Physics Letters B 733 (2014) 226–232

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Leptonic CP violation and Wolfenstein parametrization for lepton mixing

Zhuo Liu, Yue-Liang Wu *

State Key Laboratory of Theoretical Physics (SKLTP), Kavli Institute for Theoretical Physics China (KITPC), Institute of Theoretical Physics, Chinese Academy of Sciences, University of Chinese Academy of Sciences, Beijing 100190, PR China

ARTICLE INFO

Article history:
Received 28 March 2014
Accepted 27 April 2014
Available online 30 April 2014
Editor: J. Hisano

ABSTRACT

We investigate a general structure of lepton mixing matrix resulting from the $SU_F(3)$ gauge family model with an appropriate vacuum structure of $SU_F(3)$ symmetry breaking. It is shown that the lepton mixing matrix can be parametrized by using the Wolfenstein parametrization method to characterize its deviation from the tri-bimaximal mixing. A general analysis for the allowed leptonic CP-violating phase δ_e and the leptonic Wolfenstein parameters λ_e, A_e, ρ_e is carried out based on the observed lepton mixing angles. We demonstrate how the leptonic CP violation correlates to the leptonic Wolfenstein parameters. It is found that the phase δ_e is strongly constrained and only a large or nearly maximal leptonic CP-violating phase $|\delta_e| \simeq 3\pi/4 \sim \pi/2$ is favorable when $\lambda_e > 0.15$. In particular, when taking λ_e to be the Cabibbo angle $\lambda_e \simeq \lambda \simeq 0.225$, a sensible result for leptonic Wolfenstein parameters and CP violation is obtained with $A_e = 1.40$, $\rho_e = 0.20$, $\delta_e \sim 101.76°$, which is compatible with the one in quark sector. An interesting correlation between leptons and quarks is observed, which indicates a possible common origin of masses and mixing for the charged leptons and quarks.

1. Introduction

The standard model (SM) has been well established with the observation of the last particle predicted in the SM, i.e., Higgs particle, at the LHC experiment [1,2]. The neutrino oscillations with massive neutrinos [3–13] provide a strong evidence and a useful window for exploring new physics beyond the SM. In comparison with the quark masses and CKM quark mixing matrix [14] in the SM, the smallness of neutrino masses and large MNSP lepton mixing [15] have been a long-term puzzle to be understood as a possible indication for new physics. The greatest success of the SM is the gauge symmetry structure $SU_c(3) \times SU_L(2) \times U_Y(1)$ which characterizes three basic forces of strong and electroweak interactions. All the gauge symmetries are associated with the quantum numbers of quarks and leptons. $SU_c(3)$ characterizes the symmetry among three color quantum numbers of quarks, $SU_L(2)$ describes the symmetry between two isospin quantum numbers of quarks and leptons for each family, and $U_Y(1)$ is the symmetry corresponding to the hypercharge quantum number of quarks and leptons. The quark and lepton mixing matrices and CP violations reflect the properties of three family quarks and leptons. To understand the quark and lepton mixing matrices and CP violations, it is interesting to investigate the possible gauge symmetries among three family quantum numbers. Obviously, a non-abelian gauge family symmetry [16–28] for three families of quarks and leptons becomes natural as a simple extension of the SM gauge symmetry structure.

It has been shown in Ref. [16] that the $SU_F(3)$ gauge family symmetry enables us to construct a simple gauge family model for understanding the mixing and masses of leptons. The $SU_F(3)$ gauge family symmetry was first introduced in early time for estimating the top quark mass [29]. It was found in Ref. [16] that the model can provide a consistent prediction for the lepton mixing and neutrino masses when considering the appropriate vacuum structure of $SU_F(3)$ gauge symmetry breaking. Specifically, through appropriately making the $SU_F(3)$ gauge fixing condition with keeping a residual Z_2-permutation symmetry in the neutrino sector, we can obtain in the neutrino sector the so-called tri-bimaximal mixing matrix [30–34] and largely degenerate neutrino masses, while the small mixing matrix in the charged-lepton sector is resulted by requiring the vacuum structure of spontaneous symmetry breaking to possess approximate global U(1) family symmetries [16]. Thus the deviation from tri-bimaximal mixing in the lepton mixing matrix is attributed to the small mixing in the charged-lepton sector, its smallness is protected by the mechanism of approximate global U(1) family symmetries [35–38]. As the spontaneously symmetry

* Corresponding author.
E-mail addresses: liuzhuo@itp.ac.cn (Z. Liu), ylwu@itp.ac.cn (Y.-L. Wu).

http://dx.doi.org/10.1016/j.physletb.2014.04.049

breaking CP-violating phases in the vacuum [39] are not restricted by the considered symmetries, they can in principle be large and maximal. The small masses of the neutrinos and charged leptons are simply ascribed to the usual seesaw mechanism. As a simple case, when applying the Wolfenstein parametrization [40] for the CKM quark mixing matrix to the charged-lepton mixing matrix with a similar hierarchy structure as the CKM quark mixing matrix, and making a naive ansatz that all the smallness due to the approximate global U(1) family symmetries is characterized by a single Wolfenstein parameter $\lambda \simeq 0.22$, we can obtain an interesting prediction for the lepton mixing matrices with a maximal spontaneous CP violation $\delta \simeq \pi/2$ [16]

$$\sin^2\theta_{13} \simeq \frac{1}{2}\lambda^2 \simeq 0.024 \quad \left(\text{or } \sin^2 2\theta_{13} \simeq 0.094\right), \tag{1}$$

$$\sin^2\theta_{12} \simeq \frac{1}{3}, \qquad \sin^2\theta_{23} \simeq \frac{1}{2}, \tag{2}$$

which agrees with the current experimental data [41–43]. The corresponding Leptonic Jarlskog CP-violating invariant quantity [44] reaches the maximal value

$$J^e_{\mathrm{CP}} \simeq \frac{1}{6}\lambda \sin\delta \simeq 0.037. \tag{3}$$

The resulting neutrino masses are largely degenerate with the value at the order $m_{\nu_i} \simeq O(\lambda^2) \simeq 0.04$–$0.06$ eV with a total mass $\sum m_\nu \sim 0.15$ eV, which is much larger than the minimal limit $\sum m_\nu \sim 0.05$ eV and is expected to be tested by the future experiments.

It is widely expected that the leptonic CP violation can be maximal or large enough to account for the observed matter–antimatter asymmetry in the universe via the leptogenesis mechanism as the CP violation in the SM is not enough to understand the baryogenesis. In this note, we are going to make a general analysis on the leptonic CP-violating phase and its correlation with the deviation from the tri-bimaximal neutrino mixing based on the current experimental results.

2. Wolfenstein parametrization of lepton mixing matrix for characterization of deviation from tri-bimaximal mixing

Let us begin with the following general structure of MNSP lepton mixing matrix

$$\begin{aligned} V_{\mathrm{MNSP}} &= V_e^\dagger V_\nu \\ &\equiv P_\beta \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \\ &\quad \times \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \end{aligned} \tag{4}$$

which has been expressed into the standard form with $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ [41]. An interesting symmetric parametrization was discussed in [45]. Where P_β is a diagonal matrix of phase factors and can be rotated away by the redefinition of charged-lepton fields, and ϕ_i are the so-called Majorana phases for Majorana neutrinos. It is known that the lepton mixing matrix generally arises from two mixing matrices V_e and V_ν, they correspond to the charged-lepton and neutrino mixing matrices arising from diagonalizing the charged-lepton mass matrix and neutrino mass matrix respectively. When the charged-lepton mass matrix is Hermitian $M_e = P_\delta U_e m_E U_e^\dagger P_\delta^\dagger = M_e^\dagger$ with m_E the diagonal mass matrix of charged leptons $m_E = \mathrm{diag}(m_e, m_\mu, m_\tau)$, the unitary charged-lepton mixing matrix V_e can in general be written as

$$\begin{aligned} V_e^\dagger &= U_e^\dagger P_\delta^*, \\ U_e^\dagger &\equiv \begin{pmatrix} c^e_{12}c^e_{13} & s^e_{12}c^e_{13} & s^e_{13}e^{-i\delta'_e} \\ -s^e_{12}c^e_{23} - c^e_{12}s^e_{23}s^e_{13}e^{-i\delta'_e} & c^e_{12}c^e_{23} - s^e_{12}s^e_{23}s^e_{13}e^{-i\delta'_e} & s^e_{23}c^e_{13} \\ s^e_{12}s^e_{23} - c^e_{12}c^e_{23}s^e_{13}e^{-i\delta'_e} & -c^e_{12}s^e_{23} - s^e_{12}c^e_{23}s^e_{13}e^{-i\delta'_e} & c^e_{23}c^e_{13} \end{pmatrix}, \end{aligned} \tag{5}$$

$$P_\delta = \begin{pmatrix} e^{i\delta_1} & 0 & 0 \\ 0 & e^{i\delta_2} & 0 \\ 0 & 0 & e^{i\delta_3} \end{pmatrix}, \tag{6}$$

where U_e is a unitary matrix with CP-violating phase δ'_e, and mixing angles $c^e_{ij} = \cos\theta^e_{ij}$, $s^e_{ij} = \sin\theta^e_{ij}$. P_δ is a diagonal phase matrix with three phases δ_i, while only two relative phases $(\delta_i - \delta_j)$ are physically observable CP-violating phases.

For the neutrino mixing matrix, when an appropriate Z_2-symmetric neutrino mass matrix between the second and third neutrinos is considered to have three independent matrix elements, the resulting neutrino mixing matrix is completely determined to be

$$V_\nu = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}, \tag{7}$$

which is the so-called tri-bimaximal neutrino mixing matrix.

It is noticed that when three phases δ^m_{ij} $(i, j = 1, 2, 3,\ i < j)$ in the Hermitian mass matrix are not independent and they are related via $\delta^m_{ij} = \delta_i - \delta_j$, one then has

$$\delta'_e = 0. \tag{8}$$

On the other hand, when θ^e_{13} is small in comparison with θ^e_{12} and θ^e_{23}, i.e., $\theta^e_{13} \ll \theta^e_{12}, \theta^e_{23}$, it is easily seen that δ'_e will not be a dominant source of leptonic CP violation as the CP-violating phase δ'_e is associated with the mixing angle θ^e_{13}. In this situation, we may neglect the effect of CP-violating phase δ'_e and take a typical case $\delta'_e \simeq 0$ for simplicity of discussions. With these considerations, we may replace the unitary matrix U_e by an orthogonal rotation matrix $O_e = U_e(\delta'_e = 0)$

$$\begin{aligned} U_e^\dagger &\to O_e^T \\ &= \begin{pmatrix} c^e_{12}c^e_{13} & s^e_{12}c^e_{13} & s^e_{13} \\ -s^e_{12}c^e_{23} - c^e_{12}s^e_{23}s^e_{13} & c^e_{12}c^e_{23} - s^e_{12}s^e_{23}s^e_{13} & s^e_{23}c^e_{13} \\ s^e_{12}s^e_{23} - c^e_{12}c^e_{23}s^e_{13} & -c^e_{12}s^e_{23} - s^e_{12}c^e_{23}s^e_{13} & c^e_{23}c^e_{13} \end{pmatrix}. \end{aligned} \tag{9}$$

To investigate whether the leptonic CP violation can be maximally large with the present experimental measurements on the three mixing angles and the lepton mixing matrix can be characterized by the Wolfenstein parametrization method, we may make a sensible analysis by simply taking $\delta'_e = 0$. In Ref. [46], the angle θ^e_{13} is assumed to be zero, thus the effect of δ'_e automatically disappears. An alternative consideration was analyzed in [47], where the phase δ'_e was assumed to be the only CP-violating source and the phases δ_i are taken to be zero, i.e., $\delta_i = 0$.

The leptonic mixing angles θ_{ij} and mass-square differences Δm^2_{ij} have been measured by many experiments including the solar neutrino experiment, atmospheric neutrino experiment, accelerator experiment and reactor experiment. The best-fit results presented in PDG [41] are

$$\begin{aligned} &\sin^2 2\theta_{12} = 0.857 \pm 0.024, \\ &\sin^2 2\theta_{13} = 0.095 \pm 0.010, \\ &\sin^2 2\theta_{23} > 0.95, \end{aligned} \tag{10}$$

which slightly deviates from the tri-bimaximal neutrino mixing.

Note that the presently extracted mixing angles from experiments are not sensitive to the CP-violating phase due to the smallness of the effects concerning the CP violation. As the leptonic CP violation is strongly correlated to the non-zero θ_{13} which characterizes the deviation from the tri-bimaximal neutrino mixing matrix, it is then interesting to investigate the leptonic CP-violating phase and its correlation with the deviation from the tri-bimaximal neutrino mixing based on the above structure of lepton mixing matrix and the current experimental results. It is seen that the deviation from tri-bimaximal neutrino mixing is described by the charged-lepton mixing matrix U_e or orthogonal rotation matrix O_e, the smallness of the mixing angle θ_{13} indicates that $s^e_{12} \sim \mathcal{O}(0.1)$, which motivates us to parametrize the rotation matrix O_e via the Wolfenstein parametrization [40] with a hierarchy structure similar to the CKM quark mixing matrix. With the leptonic Wolfenstein parameter $s^e_{12} \sim \lambda_e \sim \mathcal{O}(0.1)$, the charged-lepton mixing matrix can be written, to the order $\mathcal{O}(\lambda_e^3)$, as the following form:

$$V_e^\dagger \simeq P_\delta^* \begin{pmatrix} 1-\frac{\lambda_e^2}{2} & \lambda_e\, e^{i(\delta_1-\delta_2)} & A_e\lambda_e^3\rho_e\, e^{i(\delta_1-\delta_3)} \\ -\lambda_e\, e^{i(\delta_2-\delta_1)} & 1-\frac{\lambda_e^2}{2} & A_e\lambda_e^2\, e^{i(\delta_2-\delta_3)} \\ A_e\lambda_e^3(1-\rho_e)\, e^{i(\delta_3-\delta_1)} & -A_e\lambda_e^2\, e^{i(\delta_3-\delta_2)} & 1 \end{pmatrix}, \tag{11}$$

where the phase matrix P_δ^* can be absorbed by the redefinitions of charged lepton fields. Note that there is no corresponding Wolfenstein parameter η_e in the above parametrization as we have neglected the CP-violating phase δ'_e. Thus the lepton mixing matrix is given by

$$V_{\mathrm{MNSP}} = \begin{pmatrix} 1-\frac{\lambda_e^2}{2} & \lambda_e\, e^{i\delta_{12}} & A_e\lambda_e^3\rho_e\, e^{i\delta_{13}} \\ -\lambda_e\, e^{-i\delta_{12}} & 1-\frac{\lambda_e^2}{2} & A_e\lambda_e^2\, e^{i\delta_{23}} \\ A_e\lambda_e^3(1-\rho_e)\, e^{-i\delta_{13}} & -A_e\lambda_e^2\, e^{-i\delta_{23}} & 1 \end{pmatrix} \times \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}, \tag{12}$$

which shows that based on the tri-bimaximal neutrino mixing, the lepton mixing matrix can be parametrized by three leptonic Wolfenstein parameters: λ_e, A_e, ρ_e, and CP-violating phases $\delta_{ij} = \delta_i - \delta_j$ $(i = 1, 2, 3)$ with $\delta_{23} = \delta_{21} - \delta_{31}$.

As indicated from Z_2 symmetry of vacuum structure in the $\mathrm{SU}(3)_F$ model [16], it is reasonable to assume that $\delta_2 \simeq \delta_3$. When expressing the lepton mixing matrix V_{MNSP} to be the standard form by requiring the matrix elements V_{11}, V_{12}, V_{23}, V_{33} be real with keeping two independent Majorana phases, we can read off the leptonic CP-violating phase from V_{13}

$$\delta_e = \delta_2 - \delta_1 \simeq \delta_3 - \delta_1. \tag{13}$$

The Wolfenstein parametrization of lepton mixing matrix is simplified to be

$$V_{\mathrm{MNSP}} = \begin{pmatrix} 1-\frac{\lambda_e^2}{2} & \lambda_e e^{-i\delta_e} & A_e\lambda_e^3\rho_e e^{-i\delta_e} \\ -\lambda_e e^{i\delta_e} & 1-\frac{\lambda_e^2}{2} & A_e\lambda_e^2 \\ A_e\lambda_e^3(1-\rho_e)e^{i\delta_e} & -A_e\lambda_e^2 & 1 \end{pmatrix} \times \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}. \tag{14}$$

In terms of the standard form Eq. (4), the lepton mixing matrix can be rewritten in terms of the leptonic Wolfenstein parameters as the following form

$$V_{\mathrm{MNSP}} = \begin{pmatrix} |V_{1,1}| & |V_{1,2}| & \frac{\lambda_e}{\sqrt{2}}(1-A_e\lambda_e^2\rho_e)e^{-i\delta_e} \\ e^{-i\phi_1}V_{2,1} & e^{-i\phi_2}V_{2,2} & \frac{1}{\sqrt{2}}(1-A_e\lambda_e^2-\lambda_e^2/2) \\ e^{-i\phi_1}V_{3,1} & e^{-i\phi_2}V_{3,2} & -\frac{1}{\sqrt{2}}(1+A_e\lambda_e^2) \end{pmatrix} \times \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{15}$$

where $V_{i,j}$ are the matrix elements of V_{MNSP} via Eq. (14). The two Majorana phases ϕ_1, ϕ_2 turn out to be

$$\phi_1 = \arg V_{1,1} = \arctan\frac{(\lambda_e + A_e\lambda_e^3\rho_e)\sin\delta_e}{2-\lambda_e^2-(\lambda_e+A_e\lambda_e^3\rho_e)\cos\delta_e},$$
$$\phi_2 = \arg V_{1,2} = \arctan\frac{-(\lambda_e + A_e\lambda_e^3\rho_e)\sin\delta_e}{1-\lambda_e^2/2+(\lambda_e+A_e\lambda_e^3\rho_e)\cos\delta_e}, \tag{16}$$

and the mixing angle θ_{ij} can be expressed in terms of the leptonic Wolfenstein parameters as

$$s_{13} = \frac{\lambda_e}{\sqrt{2}}\left|1-A_e\rho_e\lambda_e^2\right|, \tag{17}$$

$$s_{23} = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1-s_{13}^2}}\left|1-\frac{\lambda_e^2}{2}-A_e\lambda_e^2\right|, \tag{18}$$

$$s_{12} = \frac{1}{\sqrt{3}}\frac{1}{\sqrt{1-s_{13}^2}}\left|1-\frac{\lambda_e^2}{2}+\lambda_e\big(1+A_e\rho_e\lambda_e^2\big)e^{-i\delta_e}\right|, \tag{19}$$

which shows that the leptonic Wolfenstein parameters λ_e, A_e, ρ_e, and the CP-violating phase δ_e characterize the lepton mixing with deviation from the tri-bimaximal mixing.

As an illustration, it is interesting to observe that by taking the leptonic Wolfenstein parameters to be the following typical values with a maximal CP-violating phase

$$\lambda_e \sim 0.22, \qquad A_e \sim 1, \qquad \rho_e \sim 1,$$
$$\delta_e = \delta_2 - \delta_1 = \delta_3 - \delta_1 \sim \frac{\pi}{2}, \tag{20}$$

we obtain the predictions for the lepton mixing angles

$$\sin^2 2\theta_{12} \sim 0.901,$$
$$\sin^2 2\theta_{13} \sim 0.086,$$
$$\sin^2 2\theta_{23} \sim 0.986, \tag{21}$$

which are consistent with the PDG's best-fit results given in Eq. (10) [41] at 1σ level, except a small mismatch of θ_{12}. Such a consistency shows that the leptonic Wolfenstein parameters chosen in Eq. (20) are in the reasonable region of parameter space.

Alternatively, we may use the PDG's value of θ_{12} given in Eq. (10) $\sin^2 2\theta_{12} = 0.857 \pm 0.024$ to extract the leptonic CP-violating phase δ_e. With other parameters chosen as Eq. (20), it is easily found that

$$\delta_e = \big(101.94^{-6.28}_{+5.90}\big)^\circ, \tag{22}$$

which is very close to the maximal CP-violating phase $\delta_e \sim 0.57\pi$. The corresponding two Majorana phases with the input parameters as Eq. (20) are yielded to be

$$\phi_1 \sim 6.7^\circ, \qquad \phi_2 \sim -13.3^\circ. \tag{23}$$

We shall make a general constraint on the leptonic Wolfenstein parameters λ_e, A_e, ρ_e, and the CP-violating phase δ_e by a detailed analysis below.

3. Constraints on leptonic Wolfenstein parameters

As it is shown in previous section that the deviation from tri-bimaximal lepton mixing matrix can be described by three leptonic Wolfenstein parameters λ_e, A_e, ρ_e. It is seen from Eq. (17) and Eq. (18) that $\sin\theta_{13}$ depends on λ_e and $A_e\rho_e$, while $\sin\theta_{23}$ relies on λ_e and A_e. In this section, we shall take the mixing angles θ_{13} and θ_{23} indicated from the measurements as the input to provide a general constraint on leptonic Wolfenstein parameters λ_e, A_e, ρ_e.

3.1. Constraints from θ_{13}

The precise measurements on θ_{13} have been carried out by DayaBay Collaboration group [42] and RENO Collaboration group [43]. These two experiments measured the disappearance of $\bar{\nu}_e$ from the reactor. The Δm^2_{31} dominated amplitude is given by

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m^2_{31} L}{4E}, \tag{24}$$

which shows that the measured results do not sensitively correlate to the values of other two mixing angles θ_{12}, θ_{23} and CP-violating phase δ_e. From Eq. (17), it is seen that θ_{13} is also insensitive to the CP-violating phase δ_e. Thus we may use the experimental data on θ_{13} to make constraints on the leptonic Wolfenstein parameters.

Before doing that, it is noticed that when keeping the expansion of lepton mixing matrix to the order $\mathcal{O}(\lambda_e)$, we arrive at the following simple relation

$$\lambda_e \sim \frac{s_{13}}{s_{23}} \simeq 0.23, \tag{25}$$

where we have used the best fit values $\sin^2\theta_{13} \sim 0.0225$ and $\sin^2\theta_{23} \sim 0.42$ [41] to yield the numerical value $\lambda_e \sim 0.23$, which is very close to the Wolfenstein parameter of Cabibbo angle $\sin\theta_c = \lambda \simeq 0.225$ in quark sector [41]. This observation checks the consistence of the assumption that $\lambda_e \sim \mathcal{O}(10^{-1})$.

Let us now turn to make a general analysis by adopting the precisely measured mixing angle θ_{13} [41]

$$\sin^2 2\theta_{13} = 0.096 \pm 0.013 (\pm 0.040) \quad \text{at } 1\sigma\ (3\sigma), \tag{26}$$

which enables us to constrain the allowed region of the combined leptonic Wolfenstein parameters $A_e\rho_e$ for a given λ_e.

The contour plot for the input $\sin^2 2\theta_{13}$ is shown in Fig. 1. It is seen from Fig. 1 that $A_e\rho_e\lambda_e^2 < 0$ only occur for small values of λ_e. In the plot, we have restricted the region to be in the range $-1 \le A_e\rho_e\lambda_e^2 \le 0.5$, so that it satisfies the perturbative expanding of Wolfenstein parametrization. It also leads to a reasonable region for the parameter λ_e

$$\lambda_e \simeq 0.11\text{–}0.40, \tag{27}$$

which will be taken to be a possible allowed region when considering constraints from other two mixing angles θ_{12} and θ_{23}.

From Eq. (17), it is seen that $\sin^2\theta_{13}$ is an even function of λ_e. Thus for the region with $\lambda_e < 0$, the contours of $\sin^2\theta_{13}$ are just the mirror images of Fig. 1, which is omitted here.

3.2. Constraints from θ_{23} and θ_{13}

For tri-bimaximal mixing, there is a maximal mixing $\sin^2\theta_{23} = 1/2$. The small deviation to the maximal mixing indicates that

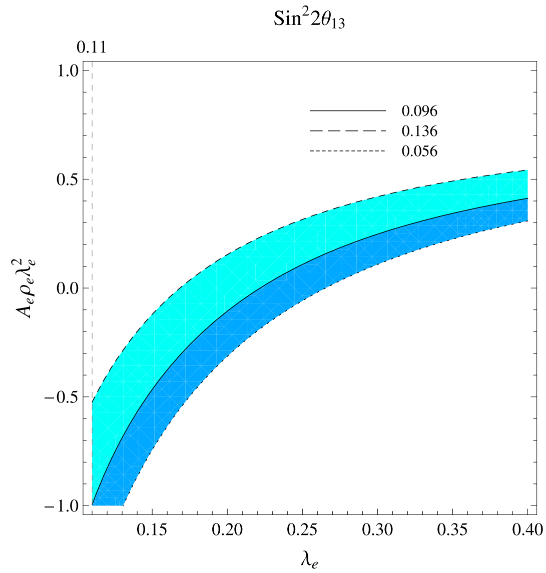

Fig. 1. The contour plot of $\sin^2 2\theta_{13}$ for $A_e\rho_e\lambda_e^2$ as a function of λ_e. The contours show the best fit value and 3σ deviations.

the leptonic Wolfenstein parameters should be small, which may still be consistent with the current data within the experimental errors. While the recent global fitting results appear to indicate a quite large deviation from the maximal mixing with $\sin^2\theta_{23} = 0.386^{+0.024}_{-0.021}$ [48], and $\sin^2\theta_{23} = 0.41^{+0.037}_{-0.025}$ [49]. For a general discussion, we may consider a constraint from a wide range of θ_{23} by covering over different global fitting results, i.e., $\sin^2\theta_{23} \simeq 0.365\text{–}0.450$. The resulting constraint is shown in the left panel of Fig. 2 for parameter A_e as a function of λ_e.

By combining the constraint from θ_{13} and θ_{23}, we are able to obtain the constraint for the allowed region of ρ_e as a function of λ_e. As shown in the right panel of Fig. 2, by taking the central value of $\sin^2\theta_{13}$ given in Eq. (26), we can obtain the allowed region for ρ_e as a function of λ_e from the given values of $\sin^2\theta_{23}$. It is seen that a wide region swept by the curve when $\sin^2\theta_{23}$ increasing from 0.365 to 0.450 is allowed.

Note that there is a special situation that for $\lambda_e = \sqrt{2}\sin\theta_{13}$, then $A_e\rho_e = 0$, namely $\rho_e = 0$ for $A_e \neq 0$. As a consequence, four curves intersect with each other at this point, as indicated in Fig. 2.

4. Leptonic CP violation and lepton–quark correlation

In this section, we should make a general analysis on the leptonic CP-violating phase and its correlation to the deviation from the tri-bimaximal neutrino mixing, which is characterized by the leptonic Wolfenstein parameters as discussed in the previous section.

4.1. Constraints from θ_{12} and leptonic CP violation

It is seen from Eq. (19) that $\sin\theta_{12}$ depends on CP-violating phase δ_e, λ_e, $A_e\rho_e$. Here $A_e\rho_e$ can be constrained from θ_{13} for a given λ_e.

The mixing angle θ_{12} is well determined from solar neutrino oscillation experiments. The measured value of θ_{12} generally correlates to the value of θ_{23}. It is convenient to obtain the values of θ_{12} by setting $\sin^2 2\theta_{23} = 1$. The global fitting results have provided us with both values of θ_{23} and θ_{12}. Although a non-maximal θ_{23} is hinted [48], there is no tension among different global fitting results on θ_{12}. For instance, $\sin^2\theta_{12} = 0.307^{+0.018}_{-0.016}$ [48], $\sin^2\theta_{12} = 0.311 \pm 0.013$ [49], and $\sin^2\theta_{12} = 0.320^{+0.016}_{-0.017}$ [50].

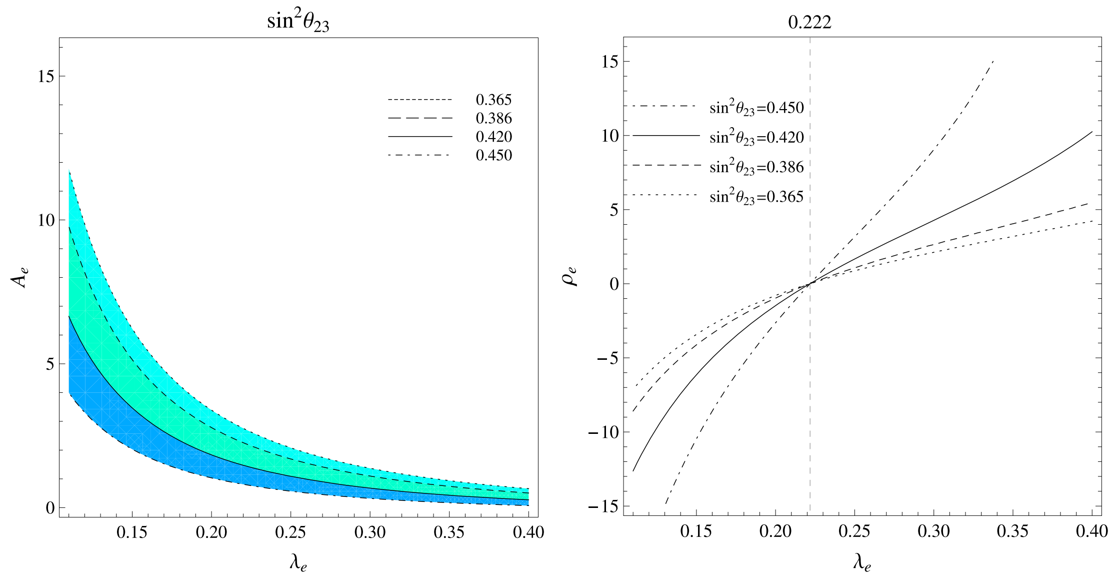

Fig. 2. The left panel is the contour plots of $\sin^2\theta_{23}$ for A_e as a function of λ_e. The right panel is the contour plots of $\sin^2\theta_{23}$ for ρ_e as a function of λ_e, where the central value of $\sin^2\theta_{13}$ has been used to fix $A_e\rho_e$ for a given λ_e. The vertical line labels a critical value of $\lambda_e = \sqrt{2}\sin\theta_{13}$, where $\rho_e = 0$.

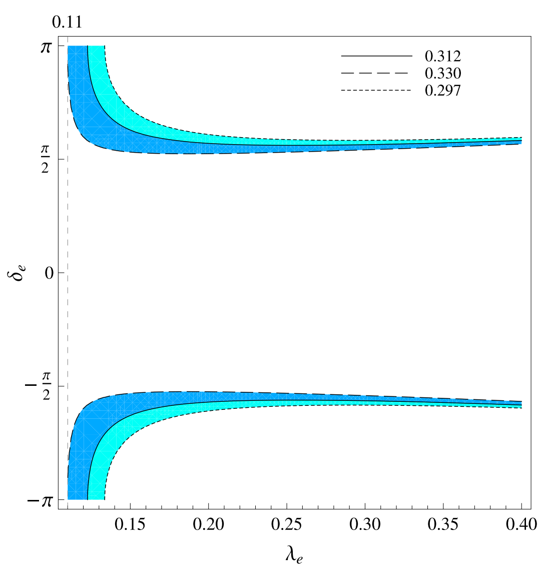

Fig. 3. The contour plot of $\sin^2\theta_{12}$ for the CP-violating phase δ_e as a function of λ_e with the best fit value of $\sin^2\theta_{12}$ and 1σ deviations. For $0.11 \le \lambda_e \le 0.15$, the allowed δ_e ranges from $\pm\pi$ to $\pm\pi/2$, and for $\lambda > 0.2$, the resulting δ_e is close to maximal $\pm\pi/2$.

Here we take the result $\sin^2\theta_{12} = 0.312^{+0.018}_{-0.015}$ given in PDG [41] to make constraints on the CP-violating phase δ_e as a function of λ_e. The value of $A_e\rho_e$ is constrained from θ_{13}. The allowed region for CP-violating phase δ_e is given as a function of λ_e in Fig. 3, where we have taken the central value $\sin^2 2\theta_{13} = 0.096$ to yield the value of $A_e\rho_e$.

It is seen from Fig. 3 that there are two special regions: for $0.11 \le \lambda_e \le 0.15$, the values of $\sin^2\theta_{12}$ is insensitive to δ_e, the allowed region of δ_e ranges from $\pm\pi$ to $\pm\pi/2$. While for $\lambda_e \ge 0.2$, the constraint on δ_e becomes very strong, the resulting CP-violating phase is near maximal $\delta_e \sim \pm\pi/2$. In this region, $\sin^2\theta_{12}$ is insensitive to the values of λ_e. Thus the leptonic CP violation favors a maximal CP violation for a large range of leptonic Wolfenstein parameter λ_e. Note that a minimal CP-violating phase $\delta_e \sim 1.08\pi$ was obtained in a global fit [48] when the atmospheric neutrino data are included, while such a fitting result corresponds to a special region in the parameter space, which does not exclude a large or nearly maximal CP violation.

4.2. Combination of all constraints and lepton–quark correlation

It is useful to combine all the constraints obtained from $\sin^2\theta_{13}$, $\sin^2\theta_{23}$ and $\sin^2\theta_{12}$ and plot them together in the same Fig. 4, so that it is easily seen the allowed values of A_e, ρ_e and δ_e for a given λ_e. A big uncertainty arises from whether $\sin^2\theta_{23}$ is largely deviate from the maximal mixing, which makes the allowed values of A_e and ρ_e become large.

It is easy to see from Fig. 4 that there are two typical regions for leptonic Wolfenstein parameters characterized with a small λ_e ($\lambda_e < 0.15$) and a large λ_e ($\lambda_e > 0.15$). For the small λ_e, we have

$$\begin{aligned} &\lambda_e \in [0.11, 0.15], \qquad A_e \in [12, 2], \\ &\rho_e \in [-12, -2], \qquad |\delta_e| \in [\pi, \pi/2), \end{aligned} \tag{28}$$

which shows that the CP-violating phase δ_e is not well constrained in this case.

A global fitting result cited in [48] corresponds to a solution of the small λ_e with $\delta_e \sim \pi$. From the results given in [48] for the normal hierarchy: $\sin^2\theta_{23} = 0.386^{+0.024}_{-0.021}$, $\sin^2\theta_{13} = 0.0241 \pm 0.0025$, $\sin^2\theta_{12} = 0.307^{+0.018}_{-0.016}$, and $\delta_e = 1.08\pi$, one can easily read from Fig. 4 the corresponding leptonic Wolfenstein parameters

$$\begin{aligned} &\lambda_e = 0.127^{-0.013}_{+0.012}, \qquad A_e = 7.27^{-1.67}_{+1.50}, \\ &\rho_e = -6.21^{-0.75}_{+0.79}. \end{aligned} \tag{29}$$

With the central values, two Majorana phases are found to be very small

$$\phi_1 = -0.29^\circ, \qquad \phi_2 = 0.61^\circ. \tag{30}$$

For the case of inverted hierarchy, the result is very close to the above one, we shall omit it here.

For large values of λ_e, we have

$$\begin{aligned} &\lambda_e \in [0.15, 0.4], \qquad A_e \in [7, 0], \\ &\rho_e \in [-10, 15], \qquad |\delta_e| \sim [3\pi/4, \pi/2), \end{aligned} \tag{31}$$

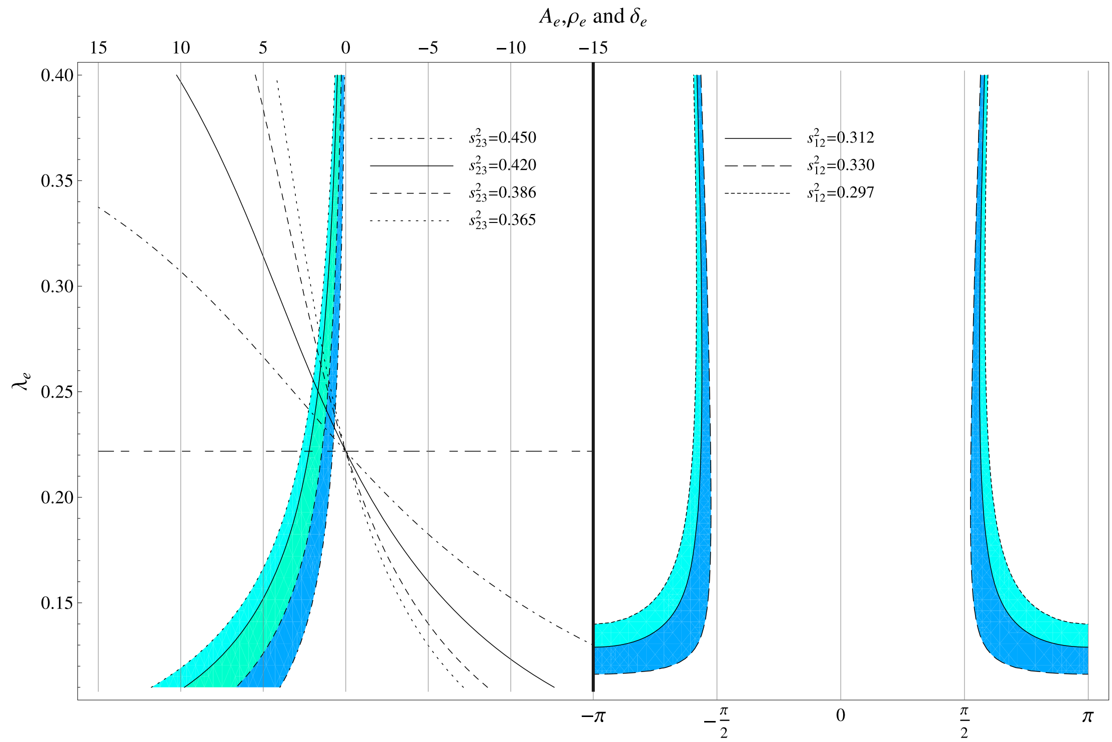

Fig. 4. The allowed parameter regions for A_e, ρ_e and δ_e for given values of λ_e.

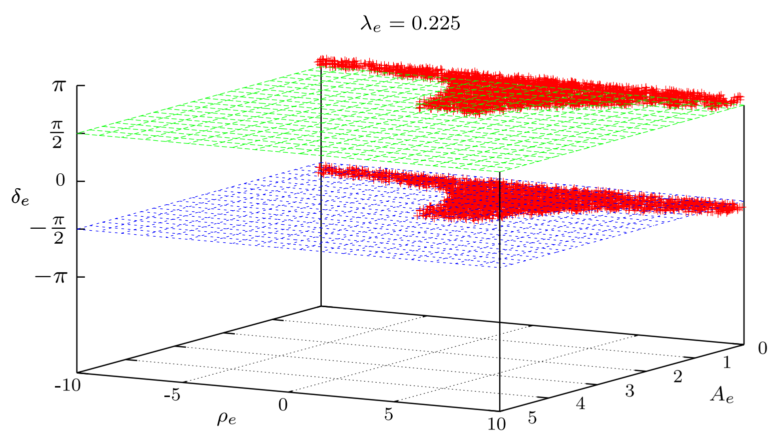

Fig. 5. The allowed regions of leptonic Wolfenstein parameters in parameter space for $\lambda_e = 0.225$.

where the CP-violating phase δ_e is strongly constrained, only a large or nearly maximal CP violation is favorable.

It is interesting to observe from Fig. 4 that when taking the value of the leptonic Wolfenstein parameter λ_e to be the same as the one in the quark sector, $\lambda_e \simeq \lambda \simeq 0.225$, and fixing the lepton mixing angles to be the central values $\sin^2 2\theta_{12} = 0.857$ and $\sin^2\theta_{23} = 0.42$, we arrive at a sensible result for the leptonic Wolfenstein parameters

$$\begin{aligned}
&\lambda_e \simeq 0.225, \qquad A_e = 1.40\\
&\rho_e = 0.20, \qquad \delta_e \sim 101.76^\circ\\
&\phi_1 = 6.40^\circ, \qquad \phi_2 = -13.56^\circ,
\end{aligned} \tag{32}$$

which is compatible with the Wolfenstein parameters in quark sector

$$\begin{aligned}
&\lambda \simeq 0.225, \qquad A = 0.811,\\
&\rho_e = 0.131, \qquad \eta = 0.345 \quad \text{or} \quad \delta \simeq 69^\circ.
\end{aligned} \tag{33}$$

In this case, the resulting lepton mixing matrix is given by

$$V_{\text{MNSP}} = \begin{pmatrix} 0.820 & 0.551 & 0.157e^{i0.57\pi} \\ -0.407-0.135i & 0.642+0.024i & 0.639 \\ -0.378+0.052i & 0.518+0.132i & -0.757 \end{pmatrix} \times \begin{pmatrix} e^{0.11i} & 0 & 0 \\ 0 & e^{-0.24i} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{34}$$

To show manifestly such an interesting situation, it is useful to plot the leptonic Wolfenstein parameters in a parameter space as shown in Fig. 5. It is easily seen that when fixing the parameter $\lambda_e \simeq \lambda \simeq 0.225$, the whole parameters space for A_e, ρ_e and δ_e is almost located on two planes with $\delta_e \sim \pm\pi/2$. The above analysis implies that only a large or nearly maximal leptonic CP violation is favorable in a large region of parameter space when $\lambda_e > 0.15$.

The above results indicate a strong correlation between charged leptons and quarks. An assumption that $V_e \simeq V_{\text{CKM}}$ and $V_{\text{MNSP}} \simeq V^{\dagger}_{\text{CKM}} V_{TB}$ was discussed early in [51–53].

5. Conclusions and remarks

We have shown that the lepton mixing can be parametrized by the Wolfenstein parametrization method based on a general structure of lepton mixing matrix, where the mixing matrix from neutrino sector is a tri-bimaximal mixing and the mixing matrix from charged lepton has small mixing. Such a structure of lepton mixing has been shown to be resulted from the $\text{SU}_F(3)$ gauge family model [16] when considering the appropriate vacuum structure of $\text{SU}_F(3)$ gauge symmetry breaking. Where the tri-bimaximal mixing can be yielded from the residual Z_2-permutation symmetry in the neutrino sector and the small mixing in the charged-lepton sector is led by requiring the vacuum structure of spontaneous symmetry breaking to possess approximate global U(1) family symmetries. We have demonstrated that the small mixing matrix in the charged-lepton sector characterizes the deviation from tri-bimaximal mixing in the lepton mixing matrix, and can be parametrized by the Wolfenstein parametrization method. As the spontaneous CP-violating phases in the vacuum are in general not restricted by the considered symmetries, so that they can in principle be large and maximal.

Based on the input values of lepton mixing angles θ_{13}, θ_{23} and θ_{12} indicated from various neutrino experiments, we have made a general analysis for the allowed leptonic CP-violating phase δ_e and leptonic Wolfenstein parameters λ_e, A_e, ρ_e. It has explicitly been shown how the leptonic CP violation correlates to the leptonic Wolfenstein parameters which characterize the deviation of tri-bimaximal lepton mixing. For a reasonable range of parameter $\lambda_e \simeq 0.11$–0.40, there appear two typical regions, i.e., one with $\lambda_e \simeq 0.11$–0.15, and other with $\lambda_e \simeq 0.15$–0.40. For the small values of $\lambda_e \simeq 0.11$–0.15, the mixing angles θ_{ij} are insensitive to δ_e, thus the CP-violating phase δ_e is not well constrained, its allowed region can range from $|\delta_e| \sim \pi$ to $|\delta_e| \sim \pi/2$. While for the large values of $\lambda_e \simeq 0.15$–0.40, the CP-violating phase δ_e has strongly been constrained, only a large or nearly maximal leptonic CP violation with $|\delta_e| \simeq (3\pi/4)$–$(\pi/2)$ is allowed.

It has been demonstrated that when taking the leptonic Wolfenstein parameter λ_e to be the Cabibbo angle in quark sector, $\lambda_e \simeq \lambda \simeq 0.225$, we are able to obtain a sensible result with $\lambda_e \simeq 0.225$, $A_e = 1.40$, $\rho_e = 0.20$, $\delta_e \sim 101.76°$, which is compatible with the Wolfenstein parameters in quark sector: $\lambda \simeq 0.225$, $A = 0.811$, $\rho_e = 0.131$, $\delta \simeq 69°$. Such a correlation implies a possible common origin of masses and mixing angles for the charged leptons and quarks.

In conclusion, the lepton mixing matrix can well be characterized by leptonic Wolfenstein parameters in the basis of tri-bimaximal neutrino mixing. The leptonic CP violation has a strong correlation to the leptonic Wolfenstein parameters, a large or nearly maximal leptonic CP violation is favorable in a large region of parameters. More precise measurements for the lepton mixing angles are very helpful. It is essential to have a direct measurement for the leptonic CP violation in near future.

Acknowledgements

The authors would like to thank Petcov and Valle for useful discussions and comments. This work is supported in part by the National Natural Science Foundation of China (NSFC) under Grants No. 10975170, No. 10905084, No. 10821504; and the Project of Knowledge Innovation Program (PKIP) of the Chinese Academy of Sciences.

References

[1] ATLAS Collaboration, G. Aad, et al., Phys. Lett. B 716 (2012) 1.
[2] CMS Collaboration, S. Chatrchyan, et al., Phys. Lett. B 716 (2012) 30.
[3] Super-Kamiokande Collaboration, Y. Fukuda, et al., Phys. Rev. Lett. 81 (1998) 1562.
[4] SNO Collaboration, Phys. Rev. Lett. 87 (2001) 71301.
[5] KamLAND Collaboration, K. Eguchi, et al., Phys. Rev. Lett. 90 (2003) 021801; Phys. Rev. Lett. 94 (2005) 081801.
[6] Soudan 2 Collaboration, M. Sanchez, et al., Phys. Rev. D 68 (2003) 113004.
[7] MARCO Collaboration, M. Ambrosio, et al., Eur. Phys. J. C 36 (2004) 323.
[8] K2K Collaboration, E. Aliu, et al., Phys. Rev. Lett. 94 (2005) 081802.
[9] SK Collaboration, Y. Ashie, et al., Phys. Rev. D 71 (2005).
[10] GNO Collaboration, M. Altmann, et al., Phys. Lett. B 616 (2005) 174.
[11] CHOOZ Collaboration, M. Apolonio, et al., Eur. Phys. J. C 27 (2003) 331.
[12] T2K Collaboration, K. Abe, et al., Phys. Rev. Lett. 107 (2011) 041801, arXiv:1106.2822 [hep-ex].
[13] MINOS Collaboration, P. Adamson, et al., Phys. Rev. Lett. 107 (2011) 181802, arXiv:1108.0015 [hep-ex].
[14] N. Cabibbo, Phys. Rev. Lett. 10 (1963) 531; M. Kobayashi, T. Maskawa, Prog. Theor. Phys. 49 (1973) 652.
[15] Z. Maki, M. Nakagawa, S. Sakata, Prog. Theor. Phys. 28 (1962) 870; B. Pontecorvo, Sov. Phys. JETP 26 (1968) 984.
[16] Y.L. Wu, Phys. Lett. B 714 (2012) 286, arXiv:1203.2382.
[17] Y.L. Wu, Phys. Rev. D 77 (2008) 113009, arXiv:0708.0867 [hep-ph].
[18] Y.L. Wu, Int. J. Mod. Phys. A 23 (2008) 3376, arXiv:0807.3847 [hep-ph].
[19] Y.L. Wu, Phys. Rev. D 60 (1999) 073010.
[20] Y.L. Wu, Nucl. Phys. B, Proc. Suppl. 85 (2000) 193.
[21] Y.L. Wu, Invited talk at the 30th International Conference on High-Energy Physics (ICHEP 2000), Osaka, Japan.
[22] Y.L. Wu, Eur. Phys. J. C 10 (1999) 491.
[23] Y.L. Wu, J. Phys. G, Nucl. Part. Phys. 26 (2000) 1131.
[24] Y.L. Wu, Sci. China Ser. A 43 (2000) 988.
[25] C. Carone, M. Sher, Phys. Lett. B 420 (1998) 83.
[26] E. Ma, Phys. Lett. B 456 (1999) 48, arXiv:hep-ph/9812344.
[27] C. Wetterich, Phys. Lett. B 451 (1999) 397, arXiv:hep-ph/9812426.
[28] R. Barbieri, L.J. Hall, G.L. Kane, G.G. Ross, arXiv:hep-ph/9901228.
[29] T. Yanagida, Phys. Rev. D 20 (1979) 2986.
[30] P.F. Harrison, D.H. Perkins, W.G. Scott, Phys. Lett. B 530 (2002) 167.
[31] Z.-Z. Xing, Phys. Lett. B 533 (2002) 85.
[32] P.F. Harrison, W.G. Scott, Phys. Lett. B 535 (2002) 163.
[33] P.F. Harrison, W.G. Scott, Phys. Lett. B 557 (2003) 76.
[34] X.G. He, A. Zee, Phys. Lett. B 560 (2003) 87.
[35] L.J. Hall, S. Weinberg, Phys. Rev. D 48 (1993) 979.
[36] Y.L. Wu, L. Wolfenstein, Phys. Rev. Lett. 73 (1994) 1762, arXiv:hep-ph/9409421.
[37] L. Wolfenstein, Y.L. Wu, Phys. Rev. Lett. 73 (1994) 2809, arXiv:hep-ph/9410253.
[38] Y.L. Wu, arXiv:hep-ph/9404241; See also: in: S.J. Seestrom (Ed.), Proceedings of 5th Conference on the Intersections of Particle and Nuclear Physics, St. Petersburg, FL, 31 May–6 June, 1994, AIP, New York, 1995, p. 338, arXiv:hep-ph/9406306.
[39] T.D. Lee, Phys. Rev. D 8 (1973) 1226; T.D. Lee, Phys. Rep. 9 (1974) 143.
[40] L. Wolfenstein, Phys. Rev. Lett. 51 (1983) 1945.
[41] Particle Data Group, J. Beringer, et al., Phys. Rev. D 86 (2012) 010001, and 2013 partial update for the 2014 edition, http://pdg.lbl.gov.
[42] DAYA-BAY Collaboration, F.P. An, Phys. Rev. Lett. 108 (2012) 171803, arXiv:1203.1669.
[43] RENO Collaboration, J.K. Ahn, Phys. Rev. Lett. 108 (2012) 191802, arXiv:1203.0626.
[44] C. Jarlskog, Phys. Rev. Lett. 55 (1985) 1039; D.D. Wu, Phys. Rev. D 33 (1986) 860.
[45] J. Schechter, J.W.F. Valle, Phys. Rev. D 22 (1980) 2227; H. Nunokawa, S. Parke, J.W.F. Valle, Prog. Part. Nucl. Phys. 60 (2008) 338; W. Rodejohann, J.W.F. Valle, Phys. Rev. D 84 (2012) 073011.
[46] D. Marzocca, S.T. Petcov, A. Romanino, M.C. Sevilla, J. High Energy Phys. 5 (2013) 73.
[47] J.A. Acosta, A. Aranda, J. Virrueta, arXiv:1402.0754 [hep-ph].
[48] G.L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo, Phys. Rev. D 86 (2012) 013012, arXiv:1205.5254.
[49] M.C. Gonzalez-Garcia, M. Maltoni, J. Salvado, T. Schwetz, J. High Energy Phys. 1212 (2012) 123, arXiv:1209.3023.
[50] D.V. Forero, M. Tortola, J.W.F. Valle, Phys. Rev. D 86 (2012) 073012, arXiv:1205.4018.
[51] F. Pentinger, W. Rodejohann, Phys. Lett. B 625 (2005) 264.
[52] A. Datta, Phys. Rev. D 78 (2008) 095004.
[53] Y. Koide, H. Nishiura, Phys. Lett. B 669 (2008) 24; Y. Koide, H. Nishiura, Phys. Rev. D 79 (2009) 093005.