A numerical and an analytical method for optimum planting date determination

Mahdi Khani a,*, Alireza Keyhani a, Reza Alimardani a, Hooman Sharifnasab b, Gholamreza Peykani c

a Department of Agricultural Machinery, Faculty of Agricultural Engineering and Technology, University of Tehran, Karaj, Iran
b Agricultural Engineering Research Institute, Ministry of Agriculture, Karaj, Iran
c Department of Agricultural Economics, Faculty of Agricultural Economics and Development, University of Tehran, Karaj, Iran

ARTICLE INFO

Article history:
Received 3 September 2014
Received in revised form
14 January 2015
Accepted 16 January 2015
Available online 7 February 2015

Keywords:
Scheduling
Timeliness index
Yield loss function
Reaching day

ABSTRACT

Apart from number and size of machines in a farm, operations scheduling can also affect timeliness costs. In this study a new equation was presented for determination of timeliness costs and it was proven that the new equation is a generalization of the well-known ASABE standard equation. Based on the new equation, a parameter namely, timeliness index was defined that indicates the distribution of an operation around the optimum time of performing the operation. By minimizing the index, the whole timeliness cost is minimized. Since the planting operation is highly sensitive to its time of accomplishment, timeliness cost was determined accordingly. In this study the optimum planting starting date (\( o_{pls} \)) regarding the concept of timeliness costs, was determined. If for any reason, the planting operation cannot be performed in the minimum planting period, an optimization method is required to determine \( o_{pls} \). For this, a numerical approach and an analytical method were presented for determination of \( o_{pls} \) and a computer model was developed based on the numerical method. \( o_{pls} \) was determined for the Research Farm of the University of Tehran in Karaj-Iran and both methods showed good agreement with each other.

© 2015 China Agricultural University. Production and hosting by Elsevier B.V. All rights reserved.

1. Introduction

1.1. Literature review of scheduling in agriculture

In an agricultural machinery system, costs are grouped into three categories: fixed (or ownership) costs, variable (or operating) costs, and timeliness costs. By changing the number and size of machines in a field, all costs categories change. Upon determination of field machines, fixed and variable costs are readily computed while timeliness costs are also influenced by field operations scheduling and are not easily attainable. Therefore, by optimum scheduling of operations, one can expect an increase in the total farm profits.

Scheduling is a decision-making process that is used on a regular basis in many manufacturing and services industries. It deals with the allocation of resources to tasks over given time periods and its goal is to optimize one or more objectives [1]. Van Elderen [2] considered operations scheduling on a farm, involving the management of the use of men and machinery for sowing, cultivating, harvesting, tilling, milking, etc.

Orfanou et al. [3] presented a planning approach for scheduling sequential tasks. Considering some factors such
as location and area of fields, available agricultural equipment, and the task time estimation, the approach determines, each machine should work in which field, in what sequence and in which period of time, to minimize the total time of operations (makespan). Considering moisture thresholds and in which period of time, to minimize the total time of each machine should work in which field, in what sequence and in which period of time, no timeliness costs are considered. Therefore, timeliness costs are considered just for this operation. A specific day as the optimum planting time (\( \text{d}_\text{ptpl} \)) can be considered for planting operation, in which the maximum crop yield is obtained with zero timeliness costs. Deviating from this specific day leads to timeliness costs accordingly.

In a specific area of a field, timeliness costs are the product of unit price of crop and yield loss due to performing the operation at an inappropriate time. Based on this definition, timeliness costs for the whole area of a field can be calculated using Eq. (1).

\[
C_{\text{ti}} = A \times Y \times L \times V
\]  

where: \( C_{\text{ti}} \) is the total timeliness costs ($), \( A \) is the area of the field (ha), \( Y \), crop yield (t ha\(^{-1}\)), \( V \), unit value of the crop ($ t^{-1} $) and \( L \), yield loss ratio (decimal).

According to ASABE standards [11], timeliness costs are calculated by Eq. (2).

\[
C_{\text{ti}} = \frac{K_t A^2 X Y V}{Z \times dh \times C_{\text{ct}} \times \text{plpwd}}
\]  

1.2. **Traditional timeliness costs determination**

In this study, minimization of timeliness costs is taken as the basis for agricultural operation scheduling. In the planting season, among plowing, disking, leveling and planting operations, the task which has the greatest sensitivity to the time of work is the planting operation. Therefore, timeliness costs are considered just for this operation. A specific day as the optimum planting time (\( \text{d}_\text{ptpl} \)) can be considered for planting operation, in which the maximum crop yield is obtained with zero timeliness costs. Deviating from this specific day leads to timeliness costs accordingly.

Other methods used in farm tasks scheduling are: stochastic programming [6], hybrid petri nets [7], metaheuristics [8], graphical evaluation and review technique [9] and hill climbing search method [10].

**Nomenclature**

- A: field area (ha)
- apls: allowable planting start date (d)
- C\(_{1}, \ldots, C_{10}\): constant parameters used in determination of opls
- C\(_{\text{ef}}\): effective field capacity of a single planter (ha h\(^{-1}\))
- C\(_{\text{ag}}\): aggregated capacity of planters (ha h\(^{-1}\))
- C\(_{\text{ct}}\): the total timeliness cost ($)
- d: Disking
- da: planted area in a time unit (dt) (ha)
- dh: number of working hours in a day (h d\(^{-1}\))
- ds: allowable disk start date (d)
- dt: time unit (d)
- ed: end day (d)
- empip: end of minimum planting period (d)
- K\(_{\text{l}}\): yield loss coefficient (d\(^{-1}\))
- l: Leveling
- L: yield loss ratio (decimal)
- minti: minimum timeliness index (M d\(^{2}\))
- mlp: minimum planting period (d)
- op: optimum planting time (d)
- opls: optimum time for start of planting (d)
- p: plowing
- pl: planting
- pls: planting start date (d)
- pltwr: planting to tractor work ratio
- plw_ps_rd: planting work from planting start to reaching day (M d)
- q: predetermined value (d)
- rd: reaching day (d)
- smplp: start date of minimum planting period (d)
- sxc: single x machine capacity. Instead of “x” which is the symbol of specific machine or operation (p, d, l, pl, s and t) are used (ha h\(^{-1}\))
- sw_pls_se: seedbed preparation work from planting start date to the end of seedbed preparation (M d)
- sw_ds_ls: seedbed preparation work from disking to leveling start dates (M d)
- sw_ps_ds: seedbed preparation work from plowing to disking start dates (M d)
- sw_ps_ls: seedbed preparation work from plowing to leveling start dates (M d)
- sw_ps_pls: whole seedbed preparation from plowing to planting start dates
- sw_ps_rd: seedbed preparation work from plowing start date to reaching day (M d)
- sw_pls_rd: seedbed preparation work from planting start date to reaching day (M d)
- sw_ds_ls: seedbed preparation work from disking to leveling start dates (M d)
- sw_ds_pls: seedbed preparation work from disking to planting start dates (M d)
- sw_pls_se: seedbed preparation work from planting start date to the end of seedbed preparation (M d)
- sr: sensitivity ratio (decimal)
- t: tractor
- ti: timeliness index (M d\(^2\))
- tlcost: timeliness cost ($)
- u: an arbitrary number used to determine the new time step size
- v: time step size (d)
- y: unit value of crop ($ t^{-1} $)
- xef: x operation field efficiency (decimal)
- xn: number of all x machines
- xpwd: probability of a working day for x operation (decimal)
- xsp: x operation work speed (km h\(^{-1}\))
- xw: the total machine work of x operation (M d)
- xref: x machine working width (m)
- y: crop yield (t ha\(^{-1}\))
- Z: indicator of work distribution around optimum planting time
- Z: indicator of work distribution around optimum planting time
- Z: indicator of work distribution around optimum planting time
- Z: indicator of work distribution around optimum planting time
where: \( plpud \) is probability of a working day for planting operation (decimal), \( dh \) number of working hours in a day (h), \( dC_{0} \) aggregated capacity of planters (ha h \(^{-1})\), \( K_{a} \) yield loss coefficient (d \(^{-1})\) and \( Z \) is an indicator of work distribution around the optimum planting time and its value is 4 when \( op \) is in the middle of the work period and is 2 when \( op \) is at the beginning or end of time period.

\[ L(t) = K_{a} \times (t - op) \]  
\[ (3) \]

(2) Number of active planters and consequently the total machines capacity is constant and after the start of all planters can work with their maximum effective capacity without any limitation. Hence, planting operation can be completed in minimum planting period \( (mplp) \) calculated by Eq. (4).

\[ mplp = \frac{A}{C_{at} \times dh \times plpud} \]  
\[ (4) \]

\( mplp \) is defined as the period in which \( op \) is located in its middle.

(3) Planting operation scheduling is performed in one of the three states as follows.

State 1, planting operation is performed in \( mplp \). The operation starts at the start of \( mplp \) \( (smplp = op - 0.5 \times mplp) \) and finishes at the end of \( mplp \) \( (emplp = op + 0.5 \times mplp) \). If planting operation is carried out in this time, the least timeliness costs are resulted and the value of \( Z \) is taken as 4.

State 2, the operation starts at \( op \) and finishes after the time duration of \( mplp \) \( (Z = 2) \).

State 3, the operation is completed at \( op \). Therefore, the operation must start before this point with time duration of \( mplp \) \( (Z = 2) \).

If planting operation can be performed in \( mplp \), the least timeliness costs are expected and there is no need to perform any optimization method for planting scheduling. However, in some circumstances, completion of the operation in this period of time is not possible, for example, if \( smplp \) stands before allowable planting start date \( (apls) \). Also, due to time limitations and low field capacities of seedbed preparation machines (plow, disk and leveler), before the completion of seedbed preparation operation, the whole already prepared area are planted. Therefore the capacity of planters is restricted by that of seedbed machines. In this condition, by commencing planting operation in \( smplp \), the operation cannot be completed in \( emlplp \). Therefore, Eq. (2) cannot be used to determine timeliness costs and methods should be developed from which one can calculate these costs for variable planter capacities. In such conditions, \( smplp \), will not be necessarily the optimum time for start of the planting operation.

The objectives of this study are: (1) Generalization of Eq. (2) to determine timeliness costs in order to create flexibility about the total planter capacities and the time of performing the operation. (2) Presenting a numerical and an analytical methods for determination of optimum time for start of planting \( opls \).

2. Materials and methods

2.1. Generalization of timeliness costs equation

In this study, the work time of a single tractor-implement set, instead of area, was considered as basis of work progress. It means that how many days are required for completion of a specified operation, in a given area of field. It is apparent that planting operation cannot be performed for whole area of a field at once and the operation is completed during a period of time. Therefore every unit area of the field is planted in a particular time. Thus, timeliness costs for each part will be different. The total timeliness costs for the whole field is obtained by summing all costs in all parts of a field. Based on Eq. (1), timeliness costs in a time unit \( (dC_{t}) \) is calculated using Eq. (5).

\[ dC_{t} = Y \times V \times L(t) \times dA \]  
\[ (5) \]

where \( dA \) is the planted area in a time unit \( (dt) \), calculated according to Eq. (6).

\[ dA = C_{at} \times dh \times plpud \times dt \]  
\[ (6) \]

Among the parameters, only the total field capacity of planters and yield loss ratio both are functions of the operation time and other parameters are constant during the operation. Thus, combining Eqs. (5) and (6), the total timeliness costs for a given period is determined by Eq. (7). This equation is considered as universal equation for determination of timeliness costs.

\[ C_{at} = dh \times plpud \times Y \times V \times \int_{t1}^{t2} C_{at} \times L(t) \times dt \]  
\[ (7) \]

where \( t1 \) and \( t2 \) are the start and the end of a given period of time, respectively.

Considering constant field capacity for planters and by setting \( smplp \) and \( emlplp \) as the start and the end of the planting period (the first case in the ASABE standards), Eq. (7) is converted to Eq. (2) in which \( Z \) is replaced with 4. At the same way, solving Eq. (7), in the second and third cases, again Eq. (2) is obtained with a constant number 2 in the denominator (Appendix).

Therefore, Eq. (2) is a special case of Eq. (7) and the method used in this study to determine timeliness costs is a generalization of the method used in the literature (ASABE Standards, 2006a).

In this study, yield loss ratio is determined by Eq. (3) as well. But the total capacity of planters and start and end time of planting operation have more flexibility. Field capacity of a single machine is constant and according to Eq. (8), the only factor which can influence the total capacity of planters is the number of active planters at any time.

\[ C_{at}(t) = pln(t) \times C_{a} \]  
\[ (8) \]

where: \( C_{a} \) is effective field capacity of a single planter \( (ha h^{-1}) \) and \( pln \) is the number of all planters. When \( pln \) is written as a function of time, it denotes the number of active planters.
Equation (9) results from Eqs. (3),(7) and (8)
\[
C_t = C_p \times dh \times plpwd \times Y \times V \times K_t \times \int_0^{t_2} p(t) \times |t - op| \times dt
\]  

(9)

Eq. (10) introduces a parameter namely timeliness index \((TI)\) which is an indicator of distribution of planting work around \(op\).
\[
TI = \int_0^{t_2} p(t) \times |t - op| \times dt
\]  

where \(TI\) is timeliness index \((M \, d^2)\).

Substituting \(TI\) into Eq. (9), results:
\[
C_t = C_p \times dh \times plpwd \times K_t \times Y \times V \times \int_0^{t_2} p(t) \times |t - op| \times dt
\]  

(11)

Timeliness index \((TI)\) is the only parameter which is affected by the time of planting operation while other parameters are constant. Therefore, by reducing this index, planting operation is performed nearer to the \(op\) and timeliness costs are reduced.

2.2. Timeliness cost determination

Calculation of \(TI\) and consequently timeliness costs will be different in constant or in variable planters capacity.

2.2.1. Constant planters capacity

Obviously, \(TI\) will be minimum when all planters are active and \(op\) is in the middle of planting period i.e. the first state (ASABE Standards, 2006a).

The start of planting operation \((ppls)\) is not possible before \(apis\). Therefore, the start and the end of planting operation \((ple)\) are determined by Eqs. (12) and (13).

\[
\begin{align*}
\text{ppls} &= \max (\text{smplp}, \text{apis}) \\
\text{ple} &= \text{ppls} + \text{mplp}
\end{align*}
\]  

(12)

(13)

Since the number of active planters is constant, the timeliness index is calculated by Eq. (14).
\[
TI = \int_0^{\text{ppls}} (op - t) \times dt + \int_{\text{op}}^{\text{ple}} (t - op) \times dt
\]  

(14)

This integral, leads to Eq. (15).
\[
TI = \frac{\text{ppls}}{2} \times ((op - \text{ppls})^2 + (ple - \text{op})^2)
\]  

(15)

2.2.2. Variable planters capacity

In circumstances in which before completion of seedbed preparation operation in the whole area of the field, the whole prepared area is planted, planters set cannot continue their work retaining their previous capacity and the progress rate of planting operation is limited by the progress rate of seedbed preparation operation. Hence, two cases are considered for planting operation scheduling. In the first case, after determining the start of planting operation, all planters, commence the job and continue their work at maximum capacity, until the whole area is finished. In this case, timeliness index is calculated using Eq. (15). In the second case, planting and seedbed preparation operations have a reaching day \((rd)\) before completion of the operations.

In order to determine whether or not the planting operation reaches to the seedbed preparation operation, the whole seedbed preparation work performed before \(ppls\) must be calculated. After \(ppls\), tractors are redistributed among planters and seedbed preparation machines, thus, the total capacity of seedbed preparation machines changes accordingly. Using the work capacity and the remained work for both operations, the end of operations are determined separately. If planting is finished after the end of seedbed preparation, the first case will occur, otherwise, planting operation reaches to the seedbed preparation operation and the second case occurs.

In this study it is assumed that at first, plowing, diskling and leveling operations commence at their allowable start time, then, limited to the number of tractors, all seedbed preparation implements start their work afterwards. At \(ppls\), all planters come into action and remaining tractors stay at seedbed preparation operation. After \(rd\), tractors are distributed among operations so that the total progress rate based on cultivated area of both operations become equal, therefore, the operations are completed simultaneously at a time called “end day” \((ed)\). These assumptions have been made in order to minimize timeliness costs.

Depending on how to determine the start time of planting, \(rd\) may be before or after \(op\). Fig. 1 illustrates an example of distribution of tractors among planters and seedbed preparation machines during the cultivation season considering \(rd\) is after \(op\). In this example the number of tractors, planters, and seedbed preparation implements are 6, 4 and 8 (3, 2 and 3 for plow, disk, and leveler, respectively).

At the first stage, the effective capacity for each machine type is determined. For example, the effective capacity of a single plow and the whole machine work required for plowing operation are determined using Eqs. (16) and (17).

\[
\text{spc} = \frac{\text{pwth} \times \text{psp} \times \text{pef}}{10}
\]  

(16)

\[
\text{pw} = \frac{A}{\text{spc} \times \text{dh} \times \text{ppwd}}
\]  

(17)

where: \(\text{spc}\) is the single plow capacity \((\text{ha} \, h^{-1})\), \(\text{pwth}\), plow working width \((\text{m})\), \(\text{psp}\), plow speed \((\text{km} \, h^{-1})\), \(\text{pef}\), plow field efficiency \((\text{decimal})\), \(\text{pw}\), the total machine work of plowing \((\text{M} \, \text{d})\), and \(\text{ppwd}\), is the probability of a working day for plowing operation \((\text{decimal})\).

Similar equations are used for other operations. The same parameter names are used for other machines except for the first letter, “p”, “d”, “l”, and “pl” are used for plow, disk, leveler and planter, respectively. Also, “t” and “s” will be indicators of tractor and seed bed preparation, respectively.

The total seedbed preparation work \((\text{sw})\) is obtained by summing the total work of plowing \((\text{pw})\), diskling \((\text{dw})\), and leveling \((\text{lu})\). The total tractor work \((\text{tu})\) is obtained by summing \(\text{sw}\) and the total planting work \((\text{plw})\). Seedbed preparation to planting work ratio \((\text{splwr})\) and planting to tractor work ratio \((\text{pltwr})\) is obtained by dividing \(\text{sw}\) to \(\text{plw}\) and \(\text{plw}\) to \(\text{tu}\), respectively.

At first, it is assumed that planting operation is started at the start of \(mplp\) \((\text{ppls} = \text{mplp})\) and based on this assumption, work progress calculations are carried out for planting and seedbed preparation operations. According to Eqs. (18)–(22),
by calculation of seedbed preparation work from ps to ds (sw\_ps\_ds), from ds to ls (sw\_ds\_ls), and from ls to pls (sw\_ls\_pls), the whole seedbed preparation operation from ps to pls (sw\_ps\_pls) and seedbed preparation work from pls to the end of seedbed preparation operation (sw\_pls\_se) are determined.

\[ sw\_ps\_ds = \min (pw, pn \times (ds - ps)) \]  
(18)

\[ Sw\_ds\_ls = \min ((dw + pw - sw\_ps\_ds), (\min (tn, pn + dn) \times (ls - ds))) \]  
(19)

\[ sw\_ls\_pls = \min (tn, sn) \times (pls - ls) \]  
(20)

\[ sw\_ps\_pls = \min (sw\_ps\_ds + sw\_ds\_ls + sw\_ls\_pls) \]  
(21)

\[ sw\_pls\_se = sw - sw\_ps\_pls \]  
(22)

From pls to the end of seedbed preparation (se), only tractors which are not at planting operation, can stay at seedbed preparation, therefore:

\[ sw\_pls\_se = (se - pls) \times (tn - pln) \]  
(23)

and for determination of se, Eq. (23) is changed to Eq. (24).

\[ se = pls + \frac{sw\_pls\_se}{tn - pln} \]  
(24)

where pn, dn, ln, pln, sn and tn are the number of plows, disks, levelers, planters, seedbed preparation machines and tractors, respectively.

If \( se < ple \), there is no reaching day (or crossing day) and timeliness index is determined by Eq. (15). Otherwise, the planting operation reaches to seedbed preparation and for determination of TI, rd and ed needs to be calculated.

The presented ratios for the total work of different machines are also valid for any unit area in which all operations are completed. At rd, all operations have the same work progress; therefore, the ratio between the seedbed preparation work from ps to rd (sw\_ps\_rd) and planting work from pls to rd (plw\_ps\_rd) is equal to splwr. Therefore:

\[ \frac{sw\_ps\_rd}{plw\_ps\_rd} = \frac{sw}{plw} \]  
(25)

Hence:

\[ \frac{sw\_ps\_pls + (rd - pls) \times (tn - pln)}{(rd - pls) \times pln} = \frac{sw}{plw} \]  
(26)

By reforming Eqs. (26) and (27) is obtained which is used for determination of rd.

\[ rd = pls + \frac{sw\_ps\_pls}{pln \times plw - tn + pln} \]  
(27)

According to Eq. (28), the total tractor work from ps to ed is obtained by summing the work performed up to pls, including the seedbed preparation work and tractor work from pls to ed.

\[ tw = sw\_ps\_pls + (ed - pls) \times tn \]  
(28)

Therefore, ed is obtained from Eq. (29).

\[ ed = pls + \frac{tw - sw\_ps\_pls}{tn} \]  
(29)

In the first case, where the reaching day of planting operation and seedbed preparation operation occurs after the optimum planting time (op \(< rd \)), the whole planting period is divided into three intervals. The first interval is from pls to op, the second is from op to rd and the third is from rd to ed.

In both first and second intervals, the number of planters at work is equal to all available planters, while in the third interval, only tractors work by their whole number. In this condition, tractor allocation to these operations is such that, the progressing rates of the operations become equal. Therefore, the number of planters in this interval is equal to the number of tractor multiplied by the work ratio of planting work to tractor work for completion of a unit area (pltwr). As a result, Eq. (10) is changed to Eq. (30) and then to Eq. (31).

\[ TI = \int_{pln}^{op} \frac{pln \times (op - t) \times dt}{ex} + \int_{rd}^{ed} \frac{pln \times (t - op) \times dt}{ex} \]  
(30)

\[ TI = \frac{pln}{2} \times ((op - pls)^2 + (rd - op)^2) \]  
(31)

rd and ed are functions of pls and they are calculated using Eqs. (27) and (29). Therefore, pls is the only independent variable in Eq. (31) and other parameters are constants. Thus, by determination of pls, TI and consequently timeliness costs are calculated.

In the second case (rd < op), the whole planting period is divided into three intervals as well. The first interval is from pls to rd, the second is from rd to op and the third is from op to ed. In this case, Eq. (10) is changed to Eq. (32) and then to Eq. (33).

\[ TI = \int_{pln}^{op} \frac{pln \times (op - t) \times dt}{ex} + \int_{rd}^{op} \frac{tn \times pltwr \times (op - t) \times dt}{ex} \]  
(32)

\[ TI = pln \times \left( \frac{pln^2 - rd^2}{2} + op \times (rd - pls) \right) \]  
(33)
2.3. Determination of optimum planting start date

In circumstances in which planting and seedbed preparation operations have a reaching day, an analytical method can be used to find optimum time for start of planting operation (ops) as well. In this method, by differentiating Eqs. (31) or (33) with respect to ops, the minimum TI is determined. In these equations, rd and ed are variables which are functions of ops. rd can be calculated using Eq. (27). One of the inputs of the equation is seedbed preparation work from ps to ops which is a function of ops and is determined using Eq. (34).

\[ Sw_{ps, ops} = Sw_{ps, ls} + Sw_{ls, ops} \]  

Equations (20) and (35) are used to determine \( Sw_{ls, ops} \) (seedbed preparation work from ls to ops) and \( Sw_{ps, ls} \) (seedbed preparation work from ps to ls), respectively.

\[ Sw_{ps, ls} = Sw_{ps, ds} + Sw_{ds, ls} \]  

To express the relation between rd and ops, Eq. (36) is obtained by combining Eqs. (27) and (35). C1 and C2 are parameters which are independent from ops and are determined using Eqs. (37) and (38).

\[ rd = C1 \times ops + C2 \]  

\[ C1 = 1 + \frac{\min(tn, sn)}{pln \times (\frac{tw}{tn} + 1) - tn} \]  

\[ C2 = \frac{sw_{ps, ls} - ls \times \min(tn, sn)}{pln \times (\frac{tw}{tn} + 1) - tn} \]  

Also, to express the relation between ed and ops, Eq. (39) is obtained by combining Eqs. (29) and (35). C3 and C4 are parameters which are independent from ops and are determined using Eqs. (40) and (41).

\[ ed = C3 \times ops + C4 \]  

\[ C3 = 1 - \frac{\min(tn, sn)}{tn} \]  

\[ C4 = \frac{tw - sw_{ps, ls} + ls \times \min(tn, sn)}{tn} \]  

If the number of seedbed preparation machines (sn) does not exceed the number of tractors (tn), the value of C3 becomes zero and ed will not be dependent on ops.

Inserting rd, ed, rd2, and ed2 values into Eq. (31), and after simplification, Eq. (42) is obtained. C5, C6, and C7 are parameters which are independent from ops and are determined using Eqs. (43)-(45), respectively.

\[ TI = C5 \times ops^2 + C6 \times ops + C7 \]  

\[ C5 = \frac{pln}{2} \times (C1^2 + 1) + tn \times \frac{plu}{tw} \times \left( \frac{C3^2 - C4^2}{2} \right) \]  

\[ C6 = \frac{pln}{2} \times C1 \times C2 \]  

\[ C7 = \frac{sw_{ps, ls}}{pln} \times (C1 + 1) \]
C6 = pln × (C1 × C2 − op − C1 × op) + tn × \( \frac{plw}{tw} \) × (C3 × C4 − C1 × C2 + op × (C1 − C3))

\( C7 = pln \times \left( \frac{op^2}{2} + \frac{C3^2}{2} \right) - op \times C2 + tn \times \frac{plw}{tw} \times \left( \frac{C4^2}{2} - \frac{C2^2}{2} + op \times (C2 - C4) \right) \)

By setting the differentiation of Eq. (42) (with respect to \( op \)), equal to zero, the optimum planting start time (opls) is determined as in Eq. (46).

\[ opls = -\frac{C6}{2 \times C5} \]

Although it seems unlikely that by applying opsls, \( rd \) occurs before \( op \), determination of opsls based on such assumption, is carried out as well. In this state, by simplification and differentiation of Eq. (33), Eq. (47) is developed to determine opsls.

\[ opsls = -\frac{C9}{2 \times C8} \]

\[ C8 = \frac{p}{2} \times (1 - C1^2) + tn \times \frac{plw}{tw} \times \left( \frac{C1^2 + C3^2}{2} \right) \]

\[ C9 = pln \times (op \times C1 - C1 \times C2 - op) + tn \times \frac{plw}{tw} \times (C1 \times C2 + C3 \times C4 - op \times (C1 + C3)) \]

2.4. Real case study

A problem is raised that, which one of Eqs. (46) or (47), is used to determine the optimum planting start time. To answer the question, real data from the research farm of College of Agriculture and Natural Resources of University of Tehran located in Karaj-Iran (35.50°N, 50.58°E) were taken into account for comparison between numerical and analytical methods using both Eqs. (46) and (47).

The field is 170 ha with two crops (wheat and forage maize). After maize harvesting at the end of summer and beginning of fall, there is very little time for tillage and planting of wheat. Regarding that the disk operation is performed two times in the planting season, for calculating the total disk work, the area for this operation was doubled.

Probability of a working day had been determined 0.48 for tillage operation [12] and was considered to be the same for all operations. Work speed and field efficiency for all operations were taken from [13]. Model inputs for the case study was displayed in Table 1. Start time of each operation is based on day number from the beginning of January.

By changing the ratio between the whole planting capacity and the whole seedbed preparation capacity, the \( rd \) changes accordingly. This ratio can be changed by changing each input parameter such as working width, work speed, field efficiency and number of machines. For changing the capacity ratio of these categories of operations, planter width was changed from very small to very large values to see if it is possible that in an optimized condition, \( rd \) occurs before \( op \). The actual planter work width (plwt) was 2.5 m but in subsequent runs, 0.5 m, 0.7 m, 1 m, 5 m, 10 m, 20 m, and 50 m work widths were tested as well.

2.5. Sensitivity analysis

When there is no crossing day between planting and seedbed preparation, Eq. (2) can be used to calculate timeliness cost. According to the equation, timeliness cost has quadratic relationship with field area (\( A \)) and direct relationship with yield (\( Y \)), yield loss coefficient (\( K_t \)) and crop value (\( V \)). Also it has inverse relation with probability of a working day (\( pw \)) and number of working hours in a day (\( dh \)). All components of effective field capacity i.e. speed, width and field efficiency have inverse relationship with timeliness cost. However if there is a crossing day between operations, Eq. (2) cannot be used for determination of timeliness cost. Thus for understanding the relationships among input data and timeliness cost, a sensitivity analysis is needed.

Sensitivity analysis was performed for \( A \) and \( dh \). By increasing and decreasing 1, 2 and 10 percent to the values of these variable, changes in timeliness cost was surveyed. \( K_t \), \( Y \) and \( V \) were taken, 0.001, 5 ton ha\(^{-1} \) and 600 $ ton\(^{-1} \). Other data are the same data in Table 1. Using Eq. (50), sensitivity ratio (SR) between timeliness cost and a variable such as field area was calculated.

\[ SR = \frac{(C_{tt} - C_{tt})}{C_{tt}} \times \frac{A - A_t}{A_t} \]

Where \( C_{tt} \) and \( A_t \) are initial values for timeliness cost and field area. Similar equation was used for \( dh \). It is apparent that variables \( pw \), width, speed and field efficiency are similar to \( dh \) in terms of their impacts on timeliness cost, therefore there is no need to perform sensitivity analysis for them.

Like no crossing day case, there is a direct relationship among \( K_t \), \( V \) and \( Y \) with timeliness cost. Because, these parameters only are used in calculation of timeliness cost (Eq. (11)) and they have no effect on timeliness index.

3. Results and discussion

3.1. Real case study

The results of running the model by different planter widths are shown in Table 2. The number of decimal points is for comparison only.
In Table 2, \( opl_{s1} \) and \( opl_{s2} \) is the optimum planting start time resulted from the numerical method, Eq. (46) \( (op < rd) \) and Eq. (47) \( (rd < op) \) respectively. Other parameters \( rd, ed, smplp, emplp \), and \( TI \) are based on planting start time resulted from the numerical method \( opl_{s} \).

By running the model by different data, it was observed that, \( opl_{s} \) is always after \( smplp \) and \( rd \) is always after \( op \). By increasing \( plwth \), the total capacity of planting operation increases and consequently the total planting work (based on machine-day unit) decreases. According to Table 2, by increasing \( plwth \), \( rd \) approaches to \( op \) \( (290) \) but never occurs before that.

Therefore, it can be concluded that if the yield loss function is expressed in the form of Eq. (3), by applying \( opl_{s} \), planting operation always reaches to seedbed preparation operation after \( op \). Thus, in the analytical method it is justified to consider \( rd \) to be after \( op \).

Results of the analytical method, (Eq. 53) is in agreement with that of numerical approach while Eq. 59 always resulted a constant value \( opl_{s2} = 276 \). Therefore, the basis assumption of Eq. 59 \( (rd < op) \) is not correct. In 0.5 m and 0.7 m width, in both numerical and analytical methods (Eq. 53), primary calculated values for \( opl_{s} \) were less than that of \( ap_{ls} \). In the computer model (based on the numerical method), values were corrected by \( ap_{ls} \). In 0.5 m width, due to very low rate of planting operation, this operation never reaches to seedbed preparation operation and \( TI \) is calculated by Eq. (15).

### 3.2. Sensitivity analysis

Tables 3 and 4 shows the effects of changes in \( A \) and \( dh \) on timeliness cost. Just by differentiating from Eq. (2), for constitutive parameters of timeliness cost can be determined. \( SR \) for \( A \) and \( dh \) will be 2 and 1, respectively. But when there is a crossing day, \( SR \) is calculated by use of sensitivity analysis.

According to Table 3, \( SR \) is about 2.6 for one percent change in \( A \). Thus, in comparison with no crossing state, changes in field area has much effects on timeliness cost in situation where there is a crossing day.

### Table 2 – Results of running the model by different planter widths.

<table>
<thead>
<tr>
<th>Planter width</th>
<th>Numerical method</th>
<th>Analytical method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( opl_{s} )</td>
<td>( rd )</td>
</tr>
<tr>
<td>0.5</td>
<td>281</td>
<td>–</td>
</tr>
<tr>
<td>0.7</td>
<td>281.001</td>
<td>322.73</td>
</tr>
<tr>
<td>1</td>
<td>282.0959</td>
<td>297.91</td>
</tr>
<tr>
<td>2.5</td>
<td>286.8389</td>
<td>293.16</td>
</tr>
<tr>
<td>5</td>
<td>284.4199</td>
<td>291.58</td>
</tr>
<tr>
<td>10</td>
<td>289.2105</td>
<td>290.79</td>
</tr>
<tr>
<td>20</td>
<td>289.6051</td>
<td>290.4</td>
</tr>
<tr>
<td>50</td>
<td>289.8424</td>
<td>290.16</td>
</tr>
</tbody>
</table>

### Table 3 – Results of sensitivity analysis for field area.

<table>
<thead>
<tr>
<th>( A )</th>
<th>( TI )</th>
<th>( C_{tt} )</th>
<th>( (A - A_{i})/A_{i} )</th>
<th>( (TI - TI_{i})/TI_{i} )</th>
<th>( (C_{tt} - C_{tt_{i}})/C_{tt_{i}} )</th>
<th>( SR )</th>
</tr>
</thead>
<tbody>
<tr>
<td>170</td>
<td>599</td>
<td>7710</td>
<td>0.01</td>
<td>0.026</td>
<td>0.026</td>
<td>2.65</td>
</tr>
<tr>
<td>171.7</td>
<td>615</td>
<td>7915</td>
<td>–0.01</td>
<td>–0.026</td>
<td>–0.026</td>
<td>2.61</td>
</tr>
<tr>
<td>168.3</td>
<td>583</td>
<td>7509</td>
<td>–0.02</td>
<td>–0.052</td>
<td>–0.052</td>
<td>2.59</td>
</tr>
<tr>
<td>173.4</td>
<td>631</td>
<td>8122</td>
<td>0.02</td>
<td>0.053</td>
<td>0.053</td>
<td>2.67</td>
</tr>
<tr>
<td>166.6</td>
<td>568</td>
<td>7310</td>
<td>0.02</td>
<td>–0.052</td>
<td>–0.052</td>
<td>2.59</td>
</tr>
<tr>
<td>187</td>
<td>767</td>
<td>9876</td>
<td>0.1</td>
<td>0.281</td>
<td>0.281</td>
<td>2.81</td>
</tr>
<tr>
<td>153</td>
<td>452</td>
<td>5819</td>
<td>–0.1</td>
<td>–0.245</td>
<td>–0.245</td>
<td>2.45</td>
</tr>
</tbody>
</table>

### Table 4 – Results of sensitivity analysis for the number of working hours in a day.

<table>
<thead>
<tr>
<th>( dh )</th>
<th>( TI )</th>
<th>( C_{tt} )</th>
<th>( (dh - dh_{i})/dh_{i} )</th>
<th>( (TI - TI_{i})/TI_{i} )</th>
<th>( (C_{tt} - C_{tt_{i}})/C_{tt_{i}} )</th>
<th>( SR )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>599</td>
<td>7710</td>
<td>0.01</td>
<td>0.026</td>
<td>–0.016</td>
<td>–1.61</td>
</tr>
<tr>
<td>10.1</td>
<td>584</td>
<td>7586</td>
<td>–0.01</td>
<td>0.027</td>
<td>0.016</td>
<td>–1.65</td>
</tr>
<tr>
<td>9.9</td>
<td>615</td>
<td>7838</td>
<td>–0.01</td>
<td>0.027</td>
<td>0.016</td>
<td>–1.65</td>
</tr>
<tr>
<td>10.2</td>
<td>569</td>
<td>7464</td>
<td>0.02</td>
<td>–0.051</td>
<td>–0.032</td>
<td>–1.6</td>
</tr>
<tr>
<td>9.8</td>
<td>632</td>
<td>7967</td>
<td>–0.02</td>
<td>0.054</td>
<td>0.033</td>
<td>–1.67</td>
</tr>
<tr>
<td>11</td>
<td>465</td>
<td>6578</td>
<td>0.1</td>
<td>–0.224</td>
<td>–0.147</td>
<td>–1.47</td>
</tr>
<tr>
<td>9</td>
<td>787</td>
<td>9120</td>
<td>–0.1</td>
<td>0.314</td>
<td>0.183</td>
<td>–1.83</td>
</tr>
</tbody>
</table>
According to Table 4, SR is about 1.6 for one percent change in \( dh \) whereas its value is one for no crossing state. Therefore in comparison with no crossing state, in situation that crossing between operations occurs, changes in parameters such \( dh, pwd \), cause major changes in timeliness cost.

4. Conclusion

The scheduling model can be used as a sub-model in models which compare different workable machinery combinations. For comparison of the total costs of machinery combinations, these models require the timeliness cost. This task can be carried out by a scheduling model. If there are a large number of machinery combinations, number of calculations becomes very high. In such conditions, analytical method can reduce the total time of computations. Also analytical method can be used in optimization models which they are based on mathematical programming.

One of the advantages of the numerical approach to analytical method is its more flexibility to the form of yield loss function and diversity of machines size. If the yield loss function is nonlinear and/or more than one type of planter with different work capacities exist, the analytical method becomes very complex. However, it seems that in such condition, application of Eq. (7) accompanied with numerical method, is a proper way for field tasks scheduling.

In this study, at first a general equation for timeliness cost was introduced (Eq. (7)). Then by determining yield loss function and diversity of machines size, if the yield loss function is linear and total capacity of planters is constant. Therefore by setting Eq. (3) instead of yield loss function, is used in the denominator.

The authors would like to thank Eng. Rafee, the manager of University of Tehran’s Research Farm for providing the valuable information about the farm and its machines and time limitations of operations.

Appendix

Deriving Eq. (2) from Eq. (7).

In the proposed equation by ASAE (Eq. (2)), it is assumed that yield loss function is linear and total capacity of planters is constant. Therefore by setting Eq. (3) instead of yield loss function in Eq. (7) and shifting total planters capacity from the integral (because of independency of total capacity from time), Eq. (A) is obtained.

\[
C_{lt} = C_{at} \times dh \times plpwd \times Y \times V \times K_t \int_{t_1}^{t_2} (t - op) \times dt \quad (A)
\]

In the first state, planting operation is performed in the minimum planting period \((mulp)\). The start, the middle and the end of the period are \( smulp, op \) and \( emulp \) respectively. Therefore Eq. (B) is attained from Eq. (A).

\[
C_{lt} = C_{at} \times dh \times plpwd \times Y \times V \times K_t \left( \int_{op-05 \times mulp}^{op} (t - op) \times dt + \int_{op}^{op+05 \times mulp} (t - op \times dt) \right) \quad (B)
\]

By solving the integral, Eq. (C) is resulted.

\[
C_{lt} = C_{at} \times dh \times plpwd \times Y \times V \times K_t \times \frac{0.25 \times mulp^2}{4 \times dh \times C_{at} \times plpwd} \quad (D)
\]

In the second state of ASAE, the start and the end of planting period are \( op \) and \( op + mulp \), respectively. In this state, Eq. (E) is derived from Eq. (A).

\[
C_{lt} = C_{at} \times dh \times plpwd \times Y \times V \times K_t \times \int_{op}^{op+mulp} (t - op) \times dt \quad (E)
\]

Solving the integral by use of Eq. (4), Eq. (F) is obtained.

\[
C_{lt} = C_{at} \times dh \times plpwd \times Y \times V \times K_t \times \frac{2}{4 \times dh \times C_{at} \times plpwd} \quad (F)
\]

By solving Eq. (A) in the third state (where the start and the end of the planting period are \( op-mulp \) and \( op \) respectively), Eq. (F) is resulted again. As it is seen, Eq. (2), is equal to Eqs. (C) and (E) where instead of numbers 2 and 4, a parameter \( Z \) is used in the denominator.

References

