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PHYSICS LETTERS B

Physics Letters B 643 (2006) 205–212

www.elsevier.com/locate/physletb

Gauge invariant formulation of massive totally symmetric fermionic fields in (A)dS space

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Received 6 September 2006; received in revised form 1 November 2006; accepted 2 November 2006

Available online 10 November 2006

Editor: N. Glover

Abstract

Massive arbitrary spin totally symmetric free fermionic fields propagating in d-dimensional (anti-)de Sitter space–time are investigated. Gauge invariant action and the corresponding gauge transformations for such fields are proposed. The results are formulated in terms of various mass parameters used in the literature as well as the lowest eigenvalues of the energy operator. We apply our results to a study of partial masslessness of fermionic fields in $(A)dS_d$, and in the case of $d = 4$ confirm the conjecture made in the earlier literature.

1. Introduction

Conjectured duality [1] of conformal $\mathcal{N} = 4$ SYM theory and superstring theory in $AdS_5 \times S^5$ Ramond–Ramond background has led to intensive study of field (string) dynamics in AdS space. By now it is clear that in order to understand the conjectured duality better it is necessary to develop powerful approaches to study of field (string) dynamics in AdS space. Light-cone approach is one of the promising approaches which might be helpful to understand AdS/CFT duality better. As is well known, quantization of Green–Schwarz superstrings propagating in flat space is straightforward only in the light-cone gauge. Since, by analogy with flat space, we expect that quantization of the Green–Schwarz AdS superstring with Ramond–Ramond flux [2] will be straightforward only in a light-cone gauge [3] we believe that from the stringy perspective of AdS/CFT correspondence the light-cone approach to field dynamics in AdS is a fruitful direction to go. Light-cone approach to dynamics of massive fields in AdS space was developed in [4,5] and a complete description of massive arbitrary spin bosonic and fermionic fields in AdS_5 was obtained in [6].

Unfortunately, this is not enough for a complete study of the AdS/CFT correspondence because in order to apply the light-cone approach to study of superstring in AdS space we need a light-cone formulation of field dynamics in $AdS_5 \times S^5$ Ramond–Ramond background. Practically useful and self-contained way to give a light-cone gauge description is to start with a Lorentz covariant and gauge invariant description of field dynamics in $AdS_5 \times S^5$ Ramond–Ramond background and then to impose the light-cone gauge. Our experience led us to conclusion that the most simple way to develop light-cone approach in $AdS_5 \times S^5$ space is to start with gauge invariant description of *fermionic fields*. It turns out, however, that gauge invariant description of *massive fermionic fields* (with fixed but arbitrary spin) even in AdS_5 is still not available in the literature. In this Letter we develop Lagrangian Lorentz covariant and gauge invariant formulation[1] for *massive totally symmetric arbitrary spin fermionic fields* in $(A)dS_d$ space. We believe that our results will be helpful to find a gauge invariant description of arbitrary spin fields in $AdS_5 \times S^5$ case. Our approach allows us to study fermionic fields in AdS_d space and dS_d space on an equal footing. In this Letter we apply our results to study of par-

[1] Sometimes, a gauge invariant approach to massive fields is referred to as a Stueckelberg approach.

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doi:10.1016/j.physletb.2006.11.002

tial masslessness of fermionic fields in $(A)dS_d$. For $d = 4$ our results confirm the conjecture made in Ref. [7].

Before proceeding to the main theme of this Letter let us mention briefly the approaches which could be used to discuss gauge invariant action for fields in (A)dS. Since the works [8–11] devoted to massless fields in AdS_d various descriptions of massive and massless arbitrary spin fields in (A)dS have been developed. In particular, an ambient space formulation was discussed in [12,13] and various BRST formulations were studied in [14–17]. The frame-like formulations of free fields which seems to be the most suitable for formulation of the theory of interacting fields in (A)dS was developed in [18,19]. Other interesting formulations of higher spin theories were also discussed recently in [20–23]. In this Letter we adopt the approach of Ref. [24] devoted to the bosonic fields in (A)dS. This approach turns out to be the most useful for our purposes.

2. Gauge invariant action of massive fermionic field in (A)dS

In d-dimensional $(A)dS_d$ space the massive totally symmetric arbitrary spin fermionic field is labelled by one mass parameter and by one half-integer spin label $s + \frac{1}{2}$, where $s > 0$ is an integer number. To discuss Lorentz covariant and gauge invariant formulation of such field we introduce Dirac complex-valued tensor-spinor spin $s' + \frac{1}{2}$ fields of the $so(d-1,1)$ Lorentz algebra $\psi^{A_1\ldots A_{s'}\alpha}$, $s' = 0, 1, \ldots, s$ (where $A = 0, 1, \ldots, d-1$ are flat vector indices of the $so(d-1,1)$ algebra), i.e. we start with a collection of the tensor-spinor fields

$$\sum_{s'=0}^{s} \bigoplus \psi^{A_1\ldots A_{s'}\alpha}. \tag{2.1}$$

In order to obtain the gauge invariant description of a massive field in an easy-to-use form, let us introduce a set of the creation and annihilation operators α^A, ζ and $\bar\alpha^A$, $\bar\zeta$ defined by the relations[2]

$$\begin{aligned} &[\bar\alpha^A, \alpha^B] = \eta^{AB}, \qquad [\bar\zeta, \zeta] = 1, \\ &\bar\alpha^A|0\rangle = 0, \qquad \bar\zeta|0\rangle = 0, \end{aligned} \tag{2.2}$$

where η^{AB} is the mostly positive flat metric tensor. The oscillators α^A, $\bar\alpha^A$ and ζ, $\bar\zeta$ transform in the respective vector and scalar representations of the $so(d-1,1)$ Lorentz algebra. The tensor-spinor fields (2.1) can be collected into a ket-vector $|\psi\rangle$ defined by

$$|\psi\rangle \equiv \sum_{s'=0}^{s} \zeta^{s-s'}|\psi_{s'}\rangle, \tag{2.3}$$

$$|\psi_{s'}\rangle \equiv \alpha^{A_1}\cdots\alpha^{A_{s'}}\psi^{A_1\ldots A_{s'}\alpha}(x)|0\rangle. \tag{2.4}$$

Here and below spinor indices are implicit. The ket-vector $|\psi_{s'}\rangle$ (2.4) satisfies the constraint

$$(\alpha^A\bar\alpha^A - s')|\psi_{s'}\rangle = 0, \quad s' = 0, 1, \ldots, s, \tag{2.5}$$

which tells us that $|\psi_{s'}\rangle$ is a degree s' homogeneous polynomial in the oscillator α^A. The tensor-spinor field $|\psi_{s'}\rangle$ is subjected the basic algebraic constraint

$$\begin{aligned} &\gamma\bar\alpha\bar\alpha^2|\psi_{s'}\rangle = 0, \quad s' = 0, 1, \ldots, s, \\ &\gamma\bar\alpha \equiv \gamma^A\bar\alpha^A, \qquad \bar\alpha^2 \equiv \bar\alpha^A\bar\alpha^A, \end{aligned} \tag{2.6}$$

which tells us that $|\psi_{s'}\rangle$ is a reducible representation of the Lorentz algebra $so(d-1,1)$.[3] Note that for $s' = 0, 1, 2$ the constraint (2.6) is satisfied automatically. In terms of the ket-vector $|\psi\rangle$ (2.3) the algebraic constraints (2.5), (2.6) take the form

$$(\alpha\bar\alpha + N_\zeta - s)|\psi\rangle = 0, \tag{2.7}$$

$$\gamma\bar\alpha\bar\alpha^2|\psi\rangle = 0, \tag{2.8}$$

$$N_\zeta \equiv \zeta\bar\zeta, \qquad \alpha\bar\alpha \equiv \alpha^A\bar\alpha^A. \tag{2.9}$$

Eq. (2.7) tells us that $|\psi\rangle$ is a degree s homogeneous polynomial in the oscillators α^A, ζ.

Lagrangian for the massive fermionic field in $(A)dS_d$ space we found takes the form

$$\mathcal{L} = \mathcal{L}_{\rm der} + \mathcal{L}_m, \tag{2.10}$$

where $\mathcal{L}_{\rm der}$ stands for a derivative depending part of $\mathcal{L}$, while $\mathcal{L}_m$ stands for a mass part of $\mathcal{L}$:[4]

$$\mathrm{i}e^{-1}\mathcal{L}_{\rm der} = \langle\psi|L|\psi\rangle, \tag{2.11}$$

$$\mathrm{i}e^{-1}\mathcal{L}_m = \langle\psi|\mathcal{M}|\psi\rangle. \tag{2.12}$$

The standard first-derivative differential operator L which enters $\mathcal{L}_{\rm der}$ (2.11) is given by

$$\begin{aligned} L \equiv{}& \not{D} - \alpha D\gamma\bar\alpha - \gamma\alpha\bar\alpha D + \gamma\alpha\not{D}\gamma\bar\alpha + \frac{1}{2}\gamma\alpha\alpha D\bar\alpha^2 \\ &+ \frac{1}{2}\alpha^2\gamma\bar\alpha\bar\alpha D - \frac{1}{4}\alpha^2\not{D}\bar\alpha^2, \end{aligned} \tag{2.13}$$

where we use the notation

$$\begin{aligned} &\gamma\alpha \equiv \gamma^A\alpha^A, \qquad \gamma\bar\alpha \equiv \gamma^A\bar\alpha^A, \\ &\alpha^2 \equiv \alpha^A\alpha^A, \qquad \bar\alpha^2 \equiv \bar\alpha^A\bar\alpha^A, \end{aligned} \tag{2.14}$$

$$\begin{aligned} &\not{D} \equiv \gamma^A D^A, \qquad \alpha D \equiv \alpha^A D^A, \\ &\bar\alpha D \equiv \bar\alpha^A D^A, \qquad D_A \equiv e_A^\mu D_\mu, \end{aligned} \tag{2.15}$$

and e_A^μ stands for inverse vielbein of $(A)dS_d$ space, while D_μ stands for the Lorentz covariant derivative

$$D_\mu \equiv \partial_\mu + \frac{1}{2}\omega_\mu^{AB}M^{AB}. \tag{2.16}$$

[2] We use oscillator formulation [10,25] to handle the many indices appearing for arbitrary spin fields. It can also be reformulated as an algebra acting on the symmetric-spinor bundle on the manifold M [21]. Note that the scalar oscillators ζ, $\bar\zeta$ arise naturally by a dimensional reduction [15,21] from flat space. Our oscillators α^A, $\bar\alpha^A$, ζ, $\bar\zeta$ are respective analogs of dx^μ, ∂_μ, du, ∂_u of Ref. [21] dealing, among other thing, with massless fermionic fields in (A)dS. We thank A. Waldron for pointing this to us.

[3] Important constraint (2.6) was introduced for the first time in [9] while study of massless fermionic fields in AdS_4. This constraint implies that the field $|\psi_{s'}\rangle$ being reducible representation of the Lorentz algebra $so(d-1,1)$ is decomposed into spin $s' + \frac{1}{2}$, $s' - \frac{1}{2}$, $s' - \frac{3}{2}$ irreps of the Lorentz algebra. Various Lagrangian formulations in terms of unconstrained fields in flat space and (A)dS space may be found e.g. in [26–30].

[4] The bra-vector $\langle\psi|$ is defined according the rule $\langle\psi| = (|\psi\rangle)^\dagger\gamma^0$.

The ω_μ^{AB} is the Lorentz connection of $(A)dS_d$ space, while a spin operator M^{AB} forms a representation of the Lorentz algebra $so(d-1,1)$:

$$M^{AB} = M_b^{AB} + \frac{1}{2}\gamma^{AB}, \qquad M_b^{AB} \equiv \alpha^A\bar\alpha^B - \alpha^B\bar\alpha^A,$$
$$\gamma^{AB} \equiv \frac{1}{2}\big(\gamma^A\gamma^B - \gamma^B\gamma^A\big). \tag{2.17}$$

We note that our derivative depending part of the Lagrangian $\mathcal{L}_{\rm der}$ (2.11) is nothing but a sum of the Lagrangians of Ref. [9] for the tensor-spinor fields (2.1).

We now proceed with discussion of the mass operator $\mathcal{M}$ (2.12). The operator $\mathcal{M}$ is given by

$$\mathcal{M} = \Big(1 - \gamma\alpha\gamma\bar\alpha - \frac{1}{4}\alpha^2\bar\alpha^2\Big)\tilde m_1 + \tilde m_4\Big(\gamma\alpha\bar\zeta - \frac{1}{2}\alpha^2\bar\zeta\gamma\bar\alpha\Big)$$
$$- \Big(\zeta\gamma\bar\alpha - \frac{1}{2}\gamma\alpha\zeta\bar\alpha^2\Big)\tilde m_4, \tag{2.18}$$

where operators $\tilde m_1$, $\tilde m_4$ do not depend on the γ-matrices and α-oscillators, and take the form

$$\tilde m_1 = \frac{2s+d-2}{2s+d-2-2N_\zeta}\kappa, \tag{2.19}$$
$$\tilde m_4 = \bigg(\frac{2s+d-3-N_\zeta}{2s+d-4-2N_\zeta}F(\kappa,s,N_\zeta)\bigg)^{1/2}. \tag{2.20}$$

Function $F(\kappa,s,N_\zeta)$ depends on a mass parameter κ, spin s and operator N_ζ, and is given by

$$F(\kappa,s,N_\zeta) = \kappa^2 + \theta\Big(s + \frac{d-4}{2} - N_\zeta\Big)^2. \tag{2.21}$$

F is restricted to be positive and throughout this Letter, unless otherwise specified, we use the convention[5]:

$$\theta = \begin{cases} -1 & \text{for AdS space,} \\ 0 & \text{for flat space,} \\ +1 & \text{for dS space.} \end{cases} \tag{2.22}$$

The mass parameter κ is a freedom of our solution, i.e. gauge invariance allows us to find Lagrangian completely by module of mass parameter as it should be for the case of massive fields.

Now we discuss gauge symmetries of the action

$$S = \int d^dx\,\mathcal{L}. \tag{2.23}$$

To this end we introduce parameters of gauge transformations $\epsilon^{A_1\dots A_{s'}\alpha}$, $s' = 0,1,\dots,s-1$ which are γ-traceless (for $s'>0$) Dirac complex-valued tensor-spinor spin $s'+\frac{1}{2}$ fields of the $so(d-1,1)$ Lorentz algebra, i.e. we start with a collection of the tensor-spinor fields

$$\sum_{s'=0}^{s-1}\bigoplus \epsilon^{A_1\dots A_{s'}\alpha}, \qquad \gamma^A\epsilon^{AA_2\dots A_{s'}} = 0, \quad \text{for } s'>0. \tag{2.24}$$

[5] Thus our Lagrangian gives description of massive fermionic fields in (A)dS space and flat space on an equal footing. Discussion of massive fermionic fields in flat space in framework of BRST approach may be found in [26,31].

As before to simplify our expressions we use the ket-vector of gauge transformations parameter

$$|\epsilon\rangle \equiv \sum_{s'=0}^{s-1}\zeta^{s-1-s'}|\epsilon_{s'}\rangle, \tag{2.25}$$
$$|\epsilon_{s'}\rangle \equiv \alpha^{A_1}\cdots\alpha^{A_{s'}}\epsilon^{A_1\dots A_{s'}\alpha}(x)|0\rangle. \tag{2.26}$$

The ket-vector $|\epsilon\rangle$ satisfies the algebraic constraints

$$(\alpha\bar\alpha + N_\zeta - s + 1)|\epsilon\rangle = 0, \tag{2.27}$$
$$\gamma\bar\alpha|\epsilon\rangle = 0. \tag{2.28}$$

The constraint (2.27) tells us that the ket-vector $|\epsilon\rangle$ is a degree $s-1$ homogeneous polynomial in the oscillators α^A, ζ, while the constraint (2.28) respects the γ-tracelessness of $|\epsilon\rangle$.

Now the gauge transformations under which the action (2.23) is invariant take the form

$$\delta|\psi\rangle = (\alpha D + \varDelta)|\epsilon\rangle, \tag{2.29}$$
$$\varDelta \equiv \zeta\tilde\varDelta_1 + \gamma\alpha\tilde\varDelta_2 + \alpha^2\tilde\varDelta_3\bar\zeta, \tag{2.30}$$

where operators $\tilde\varDelta_1$, $\tilde\varDelta_2$, $\tilde\varDelta_3$ do not depend on the γ-matrices and α-oscillators, and take the form

$$\tilde\varDelta_1 = \bigg(\frac{2s+d-3-N_\zeta}{2s+d-4-2N_\zeta}F(\kappa,s,N_\zeta)\bigg)^{1/2}, \tag{2.31}$$
$$\tilde\varDelta_2 = \frac{2s+d-2}{(2s+d-2-2N_\zeta)(2s+d-4-2N_\zeta)}\kappa, \tag{2.32}$$
$$\tilde\varDelta_3 = -\bigg(\frac{2s+d-3-N_\zeta}{(2s+d-4-2N_\zeta)^3}F(\kappa,s,N_\zeta)\bigg)^{1/2}, \tag{2.33}$$

and F is defined in (2.21). Thus we expressed our results in terms of the mass parameter κ. Since there is no commonly accepted definition of mass in (A)dS we relate our mass parameter κ with various mass parameters used in the literature. One of the most-used definitions of mass, which we denote by m_D, is obtained from the following expansion of mass part of the Lagrangian:

$$\mathrm{i}e^{-1}\mathcal{L}_m = \langle\psi_s|m_D|\psi_s\rangle + \cdots, \tag{2.34}$$

where dots stand for terms involving $|\psi_{s'}\rangle$, $s'<s$, and for contribution which vanishes while imposing the constraint $\gamma\bar\alpha|\psi\rangle$. Comparing (2.34) with (2.18), (2.19) leads then to the identification

$$\kappa = m_D. \tag{2.35}$$

Another definition of mass parameter for fermionic fields in AdS_d [5], denoted by m, can be obtained by requiring that the value of $m=0$ corresponds to the massless fields. For the case of spin $s+\frac{1}{2}$ field in AdS_d the mass parameter m is related with m_D as

$$m_D = m + s + \frac{d-4}{2} \quad \text{for } AdS_d, \tag{2.36}$$

where $m>0$ corresponds to massive unitary irreps of the $so(d-1,2)$ algebra [5,12]. Below we demonstrate that natural generalization of (2.36) which is valid for both AdS and dS spaces is given by

$$m_D = m + \sqrt{-\theta}\left(s + \frac{d-4}{2}\right) \quad \text{for } (A)dS_d. \tag{2.37}$$

Since sometimes in the case of AdS the formulation in terms of the lowest eigenvalue of energy operator E_0 is preferable we now express our results in terms of E_0. To this end we use the relation found in [5]:

$$m = E_0 - s - d + \frac{5}{2} \quad \text{for } AdS_d. \tag{2.38}$$

Making use then (2.35), (2.36) we get for the case of AdS_d the desired relations

$$\kappa = E_0 - \frac{d-1}{2}, \tag{2.39}$$

$$F = \left(E_0 - s - d + \frac{5}{2} + N_\zeta\right)\left(E_0 + s - \frac{3}{2} - N_\zeta\right). \tag{2.40}$$

3. Limit of massless fields in $(A)dS_d$

In previous section we presented the action for the massive field. In limit as the mass parameter m tends to zero our Lagrangian leads to the Lagrangian for massless field in $(A)dS_d$. Let us discuss the massless limit in detail. To realize limit of massless field in $(A)dS_d$ we take (see (2.35), (2.37))

$$m_D \to \sqrt{-\theta}\left(s + \frac{d-4}{2}\right) \quad \Longleftrightarrow \quad m \to 0. \tag{3.1}$$

We now demonstrate that this limit leads to appearance of the invariant subspace in $|\psi\rangle$ (2.3) and this invariant subspace, denoted by $|\psi^{m=0}\rangle$, is given by the leading ($s' = s$) term in (2.3):

$$\left|\psi^{m=0}\right\rangle = |\psi_s\rangle. \tag{3.2}$$

All that is required is to demonstrate that in the limit (3.1), the ket-vector $|\psi^{m=0}\rangle$ satisfies the following requirements: (i) $|\psi^{m=0}\rangle$ is invariant under action of the mass operator $\mathcal{M}$; (ii) the gauge transformation of the ket-vector $|\psi^{m=0}\rangle$ becomes the standard gauge transformation of massless field. To this end we note that an action of the mass operator $\mathcal{M}$ on $|\psi_s\rangle$ and the gauge transformation of $|\psi_s\rangle$ take the form

$$\begin{aligned}\mathcal{M}|\psi_s\rangle = {}& \left(1 - \gamma\alpha\gamma\bar\alpha - \frac{1}{4}\alpha^2\bar\alpha^2\right)\tilde m_1(0)|\psi_s\rangle \\ & + \left(\gamma\alpha - \frac{1}{2}\alpha^2\gamma\bar\alpha\right)\tilde m_4(0)|\psi_{s-1}\rangle,\end{aligned} \tag{3.3}$$

$$\delta|\psi_s\rangle = \left(\alpha D + \gamma\alpha\tilde\Delta_2(0)\right)|\epsilon_{s-1}\rangle + \alpha^2\tilde\Delta_3(0)|\epsilon_{s-2}\rangle, \tag{3.4}$$

where $\tilde m_{1,4}(0)$ (3.3) and $\tilde\Delta_{2,3}(0)$ (3.4) stand for $\tilde m_{1,4}$ (2.19), (2.20) and $\tilde\Delta_{2,3}$ (2.32), (2.33) in which we set $N_\zeta = 0$. Taking into account

$$\lim_{m\to 0}\tilde m_4(0) = 0, \qquad \lim_{m\to 0}\tilde\Delta_3(0) = 0, \tag{3.5}$$

we see that if $m = 0$ then the ket-vector $|\psi_s\rangle$ is indeed invariant under action of the mass operator $\mathcal{M}$ and a realization of the mass operator on the ket-vector $|\psi_s\rangle$ takes the form

$$\mathcal{M}^{m=0} = \sqrt{-\theta}\left(s + \frac{d-4}{2}\right)\left(1 - \gamma\alpha\gamma\bar\alpha - \frac{1}{4}\alpha^2\bar\alpha^2\right), \tag{3.6}$$

while the relations (3.4), (3.5) lead to the gauge transformation

$$\delta\left|\psi^{m=0}\right\rangle = \left(\alpha D + \frac{\sqrt{-\theta}}{2}\gamma\alpha\right)|\epsilon_{s-1}\rangle, \tag{3.7}$$

which is noting but the standard gauge transformation of massless field in (A)dS space. Thus the Lagrangian for massless spin $s + \frac{1}{2}$ fermionic field in $(A)dS_d$ space takes the form[6]

$$\mathrm{i}e^{-1}\mathcal{L} = \left\langle\psi^{m=0}\right|L + \mathcal{M}^{m=0}\left|\psi^{m=0}\right\rangle, \tag{3.8}$$

where $|\psi^{m=0}\rangle$ is given by (3.2), (2.4), while the operators L and $\mathcal{M}^{m=0}$ are defined by (2.13) and (3.6) respectively. The remaining ket-vectors $|\psi_{s-1}\rangle, \dots, |\psi_0\rangle$ in (2.3) decouple in the massless limit and they describe spin $s - \frac{1}{2}$ massive field, i.e. in the massless limit the generic field $|\psi\rangle$ is decomposed into two decoupling systems—one massless spin $s + \frac{1}{2}$ field and one massive spin $s - \frac{1}{2}$ field. Adopting (2.34) for spin $s - \frac{1}{2}$ field we find mass of the massive spin $s - \frac{1}{2}$ field: $m_D = \sqrt{-\theta}(s + (d-2)/2)$.

4. Partial masslessness of fermionic fields in $(A)dS_d$

Here we apply our results to study of partial masslessness[7] of fermionic fields in $(A)dS_d$. We confirm conjecture of Ref. [7] for $d = 4$ and obtain a generalization to the case of arbitrary $d > 4$. In this section we assume that the θ (2.22) takes the values ± 1. We start our discussion of partial masslessness of fermionic fields with simplest case of (see also Ref. [34]).

4.1. Massive spin 5/2 field

Such field is described by ket-vectors $|\psi_2\rangle$, $|\psi_1\rangle$, $|\psi_0\rangle$ (see (2.3)). For spin 5/2 field there is one critical value of m_D which leads to appearance of partial massless field. For this critical value of m_D the generic field $|\psi\rangle$ is decomposed into one partial massless field and one massive spin $\frac{1}{2}$ field. To demonstrate this we consider the gauge transformations (2.29),

$$\delta|\psi_2\rangle = \alpha D|\epsilon_1\rangle + \gamma\alpha\tilde\Delta_2(0)|\epsilon_1\rangle + \alpha^2\tilde\Delta_3(0)|\epsilon_0\rangle, \tag{4.1}$$

$$\delta|\psi_1\rangle = \alpha D|\epsilon_0\rangle + \tilde\Delta_1(0)|\epsilon_1\rangle + \gamma\alpha\tilde\Delta_2(1)|\epsilon_0\rangle, \tag{4.2}$$

$$\delta|\psi_0\rangle = \tilde\Delta_1(1)|\epsilon_0\rangle, \tag{4.3}$$

where $\tilde\Delta_{1,2,3}(n)$ are given in (2.31)–(2.33) in which we set $s = 2$ and argument n stands for an eigenvalue of the operator N_ζ. The critical value of m_D is obtained from the requirement of decoupling of the field $|\psi_0\rangle$. This requirement amounts to the equation $\tilde\Delta_1(1) = 0$ which leads to the critical value

$$m^2_{D(0)} = -\theta\left(1 + \frac{d-4}{2}\right)^2. \tag{4.4}$$

For this value of m_D the generic field $|\psi\rangle$ is decomposed into two decoupling systems—one partial massless field described

[6] Our Lagrangian (3.8) is a generalization to d-dimensions of the Lagrangian of Ref. [9] for massless field in $(A)dS_4$. Alternative Lagrangian descriptions of massless fermionic fields in $(A)dS_d$ may be found in [11,21].

[7] Partial masslessness was discovered in [32]. Recent discussion of this theme and to some extent complete list of references may be found in [33].

by $|\psi_2\rangle$, $|\psi_1\rangle$ and one massive spin $\frac{1}{2}$ field described by $|\psi_0\rangle$. We proceed with discussion of partial masslessness for

4.2. Massive spin 7/2 field

Spin 7/2 field $|\psi\rangle$ is described by ket-vectors $|\psi_3\rangle$, $|\psi_2\rangle$, $|\psi_1\rangle$, $|\psi_0\rangle$ (see (2.3)). The gauge transformations (2.29) for these ket-vectors take the form

$$\delta|\psi_3\rangle = \alpha D|\epsilon_2\rangle + \gamma\alpha\tilde{\Delta}_2(0)|\epsilon_2\rangle + \alpha^2\tilde{\Delta}_3(0)|\epsilon_1\rangle, \tag{4.5}$$

$$\delta|\psi_2\rangle = \alpha D|\epsilon_1\rangle + \tilde{\Delta}_1(0)|\epsilon_2\rangle + \gamma\alpha\tilde{\Delta}_2(1)|\epsilon_1\rangle + 2\alpha^2\tilde{\Delta}_3(1)|\epsilon_0\rangle, \tag{4.6}$$

$$\delta|\psi_1\rangle = \alpha D|\epsilon_0\rangle + \tilde{\Delta}_1(1)|\epsilon_1\rangle + \gamma\alpha\tilde{\Delta}_2(2)|\epsilon_0\rangle, \tag{4.7}$$

$$\delta|\psi_0\rangle = \tilde{\Delta}_1(2)|\epsilon_0\rangle, \tag{4.8}$$

where expressions for $\tilde{\Delta}_{1,2,3}(n)$ are given in (2.31)–(2.33) in which we set $s=3$ and argument n stands for an eigenvalue of the operator N_ζ. For the spin 7/2 field there are two critical values of m_D. For each critical value of m_D the generic field $|\psi\rangle$ is decomposed into one partial massless field and one massive field. We consider these critical values in turn.

First critical value of m_D is obtained from the requirement of decoupling of the field $|\psi_0\rangle$ (see (4.8)). This requirement amounts to the equation $\tilde{\Delta}_1(2)=0$ which leads to the critical value

$$m^2_{D(0)} = -\theta\left(1+\frac{d-4}{2}\right)^2. \tag{4.9}$$

For this value of m_D the generic field $|\psi\rangle$ is decomposed into two decoupling systems—one partial massless field described by $|\psi_3\rangle$, $|\psi_2\rangle$, $|\psi_1\rangle$ and one massive spin $\frac{1}{2}$ field $|\psi_0\rangle$.

The second critical value of m_D is obtained from the requirement of decoupling of the fields $|\psi_1\rangle$, $|\psi_0\rangle$ (see (4.6), (4.7)). This requirement amounts to the equations $\tilde{\Delta}_1(1)=0$, $\tilde{\Delta}_3(1)=0$ which lead to the critical value

$$m^2_{D(1)} = -\theta\left(2+\frac{d-4}{2}\right)^2. \tag{4.10}$$

For this m_D the generic field $|\psi\rangle$ is decomposed into one partial massless field described by $|\psi_3\rangle$, $|\psi_2\rangle$ and one massive spin $\frac{3}{2}$ field described by $|\psi_1\rangle$, $|\psi_0\rangle$. We finish with partial masslessness for

4.3. Massive arbitrary spin $s+\frac{1}{2}$ field

Such field is described by ket-vectors $|\psi_{s'}\rangle$, $s'=0,1,\ldots s$. Gauge transformations (2.29) for these ket-vectors take the form

$$\delta|\psi_{s'}\rangle = \alpha D|\epsilon_{s'-1}\rangle + \tilde{\Delta}_1(s-s'-1)|\epsilon_{s'}\rangle + \gamma\alpha\tilde{\Delta}_2(s-s')|\epsilon_{s'-1}\rangle + (s-s'+1)\alpha^2\tilde{\Delta}_3(s-s')|\epsilon_{s'-2}\rangle, \tag{4.11}$$

where $\tilde{\Delta}_{1,2,3}(n)$ are given in (2.31)–(2.33) and argument n stands for an eigenvalue of the operator N_ζ. For values $s'=0,1,s$ (4.11) we use the convention $|\epsilon_{-2}\rangle = |\epsilon_{-1}\rangle = |\epsilon_s\rangle = 0$. For the spin $s+\frac{1}{2}$ field there are $s-1$ critical values of m_D, denoted by $m_{D(n)}$, $n=0,1,\ldots,s-2$ (the case of $n=s-1$ leads to massless field and was considered in Section 3). For each $m_{D(n)}$ the generic field $|\psi\rangle$ is decomposed into one partial massless field and one spin $n+\frac{1}{2}$ massive field. To find $m_{D(n)}$ we note that the requirement of decoupling of the fields $|\psi_n\rangle,\ldots,|\psi_0\rangle$, $n=0,\ldots,s-2$ amounts to equations $\tilde{\Delta}_1(s-n-1)=0$, $\tilde{\Delta}_3(s-n-1)=0$. Solution to these equations[8]

$$m^2_{D(n)} = -\theta\left(n+1+\frac{d-4}{2}\right)^2, \tag{4.12}$$

is in agreement with conjecture made in Ref. [7] for the case of $d=4$. Thus we confirmed conjecture of Ref. [7] and obtained $m_{D(n)}$ for $d>4$. For each $m_{D(n)}$ the generic field $|\psi\rangle$ is decomposed into two decoupling systems—one partial massless field $|\psi^{(s)}_{\rm par}\rangle$ described by $|\psi_s\rangle,\ldots,|\psi_{n+1}\rangle$, and one massive spin $n+\frac{1}{2}$ field $|\psi^{(n)}_{\rm msv}\rangle$ described by $|\psi_n\rangle,\ldots,|\psi_0\rangle$. This is to say that by decomposing $|\psi\rangle$ (2.3) into the respective ket-vectors

$$\left|\psi^{(s)}_{\rm par}\right\rangle \equiv \sum_{s'=n+1}^{s}\zeta^{s-s'}|\psi_{s'}\rangle,$$

$$\left|\psi^{(n)}_{\rm msv}\right\rangle \equiv \sum_{s'=0}^{n}\zeta^{s-s'}|\psi_{s'}\rangle, \tag{4.13}$$

one can make sure that if $m_D = m_{D(n)}$ then the mass part of the Lagrangian (2.12) is factorized

$$\mathrm{i}e^{-1}\mathcal{L}_m = \left\langle\psi^{(s)}_{\rm par}\middle|\mathcal{M}\middle|\psi^{(s)}_{\rm par}\right\rangle + \left\langle\psi^{(n)}_{\rm msv}\middle|\mathcal{M}\middle|\psi^{(n)}_{\rm msv}\right\rangle. \tag{4.14}$$

For values $m_D = m_{D(n)}$ the gauge transformations (2.29) are also factorized, while $\mathcal{L}_{\rm der}$ (2.11) is factorized for arbitrary m_D.[9]

5. Uniqueness of Lagrangian for massive fermionic field

We now demonstrate that the Lagrangian and gauge transformations are uniquely determined by requiring that the action be gauge invariant. We formulate our statement. Suppose the derivative depending part of the Lagrangian is given by (2.11), while the derivative depending part of gauge transformations (2.29) is governed by αD-term. Suppose the gauge field $|\psi\rangle$ (2.3) and the gauge transformations parameter $|\epsilon\rangle$ (2.25) satisfy the respective constraints (2.8), (2.28). Then we state that the mass operator $\mathcal{M}$ given in (2.18) and the operator Δ which enters gauge transformations (2.29) are uniquely determined

[8] We note that the key point is not positivity or even reality of the mass part of action (2.12), but rather stability of the energy and unitarity of the underlying physical representations. For bosons a negative mass term is allowed in AdS (the Breitenlohner–Freedman bound, [35]), while partially massless fermions even have an imaginary mass term in their actions but are still stable and unitary in dS. Partially massless fermions are not unitary in AdS (see Refs. [7,34,36]).

[9] m_D- and m-masses of the field $|\psi^{(s)}_{\rm par}\rangle$ are given by: $m_D = \sqrt{-\theta}(n+1+(d-4)/2)$, $m = \sqrt{-\theta}(n+1-s)$, while for the field $|\psi^{(n)}_{\rm msv}\rangle$ we get $m_D = \sqrt{-\theta}(s+(d-2)/2)$, $m = \sqrt{-\theta}(s-n+1)$.

by the following requirements: (i) the action be gauge invariant; (ii) there are no invariant subspaces in $|\psi\rangle$ under action of gauge transformations. Here we outline prove of this statement. We start with general form of the mass operator $\mathcal{M}$ and the operator Δ:

$$\mathcal{M} = m_1 + \gamma\alpha m_2 \gamma\bar\alpha + \alpha^2 m_3 \bar\alpha^2 + \gamma\alpha m_4 + \alpha^2 m_5 \gamma\bar\alpha - m_4^\dagger \gamma\bar\alpha - \gamma\alpha m_5^\dagger \bar\alpha^2 + \alpha^2 m_6 + m_6^\dagger \bar\alpha^2, \quad (5.1)$$

$$\Delta = \Delta_1 + \gamma\alpha\Delta_2 + \alpha^2\Delta_3, \quad (5.2)$$

where $m_{1,\dots,6}$ and $\Delta_{1,2,3}$ do depend on γ-matrices and α-oscillators, and are given by

$$m_1 = \tilde m_1, \qquad m_2 = \tilde m_2, \qquad m_3 = \tilde m_3,$$
$$m_4 = \tilde m_4 \bar\zeta, \qquad m_4^\dagger = \zeta \tilde m_4^\dagger, \quad (5.3)$$
$$m_5 = \tilde m_5 \bar\zeta, \qquad m_5^\dagger = \zeta \tilde m_5^\dagger,$$
$$m_6 = \tilde m_6 \bar\zeta^2, \qquad m_6^\dagger = \zeta^2 \tilde m_6^\dagger, \quad (5.4)$$
$$\Delta_1 = \zeta \tilde\Delta_1, \qquad \Delta_2 = \tilde\Delta_2, \qquad \Delta_3 = \tilde\Delta_3 \bar\zeta. \quad (5.5)$$

Operators $\tilde m_{1,\dots,6}$ and $\tilde\Delta_{1,2,3}$ depend only on N_ζ (2.9). $\tilde m^\dagger_{1,\dots,6}$ stand for hermitian conjugated of $\tilde m_{1,\dots,6}$. Since $\tilde m_{1,2,3}$ are hermitian, $\tilde m^\dagger_{1,2,3} = \tilde m_{1,2,3}$, and depend only on the hermitian operator N_ζ the operators $\tilde m_{1,2,3}$ are real-valued functions of N_ζ from the very beginning. These properties of $\tilde m_{1,\dots,6}$ and $\tilde\Delta_{1,2,3}$ and expressions for $\mathcal{M}$ (5.1), Δ (5.2) are obtained by requiring that:

(i) $\mathcal{M}$ and Δ commute with the spin operator of the Lorentz algebra M^{AB} (2.17) and satisfy the commutators $[\alpha\bar\alpha + N_\zeta, \mathcal{M}] = 0$, $[\alpha\bar\alpha + N_\zeta, \Delta] = \Delta$;
(ii) $\mathcal{M}$ does not involve terms like $\alpha^2\gamma\alpha f_1$ and $f_2\bar\alpha^2\gamma\bar\alpha$, where $f_{1,2}$ are polynomial in the oscillators (such terms in view of (2.8) do not contribute to $\mathcal{L}_m$);
(iii) Δ does not involve terms like $\alpha^2\gamma\alpha f_3$, $f_4\gamma\bar\alpha$, where $f_{3,4}$ are polynomial in the oscillators (the f_3-terms lead to violation of constraint (2.8) for gauge transformed field (2.29), while the f_4-terms in view of (2.28) do not contribute to $\delta|\psi\rangle$ (2.29));
(iv) $\mathcal{M}$ and hermitian conjugated of $\mathcal{M}$ satisfy the relation $\mathcal{M}^\dagger = -\gamma^0\mathcal{M}\gamma^0$.

Thus all that is required is to find dependence of the operators $\tilde m_{1,\dots,6}$ and $\tilde\Delta_{1,2,3}$ on N_ζ. We now demonstrate that this dependence can be determined by requiring that the action be gauge invariant. We evaluate the variation of the action (2.10), (2.23) under gauge transformations (2.29), (5.2),

$$\delta S = -\mathrm{i}\int d^dx\, e\langle\psi|\big(\not{D} X_1 + \gamma\alpha\bar\alpha D X_2 + \alpha D X_3 + \gamma\alpha\not{D} X_4 + \alpha^2\bar\alpha D X_5 + \alpha^2\not{D} X_6 + \alpha D\gamma\alpha X_7 + \alpha^2\alpha D X_8 + \bar\alpha D X_9 + Y^{(0)} + \gamma\alpha\big(Y^{(1)} + Y^{(1)}_{(A)dS}\big) + \alpha^2 Y^{(2)}\big)|\epsilon\rangle + \text{h.c.}, \quad (5.6)$$

where we use the notation

$$X_1 = \Delta_1 - m_4^\dagger, \qquad X_2 = -\Delta_1 - 2m_5^\dagger, \quad (5.7)$$
$$X_3 = -(2s+d-4-2N_\zeta)\Delta_2 + m_1, \quad (5.8)$$
$$X_4 = (2s+d-4-2N_\zeta)\Delta_2 + m_2, \quad (5.9)$$
$$X_5 = \frac{1}{2}(2s+d-4-2N_\zeta)\Delta_2 + 2m_3, \quad (5.10)$$
$$X_6 = -\frac{1}{2}(2s+d-4-2N_\zeta)\Delta_3 + m_5, \quad (5.11)$$
$$X_7 = (2s+d-4-2N_\zeta)\Delta_3 + m_4, \quad (5.12)$$
$$X_8 = m_6, \qquad X_9 = 2m_6^\dagger, \quad (5.13)$$
$$Y^{(0)} = m_1\Delta_1 - (2s+d-2N_\zeta)m_4^\dagger\Delta_2, \quad (5.14)$$
$$Y^{(1)} = m_1\Delta_2 + (2s+d-2-2N_\zeta)m_2\Delta_2 + m_4\Delta_1 - 2m_4^\dagger\Delta_3 - 2(2s+d-2-2N_\zeta)m_5^\dagger\Delta_3, \quad (5.15)$$
$$Y^{(2)} = m_1\Delta_3 + 2m_2\Delta_3 + 2(2s+d-4-2N_\zeta)m_3\Delta_3 + m_4\Delta_2 + (2s+d-4-2N_\zeta)m_5\Delta_2, \quad (5.16)$$
$$Y^{(1)}_{(A)dS} = -\frac{\theta}{4}(2s+d-3-2N_\zeta)(2s+d-4-2N_\zeta). \quad (5.17)$$

Requiring this variation to vanish gives the equations

$$X_a = 0, \quad a = 1,\dots,9, \quad (5.18)$$
$$Y^{(0)} = 0, \quad (5.19)$$
$$Y^{(1)} + Y^{(1)}_{(A)dS} = 0, \quad (5.20)$$
$$Y^{(2)} = 0. \quad (5.21)$$

Solution to Eq. (5.18) is easily found to be

$$m_1 = (2s+d-4-2N_\zeta)\Delta_2,$$
$$m_4 = -(2s+d-4-2N_\zeta)\Delta_3, \quad (5.22)$$
$$m_2 = -m_1, \qquad m_3 = -\frac{1}{4}m_1, \qquad m_5 = -\frac{1}{2}m_4, \quad (5.23)$$
$$m_4^\dagger = \Delta_1, \qquad m_5^\dagger = -\frac{1}{2}\Delta_1, \qquad m_6 = m_6^\dagger = 0, \quad (5.24)$$

i.e. Eqs. (5.18) allow us to express the operators m_a, $a = 1,\dots,5$, entirely in terms of the operators Δ_2, Δ_3 which enter the gauge transformations. Moreover, the expressions for m_4 (5.22) and $m_4^\dagger$ (5.24) imply the relation

$$\Delta_1^\dagger = -(2s+d-4-2N_\zeta)\Delta_3. \quad (5.25)$$

We now analyze (5.19). Inserting m_1 (5.22) and $m_4^\dagger$ (5.24) in (5.19) we cast (5.19) in the form

$$\Delta_1\big((2s+d-6-2N_\zeta)\Delta_2^+ - (2s+d-2-2N_\zeta)\Delta_2\big) = 0, \quad (5.26)$$

where $\Delta_2 = \Delta_2(N_\zeta)$ and $\Delta_2^+ \equiv \Delta_2(N_\zeta+1)$. Solution to (5.26) $\Delta_1 = 0$ for all N_ζ (or $\Delta_1 = 0$ for some particular eigenvalues of N_ζ) leads to massless fields (or partial massless fields), i.e. such solution leads to appearance of invariant subspaces in $|\psi\rangle$. We are not interested in such solution and assume $\Delta_1 \neq 0$ for all N_ζ. Eq. (5.26) allows us then to find dependence of the operator Δ_2 on N_ζ:

$$\Delta_2 = \frac{\Delta_{2(0)}}{(2s+d-2-2N_\zeta)(2s+d-4-2N_\zeta)}, \quad (5.27)$$

where $\Delta_{2(0)}$ is dimensionfull parameter not depending on N_ζ. Inserting Δ_2 (5.27) in (5.21) one can make sure that (5.21) is satisfied automatically. All that remains then is to solve Eq. (5.20). Making use of (5.22)–(5.25) and (5.27) one can make sure that (5.20) amounts to the equation[10]

$$\begin{aligned} &Z(N_\zeta) - Z(N_\zeta - 1) \\ &\quad - \frac{(2s+d-3-2N_\zeta)(\Delta_{2(0)})^2}{(2s+d-2-2N_\zeta)^2(2s+d-4-2N_\zeta)^2} \\ &\quad - \frac{\theta}{4}(2s+d-3-2N_\zeta) = 0, \end{aligned} \tag{5.28}$$

where we use the notation

$$Z(N_\zeta) \equiv (2s+d-4-2N_\zeta)(N_\zeta+1)(\tilde{\Delta}_3)^2. \tag{5.29}$$

The relation (5.29) implies a condition $Z(-1) = 0$. This condition and (5.28) lead to the initial condition

$$Z(0) = \frac{(2s+d-3)(\Delta_{2(0)})^2}{(2s+d-2)^2(2s+d-4)^2} + \frac{\theta}{4}(2s+d-3). \tag{5.30}$$

Eq. (5.28) and the initial condition (5.30) allow us to find $Z(N_\zeta)$ uniquely and taking into account (5.29) we obtain

$$\begin{aligned} (\tilde{\Delta}_3)^2 = {} & \frac{2s+d-3-N_\zeta}{2s+d-4-2N_\zeta} \\ & \times \left(\frac{(\Delta_{2(0)})^2}{(2s+d-2)^2(2s+d-4-2N_\zeta)^2} + \frac{\theta}{4} \right). \end{aligned} \tag{5.31}$$

Thus we satisfied all equations imposed on $\mathcal{M}$ and Δ by the requirement of gauge invariance of the action and the expressions (5.22)–(5.25), (5.27), (5.31) determine $\mathcal{M}$ and Δ uniquely. In view of the first relation in (5.22) and (5.27) the $\Delta_{2(0)}$ is real-valued and introducing the mass parameter κ (which is assumed to be positive) by relation

$$\Delta_{2(0)} = (2s+d-2)\kappa, \tag{5.32}$$

we arrive at the expressions for $\mathcal{M}$ and Δ given in Section 2.

To summarize, we found the gauge invariant action for the fermionic fields in $(A)dS_d$. All that remains to construct action for fermionic fields in $AdS_5 \times S^5$ Ramond–Ramond background is to add appropriate dependence of S^5-coordinates and take into account contribution of Ramond–Ramond background fields.[11] The result will be reported elsewhere.

Acknowledgements

This work was supported by the INTAS project 03-51-6346, by the RFBR Grant No. 05-02-17654, RFBR Grant for Leading Scientific Schools, Grant No. 1578-2003-2 and Russian Science Support Foundation.

Appendix A. Notation and commutators of oscillators and covariant derivative

We use $2^{[d/2]} \times 2^{[d/2]}$ Dirac gamma matrices γ^A in d-dimensions, $\{\gamma^A, \gamma^B\} = 2\eta^{AB}$, $\gamma^{A\dagger} = \gamma^0\gamma^A\gamma^0$, where η^{AB} is mostly positive flat metric tensor and flat vectors indices of the $so(d-1,1)$ algebra take the values $A, B = 0, 1, \ldots, d-1$. To simplify our expressions we drop η_{AB} in scalar products, i.e. we use $X^AY^A \equiv \eta_{AB}X^AY^B$. Indices $\mu, \nu = 0, 1, \ldots d-1$ stand for indices of space–time base manifold.

We use the algebra of commutators for operators that can be constructed out the oscillators α^A, $\bar{\alpha}^A$ (2.2) and derivative D^A (2.15), (2.16) (see also Appendix A in Ref. [4]). Starting with

$$\begin{aligned} &[\hat{\partial}_A, \hat{\partial}_B] = \Omega_{AB}{}^C\hat{\partial}_C, \qquad \Omega^{ABC} \equiv -\omega^{ABC} + \omega^{BAC}, \\ &\omega_A{}^{BC} \equiv e_A^\mu \omega_\mu^{BC}, \end{aligned} \tag{A.1}$$

where $\hat{\partial}_A \equiv e_A^\mu \partial_\mu$, $\partial_\mu \equiv \partial/\partial x^\mu$, and Ω^{ABC} is a contorsion tensor we get the basic commutator

$$\begin{aligned} [D^A, D^B] &= \Omega^{ABC}D^C + \frac{1}{2}R^{ABCD}M^{CD} \\ &= \Omega^{ABC}D^C + \theta M^{AB}, \end{aligned} \tag{A.2}$$

and R^{ABCD} is a Riemann tensor which for $(A)dS_d$ geometry takes the form

$$R^{ABCD} = \theta(\eta^{AC}\eta^{BD} - \eta^{AD}\eta^{BC}). \tag{A.3}$$

The spin operator M^{AB} is given in (2.17). For flexibility in (A.2) and below we present our relations for a space of arbitrary geometry and for $(A)dS_d$ space. Using (A.2) and the commutators

$$\begin{aligned} &[D^A, \alpha^B] = -\omega^{ABC}\alpha^C, \qquad [D^A, \bar{\alpha}^B] = -\omega^{ABC}\bar{\alpha}^C, \\ &[D^A, \gamma^B] = -\omega^{ABC}\gamma^C, \end{aligned} \tag{A.4}$$

we find straightforwardly

$$[D^A, \alpha^2] = 0, \qquad [\bar{\alpha}^2, D^A] = 0, \qquad [D^A, \gamma\alpha] = 0, \tag{A.5}$$

$$\begin{aligned} &[\bar{\alpha}^2, \alpha D] = 2\bar{\alpha}D, \qquad [\gamma\bar{\alpha}, \alpha D] = \not{D}, \\ &\{\not{D}, \gamma\alpha\} = 2\alpha D, \end{aligned} \tag{A.6}$$

$$\begin{aligned} \not{D}^2 &= D^AD^A + \omega^{AAB}D^B + \frac{1}{4}\gamma^{AB}R^{ABCD}M^{CD} \\ &= D^AD^A + \omega^{AAB}D^B + \frac{\theta}{2}\gamma^{AB}M^{AB}, \end{aligned} \tag{A.7}$$

$$\begin{aligned} [\bar{\alpha}D, \alpha D] &= D^AD^A + \omega^{AAB}D^B - \frac{1}{4}R^{ABCD}M_b^{AB}M^{CD} \\ &= D^AD^A + \omega^{AAB}D^B - \frac{\theta}{2}M_b^{AB}M^{AB}, \end{aligned} \tag{A.8}$$

$$\begin{aligned} [\not{D}, \alpha D] &= \frac{1}{2}\gamma^A\alpha^B R^{ABCD}M^{CD} \\ &= \theta\left(\gamma\alpha\left(\alpha\bar{\alpha} + \frac{d-1}{2}\right) - \alpha^2\gamma\bar{\alpha}\right), \end{aligned} \tag{A.9}$$

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10 It is easy to demonstrate that making use of field redefinitions, the phase factors of $\tilde{\Delta}_3$ can be normalized to be equal to -1. Therefore in (5.28) and below $\tilde{\Delta}_3$ is assumed to be real-valued and negative. Relations (5.5) and Eq. (5.25) imply then that $\tilde{\Delta}_1$ is real-valued and positive.

11 Study of some leading contributions of Ramond–Ramond background fields to mass operator of the bosonic fields in $AdS_5 \times S^5$ Ramond–Ramond background may be found in [37]. Precise form of mass operator for bosonic fields is still to understood.

$$[\bar{\alpha} D, \not{p}] = -\frac{1}{2}\gamma^A \bar{\alpha}^B R^{ABCD} M^{CD}$$
$$= \theta\left(\left(\alpha\bar{\alpha} + \frac{d-1}{2}\right)\gamma\bar{\alpha} - \gamma\alpha\bar{\alpha}^2\right). \quad \text{(A.10)}$$

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