relevant intermediate expressions.

2. Preliminaries

Let us review some of the basic ingredients in Type IIB orientifold compactifications with O7- and O3-planes. Here we focus on those aspects which will become relevant in our investigation of axions in the LARGE volume scenario.

¹ See [44] for applications of axions as quintessence fields, and [45] for an interesting field theoretic attempt of combining the three axionic scenarios (QCD axion, inflaton and quintessence axion) into a single framework.

Table 1
Orientifold invariant states.

	$(-)^{F_L}$	Ω_p	σ^*
ϕ	+	+	+
C_0	–	–	+
$g_{\mu\nu}$	+	+	+
B_2	+	–	–
C_2	–	+	–
C_4	–	–	+

2.1. Type IIB orientifolds

We consider Type IIB superstring theory compactified on an orientifold of a Calabi–Yau three-fold X . The admissible orientifold projections fall into two categories, which are distinguished by their action on the Kähler form J and the holomorphic three-form Ω_3 of the Calabi–Yau:

$$\mathcal{O} = \begin{cases} \Omega_p \sigma & \text{with } \sigma^*(J) = J, \sigma^*(\Omega_3) = \Omega_3, \\ (-)^{F_L} \Omega_p \sigma & \text{with } \sigma^*(J) = J, \sigma^*(\Omega_3) = -\Omega_3, \end{cases} \tag{1}$$

where Ω_p is the world-sheet parity transformation and F_L denotes the left-moving space–time fermion number. Moreover, σ is a holomorphic, isometric involution. The first choice leads to orientifold O9- and O5-planes whereas the second choice to O7- and O3-planes. The generated R–R tadpoles need to be canceled by the introduction of the corresponding D-branes. For latter case, the one of primary interest here, these are in general D7-branes carrying addition gauge flux and D3-branes. The $(-)^{F_L} \Omega_p \sigma$ invariant states in four-dimensions are listed in Table 1.

Therefore, the massless states are in one-to-one correspondence with harmonic forms which are either even or odd under the action of σ . These do generate the equivariant cohomology groups $H_{\pm}^{p,q}(X)$. Therefore, the Kähler form J , the two-forms B_2, C_2 and the R–R four-form C_4 can be expanded as

$$\begin{aligned} J &= t^\alpha \omega_\alpha, \\ C_2 &= c^a \omega_a, \quad B_2 = b^a \omega_a, \\ C_4 &= D_2^\alpha \wedge \omega_\alpha + V^{\tilde{\alpha}} \wedge \alpha_{\tilde{\alpha}} + U_{\tilde{\alpha}} \wedge \beta^{\tilde{\alpha}} + \rho_\alpha \tilde{\omega}^\alpha, \end{aligned} \tag{2}$$

where ω_α and ω_a denote a bases for $H_+^{1,1}(X)$ and $H_-^{1,1}(X)$, respectively. Similarly, $\tilde{\omega}^\alpha$ and $\tilde{\omega}^a$ is a basis of $H_+^{2,2}(X)$ and $H_-^{2,2}(X)$, while $(\alpha_{\tilde{\alpha}}, \beta^{\tilde{\alpha}})$ is a real symplectic basis of $H_+^3(X)$.

Since σ^* reflects the holomorphic three-form, in the orientifold one keeps $h_-^{2,1}(X)$ complex structure moduli $z^{\tilde{a}}$, which are complex scalars. Moreover, b^a, c^a and ρ^α are also scalars, while $V^{\tilde{\alpha}}$ and $U_{\tilde{\alpha}}$ are space–time one-forms and D_2^α a space–time two-form. Due to the self-duality of the R–R four-form, half of the degrees of freedom of C_4 are removed. Note that the even component of the Kalb–Ramond field $B_+ = b^\alpha \omega_\alpha$, though not a continuous modulus, can take the two discrete values $b^\alpha \in \{0, 1/2\}$. The resulting $\mathcal{N} = 1$ supersymmetric massless bosonic spectrum is summarized in Table 2.

By performing the detailed dimensional reduction from ten to four dimensions [6], one realizes that the complex bosons in the chiral superfields are given by the combinations

Table 2
 $\mathcal{N} = 1$ massless bosonic spectrum of Type IIB Calabi–Yau orientifold.

Chiral multiplets	$h_-^{2,1}$	$z^{\tilde{a}}$
	$h_+^{1,1}$	(t^α, ρ_α)
	$h_-^{1,1}$	(b^a, c^a)
	1	(ϕ, C_0)
Vector multiplet	$h_+^{2,1}$	$V^{\tilde{a}}$
Gravity multiplet	1	$g_{\mu\nu}$

$$\begin{aligned}
 S &= iC_0 + e^{-\phi}, \\
 G^a &= ic^a - Sb^a, \\
 T_\alpha &= \frac{1}{2}\kappa_{\alpha\beta\gamma}t^\beta t^\gamma + i\left(\rho_\alpha - \frac{1}{2}\kappa_{\alpha ab}c^a b^b\right) - \frac{1}{4}e^\phi\kappa_{\alpha ab}\bar{G}^a(G + \bar{G})^b.
 \end{aligned}
 \tag{3}$$

The low energy effective action at second order in derivatives is given by a supergravity theory, whose dynamics is encoded in a Kähler potential K , a holomorphic superpotential W and the holomorphic gauge kinetic functions. In our case, the Kähler potential can be expanded as

$$K = -\ln(S + \bar{S}) - \ln\left(-i \int_X \Omega_3 \wedge \bar{\Omega}_3\right) - 2\ln(\mathcal{Y}(S, G^a, T_\alpha, \dots))
 \tag{4}$$

where $\mathcal{Y} = \frac{1}{6}\mathcal{K}_{ABC}t^A t^B t^C$ is the volume of the Calabi–Yau manifold expressed in terms of two-cycle volumes t^A . The dots in (4) denote the potential appearance of other moduli like D3/D7-brane fluctuations (and hence complex structure moduli which get coupled after including brane-fluctuations) or Wilson line moduli. Unfortunately, \mathcal{Y} is only implicitly given in terms of the chiral superfields. It is in general non-trivial to invert the last relation in (3).

As we will review in more detail below, the general schematic form of the superpotential W is given as

$$\begin{aligned}
 W &= \int_X G_3 \wedge \Omega + \sum_E \mathcal{A}_E(z^{\tilde{a}}, G^a, \mathcal{F}_E, \dots)e^{-\pi a_E^g T_\alpha} \\
 &= W_0 + W_{np}
 \end{aligned}
 \tag{5}$$

where the first term is the Gukov–Vafa–Witten (GVW) three-form flux induced tree-level superpotential [3] (see [49,50] also for related work). The second term denotes a sum over non-perturbative corrections coming from Euclidean D3-brane instantons or gaugino condensation on D7-branes [51]. Here, the pre-factor does not only contain the one-loop Pfaffian for fluctuations around the instanton background but also contributions from so-called (gauge-)fluxed instantons and Euclidean D1-brane instantons. Again the dots indicate a further dependence on e.g. D3/D7-brane fluctuations or Wilson line moduli. From the Kähler- and the superpotential one can compute the $\mathcal{N} = 1$ scalar potential

$$V = e^K \left(\sum_{I,J} K^{I\bar{J}} \mathcal{D}_I W \bar{\mathcal{D}}_{\bar{J}} \bar{W} - 3|W|^2 \right)
 \tag{6}$$

where the sum runs over all moduli.

2.2. Fluxed D3-brane instantons

Let us provide some more relevant information about the fluxed D3-brane instanton contributions to the superpotential. Here we essentially follow [21,52].²

For a single Euclidean D3-brane instanton to contribute to the superpotential it needs to carry the right zero-mode structure. In particular, the instanton has to be a so-called $O(1)$ instanton, which means that it has to wrap an orientifold invariant four-cycle, i.e. $\sigma(D_E) = D_E$. In addition one has the freedom to turn on a gauge flux $\tilde{\mathcal{F}}_E = 2\pi\alpha' \mathcal{F}_E - \iota^* B$ on the brane, where $\iota : D_E \rightarrow X$ denotes the inclusion map of the four-cycle into the Calabi–Yau threefold. Since the gauge flux is anti-invariant under the world-sheet parity transformation Ω_p , the instanton remains to be $O(1)$ only if the gauge flux is supported on a σ -odd two-cycle. Therefore, the gauge flux is supported on two-cycles in $H_-^{1,1}(D_E)$ which can be expanded as

$$2\pi\alpha' \mathcal{F}_E = \mathcal{F}_E^a \iota^* \omega_a + \mathcal{F}_E^v \tag{7}$$

where $\iota^* \omega_a$ denotes a σ -odd basis of harmonic two-forms lying in the image of the pullback ι^* . The second component \mathcal{F}_E^v is given by fluxes supported on two-cycles inside the divisor D_E , which are trivial in the bulk, i.e. they are in the co-kernel of ι^* .

Such a family of fluxed D3-brane instantons, all wrapping the divisor D_E , contributes to the superpotential as

$$W_{np} \sim \sum_{\mathcal{F}_E} e^{-S_E}. \tag{8}$$

Dimensionally reducing the corresponding DBI and CS actions, one finds

$$S_E = \pi (a_E^\alpha (T_\alpha + \Delta_{E\alpha}) + \Delta_E^v) \tag{9}$$

with

$$\begin{aligned} \Delta_{E\alpha} &= \kappa_{abc} G^b \mathcal{F}_E^c + \frac{S}{2} (\kappa_{abc} \mathcal{F}_E^b \mathcal{F}_E^c), \\ \Delta_E^v &= S \int_{D_E} \mathcal{F}_E^v \wedge \mathcal{F}_E^v \end{aligned} \tag{10}$$

where $a_E^\alpha = \int_{D_E} \tilde{w}^\alpha$. Collecting the various terms, the superpotential can be written as

$$W_{np} = \sum_{\mathcal{F}_E} \mathcal{A}_E(\mathcal{F}_E) e^{-\pi a_E^\alpha T_\alpha - \tilde{q}_{Ea} G^a} \tag{11}$$

where

$$\begin{aligned} \mathcal{A}_E(\mathcal{F}_E) &= A \exp\left(-\frac{S}{2} \left[\pi a_E^\alpha \kappa_{abc} \mathcal{F}_E^b \mathcal{F}_E^c + 2\pi \int_{D_E} \mathcal{F}_E^v \wedge \mathcal{F}_E^v \right]\right), \\ \tilde{q}_{Ea} &= \pi \kappa_{\alpha ab} a_E^\alpha \mathcal{F}_E^b. \end{aligned} \tag{12}$$

² For more on instanton-corrections to the superpotential, see [53]. For recent progress towards the possibility of new-instanton corrections; see [54] for rigidifying the deformation zero modes of a divisor wrapped by E3-instanton, and [55] for avoiding the strong constraints from Freed–Witten anomaly [16] relevant for a non-spin divisor wrapping an E3-instanton.

Here A denotes the one-loop determinant for fluctuations around the instanton, which only depends on the complex structure moduli and the D7-brane deformations. Thus, it can be assumed to be a constant for our analysis.

Following the discussion in [52], one now introduces a basis of two-forms of $H_{-}^{1,1}(D_E)$ satisfying $\int_{D_E} \omega_M \wedge \omega_N = 2\delta_{MN}$ with the index $M = \{m, \hat{m}\}$. The extra factor of two is due to the orientifold projection (see [52]). The two-cycles ω_m are related to the pull-back two-cycles in $\iota^*H_{-}^{1,1}(X)$ as $\omega_a = M_a^m \omega_m$ where the matrix M_a^m satisfies

$$a_E^\alpha \kappa_{\alpha bc} = \int_{D_E} \omega_b \wedge \omega_c = 2M_a^m M_b^n \delta_{mn}. \tag{13}$$

The $\omega_{\hat{m}}$ denote a basis of the orthogonal complement of $\iota^*H_{-}^{1,1}(X)$ in $H_{-}^{1,1}(D_E)$.

Now, one can expand the instanton fluxes as $\mathcal{F}_E = \sum_M f^M \omega_M$ with $f^M \in \mathbb{Z}$ so that the entire instanton generated superpotential can be written as

$$W_{np} = A \sum_E e^{-\pi a_E^\alpha T_\alpha} \sum_{f^M \in \mathbb{Z}} \exp(-\pi S f^M f^N \delta_{MN} - 2\pi G^m f^n \delta_{mn}). \tag{14}$$

Due to the diagonal form in the exponential part of Eq. (14), it can be further simplified as:

$$W_{np} = A_s \sum_E e^{-\pi a_E^\alpha T_\alpha} \sum_{f^m \in \mathbb{Z}} e^{-\pi S f^{m^2} - 2\pi f^m G^m} \tag{15}$$

where $A_s = A \sum_{f^{\hat{m}} \in \mathbb{Z}} e^{-\pi S f^{\hat{m}^2}}$ is just a $\mathcal{O}(1)$ constant. This is the form of the superpotential to be heavily utilized in the subsequent analysis of axion moduli stabilization. There we will assume that the zero mode structure of such an instanton is just right to guarantee a contribution to W .

In general, for stabilizing the odd moduli in a realistic setup which concretely supports a MSSM-like visible sector, one should also examine for the possible D-term potential coming from the D7-brane fluxes \mathcal{F}_A turned-on on a stack of D7-branes along the holomorphic divisor D_A and its orientifold image $D_{A'}$. Such a D-term reads as

$$\begin{aligned} \mathcal{D}_A &= \frac{l_s^2}{2\pi\mathcal{V}} \int_{D_A} J \wedge (\iota^* B_2 - 2\pi\alpha' \mathcal{F}_A) \\ &= \frac{l_s^2}{4\pi\mathcal{V}} t^\alpha (\kappa'_{\alpha bc} (b^b - \mathcal{F}_A^b) C_A^c - \kappa_{\alpha\beta\gamma} \tilde{\mathcal{F}}_A^\beta C_A^\gamma) \end{aligned} \tag{16}$$

where $C_A^{\alpha,a} = N_A \int_{D_A^\pm} \tilde{\omega}^{\alpha,a}$ ($D_A^\pm = D_A \cup (\pm D_{A'})$) are the wrapping number along the basis of $H_4^\pm(X)$ and $\kappa_{ABC} := \int_{CY_3} \omega_A \wedge \omega_B \wedge \omega_C$ with $A = \{\alpha, a\}$ gives the intersection numbers for even/odd sectors. In general, several $\kappa_{\alpha bc}$ intersection numbers can be non-zero, then b^a moduli are stabilized at tree level by requiring D-flatness condition.³ However, this may not be always the case; for example, to generate the FI-term as given in (16), one requires a $U(1)$ group on the D7-brane configuration which may not be necessarily met. In case of a $U(1)$ gauge group

³ For example, one way to impose the D-flatness conditions is to set two-cycle volumes t^α appearing in (16) to zero. This leads to models of D-branes at singularity in [20] where only self-intersecting (shrinkable) del-Pezzo divisors have been exchanged under involution σ .

being present with certain brane configuration, it can also happen that the two-cycle ω_b intersects with the ω_c dual to D_A^- divisor trivially, i.e. $\kappa'_{abc} = 0$. It is worth to mention that $\kappa'_{abc} = 0$ does not mean that the intersection number between E3-instanton divisor and odd cycle has to vanish ($\kappa_{abc} \neq 0$) unless the D7-brane appearing in (16) wraps the same divisor. *Here, we stress that our main motivation is to investigate F-term moduli stabilization with the inclusion of instanton flux effects in a toy model, and therefore in the present work, we assume that concrete setups with desired divisor intersections and allowed fluxes can be constructed. Further, as our investigations are based on a phenomenologically oriented approach, we do not intend to explicitly address the issue of supporting a concrete MSSM-like visible sector.* Therefore, we can choose the configuration in which D7-brane does not wrap the instanton divisor and results in $\tilde{q}_{E_a} \neq 0$ while $\kappa'_{abc} = 0$.

3. Extended LARGE volume scenario

In this section, we discuss the moduli stabilization in an extended LARGE volume setup with the inclusion of involutively odd axions in the context of Type IIB orientifold compactification. Let us start with briefly reviewing the standard features of the minimal LARGE volume setup. We assume that all the complex structure moduli as well as axion–dilaton are supersymmetrically stabilized at the perturbative stage by background-flux superpotential W_0 via $\mathcal{D}_{c.s.}W_0 = 0 = \mathcal{D}_S W_0$. This remains justified with the inclusion of non-perturbative contributions as long as the overall volume of the Calabi–Yau space remains sufficiently large. For stabilizing the Kähler moduli, one starts with the following form of Kähler potential and superpotential,

$$K = -2 \ln \mathcal{V}, \quad W = W_0 + \sum_{s=2}^{h_+^{1,1}} A_s e^{-a_s T_s}, \tag{17}$$

where $\mathcal{V} = \mathcal{V}(T_\alpha) + C_{\alpha'}$ such that

$$\mathcal{V} = \xi_b (T_b + \bar{T}_b)^{\frac{3}{2}} - \sum_{s=2}^{h_+^{1,1}} \xi_s (T_s + \bar{T}_s)^{\frac{3}{2}} + C_{\alpha'}.$$

Here, we consider the ansatz of multi-hole swiss-cheese structure in the Calabi–Yau volume with a shift $C_{\alpha'}$ which denotes the perturbative α'^3 -correction given as $C_{\alpha'} = -\frac{\chi(\mathcal{M})(\tau - \bar{\tau})^{\frac{3}{2}} \zeta(3)}{4(2\pi)^3 (2i)^{\frac{3}{2}}}$. This α'^3 -correction breaks the no-scale structure,⁴ and the ansatz (17) results in the following form of F-term effective scalar potential

$$\begin{aligned} V^{\text{LVS}}(\mathcal{V}; \{\tau_s, \rho_s\}) &= \frac{3C_{\alpha'} |W_0|^2}{2\mathcal{V}^3} + \sum_{s=2}^{h_+^{1,1}} \frac{2\sqrt{2} a_s^2 A_s^2 e^{-2a_s \tau_s} \sqrt{\tau_s}}{3\xi_s \mathcal{V}} \\ &+ \sum_{s=2}^{h_+^{1,1}} \frac{4a_s A_s e^{-a_s \tau_s} \tau_s \cos[a_s \rho_s] W_0}{\mathcal{V}^2}. \end{aligned} \tag{18}$$

⁴ In the meantime, there have been proposals for string-loop corrections [56,57] as well as ‘new’ α' -corrections [58–60] (see also [61] for a related progress in $\mathcal{N} = 2$ F-theory compactifications). However, an ‘extended’ no-scale structure has been observed making the LARGE volume scenarios more robust. From a field theoretic approach, similar structure has been observed earlier for certain form of corrections to the Kähler potential [62].

This potential stabilizes the overall volume mode at exponentially large value in terms of stabilized value of the ‘small’ divisor volume $\bar{\mathcal{V}} \sim |W_0| \exp(a_s \bar{\tau}_s)$ where $\bar{\tau}_s \sim (C_{\alpha'})^{2/3}$. One realizes a non-supersymmetric AdS minimum which can be uplifted to a de-Sitter minimum via various uplifting mechanisms [1,63–67]. Further, the leading order contributions to the decay constants for all the axions can be estimated in large volume limit to be

$$f_{\rho_b} = \frac{\sqrt{6} \xi_b^{2/3}}{\mathcal{V}^{2/3}} \sim \mathcal{V}^{-2/3}, \quad \text{and} \quad f_{\rho_s} = \frac{\sqrt{3} \xi_s}{(2\tau_s)^{1/4} \mathcal{V}^{1/2}} \sim \mathcal{V}^{-1/2} \quad \forall s \in \{2, \dots, h_+^{11}\}. \quad (19)$$

After looking at the eigenvalues of the mass-squared matrix $M_{ij} \equiv \sum_k \frac{1}{2} (K^{-1})_{ik} V_{kj}$, one gets the following leading order contributions for moduli masses (evaluated at the minimum),

$$M_{\mathcal{V}} \sim \frac{\delta}{\mathcal{V}^{3/2}}, \quad M_{\tau_s} \sim \frac{\delta}{\mathcal{V}}, \quad M_{\rho_b} = 0, \quad M_{\rho_s} \sim \frac{\delta}{\mathcal{V}} \quad \forall s \in \{2, \dots, h_+^{11}\}, \quad (20)$$

where $\delta \sim \sqrt{\frac{g_s e^{K_{CS}} |W_0|^2}{8\pi}}$. It is important to note that the axionic direction ρ_b corresponding to the non-local (so-called ‘big’) divisor remains flat. Furthermore, one can lower the decay constant f_{ρ_b} naturally (in large volume limit) to get the correct order of magnitude for QCD axion, and so the ρ_b axion has appeared to be quite attractive for this purpose [68].⁵

Now, let us consider an extension of the simplest LARGE volume scenario with the inclusion of a single involutively odd modulus G^1 . In order to support the odd modulus, a non-zero component in (1, 1)-cohomology class on the Calabi–Yau threefold under some homomorphic involution σ is needed, i.e. $h_-^{1,1}(CY_3/\sigma) \neq 0$. In [33], we scanned through the toric Calabi–Yau threefolds with $h_-^{1,1}(CY_3) \leq 4$ and studied two kinds of involutions, namely divisor exchange involution and divisor reflection, which can result a non-trivial odd (1, 1)-cohomology. In the presence of a single odd modulus G^1 , the superpotential (15) including the non-perturbative fluxed-instanton contribution becomes

$$W = \int_X G_3 \wedge \Omega + A_s e^{-a_s T_s} \sum_{f^1 \in \mathbb{Z}} e^{-\pi S f^1{}^2 - 2\pi f^1 G^1} \quad (21)$$

where $A_s = A \sum_{f^{\tilde{m}} \in \mathbb{Z}} e^{-\pi S f^{\tilde{m}}{}^2}$. Again, we assume that all the complex structure moduli and axion–dilaton are stabilized by Gukov–Vafa–Witten superpotential perturbatively. For simplicity, we also assume that the background flux are tuned such that RR scalar is set to zero, $\bar{C}_0 = 0$. Subsequently, the non-perturbative term in the superpotential (21) can be written in terms of simplified elliptic theta function $\theta(G^1)$. The appearance of theta-function as a holomorphic pre-factor of a standard instanton correction (to the superpotential) has been also argued in [71]. The same was based on modular completion arguments assuming that a subgroup of $SL(2, \mathbb{Z})$ survives after orientifold truncation.

For a generalized LARGE volume setup, we proceed with the following ansatz for the Kähler potential and the superpotential

$$K = -2 \ln \mathcal{V} = -2 \ln(\xi_b \Sigma_b^{3/2} - \xi_s \Sigma_s^{3/2} + C_{\alpha'}),$$

$$W = W_0 + A_s \sqrt{g_s} e^{-a_s T_s} (e^{g_s \pi G^1} \theta_3[g_s \pi G^1, e^{-g_s \pi}]) \quad (22)$$

where

⁵ See [9,69,70] also for recent progress with more phenomenological approach.

$$\begin{aligned} \Sigma_b &= T_b + \bar{T}_b + \frac{\kappa_{b11}}{2(S + \bar{S})} (G^1 + \bar{G}^1)(G^1 + \bar{G}^1), \\ \Sigma_s &= T_s + \bar{T}_s + \frac{\kappa_{s11}}{2(S + \bar{S})} (G^1 + \bar{G}^1)(G^1 + \bar{G}^1). \end{aligned} \tag{23}$$

Depending on the possible intersections of various even/odd four-cycles, we consider two cases for stabilizing *all* the even/odd moduli using F-term contributions. These two cases are also of interest because of the different volume scaling in the leading order axion decay constants as we will see later.

3.1. Case I: $\kappa_{b11} \neq 0$

Let us assume that the so-called big divisor has non-zero intersections with the odd cycles, i.e. $\kappa_{b11} \neq 0$. Utilizing the ansatz (22)–(23) for the Kähler potential and superpotential, the leading order contributions to the F-term scalar potential can be collected in three types of terms as under

$$\begin{aligned} V(\mathcal{V}, \tau_s; \rho_s, b^1, c^1) &= V_{\alpha'} + V_{np1} + V_{np2} \quad \text{where} \\ V_{\alpha'} &= \frac{3C_{\alpha'}|W_0|^2}{2\mathcal{V}^3}, \\ V_{np1} &= \frac{2a_s A_s \tau_s \sqrt{g_s} W_0}{\mathcal{V}^2} \times \exp\left[\frac{a_s}{2g_s}(\kappa_{s11} b^1 b^1 - 2g_s \tau_s) + \frac{\pi}{g_s}(b^1 - i g_s c^1)^2\right] \\ &\quad \times \{\cos(a_s \rho_s)(\bar{\Theta}(b^1, c^1)e^{i4\pi a_s b^1 c^1} + \Theta(b^1, c^1)) \\ &\quad - i \sin(a_s \rho_s)(\bar{\Theta}(b^1, c^1)e^{i4\pi a_s b^1 c^1} - \Theta(b^1, c^1))\}, \\ V_{np2} &= \frac{2\sqrt{2}\tau_s g_s a_s^2 A_s^2}{3\xi_s} |\Theta(b^1, c^1)|^2 \\ &\quad \times \exp\left[\frac{a_s(\kappa_{s11} b^1 b^1 - 2g_s \tau_s)}{g_s} + \frac{2\pi}{g_s}(c^{12} g_s^2 - b^{12})\right] \end{aligned} \tag{24}$$

where $\Theta(b^1, c^1) = \theta_3[-b^1\pi + ic^1 g_s \pi, e^{-g_s \pi}]$. There are several extrema in the axionic directions due to periodicities appearing in the potential (24), and the generic extremization conditions are quite coupled. However, after utilizing one extremizing condition into another, the most simplest local extremum of the scalar potential (24) can be collectively described by intersection of the following hypersurfaces in moduli space:

$$\begin{aligned} a_s \rho_s &= N\pi, \quad b^1 = 0, \quad c^1 = 0, \quad C_{\alpha'} = \frac{32\sqrt{2} a_s \xi_s \tau_s^{\frac{5}{2}} (-1 + a_s \tau_s)}{(-1 + 4a_s \tau_s)^2}, \\ W_0 &= -\frac{a_s A_s e^{-a_s \tau_s} \mathcal{V}(-1 + 4a_s \tau_s) \sqrt{g_s} \Theta(0)}{6\sqrt{2} \xi_s \sqrt{\tau_s} (-1 + a_s \tau_s)} \end{aligned} \tag{25}$$

where $\Theta(0) = \theta_3[x, e^{-g_s \pi}]_{x=0}$. From Eq. (25), one finds that similar to the standard LARGE volume scenario, the τ_s stabilization condition can get decoupled from \mathcal{V} dependence and results in $\bar{\tau}_s \sim (C_{\alpha'})^{2/3}$. Subsequently, the overall volume \mathcal{V} gets stabilized at an exponential large value via $\bar{\mathcal{V}} \sim \exp(a_s \bar{\tau}_s)$. The scalar potential at this non-susy AdS minimum (25) is simply given as

$$V_{\text{AdSmin}} = -\frac{24\sqrt{2} \xi_s \tau_s^{3/2} |W_0|^2 (a_s \tau_s - 1)}{\mathcal{V}^3 (1 - 4a_s \tau_s)^2}. \tag{26}$$

Table 3
Sampling of the model dependent parameters.

Model	$C_{\alpha'}$	κ_{b11}	κ_{s11}	ξ_b	ξ_s	W_0	a_s	A_s	g_s
B1	1.697	-1	-1	$\frac{1}{9\sqrt{2}}$	$\frac{1}{9\sqrt{2}}$	-0.1	2π	1	0.5
B2	1.697	-1	-1	$\frac{1}{9\sqrt{2}}$	$\frac{1}{9\sqrt{2}}$	-0.1	2π	0.5	0.1

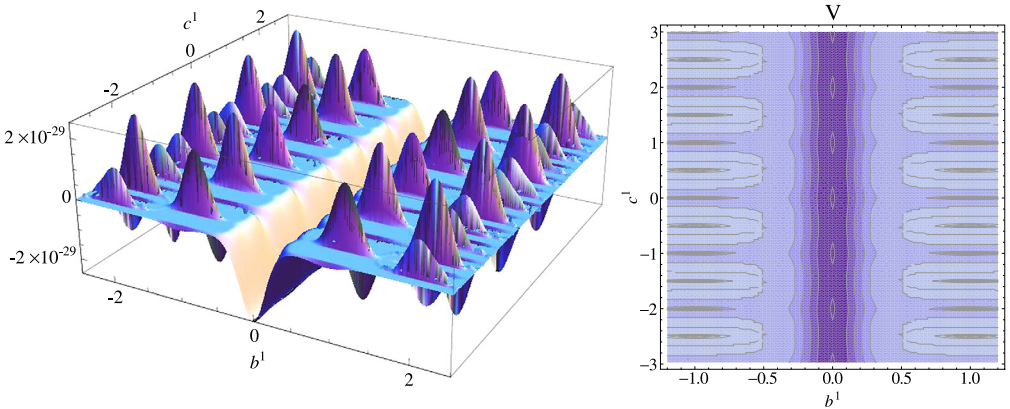


Fig. 1. The quasi-periodicity of the scalar potential in the odd-axion direction b^1 and c^1 after stabilizing all the other even moduli in a consistent way.

It is worth to recall that in the above discussion, we have considered only the simplest minimum for which the extremization conditions could be analytically solved. In fact, there are many extrema in the odd moduli directions due to the quasi-periodic property of the inverse elliptic theta function. An easy way to illustrate such property is to show the section of the scalar potential as a function of odd-axionic modulus after stabilizing all the other even moduli in a consistent way.⁶ Using the following sampling of model dependent parameters in Table 3, the scalar potential (24) are shown in Figs. 1, 2 where the quasi-periodicity in both (b^1 and c^1) directions are observed.

3.2. Axion decay constant and mass matrix

Let us look at the axion decay constant and the masses of various moduli at the non-supersymmetric minimum. Utilizing the Kähler metric, all kinetic terms for the respective moduli can be written as

$$\mathcal{L}_{\text{kinetic}}(\mathcal{V}, \tau_s, \rho_b, \rho_s; b^1, c^1) \equiv K_{I\bar{J}}(D_\mu T_I)(\bar{D}^\mu \bar{T}_{\bar{J}}). \tag{27}$$

In the basis of real moduli $\{\mathcal{V}, \tau_s, \rho_b, \rho_s; b^1, c^1\}$, the kinetic matrix (see Appendix A) is found to be block diagonal in both even and odd sector. The axionic sector of the kinetic matrix implies

⁶ From Eq. (24) one can see that the volume and τ_s moduli couples to b^1 and c^1 in a complicated way. In general, it is hard to get a simple expression to show explicitly how the stabilization conditions for moduli \mathcal{V} and τ_s depend on the odd moduli. However, we can solve these coupled extremization conditions numerically to get a potential for b^1 and c^1 moduli.

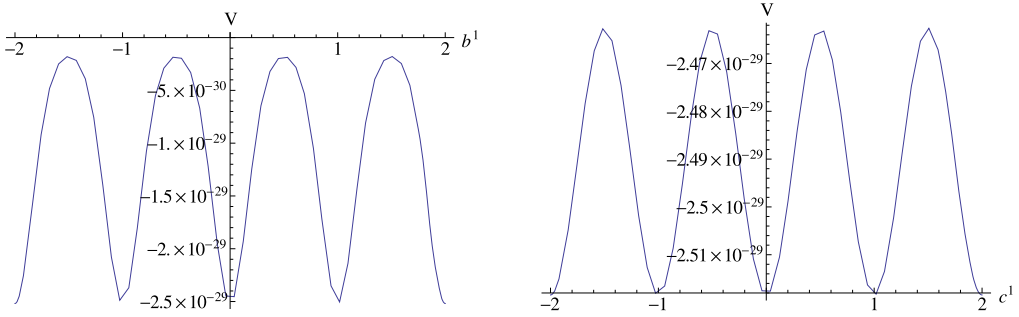


Fig. 2. The periodicity of the scalar potential in b^1 direction for $c^1 = 0$, and in c^1 direction for $b^1 = 0$ after stabilizing all the other even moduli.

that the leading order contributions to the decay constants for all the axions can be estimated to be

$$\begin{aligned}
 f_{\rho_b} &= \frac{\sqrt{6}\xi_b^{2/3}}{\mathcal{V}^{2/3}} \sim \mathcal{V}^{-2/3}, & f_{\rho_s} &= \frac{\sqrt{3\xi_s}}{(2\tau_s)^{1/4}\mathcal{V}^{1/2}} \sim \mathcal{V}^{-1/2}, \\
 f_{b^1} &= \frac{\sqrt{-3\kappa_{b11}}\xi_1^{1/3}}{\sqrt{2g_s}\mathcal{V}^{1/3}} \sim \mathcal{V}^{-1/3}, & f_{c^1} &= \frac{\sqrt{-3g_s\kappa_{b11}}\xi_1^{1/3}}{\sqrt{2}\mathcal{V}^{1/3}} \sim \mathcal{V}^{-1/3}.
 \end{aligned}
 \tag{28}$$

This shows that the positive definiteness of the kinetic matrix of odd axionic sector demands that $\kappa_{b11} < 0$. Now, let us investigate the squared-mass matrix evaluated at the minimum (25),

$$M_{ij} = \left(\begin{array}{ccc|cc}
 \frac{\beta_1}{\mathcal{V}^3} & \frac{\beta_2}{\mathcal{V}^2} & 0 & 0 & 0 \\
 \frac{\beta_4}{\mathcal{V}^3} & \frac{\beta_5}{\mathcal{V}^2} & 0 & 0 & 0 \\
 0 & 0 & \frac{\gamma_1}{\mathcal{V}^2} & 0 & 0 \\
 \hline
 0 & 0 & 0 & M_{b^1 b^1} & 0 \\
 0 & 0 & 0 & 0 & M_{c^1 c^1}
 \end{array} \right).
 \tag{29}$$

The upper left 3×3 block corresponds to the even-moduli sector $\{\mathcal{V}, \tau_s, \rho_s\}$ and reproduces the standard LARGE volume results without odd-axions. The lower right 2×2 block corresponds to the odd-moduli sector $\{b^1, c^1\}$. Before we analyze the odd axion mass in detail, let us recall from the superpotential expression (21), that in the absence of instanton-flux, the c^1 modulus direction is flat as the theta-function appearance in the superpotential disappears, however, the b^1 axionic flatness is lifted even in the absence of instanton-flux because of its implicit appearance in chiral coordinate T_s through the non-perturbative exponential suppression. For the eigenvalues of the mass-squared matrix (29), one gets the following volume scalings in the moduli masses evaluated at the minimum,

$$\begin{aligned}
 M_{\mathcal{V}} &\sim \frac{\delta}{\mathcal{V}^{3/2}}, & M_{\tau_s} &\sim \frac{\delta}{\mathcal{V}}; & M_{\rho_b} &= 0, & M_{\rho_s} &\sim \frac{\delta}{\mathcal{V}}, \\
 M_{b^1} &\sim \frac{\delta\Delta_1(\mathcal{F})}{\mathcal{V}^{7/6}}, & M_{c^1} &\sim \frac{\delta\Delta_2(\mathcal{F})}{\mathcal{V}^{7/6}}
 \end{aligned}
 \tag{30}$$

where $\delta \sim \sqrt{\frac{g_s|W_0|^2}{8\pi}}$. The first line represents the expected results of even-moduli sector and volume scalings are as per expectations [8]. The $\Delta_i(\mathcal{F})$'s appearing in odd axionic masses are

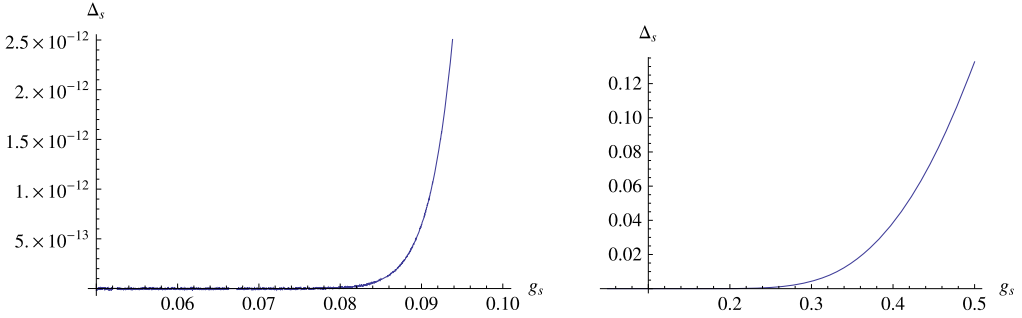


Fig. 3. The estimate of suppression factor Δ_s with different values of string coupling g_s .

introduced in place of multiplicative factors having a theta-function dependence, and are given as

$$\begin{aligned} \Delta_1(\mathcal{F}) &\sim \mathcal{O}(1)(g_s \pi^2 \Theta''(0) + 2\pi \Theta(0) + a_s \kappa_{s11} \Theta(0))^{1/2}, \\ \Delta_2(\mathcal{F}) &\sim \mathcal{O}(1)(g_s \pi^2 \Theta''(0) + 2\pi \Theta(0))^{1/2} \end{aligned} \tag{31}$$

where $\Theta''(0) = \partial_x^2 \theta_3(x, e^{-\pi g_s})|_{x=0}$. Here, it is worth to mention that $\Delta_i(\mathcal{F})$ does not have explicit flux dependence as fluxes on the instanton divisor are already summed over, even then we mention \mathcal{F} to keep one reminded that such theta-function contributions are rooted into the instanton flux effects. Further, the naive volume scalings in mass estimates (30) imply that odd-axions are heavier than the overall volume mode. However, the analytic expressions (31) of $\Delta_i(\mathcal{F})$ s show that $|\Delta_1(\mathcal{F})| \sim \mathcal{O}(1)$ while $|\Delta_2(\mathcal{F})| \ll \mathcal{O}(1)$ for natural model dependent parameters. The reason for the same is a crucial multiplicative factor appearing in $\Delta_2(\mathcal{F})$, which is $(2\theta_3[0, e^{-\pi g_s}] + g_s \pi \theta_3''[0, e^{-\pi g_s}])^{1/2}$, as seen from (31). This is a reasonable amount of suppression of the order 10^{-12} for $g_s \sim 0.1$ in mass-squared value of the c^1 axion. This happens because of a fine cancellation in two pieces of Δ_2 . Let us make it explicit by taking some numerical values:

g_s	0.05	0.1	0.2	0.3	0.4	0.5
$2\theta_3[0, e^{-\pi g_s}]$	8.94427	6.32456	4.47214	3.65169	3.16473	2.83899
$g_s \pi \theta_3''[0, e^{-\pi g_s}]$	-8.94427	-6.32456	-4.47209	-3.64736	-3.12617	-2.70624

The suppression factor in the squared-mass values of c^1 , which is given as $\Delta_s = (2\theta_3[0, e^{-\pi g_s}] + g_s \pi \theta_3''[0, e^{-\pi g_s}])$, gets more clear from the Fig. 3.

However, for larger values of string coupling, the factor Δ_s becomes order one implying that the mass of c^1 axion will be larger than that of overall volume mode. Thus, for c^1 axion to be the lightest, one has to keep string coupling g_s small enough such that $\Delta_s < \mathcal{O}(V^{-2/3})$. Utilizing the sampling given in Table 3, the eigenvalues of mass-squared matrix are shown in Table 4.

Note that, the b^1 axion is always heavier than the overall volume mode and lighter than small divisor volume mode in large volume limit. This is quite expected because b^1 flatness is expected to get lifted with the standard (unfluxed) E3-instanton correction due to an implicit appearance of b^1 (and not c^1) in the chiral coordinate T_α . The same is reflected through $\Delta_1(\mathcal{F})$ in (31) which has an additional piece $a_s \kappa_{s11} \Theta(0)$, directly coming from $e^{-a_s T_s}$ as just have been argued above. This additional piece, in general, causes an imbalance in the fine cancellation of other two terms

Table 4

Stabilized values of (divisor) volume moduli along with the eigenvalues of mass-squared matrix (evaluated at the minimum) in $Mp = 1$ units. The respective stabilized values for the axions are $\bar{\rho}_s = 0 = b^1 = c^1$.

Model	$\bar{\mathcal{V}}$	$\bar{\tau}_s$	$m_{\mathcal{V}}^2$	$m_{\tau_s}^2 \sim m_{\rho_s}^2$	$m_{b^1}^2$	$m_{c^1}^2$
B1	3×10^8	3.93	3.4×10^{-28}	3.2×10^{-16}	6.9×10^{-22}	3.4×10^{-23}
B2	1×10^8	3.93	3.3×10^{-28}	3.1×10^{-16}	7.2×10^{-22}	1.5×10^{-36}

(we discussed earlier), and results in an order one value. Note that, to nullify this extra pieces $a_s \kappa_{s11} \Theta(0)$ via $\kappa_{s11} = 0$ is not sensible as that would mean that small divisor does not have intersection with odd four-cycle and so no odd moduli can be supported on that divisor, and thus things would be too trivialized.

Another important observation in this setup is the fact that there are no tachyons present. It has been argued in [68,13] that in a setup equipped with supersymmetric moduli stabilization, in the presence of flat axionic directions, there are always tachyons. However, such a No-Go theorem does not hold for large volume model in which moduli stabilization is done in a non-supersymmetric manner [68], and hence there is no conflict in having a flat ρ_b -direction and no tachyons.

3.3. Case II: $\kappa_{b11} = 0$

Let us assume that the big divisor does not intersect with the odd four-cycles, i.e. $\kappa_{b11} = 0$. This is also common when one considers the holomorphic involution which permutes two “non-trivial identical” shrinkable del-Pezzo surfaces [33].

Using $\kappa_{b11} = 0$ in the ansatz Eq. (23), the volume form appearing in the Kähler potential Eq. (23) simplifies to⁷

$$\mathcal{V} = \xi_b (T_b + \bar{T}_b)^{3/2} - \xi_s \left((T_s + \bar{T}_s) + \frac{\kappa_{s11}}{2(S + \bar{S})} (G^1 + \bar{G}^1)(G^1 + \bar{G}^1) \right)^{3/2} + \mathcal{C}_{\alpha'}. \quad (32)$$

The leading order contributions to the F-term scalar potential can be again collected as three types of terms,

$$V_{\kappa_{b11}=0} \equiv V_{\alpha'} + V_{np1} + V_{np2}$$

where $V_{\alpha'}$ and V_{np1} are the same as in Eq. (24) while V_{np2} is modified, and is given as under

$$\begin{aligned} V_{np2} = & \frac{\sqrt{2} A_s^2}{3 \xi_s \mathcal{V} \sqrt{\tau_s}} \times \exp \left[\frac{a_s (\kappa_{s11} b^1 b^1 - 2 g_s \tau_s)}{g_s} + \frac{2\pi}{g_s} (c^{12} g_s^2 - b^{12}) \right] \\ & \times \left\{ 2 a_s^2 g_s \tau_s |\Theta(b^1, c^1)|^2 + a_s g_s b^1 (\cos(a_s \rho_s) + i \sin(a_s \rho_s)) \right. \\ & \times (-g_s \pi \bar{\Theta}'(b^1, c^1) \Theta(b^1, c^1) + \bar{\Theta}(b^1, c^1) (-g_s \pi \Theta'(b^1, c^1) \\ & + a_s b^1 \kappa_{s11} \Theta(b^1, c^1) + 4 g_s \pi b^1 \Theta(b^1, c^1)) \cos(a_s \rho_s)) \\ & \left. - i (g_s \pi \bar{\Theta}'(b^1, c^1) \Theta(b^1, c^1) + \bar{\Theta}(b^1, c^1) (-g_s \pi \Theta'(b^1, c^1) + a_s b^1 \kappa_{s11} \Theta(b^1, c^1)) \right\} \end{aligned}$$

⁷ We thank T. Higaki for bringing our notice to [69] where volume form of type (32) with $\kappa_{b11} = 0$ has been considered (without odd moduli stabilization via F-term scalar potential).

Table 5

Stabilized values of (divisor) volume moduli along with the eigenvalues of mass-squared matrix (evaluated at the minimum) in $Mp = 1$ units. The respective stabilized values for the axion are $\bar{\rho}_s = 0 = b^1 = c^1$.

Model	$\bar{\mathcal{V}}$	$\bar{\tau}_s$	$m_{\mathcal{V}}^2$	$m_{\tau_s}^2 \sim m_{\rho_s}^2$	$m_{b^1}^2$	$m_{c^1}^2$
S1	3.7×10^8	3.93	3.4×10^{-28}	3.1×10^{-16}	-8.4×10^{-20}	1.7×10^{-20}
S2	1.0×10^8	3.93	3.3×10^{-28}	3.1×10^{-16}	-3.5×10^{-20}	2.0×10^{-34}

$$\begin{aligned}
 & -4ic^1 g_s^2 \pi \Theta(b^1, c^1) \sin(a_s \rho_s) - \frac{g_s^2 \pi^2}{\kappa_{s11}} (\bar{\Theta}'(b^1, c^1) - 2\bar{\Theta}(b^1, c^1)(b^1 + ic^1 g_s)) \\
 & \times (\cos(2a_s \rho_s) + i \sin(2a_s \rho_s)) \Big\} \tag{33}
 \end{aligned}$$

where $\Theta'(b^1, c^1) = \theta_3'[-b^1 \pi + ic^1 g_s \pi, e^{-g_s \pi}]$ and $\theta_3'(u, q)$ gives the derivative with respect u . It is important to mention that the simplest critical point which minimizes the potential (33) is the same as what was in Case I (25). The reason for the same is the fact that the difference between Eqs. (33) and (24) effectively vanish at this critical point. So, it realizes the same LARGE volume non-susy AdS minimum as given by Eq. (26).

3.4. Axion decay constant and mass matrix

The axionic sector of the kinetic matrix results in the following estimates for the leading order contributions to the axion decay constants,

$$\begin{aligned}
 f_{\rho_b} & \simeq \frac{\sqrt{6} \xi_b^{2/3}}{\mathcal{V}^{2/3}} \sim \mathcal{V}^{-2/3}, & f_{\rho_s} & \simeq \frac{\sqrt{3} \xi_s}{(2\tau_s)^{1/4} \mathcal{V}^{1/2}} \sim \mathcal{V}^{-1/2}, \\
 f_{b^1} & \simeq \frac{\sqrt{3} \xi_s \kappa_{s11} \sqrt{2\tau_s}}{\sqrt{g_s} \mathcal{V}^{1/2}} \sim \mathcal{V}^{-1/2}, & f_{c^1} & \simeq \frac{\sqrt{3} g_s \kappa_{s11} \xi_s \sqrt{2\tau_s}}{\mathcal{V}^{1/2}} \sim \mathcal{V}^{-1/2}.
 \end{aligned} \tag{34}$$

Here, we observe two things; first, the positive definiteness of the kinetic matrix of odd axionic sector demands that $\kappa_{s11} > 0$ and second, the volume scalings in decay constants for odd-axions are different from the previous case (28). For the present case, it has an extra volume suppression of order $\mathcal{V}^{-1/6}$. However, there is a crucial observation that in this case b^1 -axionic direction is tachyonic. After looking at the eigenvalues of the mass-squared matrix, one gets the following estimates,

$$\begin{aligned}
 M_{\mathcal{V}} & \sim \frac{\delta}{\mathcal{V}^{3/2}}, & M_{\tau_s} & \sim \frac{\delta}{\mathcal{V}}, & M_{\rho_b} & = 0, & M_{\rho_s} & \sim \frac{\delta}{\mathcal{V}}, \\
 M_{b^1} & \sim \frac{\delta \Delta_3(\mathcal{F})}{\mathcal{V}}, & M_{c^1} & \sim \frac{\delta \Delta_4(\mathcal{F})}{\mathcal{V}}
 \end{aligned} \tag{35}$$

where $\delta \sim \sqrt{\frac{g_s |W_0|^2}{8\pi}}$. The above volume scaling differs in odd-axionic sector from those of Case I (30) while the scaling for the even-moduli sector remain the same. In odd-moduli sector, $|\Delta_3(\mathcal{F})| \sim \mathcal{O}(1)$ while $|\Delta_4(\mathcal{F})| < \mathcal{O}(1)$ for natural model dependent parameters the same as Table 3 except that $\kappa_{b11} = 0$ and $\kappa_{s11} = 1$. From Table 5, one realize that b^1 modulus direction is always tachyonic for a generic volume form (32) with $\kappa_{b11} = 0$.

Note that, in both cases with different volume forms (23) and (32) studied in this section, the model dependent parameters are chosen such that volume mode avoids the cosmological

moduli problem. Further, it is observed that c^1 axion is not the lightest for generic values of string coupling. For string coupling $g_s \sim 0.5$, we find that overall volume mode is the lightest, and thus a mass hierarchy is not very generic. For building inflationary model of odd axion c^1 , one has to consider a small enough string coupling which is well consistent and very natural in the large volume limits. In the next section, we will investigate a poly-instanton LARGE volume setup in which a mass hierarchy (in a part of even and odd sector) is manifestly present via a subdominant poly-instanton correction on top of standard non-perturbative effect.

4. Extended poly-instanton LARGE volume scenario

In this section, we start with a short review of the moduli stabilization mechanism after implementing the poly-instanton corrections in the standard LARGE volume scenario. The hierarchy of the poly-instanton contribution at the level of superpotential appears in the F-term scalar potential also. This hierarchial contribution facilitates the moduli stabilization process to be performed in three steps [30]. After stabilizing all the complex structure moduli and axion–dilaton by the GVW superpotential, the Kähler moduli along with respective C_4 axions are stabilized in the next two steps with/without poly-instanton effects. Unlike previous cases studied regarding odd-moduli stabilization, we expect to have decoupled standard LARGE volume framework from some of the (lighter) moduli. Recall that in the earlier cases, the stabilization process of all the even/odd moduli was coupled because the leading contribution for the odd moduli was originated on top of the standard E3-instanton correction to the superpotential which is responsible for stabilizing ‘small’ divisor volume mode. Hence, from the point of view of volume scaling, the masses of the odd axions were found to be larger than that of the overall volume mode. We will investigate if there is some improvement in this regard with the inclusion of fluxed poly-instanton corrections. The same is expected due to the appearance of a new hierarchy among standard E3-instanton and the poly-instanton corrections to the superpotential.

Before analyzing the poly-instanton setup with presence of odd moduli, let us briefly recall the relevant features of the moduli stabilization mechanism in the standard poly-instanton setup in LARGE volume scenario. We consider one C_4 axion corresponding to the complexified divisor volume of a del-Pezzo ‘small’ divisor and another C_4 axion complexifying the volume mode of a so-called ‘Wilson’ divisor relevant for generating poly-instanton contributions to the superpotential [28]. The expressions for the Kähler potential and the superpotential are

$$\begin{aligned}
 K &= -2 \ln \mathcal{V}, \\
 W &= W_0 + A_s e^{-a_s T_s} + A_s A_w e^{-a_s T_s - a_w T_w} - B_s e^{-b_s T_s} - B_s B_w e^{-b_s T_s - b_w T_w},
 \end{aligned}
 \tag{36}$$

where

$$\mathcal{V} = \xi_b (T_b + \bar{T}_b)^{\frac{3}{2}} - \xi_s (T_s + \bar{T}_s)^{\frac{3}{2}} - \xi_{sw} ((T_s + \bar{T}_s) + (T_w + \bar{T}_w))^{\frac{3}{2}} + C_{\alpha'}.
 \tag{37}$$

Here, we consider a racetrack form of the superpotential as it has been realized that with a superpotential ansatz without racetrack form, one does not get a minimum which could be trusted within the regime of validity of effective field theory description [30]. One can show that the same happens for the present case also. Now, in the large volume limit, (sub)leading contributions to the generic scalar potential $\mathbf{V}(\tau_b, \tau_s, \tau_w; \rho_s, \rho_w)$ are simply given as:

$$\mathbf{V}(\mathcal{V}, \tau_s, \tau_w; \rho_s, \rho_w) \simeq \mathbf{V}_{\text{racetrack}}^{\text{LVS}}(\mathcal{V}, \tau_s; \rho_s) + \mathbf{V}_{\text{poly}}(\mathcal{V}, \tau_s, \tau_w; \rho_s, \rho_w).
 \tag{38}$$

As expected, the first term $\mathbf{V}_{\text{racetrack}}^{\text{LVS}} \sim \mathcal{V}^{-3}$ in the scalar potential (38) does not depend on the Wilson line divisor volume modulus τ_w (along with its respective C_4 axion ρ_w). So these

directions remain flat at leading order and get lifted via subdominant poly-instanton effects $V_{\text{poly}} \sim \mathcal{V}^{-3-p}$, where p is a model dependent parameter. All the Kähler moduli are stabilized with this form of potential (38) resulting in a non-susy AdS minimum, and the details for the same can be found in [30,31]. Further, in large volume limits, the estimates for axionic decay constants are as

$$f_{\rho_b} \simeq \mathcal{V}^{-2/3}, \quad f_{\rho_s} \simeq \mathcal{V}^{-1/2} \simeq f_{\rho_w} \tag{39}$$

while the various masses scale as

$$\begin{aligned} M_{\mathcal{V}} &\sim \frac{\delta}{\mathcal{V}^{3/2}}, & M_{\tau_s} &\sim \frac{\delta}{\mathcal{V}}, & M_{\tau_w} &\sim \frac{\delta}{\mathcal{V}^{\frac{2+p}{2}}}, \\ M_{\rho_b} &= 0, & M_{\rho_s} &\sim \frac{\delta}{\mathcal{V}}, & M_{\rho_w} &\sim \frac{\delta}{\mathcal{V}^{\frac{2+p}{2}}} \end{aligned} \tag{40}$$

where $\delta \sim \sqrt{\frac{g_s |W_0|^2}{8\pi}}$. Thus, we observe that the masses of axionic directions lifted by a non-perturbative term in the superpotential come out to be of the same order as those of the respective saxion divisor volume modulus appearing in complexified chiral coordinate $T_\alpha = \tau_\alpha + i\rho_\alpha$.

4.1. Fluxed poly-instanton corrections

Assuming that the desired mathematical conditions relevant for generating the poly-instanton corrections can be satisfied, let us consider the following ansatz for the Kähler potential and the superpotential

$$\begin{aligned} K &= -2 \ln \mathcal{V} = -2 \ln (\xi_b \Sigma_b^{3/2} - \xi_s \Sigma_s^{3/2} - \xi_{sw} \Sigma_{sw}^{3/2} + C_{\alpha'}), \\ W &= \int_X G_3 \wedge \Omega + A_s e^{-a_s T_s} - B_s e^{-b_s T_s} + (A_s A_w e^{-a_s T_s} e^{-a_w T_w} \\ &\quad - B_s B_w e^{-b_s T_s} e^{-b_w T_w}) (\sqrt{g_s} e^{g_s \pi G^1 G^1} \theta_3 [g_s \pi G^1, e^{-g_s \pi}]) \end{aligned} \tag{41}$$

where

$$\begin{aligned} \Sigma_b &= T_b + \bar{T}_b + \frac{\kappa_{b11}}{2(S + \bar{S})} (G^1 + \bar{G}^1)(G^1 + \bar{G}^1), \\ \Sigma_s &= T_s + \bar{T}_s + \frac{\kappa_{s11}}{2(S + \bar{S})} (G^1 + \bar{G}^1)(G^1 + \bar{G}^1), \\ \Sigma_{sw} &= T_s + T_w + \bar{T}_s + \bar{T}_w + \frac{(\kappa_{s11} + \kappa_{w11})}{2(S + \bar{S})} (G^1 + \bar{G}^1)(G^1 + \bar{G}^1). \end{aligned} \tag{42}$$

Here, we assume that the E3-instanton wrapping the ‘Wilson’ divisor is fluxed and hence correct the holomorphic pre-factor via a odd-modulus dependent theta function. Further, the E3-instanton wrapping the ‘small’ divisor is not fluxed.

In large volume limit, the F-term scalar potential with the aforementioned ansatz (41) comes out to be in the following form:

$$V(\mathcal{V}, \tau_s, \tau_w; \rho_s, \rho_w, b^1, c^1) = V_{\text{LVS}}^{\text{ex}}(\mathcal{V}, \tau_s; \rho_s, b^1) + V_{\text{poly}}^{\text{ex}}(\mathcal{V}, \tau_s, \tau_w; \rho_s, \rho_w, b^1, c^1) \tag{43}$$

where

- $V_{\text{LVS}}^{\text{ex}}(\mathcal{V}, \tau_s; \rho_s; b^1)$ denotes the *extended* version of LARGE volume potential with the inclusion of b^1 odd axion.⁸ This potential scales as \mathcal{V}^{-3} in large volume limit and stabilizes the heavier moduli $\{\mathcal{V}, \tau_s; \rho_s, b^1\}$.
- The leading corrections to c^1 axion along with the Wilson divisor volume mode τ_w (and its respective axion ρ_w) come from the subdominant contributions $V_{\text{poly}}^{\text{ex}}(\mathcal{V}, \tau_s, \tau_w; \rho_s, \rho_w; b^1, c^1)$ which scales as \mathcal{V}^{-3-p} . Here, the parameter p is model dependent and in the absence of instanton-fluxes, $p > 1$ is required for the ‘Wilson’ divisor volume mode to be the lightest volume modulus [30].

The general expressions for $V_{\text{LVS}}^{\text{ex}}(\mathcal{V}, \tau_s; \rho_s; b^1)$ and $V_{\text{poly}}^{\text{ex}}(\tau_w; \rho_w, c^1)$ are given in [Appendix B](#). After stabilizing the heavier moduli at their respective minimum, the subleading scalar potential can be simplified to⁹

$$\begin{aligned} \mathbf{V}_{\text{poly}}^{\text{ex}}(\tau_w, \rho_w, c^1) &= e^{-a_w \tau_w} (\lambda_1 + \lambda_2 \tau_w) e^{-\pi g_s c^1{}^2} [(\Theta(c^1) + \bar{\Theta}(c^1)) \cos[a_w \rho_w] \\ &\quad - i(\Theta(c^1) - \bar{\Theta}(c^1)) \sin[a_w \rho_w]] \end{aligned} \tag{44}$$

where λ_0, λ_1 and λ_2 depend on the stabilized values of the heavier moduli and other model dependent parameters. The expressions for the same are given by Eq. (B.5) in [Appendix B](#). The potential (44) has the following minimum,

$$c^1 = 0, \quad a_w \bar{\rho}_w = N\pi, \quad \bar{\tau}_w = \frac{1}{a_w} - \frac{\lambda_1}{\lambda_2}$$

Recall that in the absence of racetrack form of the superpotential, the parameters λ_i ’s are such that one can not have a minimum which could be trusted in the regime of validity of the effective field theory. However, the presence of racetrack form introduces more parameters in the picture which facilitates more freedom in model dependent parameters such that $\lambda_1 > 0$ and $\lambda_2 < 0$, and $\bar{\tau}_w > 1$ can be easily realized. Further, following the same strategy for computing the axion decay constants as well as moduli masses, in large volume limit, we find

$$f_{\rho_b} \simeq \mathcal{V}^{-2/3}, \quad f_{\rho_s} \simeq \mathcal{V}^{-1/2} \simeq f_{\rho_w}, \quad f_{b^1} \simeq \frac{1}{\sqrt{g_s} \mathcal{V}^{1/3}}, \quad f_{c^1} \simeq \frac{\sqrt{g_s}}{\mathcal{V}^{1/3}} \tag{45}$$

and

$$\begin{aligned} M_{\mathcal{V}} &\sim \frac{\delta}{\mathcal{V}^{3/2}}, & M_{\rho_b} &= 0, & M_{\tau_s} &\sim \frac{\delta}{\mathcal{V}} \sim M_{\rho_s}, & M_{b^1} &\sim \frac{\delta}{\mathcal{V}^{7/6}}; \\ M_{\tau_w} &\sim \frac{\delta}{\mathcal{V}^{1+\frac{p}{2}}} \sim M_{\rho_w}, & M_{c^1} &\sim \frac{\delta \Delta_5(\mathcal{F})}{\mathcal{V}^{\frac{2}{3}+\frac{p}{2}}} \end{aligned} \tag{46}$$

where $\delta \sim \sqrt{\frac{g_s |W_0|^2}{8\pi}}$. The first line represents the masses for those moduli which have been stabilized at the leading order in the absence of poly-instanton corrections, while the second line represents masses for those moduli which have been stabilized by fluxed poly-instanton corrections. For obvious reasons, the moduli masses in the even sector are same as in [30]. For justifying the two-step Kähler moduli stabilization, we have to choose model dependent parameters such

⁸ Recall that c^1 flatness is present in the absence of fluxes turned-on on the instanton divisor.

⁹ In the absence of odd moduli sector, this potential (44) reduces to a two field poly-instanton setup which has been used to study the possibility of realizing the non-Gaussianities signatures in [31].

that $p > 1$. Then the lighter ones which remain flat in the absence of poly-instanton effects get stabilized after including the same, and we observe different volume scalings for τ_w , ρ_w and c^1 moduli masses. However, similar to the previous case, we again realize the same suppression factor $\Delta_s \equiv (2\theta_3[0, e^{-\pi g_s}] + g_s \pi \theta'_3[0, e^{-\pi g_s}])$ appearing inside $\Delta_5(\mathcal{F})$ for c^1 axion mass, and for natural model dependent parameters, $\Delta_s \ll \mathcal{O}(1)$. To get an idea about the numerics, with the following sampling of parameters (similar to the ones used in [30,31]),

$$\begin{aligned} \xi_b &= \frac{1}{36}, & \xi_s &= \frac{1}{6\sqrt{2}}, & \xi_{sw} &= \frac{1}{6\sqrt{2}}, & C_{\alpha'} &= \frac{0.165}{g_s^{3/2}}, \\ W_0 &= -20, & g_s &= 0.12, & a_s &= \frac{2\pi}{7}, & b_s &= \frac{2\pi}{6}, & a_w &= 2\pi = b_w, \\ A_s &= 3, & A_w &= 0.5, & B_s &= 2, & B_w &= 1.749, & \kappa_{b11} &= -1 = \kappa_{s11} \end{aligned} \quad (47)$$

we have the stabilized values of moduli in the simplest minimum as

$$\bar{\mathcal{V}} \sim 904.86, \quad \bar{\tau}_s \sim 5.68, \quad \bar{\tau}_w \sim 1.73, \quad \bar{\rho}_s = 0 = \bar{\rho}_w, \quad \bar{b}^1 = 0 = \bar{c}^1. \quad (48)$$

Further, utilizing the (47) and (48), we have the following estimates for masses of the canonically normalized moduli and the axions (in $M_p = 1$ units)

$$\begin{aligned} M_{\mathcal{V}} &\sim 2.0 \times 10^{-4}, & M_{\tau_s} &\sim 1.8 \times 10^{-2}, & M_{\tau_w} &\sim 2.0 \times 10^{-5}, \\ M_{\rho_b} &= 0, & M_{\rho_s} &\sim 1.8 \times 10^{-2}, & M_{\rho_w} &\sim 2.0 \times 10^{-5}, \\ M_{b^1} &\sim 2.4 \times 10^{-3}, & M_{c^1} &\sim 3.3 \times 10^{-11} \end{aligned} \quad (49)$$

which reflects the following mass hierarchies

$$M_{\tau_s} \sim M_{\rho_s} > M_{b^1} > M_{\mathcal{V}} > M_{\tau_w} \sim M_{\rho_w} > M_{c^1}$$

as discussed earlier. Further, although in the present case, the risk of c^1 axion being heavier than volume mode can be easily avoided for $p > 1$, however, now the problem with lowering of axion mass along with those of saxion in model dependent way reappears through the lowering the mass of ‘Wilson’ divisor volume mode. This can be seen from (46) that if $\Delta_5(\mathcal{F})$ becomes order one by increasing the string coupling as seen from the plots 3, then simple volume scaling shows that $M_{\tau_w} \sim M_{\rho_w} > M_{c^1}$.

5. Conclusion

In this article, we revisited the F-term moduli stabilization in an extended LARGE volume setup equipped with the involutively odd moduli. First, we considered a simple extension of large volume setup with the inclusion of a single odd modulus, and investigated the odd axion stabilization with the inclusion of instanton flux effects. Then, we extended the analysis into a poly-instanton LARGE volume framework and revisited the moduli stabilization in the presence of odd moduli. We also computed the masses and decay constants for various even/odd axions present in the respective setups. Subsequently, we realized a mass hierarchy among the divisor volume moduli masses and even/odd axion masses which might be helpful in exploring the inflationary implications of odd-axions. Further, it is also desired to implement this moduli stabilization process in a less simple Type IIB orientifold setup which supports an ‘explicit’ MSSM-like visible sector and subsequently explore the utility of odd axions for studying various cosmo/pheno aspects in the regime of Axion Physics.

Acknowledgements

We are grateful to Ralph Blumenhagen for several enlightening clarifications, discussions and suggestions. We also thank Thomas Grimm, Sebastian Halter for useful discussions, and Arthur Hebecker, Savdeep Sethi for useful comments. X.G. is supported by the MPG–CAS Joint Doctoral Promotion Programme and P.S. is supported by a postdoctoral research fellowship from the Alexander von Humboldt Foundation.

Appendix A. Kinetic matrices for various K ansatz

All the kinetic terms for the respective moduli in a given ansatz for the Kähler potential can be written out utilizing the Kähler metric as below,

$$\mathcal{L}_{\text{kinetic}}(\mathcal{V}, \tau_s, \dots; \rho_b, \rho_s, \dots; b^1, c^1, \dots) \equiv K_{I\bar{J}}(D_\mu T_I)(\bar{D}^\mu \bar{T}_{\bar{J}}), \tag{A.1}$$

where the Kähler potential is generically defined as $K \equiv -2 \ln \mathcal{V} = -2 \ln(\mathcal{V} + C_{\alpha'})$, \mathcal{V} being overall volume of the Calabi–Yau. Neglecting the α' -corrections $C_{\alpha'}$, in this section, we present the kinetic matrix for each of the Kähler potential ansatz studied in this article. The kinetic matrix when evaluated at the respective minimum of the potential is found to be of block diagonal form with three blocks corresponding to divisor volume moduli, respective C_4 axions and odd (B_2, C_2) axions, if present in the ansatz for K . The relevant volume scalings in the even/odd axion decay constants can be easily estimated utilizing large volume limit.

A.1. Standard LARGE volume setup

For the volume form of type

$$\mathcal{V} = \xi_b(T_b + \bar{T}_b)^{\frac{3}{2}} - \sum_{s=2}^{h_+^{1,1}} \xi_s(T_s + \bar{T}_s)^{\frac{3}{2}},$$

the non-zero components of the kinetic matrix are as under

$$\begin{aligned} K_{\mathcal{V}\mathcal{V}} &= \frac{1}{3\mathcal{V}^2}, & K_{\mathcal{V}\tau_s} &= -\frac{3\xi_s\sqrt{\tau_s}}{\sqrt{2}\mathcal{V}^2} = K_{\tau_s\mathcal{V}}, \\ K_{\tau_s\tau_s} &= \frac{3\xi_s}{2\sqrt{2}\sqrt{\tau_s}\mathcal{V}}, & K_{\tau_s\tau_r} &= -\frac{3\xi_s\xi_r\sqrt{\tau_s\tau_r}}{\mathcal{V}^2} = K_{\tau_r\tau_s}, \\ K_{\rho_b\rho_b} &= \frac{3\xi_b^{4/3}}{\mathcal{V}^{4/3}}, & K_{\rho_b\rho_s} &= -\frac{9\xi_b^{2/3}\xi_s\sqrt{\tau_s}}{\sqrt{2}\mathcal{V}^{5/3}} = K_{\rho_s\rho_b}, \\ K_{\rho_s\rho_s} &= \frac{3\xi_s}{2\sqrt{2}\sqrt{\tau_s}\mathcal{V}}, & K_{\rho_s\rho_r} &= \frac{9\xi_s\xi_r\sqrt{\tau_s\tau_r}}{\mathcal{V}^2} = K_{\rho_r\rho_s} \end{aligned} \tag{A.2}$$

where $\{s, r\} \in \{2, \dots, h_+^{1,1}\}$ and $s \neq r$ is considered in cross terms.

A.2. Extended LARGE volume setup

For the volume form of type

$$\mathcal{V} = \xi_b \left((T_b + \bar{T}_b) + \frac{\kappa_{b11}}{2(S + \bar{S})} (G^1 + \bar{G}^1)(G^1 + \bar{G}^1) \right)^{\frac{3}{2}} - \xi_s \left((T_s + \bar{T}_s) + \frac{\kappa_{s11}}{2(S + \bar{S})} (G^1 + \bar{G}^1)(G^1 + \bar{G}^1) \right)^{\frac{3}{2}}, \tag{A.3}$$

the non-zero components of the kinetic matrix are as under

$$\begin{aligned} K_{\mathcal{V}\mathcal{V}} &= \frac{1}{3\mathcal{V}^2}, & K_{\mathcal{V}\tau_s} &= -\frac{3\xi_s\sqrt{\tau_s}}{\sqrt{2}\mathcal{V}^2}, & K_{\mathcal{V}b^1} &= \frac{\xi_b^{2/3}\kappa_{b11}b^1}{g_s\mathcal{V}^{5/3}} = -\frac{1}{g_s}K_{\mathcal{V}c^1}, \\ K_{\tau_s\tau_s} &= \frac{3\xi_s}{2\sqrt{2}\sqrt{\tau_s}\mathcal{V}}, & K_{\tau_sb^1} &= \frac{3\xi_s\kappa_{s11}b^1}{2\sqrt{2}g_s\sqrt{\tau_s}\mathcal{V}} = -\frac{1}{g_s}K_{\tau_sc^1}, \\ K_{b^1b^1} &= -\frac{3\kappa_{b11}\xi_b^{2/3}}{2g_s\mathcal{V}^{2/3}} = \frac{1}{g_s^2}K_{c^1c^1}, & K_{\rho_b\rho_b} &= \frac{3\xi_b^{4/3}}{\mathcal{V}^{4/3}}, \\ K_{\rho_b\rho_s} &= -\frac{9\xi_b^{2/3}\xi_s\sqrt{\tau_s}}{\sqrt{2}\mathcal{V}^{5/3}}, & K_{\rho_bb^1} &= \frac{3\xi_b^{4/3}\kappa_{b11}b^1}{g_s\mathcal{V}^{4/3}} = -\frac{1}{g_s}K_{\rho_bc^1}, \\ K_{\rho_s\rho_s} &= \frac{3\xi_s}{2\sqrt{2}\sqrt{\tau_s}\mathcal{V}}, & K_{\rho_sb^1} &= \frac{3\xi_s\kappa_{s11}b^1}{2\sqrt{2}g_s\sqrt{\tau_s}\mathcal{V}} = -\frac{1}{g_s}K_{\rho_sc^1}. \end{aligned} \tag{A.4}$$

Therefore, the kinetic matrix is block diagonal in even/odd sector only when evaluated at the minimum which requires $b^1 = 0$.

A.3. Poly-instanton setup

For the volume form of type

$$\mathcal{V} = \xi_b(T_b + \bar{T}_b)^{\frac{3}{2}} - \xi_s(T_s + \bar{T}_s)^{\frac{3}{2}} - \xi_{sw}((T_s + \bar{T}_s) + (T_w + \bar{T}_w))^{\frac{3}{2}},$$

the non-zero independent components of the kinetic matrix are as under

$$\begin{aligned} K_{\mathcal{V}\mathcal{V}} &= \frac{1}{3\mathcal{V}^2}, & K_{\mathcal{V}\tau_s} &= -\frac{3(\sqrt{\tau_s}\xi_s + \xi_{sw}\sqrt{\tau_s + \tau_w})}{\sqrt{2}\mathcal{V}^2}, \\ K_{\mathcal{V}\tau_w} &= -\frac{3\xi_{sw}\sqrt{\tau_s + \tau_w}}{\sqrt{2}\mathcal{V}^2}, & K_{\tau_s\tau_s} &= \frac{3\left(\frac{\xi_s}{\sqrt{\tau_s}} + \frac{\xi_{sw}}{\sqrt{\tau_s + \tau_w}}\right)}{2\sqrt{2}\mathcal{V}}, \\ K_{\tau_s\tau_w} &= -\frac{3\xi_{sw}}{2\sqrt{2}\mathcal{V}\sqrt{\tau_s + \tau_w}}, & K_{\tau_w\tau_w} &= \frac{3\xi_{sw}}{2\sqrt{2}\mathcal{V}\sqrt{\tau_s + \tau_w}}, \\ K_{\rho_b\rho_b} &= 3\left(\frac{\xi_b}{\mathcal{V}}\right)^{4/3}, & K_{\rho_b\rho_s} &= -\frac{9\xi_b^{2/3}(\sqrt{\tau_s}\xi_s + \xi_{sw}\sqrt{\tau_s + \tau_w})}{\sqrt{2}\mathcal{V}^{5/3}}, \\ K_{\rho_b\rho_w} &= -\frac{9\xi_b^{2/3}\xi_{sw}\sqrt{\tau_s + \tau_w}}{\sqrt{2}\mathcal{V}^{5/3}}, & K_{\rho_s\rho_s} &= \frac{3\left(\frac{\xi_s}{\sqrt{\tau_s}} + \frac{\xi_{sw}}{\sqrt{\tau_s + \tau_w}}\right)}{2\sqrt{2}\mathcal{V}}, \\ K_{\rho_s\rho_w} &= \frac{3\xi_{sw}}{2\sqrt{2}\mathcal{V}\sqrt{\tau_s + \tau_w}}, & K_{\rho_w\rho_w} &= \frac{3\xi_{sw}}{2\sqrt{2}\mathcal{V}\sqrt{\tau_s + \tau_w}}. \end{aligned} \tag{A.5}$$

A.4. Extended poly-instanton setup

For the volume for os type

$$\mathcal{V} = \xi_b \Sigma_b^{3/2} - \xi_s \Sigma_s^{3/2} - \xi_{sw} \Sigma_{sw}^{3/2},$$

where

$$\Sigma_b = T_b + \bar{T}_b + \frac{\kappa_{b11}}{2(S + \bar{S})} (G^1 + \bar{G}^1)(G^1 + \bar{G}^1),$$

$$\Sigma_s = T_s + \bar{T}_s + \frac{\kappa_{s11}}{2(S + \bar{S})} (G^1 + \bar{G}^1)(G^1 + \bar{G}^1),$$

$$\Sigma_{sw} = T_s + T_w + \bar{T}_s + \bar{T}_w + \frac{(\kappa_{s11} + \kappa_{w11})}{2(S + \bar{S})} (G^1 + \bar{G}^1)(G^1 + \bar{G}^1),$$

the non-zero independent components of the kinetic matrix are as in Eq. (A.5) along with the following extra components in odd sector,

$$\begin{aligned} K_{\mathcal{V}b^1} &= \frac{\xi_b^{2/3} \kappa_{b11} b^1}{g_s \mathcal{V}^{5/3}} = -\frac{1}{g_s} K_{\mathcal{V}c^1}, \\ K_{\tau_s b^1} &= \frac{3b^1 \left(\frac{\xi_s \kappa_{s11}}{\sqrt{\tau_s}} + \frac{\xi_{sw} (\kappa_{s11} + \kappa_{w11})}{\sqrt{\tau_s + \tau_w}} \right)}{2\sqrt{2} g_s \mathcal{V}} = -\frac{1}{g_s} K_{\tau_s c^1}, \\ K_{\tau_w b^1} &= \frac{3\xi_{sw} (\kappa_{s11} + \kappa_{w11}) b^1}{2\sqrt{2} g_s \sqrt{\tau_s + \tau_w} \mathcal{V}} = -\frac{1}{g_s} K_{\tau_w c^1}, \\ K_{b^1 b^1} &= -\frac{3\kappa_{b11} \xi_b^{2/3}}{2g_s \mathcal{V}^{2/3}} = \frac{1}{g_s^2} K_{c^1 c^1}, \\ K_{\rho_b b^1} &= \frac{3\xi_b^{4/3} \kappa_{b11} b^1}{g_s \mathcal{V}^{4/3}} = -\frac{1}{g_s} K_{\rho_b c^1}, \\ K_{\rho_s b^1} &= \frac{3b^1 \left(\frac{\xi_s \kappa_{s11}}{\sqrt{\tau_s}} + \frac{\xi_{sw} (\kappa_{s11} + \kappa_{w11})}{\sqrt{\tau_s + \tau_w}} \right)}{2\sqrt{2} g_s \mathcal{V}} = -\frac{1}{g_s} K_{\rho_s c^1}, \\ K_{\rho_w b^1} &= \frac{3\xi_{sw} (\kappa_{s11} + \kappa_{w11}) b^1}{2\sqrt{2} g_s \sqrt{\tau_s + \tau_w} \mathcal{V}} = -\frac{1}{g_s} K_{\rho_w c^1}. \end{aligned} \tag{A.6}$$

Appendix B. Scalar potential and moduli stabilization

In the large volume limit, (sub)leading contributions to the generic scalar potential $\mathbf{V}(\mathcal{V}, \tau_s, \tau_w; \rho_s, \rho_w, b^1, c^1)$ are simply given as:

$$\begin{aligned} \mathbf{V}(\mathcal{V}, \tau_s, \tau_w; \rho_s, \rho_w, b^1, c^1) \\ \simeq \mathbf{V}_{\text{LVS}}^{\text{ex}}(\mathcal{V}, \tau_s; \rho_s, b^1) + \mathbf{V}_{\text{poly}}^{\text{ex}}(\mathcal{V}, \tau_s, \tau_w; \rho_s, \rho_w, b^1, c^1). \end{aligned} \tag{B.1}$$

In Eq. (B.1), the leading contributions for the two types of terms are given as under

$$\mathbf{V}_{\text{LVS}}^{\text{ex}}(\mathcal{V}, \tau_s; \rho_s, b^1) = \frac{3C_{\alpha'} |W_0|^2}{2\mathcal{V}^3} + \frac{2\sqrt{2} a_s^2 \mu_1^2 \sqrt{\tau_s}}{3\xi_s \mathcal{V}} + \frac{2\sqrt{2} b_s^2 \mu_2^2 \sqrt{\tau_s}}{3\xi_s \mathcal{V}}$$

$$\begin{aligned}
 & + \frac{4a_s W_0 \mu_1 \tau_s \cos[a_s \rho_s]}{\mathcal{V}^2} - \frac{4b_s W_0 \mu_2 \tau_s \cos[b_s \rho_s]}{\mathcal{V}^2} \\
 & - \frac{4\sqrt{2} a_s b_s \mu_1 \mu_2 \sqrt{\tau_s} \cos[(a_s - b_s) \rho_s]}{3\xi_s \mathcal{V}}
 \end{aligned} \tag{B.2}$$

where $\mu_1 = A_s e^{-a_s(\tau_s - \frac{\kappa_{s11} b^1 b^1}{2g_s})}$ and $\mu_2 = B_s e^{-b_s(\tau_s - \frac{\kappa_{s11} b^1 b^1}{2g_s})}$. The extrema of the above leading order scalar potential can be collectively described by the intersection of the following hypersurfaces in moduli space

$$\begin{aligned}
 & b^1 = 0, \quad a_s \bar{\rho}_s = N\pi, \quad \text{where } N \in \mathbb{Z}, \\
 & W_0 \simeq \frac{\bar{\mathcal{V}}(b_s \bar{\mu}_2 - a_s \bar{\mu}_1)[b_s \bar{\mu}_2(-1 + 4b_s \bar{\tau}_s) - a_s \bar{\mu}_1(-1 + 4a_s \bar{\tau}_s)]}{6\sqrt{2} \xi_s \sqrt{\bar{\tau}_s} [b_s \bar{\mu}_2(-1 + b_s \bar{\tau}_s) - a_s \bar{\mu}_1(-1 + a_s \bar{\tau}_s)]}, \\
 & C_{\alpha'} \simeq \frac{32\sqrt{2} \xi_s \bar{\tau}_s^{\frac{5}{2}} (b_s^2 \bar{\mu}_2 - a_s^2 \bar{\mu}_1)[b_s \bar{\mu}_2(-1 + b_s \bar{\tau}_s) - a_s \bar{\mu}_1(-1 + a_s \bar{\tau}_s)]}{[a_s \bar{\mu}_1(-1 + 4a_s \bar{\tau}_s) - b_s \bar{\mu}_2(-1 + 4b_s \bar{\tau}_s)]^2}, \\
 & \text{with } \bar{\mu}_1 \equiv \mu_1(b^1 = 0) = A_s e^{-a_s \bar{\tau}_s} \quad \text{and} \quad \bar{\mu}_2 \equiv \mu_2(b^1 = 0) = B_s e^{-b_s \bar{\tau}_s}.
 \end{aligned} \tag{B.3}$$

After stabilizing the (heavier) moduli $\{\mathcal{V}, \tau_s, \rho_s, b^1\}$ via the aforementioned extremization conditions (B.3), the second part of the expression (B.1) which is subleading contribution coming from the poly-instanton corrections simplifies to the form below

$$\begin{aligned}
 \mathcal{V}_{\text{poly}}^{\text{ex}}(\tau_w, \rho_w, c^1) &= e^{-a_w \tau_w} (\lambda_1 + \lambda_2 \tau_w) e^{-\pi g_s c^{12}} \\
 & \times [(\Theta(c^1) + \bar{\Theta}(c^1)) \cos[a_w \rho_w] - i(\Theta(c^1) - \bar{\Theta}(c^1)) \sin[a_w \rho_w]]
 \end{aligned} \tag{B.4}$$

where

$$\begin{aligned}
 \lambda_1 &= \lambda_0 \left[4\bar{\tau}_s \left((a_s - a_w) A_w \bar{\mu}_1 - (b_s - a_w) B_w \bar{\mu}_2 \right) \right. \\
 & \left. + \frac{\bar{\tau}_s (b_s B_w \bar{\mu}_2 - a_s A_w \bar{\mu}_1) (a_s \bar{\mu}_1(-1 + 4a_s \bar{\tau}_s) - b_s \bar{\mu}_2(-1 + 4b_s \bar{\tau}_s))}{a_s \bar{\mu}_1(-1 + a_s \bar{\tau}_s) - b_s \bar{\mu}_2(-1 + b_s \bar{\tau}_s)} \right], \\
 \lambda_2 &= \lambda_0 a_w \left[\frac{(B_w \bar{\mu}_2 - A_w \bar{\mu}_1) (a_s \bar{\mu}_1(-1 + 4a_s \bar{\tau}_s) - b_s \bar{\mu}_2(-1 + 4b_s \bar{\tau}_s))}{a_s \bar{\mu}_1(-1 + a_s \bar{\tau}_s) - b_s \bar{\mu}_2(-1 + b_s \bar{\tau}_s)} \right]
 \end{aligned} \tag{B.5}$$

with

$$\lambda_0 = \frac{\sqrt{g_s} (a_s \bar{\mu}_1 - b_s \bar{\mu}_2)}{3\sqrt{2} \xi_s \bar{\mathcal{V}} \sqrt{\bar{\tau}_s}}.$$

Appendix C. Expressions for odd-axion masses

- The expressions for squared-mass values of the odd-moduli for $\kappa_{b11} \neq 0$ case are given as

$$\begin{aligned}
 M_{b^1, b^1} &= \frac{48\sqrt{2} \xi_s W_0^2 \tau_s^{3/2} (-1 + a_s \tau_s) (g_s \pi^2 \Theta''(0) + 2\pi \Theta(0) + a_s \kappa_{s11} \Theta(0))}{\xi_b^{2/3} \kappa_{b11} \Theta(0) \mathcal{V}^{7/3} (1 - 4a_s \tau_s)^2}, \\
 M_{c^1, c^1} &= -\frac{48\sqrt{2} \xi_s W_0^2 \tau_s^{3/2} (-1 + a_s \tau_s) (g_s \pi^2 \Theta''(0) + 2\pi \Theta(0))}{\xi_b^{2/3} \kappa_{b11} \Theta(0) \mathcal{V}^{7/3} (1 - 4a_s \tau_s)^2}.
 \end{aligned} \tag{C.1}$$

- For $\kappa_{b11} = 0$, the squared-mass values of odd-moduli become:

$$\begin{aligned}
 M_{b^1, b^1} &= -\frac{4W_0^2(-1 + a_s \tau_s)}{a_s^2 \kappa_{s11}^2 \mathcal{V}^2 \Theta(0)^2 (1 - 4a_s \tau_s)} \times \left\{ -2g_s^2 \pi^2 (\pi \Theta''(0) + 2\Theta(0))^2 \right. \\
 &\quad \times (-1 + a_s \tau_s) + a_s g_s \kappa_{s11} \pi \Theta(0) (\pi \Theta'(0) (4 - a_s \tau_s) - 8\Theta(0) \\
 &\quad \times (-1 + a_s \tau_s)) + a_s^2 \kappa_{s11} \Theta(0)^2 (6\pi \tau_s + \kappa_{s11} (2 + a_s \tau_s)) \left. \right\}, \\
 M_{c^1, c^1} &= \frac{4\pi W_0^2(-1 + a_s \tau_s)}{a_s^2 \kappa_{s11}^2 \mathcal{V}^2 \Theta(0)^2 (1 - 4a_s \tau_s)} \times \left\{ 3a_s^2 g_s \tau_s \kappa_{s11} \pi \Theta''(0) \Theta(0) \right. \\
 &\quad \left. + 6a_s^2 \tau_s \kappa_{s11} \Theta(0)^2 + 2g_s \pi (\pi g_s \Theta''(0) + 2\Theta(0))^2 (-1 + a_s \tau_s) \right\}. \quad (C.2)
 \end{aligned}$$

- The squared-mass expression for odd moduli in extended poly-instanton setup are as under,

$$\begin{aligned}
 M_{b^1, b^1} &= \frac{48\sqrt{2} \xi_s \kappa_{s11} (b_s^2 \bar{\mu}_2 - a_s^2 \bar{\mu}_1) |W_0|^2 \bar{\tau}_s^{3/2}}{\xi_b^{2/3} \kappa_{b11} \bar{\mathcal{V}}^{7/3} (a_s \bar{\mu}_1 (-1 + 4a_s \bar{\tau}_s) - b_s \bar{\mu}_2 (-1 + 4b_s \bar{\tau}_s))^2} \\
 &\quad \times (-a_s \bar{\mu}_1 (-1 + a_s \bar{\tau}_s) + b_s \bar{\mu}_2 (-1 + b_s \bar{\tau}_s)), \\
 M_{c^1, c^1} &= \frac{g_s \lambda_2 e^{-1 + \frac{a_w \lambda_1}{\lambda_2}} (2\Theta(0) + \pi g_s \Theta''(0))}{3a_w \kappa_{b11}} \quad (C.3)
 \end{aligned}$$

where λ_i s and $\bar{\mu}_i$ s are defined in previous [Appendix B](#).

In all above expressions, $\Theta'(0) = \theta'_3[-b^1 \pi + ic^1 g_s \pi, e^{-g_s \pi}]|_{b^1=0, c^1=0}$. The explicit expressions for masses of even sector moduli can be found in earlier work [\[31\]](#).

References

- [1] S. Kachru, R. Kallosh, A.D. Linde, S.P. Trivedi, De Sitter vacua in string theory, *Phys. Rev. D* 68 (2003) 046005, arXiv:hep-th/0301240.
- [2] V. Balasubramanian, P. Berglund, J.P. Conlon, F. Quevedo, Systematics of moduli stabilisation in Calabi–Yau flux compactifications, *J. High Energy Phys.* 0503 (2005) 007, arXiv:hep-th/0502058.
- [3] S. Gukov, C. Vafa, E. Witten, CFT’s from Calabi–Yau four folds, *Nucl. Phys. B* 584 (2000) 69–108, arXiv:hep-th/9906070.
- [4] K. Becker, M. Becker, M. Haack, J. Louis, Supersymmetry breaking and alpha-prime corrections to flux induced potentials, *J. High Energy Phys.* 0206 (2002) 060, arXiv:hep-th/0204254.
- [5] M. Grana, Flux compactifications in string theory: A comprehensive review, *Phys. Rep.* 423 (2006) 91–158, arXiv:hep-th/0509003.
- [6] D. Lust, S. Reffert, E. Scheidegger, W. Schulgin, S. Stieberger, Moduli stabilization in type IIB orientifolds (II), *Nucl. Phys. B* 766 (2007) 178–231, arXiv:hep-th/0609013.
- [7] J.P. Conlon, F. Quevedo, Kahler moduli inflation, *J. High Energy Phys.* 0601 (2006) 146, arXiv:hep-th/0509012.
- [8] J.P. Conlon, F. Quevedo, Astrophysical and cosmological implications of large volume string compactifications, *J. Cosmol. Astropart. Phys.* 0708 (2007) 019, arXiv:0705.3460.
- [9] M. Cicoli, M. Goodsell, A. Ringwald, The type IIB string axiverse and its low-energy phenomenology, arXiv:1206.0819.
- [10] K. Hristov, Axion stabilization in type IIB flux compactifications, *J. High Energy Phys.* 0901 (2009) 046, arXiv:0810.3329.
- [11] T.W. Grimm, Axion inflation in type II string theory, *Phys. Rev. D* 77 (2008) 126007, arXiv:0710.3883.
- [12] L. McAllister, E. Silverstein, A. Westphal, Gravity waves and linear inflation from axion monodromy, *Phys. Rev. D* 82 (2010) 046003, arXiv:0808.0706.
- [13] R. Flauger, L. McAllister, E. Pajer, A. Westphal, G. Xu, Oscillations in the CMB from axion monodromy inflation, *J. Cosmol. Astropart. Phys.* 1006 (2010) 009, arXiv:0907.2916.

- [14] R. Blumenhagen, V. Braun, T.W. Grimm, T. Weigand, GUTs in type IIB orientifold compactifications, *Nucl. Phys. B* 815 (2009) 1–94, arXiv:0811.2936.
- [15] R. Blumenhagen, S. Moster, E. Plauschinn, Moduli stabilisation versus chirality for MSSM like type IIB orientifolds, *J. High Energy Phys.* 0801 (2008) 058, arXiv:0711.3389.
- [16] D.S. Freed, E. Witten, Anomalies in string theory with D-branes, *Asian J. Math.* 3 (1999) 819, arXiv:hep-th/9907189.
- [17] A. Collinucci, M. Kreuzer, C. Mayrhofer, N.-O. Walliser, Four-modulus ‘Swiss Cheese’ chiral models, *J. High Energy Phys.* 0907 (2009) 074, arXiv:0811.4599.
- [18] M. Cicoli, C. Mayrhofer, R. Valandro, Moduli stabilisation for chiral global models, *J. High Energy Phys.* 1202 (2012) 062, arXiv:1110.3333.
- [19] V. Balasubramanian, P. Berglund, V. Braun, I. Garcia-Etxebarria, Global embeddings for branes at toric singularities, *J. High Energy Phys.* 1210 (2012) 132, arXiv:1201.5379.
- [20] M. Cicoli, S. Krippendorf, C. Mayrhofer, F. Quevedo, R. Valandro, D-branes at del Pezzo singularities: Global embedding and moduli stabilisation, *J. High Energy Phys.* 1209 (2012) 019, arXiv:1206.5237.
- [21] T.W. Grimm, M. Kerstan, E. Palti, T. Weigand, On fluxed instantons and moduli stabilisation in IIB orientifolds and F-theory, *Phys. Rev. D* 84 (2011) 066001, arXiv:1105.3193.
- [22] R. Blumenhagen, M. Schmidt-Sommerfeld, Power towers of string instantons for $N = 1$ vacua, *J. High Energy Phys.* 0807 (2008) 027, arXiv:0803.1562.
- [23] C. Petersson, P. Soler, A.M. Uranga, D-instanton and polyinstanton effects from type I’ D0-brane loops, *J. High Energy Phys.* 1006 (2010) 089, arXiv:1001.3390.
- [24] R. Blumenhagen, S. Moster, E. Plauschinn, String GUT scenarios with stabilised moduli, *Phys. Rev. D* 78 (2008) 066008, arXiv:0806.2667.
- [25] M. Cicoli, C. Burgess, F. Quevedo, Anisotropic modulus stabilisation: Strings at LHC scales with micron-sized extra dimensions, *J. High Energy Phys.* 1110 (2011) 119, arXiv:1105.2107.
- [26] M. Cicoli, F.G. Pedro, G. Tasinato, Poly-instanton inflation, *J. Cosmol. Astropart. Phys.* 1112 (2011) 022, arXiv:1110.6182.
- [27] M. Cicoli, G. Tasinato, I. Zavala, C. Burgess, F. Quevedo, Modulated reheating and large non-Gaussianity in string cosmology, arXiv:1202.4580.
- [28] R. Blumenhagen, X. Gao, T. Rahn, P. Shukla, A note on poly-instanton effects in type IIB orientifolds on Calabi–Yau threefolds, *J. High Energy Phys.* 1206 (2012) 162, arXiv:1205.2485.
- [29] M. Cicoli, F.G. Pedro, G. Tasinato, Natural quintessence in string theory, arXiv:1203.6655.
- [30] R. Blumenhagen, X. Gao, T. Rahn, P. Shukla, Moduli stabilization and inflationary cosmology with poly-instantons in type IIB orientifolds, arXiv:1208.1160.
- [31] X. Gao, P. Shukla, On non-Gaussianities in two-field poly-instanton inflation, *J. High Energy Phys.* 1303 (2013) 061, arXiv:1301.6076.
- [32] D. Lust, X. Zhang, Four Kahler moduli stabilisation in type IIB orientifolds with K3-fibred Calabi–Yau threefold compactification, *J. High Energy Phys.* 1305 (2013) 051, arXiv:1301.7280.
- [33] X. Gao, P. Shukla, On classifying the divisor involutions in Calabi–Yau threefolds, arXiv:1307.1139.
- [34] S. Weinberg, A new light boson?, *Phys. Rev. Lett.* 40 (1978) 223–226.
- [35] R. Peccei, H.R. Quinn, CP conservation in the presence of instantons, *Phys. Rev. Lett.* 38 (1977) 1440–1443.
- [36] F. Wilczek, Problem of strong p and t invariance in the presence of instantons, *Phys. Rev. Lett.* 40 (1978) 279–282.
- [37] J.E. Kim, Light pseudoscalars, particle physics and cosmology, *Phys. Rep.* 150 (1987) 1–177.
- [38] M.S. Turner, Windows on the axion, *Phys. Rep.* 197 (1990) 67–97.
- [39] J.E. Kim, G. Carosi, Axions and the strong CP problem, *Rev. Mod. Phys.* 82 (2010) 557–602, arXiv:0807.3125.
- [40] K. Freese, J.A. Frieman, A.V. Olinto, Natural inflation with pseudo-Nambu–Goldstone bosons, *Phys. Rev. Lett.* 65 (1990) 3233–3236.
- [41] P. Sikivie, Axion Cosmology, *Lect. Notes Phys.*, vol. 741, 2008, pp. 19–50, arXiv:astro-ph/0610440.
- [42] G.G. Raffelt, Stars as Laboratories for Fundamental Physics. The Astrophysics of Neutrinos, Axions, and Other Weakly Interacting Particles, University of Chicago Press, 1996.
- [43] WMAP Collaboration, E. Komatsu, et al., Five-year Wilkinson microwave anisotropy probe (WMAP) observations: Cosmological interpretation, *Astrophys. J. Suppl. Ser.* 180 (2009) 330–376, arXiv:0803.0547.
- [44] J.E. Kim, H.P. Nilles, A quintessential axion, *Phys. Lett. B* 553 (2003) 1–6, arXiv:hep-ph/0210402.
- [45] A. Chatzistavrakidis, E. Erfani, H.P. Nilles, I. Zavala, Axiology, *J. Cosmol. Astropart. Phys.* 1209 (2012) 006, arXiv:1207.1128.
- [46] A. Arvanitaki, S. Dimopoulos, S. Dubovsky, N. Kaloper, J. March-Russell, String axiverse, *Phys. Rev. D* 81 (2010) 123530, arXiv:0905.4720.

- [47] A. Arvanitaki, S. Dubovsky, Exploring the string axiverse with precision black hole physics, *Phys. Rev. D* 83 (2011) 044026, arXiv:1004.3558.
- [48] E.M. Burbidge, G.R. Burbidge, W.A. Fowler, F. Hoyle, Synthesis of the elements in stars, *Rev. Mod. Phys.* 29 (1957) 547–650.
- [49] K. Dasgupta, G. Rajesh, S. Sethi, M theory, orientifolds and G-flux, *J. High Energy Phys.* 9908 (1999) 023, arXiv:hep-th/9908088.
- [50] T.R. Taylor, C. Vafa, R–R flux on Calabi–Yau and partial supersymmetry breaking, *Phys. Lett. B* 474 (2000) 130–137, arXiv:hep-th/9912152.
- [51] E. Witten, Non-perturbative superpotentials in string theory, *Nucl. Phys. B* 474 (1996) 343–360, arXiv:hep-th/9604030.
- [52] M. Kerstan, T. Weigand, Fluxed M5-instantons in F-theory, *Nucl. Phys. B* 864 (2012) 597–639, arXiv:1205.4720.
- [53] R. Blumenhagen, M. Cvetič, S. Kachru, T. Weigand, D-brane instantons in type II orientifolds, *Annu. Rev. Nucl. Part. Sci.* 59 (2009) 269–296, arXiv:0902.3251.
- [54] M. Bianchi, A. Collinucci, L. Martucci, Magnetized E3-brane instantons in F-theory, *J. High Energy Phys.* 1112 (2011) 045, arXiv:1107.3732.
- [55] P. Berglund, I. Garcia-Etxebarria, D-brane instantons on non-spin cycles, *J. High Energy Phys.* 1301 (2013) 056, arXiv:1210.1221.
- [56] M. Berg, M. Haack, E. Pajer, Jumping through loops: On soft terms from large volume compactifications, *J. High Energy Phys.* 0709 (2007) 031, arXiv:0704.0737.
- [57] M. Cicoli, J.P. Conlon, F. Quevedo, General analysis of LARGE volume scenarios with string loop moduli stabilisation, *J. High Energy Phys.* 0810 (2008) 105, arXiv:0805.1029.
- [58] L. Anguelova, C. Quigley, S. Sethi, The leading quantum corrections to stringy Kahler potentials, *J. High Energy Phys.* 1010 (2010) 065, arXiv:1007.4793.
- [59] T.W. Grimm, R. Savelli, M. Weissenbacher, On α' corrections in $\mathcal{N} = 1$ F-theory compactifications, arXiv:1303.3317.
- [60] F.G. Pedro, M. Rummel, A. Westphal, Extended no-scale structure and α'^2 corrections to the type IIB action, arXiv:1306.1237.
- [61] I. Garcia-Etxebarria, H. Hayashi, R. Savelli, G. Shiu, On quantum corrected Kähler potentials in F-theory, *J. High Energy Phys.* 1303 (2013) 005, arXiv:1212.4831.
- [62] G. von Gersdorff, A. Hebecker, Kahler corrections for the volume modulus of flux compactifications, *Phys. Lett. B* 624 (2005) 270–274, arXiv:hep-th/0507131.
- [63] V. Balasubramanian, P. Berglund, Stringy corrections to Kahler potentials, SUSY breaking, and the cosmological constant problem, *J. High Energy Phys.* 0411 (2004) 085, arXiv:hep-th/0408054.
- [64] A. Westphal, de Sitter string vacua from Kahler uplifting, *J. High Energy Phys.* 0703 (2007) 102, arXiv:hep-th/0611332.
- [65] C. Burgess, R. Kallosh, F. Quevedo, De Sitter string vacua from supersymmetric D terms, *J. High Energy Phys.* 0310 (2003) 056, arXiv:hep-th/0309187.
- [66] A. Saltman, E. Silverstein, The scaling of the no scale potential and de Sitter model building, *J. High Energy Phys.* 0411 (2004) 066, arXiv:hep-th/0402135.
- [67] M. Cicoli, A. Maharana, F. Quevedo, C. Burgess, De Sitter string vacua from dilaton-dependent non-perturbative effects, *J. High Energy Phys.* 1206 (2012) 011, arXiv:1203.1750.
- [68] J.P. Conlon, The QCD axion and moduli stabilisation, *J. High Energy Phys.* 0605 (2006) 078, arXiv:hep-th/0602233.
- [69] T. Higaki, T. Kobayashi, Note on moduli stabilization, supersymmetry breaking and axiverse, *Phys. Rev. D* 84 (2011) 045021, arXiv:1106.1293.
- [70] D.J. Marsh, E.R. Tarrant, E.J. Copeland, P.G. Ferreira, Cosmology of axions and moduli: A dynamical systems approach, *Phys. Rev. D* 86 (2012) 023508, arXiv:1204.3632.
- [71] T.W. Grimm, Non-perturbative corrections and modularity in $N = 1$ type IIB compactifications, *J. High Energy Phys.* 0710 (2007) 004, arXiv:0705.3253.