Amplitude Transformation-Based Blind Equalization Part II: Suitable for High-Order QAM Signals

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Abstract

In the digital transmission system, constant modulus algorithm (CMA) is a famous blind equalization to overcome the inter-symbol interference without the aid of training sequences. But for the non-constant modulus signals such as higher-order QAM signals, the CMA just achieve moderate steady-state mean square error (MSE) which may not be sufficient for the system to obtain adequate performance. By studying the amplitude relationship between different constellations of QAM signals, a new amplitude transformation method is proposed. According to such method the non-constant modulus signals can be changed to be constant modulus, and then using it to the CMA the zero steady-state MSE can be obtained. The efficiency of the proposed method is proved by computer simulations.

Keywords: blind equalization; amplitude transformation; constant modulus algorithm; QAM.

1. Introduction

In digital communication systems, Inter-Symbol Interference (ISI) due to bandwidth limited channels or multipath propagation is the main factor which affects the performance of communication systems seriously. Channel equalization is one of the techniques to mitigate these effects. Unlike traditional trained equalization algorithm, the blind channel equalization operating without the aid of a training sequence can save the bandwidth and improve the validity and reliability of communications, so it is more suitable for channel equalization.

The constant modulus algorithm, first introduced in [1], is by far the most popular blind equalization algorithm because of its robustness and because it can be easily implemented [2]. For the constant modulus signals such as 2-PAM and 4-QAM signals, the CMA can obtain perfect performance such as zero steady-state MSE and faster convergence rate. However, for the non-constant modulus signals such as
as 4-PAM and 16-QAM signals, the CMA only achieves moderate residual error, which may not be sufficient for the system to obtain adequate performance [3-6].

In part I, the amplitude relationship between different constellations of PAM signals was studied, and then a new amplitude transformation method for higher-order PAM signals was proposed. According to such amplitude transformation method the non-constant modulus PAM signals can be changed to be a constant modulus PAM signal that is 2-PAM signal. In the CMA, such method is used and then for the non-constant modulus PAM signals it can get the same perfect performance as for constant modulus signals.

In this paper, we use amplitude transformation to the QAM signals and propose the blind equalization based on amplitude transformation suitable for high-order QAM signals. The efficiency of the method is proved by computer simulations.

2. Problem formulations

We start our analysis with CMA2-2[1]. Figure 1 shows a block diagram of the CMA equalizer.

![Fig. 1. Structure of the CMA equalizer](image)

where $s(k)$ is the transmitted symbol, $C(k)$ is the impulse response of the channel, $n(k)$ is the channel noise, $x(k)$ is the equalizer input, $y(k)$ is the equalizer output and $\{\hat{s}(k)\}$ is the output of the decision device. The equalizer N-taps weight vector is defined by $f(k)=[f_0(k), f_1(k), \ldots, f_{N-1}(k)]^T$. The N-taps input vector is defined by $x(k)=[x(k), x(k-1), \ldots, x(k-N+1)]^T$. Equalizer output can be expressed as

$$y(k) = f^T(k)x(k) \quad (1)$$

The cost function of CMA is defined by

$$J_{CMA}(f) = E[|e(k)|^2] \quad (2)$$

where $E[\cdot]$ indicates statistical expectation and $e(k)$ is the error function of CMA, defined by

$$e(k) = |y(k)|^2 - R^2 \quad (3)$$

where $R^2$ is a constant modulus which is defined by

$$R^2 = E[|s(k)|^4]/E[|s(k)|^2] = 1 \quad (4)$$

Using a stochastic gradient algorithm, the weight vector of CMA equalizer is updated by

$$f(k+1) = f(k) - \mu \mathbf{x}^\ast(k)y(k)(|y(k)|^2 - R^2) \quad (5)$$

where $\mu$ is the step size.
The CMA is widely used in practice for its robustness and the capability of opening “initially closed
eye”, and the key of bind equalization is the statistics information $R^2$. And the different order PAM
signals have different statistics information.

For the 4-QAM signal with coordinates $\{\pm 1 \pm j\}$, the amplitudes of its every constellation is $\{\sqrt{2}\}$
which is a constant value. So it belongs to constant modulus signals. And the constant modulus $R$ in (4)
is equal to $\sqrt{2}$ which is the same as its amplitude. In this situation the CMA can obtain zero steady-state
MSE with T/2 fraction scheme [7].

But for the 16-QAM signal with coordinates $\{\pm 1 \pm j, \pm 1 \pm 3 j, \pm 3 \pm j, \pm 3 \pm j 3\}$, the amplitudes of its every
constellation are $\{\sqrt{2}, \sqrt{10}, 3\sqrt{2}\}$ which are not a constant value. So it belongs to non-constant modulus
signals. And the constant modulus $R$ in (4) is equal to $\sqrt{13.2}$ which is the same as none of its amplitudes.
In this situation the steady-state MSE of CMA is very large. Namely, for the non-constant modulus
signals, the cost function of CMA can not become exactly zero even when the channel is perfectly
equalized [7-9].

3. Amplitude transformation method for higher-order QAM signals

Take the 16-QAM signal for example, its coordinates are $\{\pm 1 \pm j, \pm 1 \pm 3 j, \pm 3 \pm j, \pm 3 \pm j 3\}$ and its
amplitudes are $\{\sqrt{2}, \sqrt{10}, 3\sqrt{2}\}$. But the amplitudes of the real parts and the imaginary parts of its
coordinates are $\{1,3\}$. They are set to be $x_r$ and $x_i$ respectively, now we define the amplitude
transformation rule for the 16-QAM signal

$$|x_r - 2|$$

$$|x_i - 2|$$

Then the amplitudes of its real parts and imaginary parts coordinates $\{1,3\}$ are changed to be $\{1\}$ and
$\{1\}$ respectively which is the same as the 4-QAM.

For the 64-QAM signal, its coordinates are $\{\pm 1 \pm j, \pm 1 \pm 3 j, \pm 5 \pm j 3, \pm 3 \pm j, \pm 3 \pm j 3, \pm 3 \pm j 5, \pm 3 \pm j 7, \pm 5 \pm j, \pm 5 \pm 3 j, \pm 3 \pm 5 j, \pm 5 \pm 7 j, \pm 3 \pm 7 j, \pm 7 \pm 3 j, \pm 7 \pm 5 j, \pm 7 \pm 7 j\}$ and its amplitudes of the real parts
and the imaginary parts are $\{1,3,5,7\}$. Now we define an amplitude transformation rule for the 64-QAM
signal

$$\|x_r - 4|-2\|$$

$$\|x_i - 4|-2\|$$

Then the amplitudes $\{1,3,5,7\}$ is changed to be $\{1\}$.

4. Proposed algorithm

For the 16-PAM signal, the cost function suitable for it is defined by

$$J_{16\text{-QAM}}(f) = E[\|y_r(k)\| - 2|^2 - 1|^2] + E[\|y_i(k)\| - 2|^2 - 1|^2]$$

And the cost function suitable for 8-PAM signal is defined by

$$J_{64\text{-QAM}}(f) = E[\|y_r(k)\| - 4|^2 - 1|^2] + E[\|y_i(k)\| - 4|^2 - 1|^2]$$

(10)

(11)
Using the stochastic gradient algorithm to minimize above cost functions, the equalizer weight vector of 16-QAM and 64-QAM signals are updated by

\[
f(k + 1) = f(k) - \mu \mathbf{x}^*(k) \{[|y_r(k)| - 2]^2 - 1][|y_r(k)| - 2] \text{sign}[y_r(k)] + \mu f(|y_i(k)| - 2)^2 - 1][|y_i(k)| - 2] \text{sign}[y_i(k)]\}
\]

and

\[
f(k + 1) = f(k) - \mu \mathbf{x}^*(k) \{[|y_r(k)| - 4]^2 - 1][|y_r(k)| - 4] \text{sign}[y_r(k)] + \mu f(|y_i(k)| - 4)^2 - 1][|y_i(k)| - 4] \text{sign}[y_i(k)]\}
\]

respectively.

5. Simulation study

Here, we present simulation results to demonstrate performances of the proposed algorithm by comparing with conventional CMA in the nonoise channels.

64-QAM data symbols were transmitted through an underwater acoustic channel with the channel impulse response \( \mathbf{c} = [0.3132, -0.1040, 0.8908, 0.3134]^T \). For the CMA and proposed algorithm, they all have 6 taps with a center-spike initialization. Their step sizes were set to 0.000001 and 0.0004 respectively. The results of this simulation are shown in figure 2, 3 and 4. The learning curves presented with the CMA and proposed algorithm in terms of the convergence rate and mean square error are depicted in Figure 2, and the outputs of them after convergence are depicted in figure 3 and figure 4 respectively. It is clear that the steady-state MSE of proposed algorithm is zeros approximately.

![Fig. 2. Learning curves with MSE of algorithms](image)

![Fig. 3. Output of CMA equalizer](image)
6. Conclusions

In this paper, by adjusting the amplitude of received signals the new blind equalization based on amplitude transformation suitable for 16-QAM and 64-QAM signals is proposed. Furthermore, such method can be suitable for other orders QAM signals too, for example we modified the amplitude transformation rule

\[ \| x_r - 8 \| \| x_i - 4 \| \| -2 \| \]  
\[ \| x_r - 8 \| \| x_i - 4 \| \| -2 \| \]  

then it can be suitable for 128-QAM signal.

Compared with the CMA, the proposed algorithm is attractive as it has zero steady-state MSE for higher-order QAM signals.

References