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Contouring accuracy improvement using an adaptive feedrate planning method for CNC machine tools

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Abstract

The reduction of contour error plays an important role in achieving high accuracy machining. To reduce contour error, most of previous studies have focused on developing advanced control strategies. As an alternative strategy, contouring accuracy improvement using an adaptive feedrate planning method is proposed in this paper. First, a typical PID controller is adopted to build the contour error model, from which the feedrate can be scheduled in the contour error violated zones. Then, the relations between each constraint and the cutter tip feedrate are derived. After that, a linear programming model is applied to obtain the optimal feedrate profile on the sampling positions of the given tool path. Finally, illustrated examples are given to validate the feasibility and applicability of the proposed feedrate planning method. The comparison results show that the proposed method has a significant effect on improving contouring accuracy.

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Keywords: Contour error, feedrate planning, linear programming;

1. Introduction

In modern CNC machining, contouring accuracy is one of the most important indexes in evaluating the accuracy dimension of manufactured part. Due to increasing requirements for high efficient and high precision machining, contouring accuracy improvement has become an important issue to be solved in contour-following applications. To handle this issue, many scholars devote to investigating the fields such as feedrate profile optimization, real-time parametric curve interpolation and contour control strategies. Generally, existing approaches can mainly be classified into two categories: reducing contour error directly with advanced control strategies and contouring accuracy improvement using adaptive feedrate planning methods.

Contouring accuracy is influenced by many factors such as the mismatch between axial servo dynamics and the non-linear friction and backlash. To reduce the contour error induced by servo system, several advanced control strategies have been proposed. For instance, conventional servo controller have been developed in Ref. [1-4] to reduce individual axial tracking errors. Although the tracking error can be reduced, feedforward control can also generate high frequency control signals, and the performance relies on the accuracy of the plant model. Other researches explored the direct contour error control methods rather than reducing individual axial tracking errors. For example, the Cross Couple Control (CCC) strategy, which consists of an algorithm to predict contour error and a control law to coordinate the drive axes motion, have been proposed in Ref. [5-9]. Meanwhile, the dynamics matching method strategies can be also found in Ref. [10-12]. Those strategies are effective in reducing contour error, but it is difficult to realize the control strategy for a given commercial machine due to unknown information protected by suppliers.

The second category focused on improving contouring

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accuracy is to optimize the feedrate profile with contour error related constraints. Recently, Sencer and Altinta et al. [13] proposed a nonlinear optimization strategy respecting the drive constraints such as axis velocity, acceleration and jerk. Dong et al. [14] gave a discrete greedy algorithm with the constraints of tangential acceleration and jerk. However, it is generally time-consuming and difficult to reach the global optimum by solving nonlinear programing problems. Gao et al. [15] developed a linear programming program by using a linear function to approximate the nonlinear jerk constraint. Zhou et al. [16] proposed a multiconstraints feedrate optimization method with a linear programming algorithm, in which the geometric error constraints, cutting performance and drive constraints are considered. Although it is more efficient to schedule feedrate, contour error constraint isn't included. Lin et al. [17] developed a dynamics based interpolation algorithm with look-ahead scheme to generate a smooth feedrate profile, where a dynamics feedrate modification equation (DFME) is utilized to estimate contour errors at the sharp corners and adjusts the feedrate at the locations of the sharp corners. Dong et al. [18] proposed a new real-time smooth feedrate planning algorithm for short line tool path, in which the servo system was approximated by a first order transfer function to regulate the target feedrate adaptively. Cheng et al. [19] developed a fuzzy logic-based federate regulator by using the approximate contour error information to adjusts the value of the desired feedrate. As mentioned before, planning feedrate with contour error constraint can effectively improve contouring accuracy. Additionally, in the actual machining process, if axis acceleration out of the allowable range, it may result in poor machining quality and deteriorated accuracy. Therefore, it is also necessary to take the drive constraints into account. However, the feedrate planning methods constrained by chord error, contour error and drive performances are hardly mentioned in existing works.

In this paper, an adaptive feedrate planning method is proposed to improve contouring accuracy with constraints of chord error, contour error and drive constraints. In the method, a typical PID controller is adopted to build the contour error model. The relations between each constraint and the cutter tip feedrate are derived. Meanwhile, a linear programming model was applied to obtain the final feedrate profile. The remainder of this paper is structured as follows: Section 2 establishes the contour error model. Section 3 derives the relations between each constraint and the cutter tip feedrate and describes detailed feedrate planning algorithm. Illustrative examples are performed in Section 4 to validate the effectiveness of the proposed method. Section 5 concludes the paper.

2. Contour error model

Generally speaking, good tracking performance is helpful for improving contouring accuracy. Due to the advantages of high precise feeding and good dynamic characteristic, PID controllers are widely used in contour machining applications. Therefore, a PID controller is used here to build the contour error model and its transfer function can be given as follows

$$G_{PID}(s) = k_p + \frac{k_i}{s} + k_d s \tag{1}$$

where k_p, k_i and k_d are the proportional, integral and derivative gains of the controller. On this basis, the feed servo model is built as shown in Fig. 1

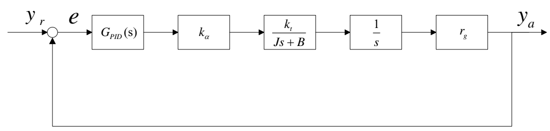

Fig. 1. The servo control system for feed axis.

where k_a , k_t , J , B and r_g represent current amplifier, torque constant of motor, inertial of drive axis, viscous damping constant and screw lead. The transfer function between actual position and reference input can be expressed in the Laplace domain as follows

$$G(s) = \frac{y_r(s)}{y_a(s)} = \frac{K(k_d s^2 + k_p + k_i)}{Js^3 + (B + Kk_d)s^2 + Kk_p s + Kk_i} \tag{2}$$

For an arbitrary position of a given continuous parametric curve, its local curve segment can be well approximated by a circular arc with the same curvature radius as that of the curve at that position point. Therefore, a contour error model based on circular arc tool path is first built. Supposing the desired radius in the circular contouring is ρ_d , the angular speed of contouring is ω and the contouring time is t. Then, the reference commands for X- and Y- axis can be expressed as

$$\begin{cases} X_r = \rho_d \cos \omega t \\ Y_r = \rho_d \sin \omega t \end{cases} \tag{3}$$

The corresponding actual position of the feed axis in the steady-state can be given as

$$\begin{cases} X_a = M_x(\omega)\rho_d \cos\left[\omega t + \varphi_x(\omega)\right] \\ Y_a = M_y(\omega)\rho_d \sin\left[\omega t + \varphi_y(\omega)\right] \end{cases} \tag{4}$$

where $M_*(\omega)(* = x, y)$ are the gains of magnitudes, $\varphi_*(\omega)(* = x, y)$ represent the phase shifts of the responses. If there is no dynamic mismatch between axes, the actual position of the feed axis can be expressed as

$$\begin{cases} X_a = \rho_d M(\omega)\cos\left[\omega t + \varphi(\omega)\right] \\ Y_a = \rho_d M(\omega)\sin\left[\omega t + \varphi(\omega)\right] \end{cases} \tag{5}$$

where $M(\omega) = M_x(\omega) = M_y(\omega)$, $\varphi(\omega) = \varphi_x(\omega) = \varphi_y(\omega)$ .The steady-state contour error ε_c can be further calculated by

$$\varepsilon_c = \left|\rho_d - \rho_a\right| = \rho_d \left|\left(1 - \left|G(j\omega)\right|\right)\right| \tag{6}$$

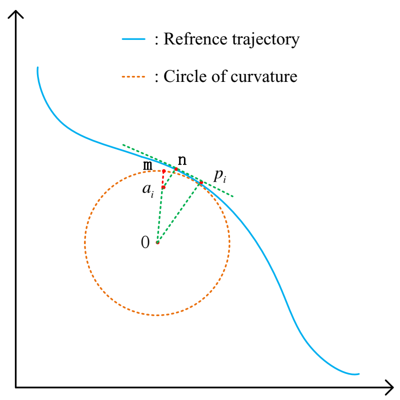

Fig. 2. Contour error model for a parameter curve.

Once the contour error model for a circular path is built, it can be used to well approximate that for a given parametric curve as illustrated in Fig. 2. As shown in the figure, p_i , a_i , na_i and ma_i represent the reference point, actual point, actual contour error and estimated contour error. The radius of curvature in point p_i is ρ, then the contour error can be estimated as

$$\varepsilon_c = \rho\left|\left(1-\left|G(j\omega)\right|\right)\right| \tag{7}$$

3. The proposed feedrate planning method

In this section, the relations between each constraint and the cutter tip feedrate are derived and the linear programming model is built to obtain the optimal feedrate profile.

3.1. Constraints

The relations between each constraint and the cutter tip feedrate will be derived in this section. These constraints include contour error, chord error and drive constraints. In CNC machine, chord error often needs to be limited for ensuring machining accuracy. Meanwhile, due to the avoidable of excessive axis velocity and acceleration, actuators can be in the capacities and machines can avoid the chattering or vibration, which have also brought serious influence on the contouring accuracy.

3.1.1 Contour error constraint

To ensure good contouring accuracy, the contour error ε_c should satisfy the given contour error tolerance ε_0

$$\varepsilon_c \le \varepsilon_o \tag{8}$$

Further, Eq.(8) can be written as

$$1-\kappa\varepsilon_0 \le \left|G(j\omega)\right| \le 1+\kappa\varepsilon_0 \tag{9}$$

where κ is the curvature in point p_i. For further analysis, we adopt the the dynamic control system parameters provided in [20], which have been listed in Table 1. On this basis, the high order transfer function can be written as

$$G(s) = \frac{A_1 s^2 + A_2 s + A_3}{A_4 s^3 + A_5 s^2 + A_6 s + A_7} \tag{10}$$

The parameters $A_i (i = 1,2,...,7)$ are listed in Table 2.

Table 1. Parameters of PID controller and servo control system.

Parameter	X-axis	Y-axis	Unit
k_p	10	12.344	V/mm
k_i	50	61.718	V/(mm s)
k_d	0.3	0.367	V/(mm/s)
k_α	6.5723	6.2274	A/V
k_t	0.4769	0.4769	Nm/A
J	7.0028e-3	8.1904e-3	kg m^2
B	0.023569	0.043009	kg m^2/s
r_g	1.5915	1.5915	mm/rad

Table 2. Parameters of the system transfer function.

Parameter	Value	Parameter	Value
A_1	1.4965	A_3	0.007
A_2=A_6	49.883	A_5	1.52
A_3=A_7	249.415		

Generally, the most suitable angular frequency value ω_0 should be kept lower than the natural frequency ω_a of the machine tool. Therefore, we can obtain ω_0 by the following formulas

$$\begin{cases} 1-\kappa\varepsilon_0 = \left|G(j\omega_0)\right| \\ 1+\kappa\varepsilon_0 = \left|G(j\omega_0)\right| \\ \quad 0 < \omega_0 \le \omega_a \end{cases} \tag{11}$$

Then the feedrate f can be determined by the following formula

$$f = \omega_0(u_i)\rho(u_i) \tag{12}$$

Furthermore, the feasible feedrate respecting contour error constraint can be derived as

$$f(u_i) \le \omega_0(u_i)\rho(u_i) \tag{13}$$

3.1.2 Chord error constraint

For curve path $\boldsymbol{P}(u)$, the chord error is normally approximated as

$$\delta(u_i) = \rho(u_i) - \sqrt{\rho(u_i)^2 - \left(\frac{f(u_i)T}{2}\right)^2} \tag{14}$$

where $\rho(u_i)$ is the curvature radius at the parameter position u. Given a maximum chord error δ_{max}, the feasible feedrate respecting chord error constraint can be derived as

$$f(u_i) \le 2\sqrt{\rho(u_i)^2 - (\rho(u_i) - \delta_{\max})^2} / T \tag{15}$$

3.1.3 Drive constraints

For a given path curve $\boldsymbol{P}(u)$, the corresponding path feedrate $f(u_i)$ in parameter position u can be derived as

$$f(u) = \left\| \frac{d\boldsymbol{P}(u)}{dt} \right\| = \left\| \frac{d\boldsymbol{P}(u)}{du} \right\| \frac{du}{dt} = \psi \frac{du}{dt} \tag{16}$$

where ψ is the total length of the path $\boldsymbol{P}(u)$ and $\Delta s = \psi \Delta u$. Setting $f_{\max}(f_{\max} > 0)$ as the maximum feedrate of the cutter tip, then feedrate should satisfy the following formula

$$0 < f(u_i) \le f_{\max} \tag{17}$$

The drive velocity and acceleration can be calculated as

$$f^\tau(u) = \frac{d\boldsymbol{Q}^\tau(u)}{dt} = \boldsymbol{Q}_u^\tau(u)\frac{du}{dt} \tag{18}$$

$$a^\tau(u) = \frac{df^\tau(u)}{dt} = \boldsymbol{Q}_{uu}^\tau(u)\left(\frac{du}{dt}\right)^2 + \boldsymbol{Q}_u^\tau(u)\frac{d^2u}{dt^2} \tag{19}$$

From Eq. (16) we can get

$$\frac{du}{dt} = \frac{f(u)}{\psi} \tag{20}$$

$$\frac{d^2u}{dt^2} = \frac{a(u)}{\psi} = \frac{f(u_{i+1})^2 - f(u_i)^2}{2\psi^2 \Delta u} \tag{21}$$

Substituting du/dt and d^2u/dt^2 into formula (20) and (21), drive constraints can be written as the following form

$$0 < f(u_i) \le \frac{\psi f_{\max}^\tau}{\left\| \boldsymbol{Q}_u^\tau(u_i) \right\|} \tag{22}$$

$$-a_{\max}^\tau \le \frac{\boldsymbol{Q}_u^\tau(u_i)}{2\psi^2(u_{i+1} - u_i)} f(u_{i+1})^2 - \left(\frac{\boldsymbol{Q}_u^\tau(u_i) - 2\boldsymbol{Q}_{uu}^\tau(u_i)(u_{i+1} - u_i)}{2\psi^2(u_{i+1} - u_i)} \right) f(u_i)^2 \le a_{\max}^\tau$$
(23)

where $\boldsymbol{Q}^\tau(\tau = x, y)$ represents the corresponding axis displacements, $\boldsymbol{Q}_u^\tau$ 、 $\boldsymbol{Q}_{uu}^\tau$ are the derivatives along the tool path with respect to parameter *u*.

3.2. Linear programming model

Although the federate optimization can be transformed as a standard linear programming problem according to the inequity formulas (13), (15), (17), (22) and (23), the computation complexity can be further simplified for those formulas. For the contour error limit, chord error limit, and maximum velocity limits of each drive axes, it is only related to the feedrate itself and doesn't involve the differential properties of federate profile at the given position. Therefore, after giving the initial feedrate $f_{\max}$, the maximum feasible feedrate satisfying those constraints mentioned before can be easily obtained by performing a simple comparison with following expression

$$f(u_i) \le f^*(u_i) = \min\left[\frac{2\sqrt{2\rho(u_i)\delta_{\max}}}{T}, \omega_{\max}\rho(u_i), f_{\max}, \frac{\psi f_{\max}^\tau}{\left\| \boldsymbol{Q}_u^\tau(u_i) \right\|} \right]$$
(24)

Taking $f^*(u_i)$ as the up boundary of feedrate in the feed optimization model solved by a linear programming algorithm, the feedrate planning problem can be finally arranged as

$$Max \ \sum_{i=1}^{m} f(u_i)^2$$

$$s.t. \begin{cases} -a_{\max}^\tau \le \dfrac{\boldsymbol{Q}_u^\tau(u_i)}{2\psi^2(u_{i+1} - u_i)} f(u_{i+1})^2 - \left(\dfrac{\boldsymbol{Q}_u^\tau(u_i) - 2\boldsymbol{Q}_{uu}^\tau(u_i)(u_{i+1} - u_i)}{2\psi^2(u_{i+1} - u_i)} \right) f(u_i)^2 \le a_{\max}^\tau \\ 0 \le f(u_i)^2 \le f^*(u_i)^2 \end{cases}$$
(25)

Thus, from the above formula, it can be seen that the feedrate optimization is transformed as a standard linear programming problem, and it can be solved by the existing well developed algorithm. The procedure of computing the optimal feedrate at sampling positions is summarized as follows

Input: NURBS path curve $\boldsymbol{P}(u)$, the preset values of all constraints, interpolator period, parameters of PID controller and servo control system, and the number of sampling positions.

- According to the parameters of PID controller and servo control system, build the contour error model.
- Divide parameter *u* of the path curve with a suitable parameter interval, and then calculate the first and second derivatives with respect to parameter *u*, as well as the curvatures associated with all sampled position.
- Using related formulas (13), (15), (17), (22) and (23) to build linear programming model, then calculate matrix of optimization objective function and constrained conditions.
- Make use of the linear programming algorithm to get the optimal feedrate at each sampled position, and then fit

NURBS spline to the sampled feedrate data with respect to parameter *u*.

4. Illustrative examples

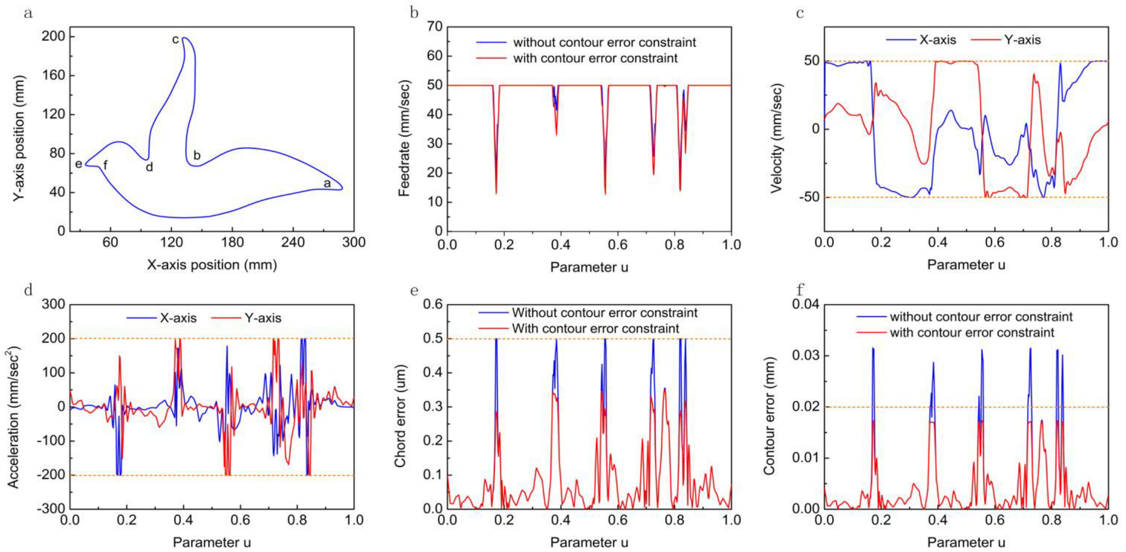

Fig. 3. (a) white-dove curve; (b) feedrate ; (c) axis velocity; (d) axis acceleration; (e) chord error ;(f) contour error.

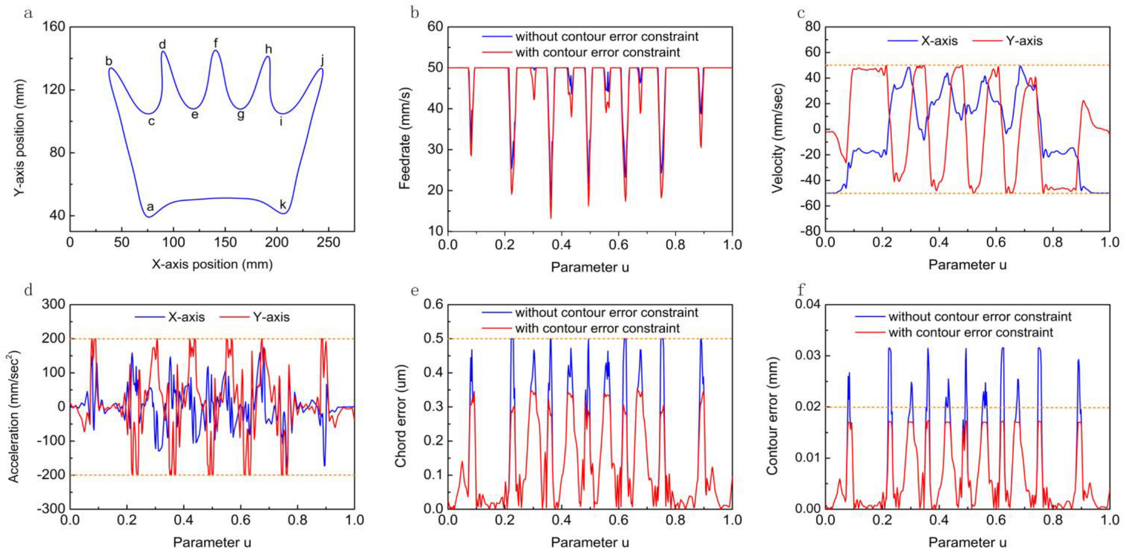

Fig. 4. (a) imperial-crown curve ; (b) feedrate ; (c) axis velocity; (d) axis acceleration; (e) chord error ;(f) contour error.

In the following, two illustrative examples were adopted to validate the proposed method. In order to distinguish the items of constraints for each example intuitively, the corresponding constraints and sampling time are given in Table 3.

Table 3. Corresponding constraints in feedrate planning.

Parameters	Case 1	Case 2	Unit
Sampling time	0.004	0.004	*s*
Chord error	0.0005	0.0005	*mm*
Contour error	0.02	0.02	*mm*
Drive axis velocity	50	50	*mm/s*
Drive axis acceleration	200	200	mm/s^2

The first case is a white-dove curve, and the other is an imperial-crown curve. Both of them have high curvature and low curvature zones. The initial feedrate f is given as the maximum value 50 mm/s and the sampling time set as T=0.004s in the process.

4.2.1 Case 1

As shown in Fig. 3(a), a white-dove curve was chosen in case 1, which has 100 control points and its total length is 793 mm. From Fig. 3(a) and Fig. 3(b), the feedrate profile has six low value regions, which was corresponding to six high-curvature zones marked as a, b, c, d, e and f in parameter curve path. Fig. 3(c) and Fig. 3(d) show that velocity and acceleration of individual axis fall into the allowable range [-50mm/s, 50mm/s] and [-200mm/s^2, 200mm/s^2]. As shown in Fig. 3(e) and Fig. 3(f), it can be seen that confining the chord error and drive performances is helpful for reducing the contour error, but it is not enough to control the contour error within predefined value. If without contour error limit, chord error obviously exceeds its limit 0.02mm in the six high-curvature regions. Especially at parameter position u=0.17, the contour error reaches its maximum peak value 0.0316 mm, which exceed the contour error limit of 58%. However, the contour error in the six high-curvature regions are all decreased significantly and fall into the preset allowable range after adopting proposed method. Illustrate results validate the feasibility and applicability of the proposed method.

4.2.1 Case 2

To demonstrate the validity of the proposed feedrate planning method, a more complicated imperial-crown curve is chosen as case 2. The imperial-crown curve shown in Fig. 4(a) has 100 control points and its total length is 716.22 mm. As shown in Fig. 4(a), curve path has eleven high-curvature regions, which is corresponding to eleven low feedrate zones in the feedrate profile Fig. 4(b). From Fig. 4(c) and Fig. 4(d), the drive axis velocity and acceleration are all confined below the preset limits of 50 mm/s and 200mm/s^2. Fig. 4(e) and Fig. 4(f) show that contour error exceed the default allowable limit without contour error limit even if chord error is helpful for reducing it. The maximum contour error value has reached 0.0316 mm at parameter position u=0.75, which is also beyond the contour error limit of 58%. However, contour error in the eleven high-curvature regions are all decreased significantly and fall into the preset allowable range after adopting the proposed feedrate planning method. The results further validate that the proposed feedrate planning method is capable of confining the contour error below the preset value.

5. Conclusion

This paper proposed an adaptive feedrate planning method constrained by chord error, contour error and drive constraints. As an alternative strategy for reducing contour error, a typical PID controller is first adopted to build the contour error model. Besides, chord error constraint is included because of its contribution to ensuring the machining accuracy. Meanwhile, due to the physical saturation limits of the servo drive system, drive velocity and acceleration have to be confined to suppress the vibrations and shocks of machine tools, which have also brought serious influence on the machining surface quality. On this basis, the relations between each constraint and the cutter tip feedrate are derived. After that, a linear programming model is applied to obtain the optimal feedrate profile. Finally, illustrated examples are given to validate the feasibility and applicability of the proposed method. The comparison results show that proposed method has a significant effect on improving contouring accuracy. Moreover, this new method has a potential to be applied in five-axis tool path and further research is necessary for this idea.

Acknowledgements

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