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Pion electroproduction in a nonrelativistic theory

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ABSTRACT

A nonrelativistic effective theory to describe the electroproduction reaction of a single pion on the nucleon at leading order in the electromagnetic coupling is constructed. The framework is tailored to accurately describe the cusp generated by the pion and nucleon mass differences. The S- and P-wave multipole amplitudes at two loops for all four reaction channels are provided. As an application, a new low energy theorem is discussed.

1. Pion electroproduction is one of the reactions in particle physics which has been studied ever since almost the beginning of the field. Since the first experiment over fifty years ago [1], the accuracy has been constantly increased and the threshold region at low values of the photon virtuality became accessible [2–4]. It has been known for a long time that at low energies, this process allows one to study pion–nucleon physics. The strength of the pronounced cusp in the production channel of neutral pions which appears at the $n\pi^+$ threshold is intimately related to the pion–nucleon scattering lengths [5]. One of the aims of the present Letter is to derive this connection in an effective field theory with a suitable perturbative expansion. The standard tool to study any hadronic low-energy process in the Standard Model is chiral perturbation theory (ChPT) [6–9], which has been applied to the reaction in question in a series of articles [10–13]. Here, we propose a nonrelativistic effective theory which has already been successfully applied to hadronic atoms (see Ref. [14] and references therein), $K, \eta, \eta' \to 3\pi$ decays [15–17], K_{e4} decays [18] and which was recently also formulated for pion photoproduction [19].

The strength of the nonrelativistic framework in comparison to ChPT is that the fundamental quantities of pion–nucleon interactions at low energies, i.e. the coefficients of the effective range expansion (scattering lengths, effective range parameters, ...), are *free parameters* of the theory. There is no expansion of these parameters in terms of M_π/Λ_{QCD} or M_π/m_p. On the other hand, the drawback is a more restricted region of validity than in ChPT. However, it was noted in Ref. [11] that especially for the S-wave multipoles, the expansion in terms of M_π/m_p converges rather slowly. Therefore, a limited region of applicability might be a price worth to pay to avoid this expansion.

The Letter is organized as follows: After collecting the notation, the kinetic part of the Lagrangian is written down and the power counting is discussed. Then the interaction terms are constructed and the matching relations of the coupling constants are discussed. The calculation of the multipoles up to and including two-loop corrections is then straightforward. The Letter concludes with a discussion of a low energy theorem.

2. We calculate the electroproduction reaction at leading order in the electromagnetic coupling. Therefore, only one photon is exchanged between the electron and the nucleon and all the hadronic physics that we are interested in is contained in the transition matrix element for the process $N(p_1) + \gamma^\star(k) \to N(p_2) + \pi^{0,\pm}(q)$, where $\gamma^\star$ denotes an off-shell photon with $k^2 < 0$. A discussion of pion electroproduction including the leptonic part can be found for instance in Ref. [20]. In the following, the four reaction channels will be abbreviated as

$$\gamma^\star p \to p\pi^0\colon\ (p0), \qquad \gamma^\star p \to n\pi^+\colon\ (n+),$$
$$\gamma^\star n \to n\pi^0\colon\ (n0), \qquad \gamma^\star n \to p\pi^-\colon\ (p-).$$

All the observables of electroproduction experiments can be expressed in terms of electric, magnetic and scalar multipoles. The transition current matrix element $\epsilon_\mu J^\mu$, where ϵ_μ is the photon polarization vector, is conveniently written in terms of two com-

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ponent spinors ξ_t and Pauli matrices τ^k [21] and Coulomb gauge, $\nabla\mathbf{A}=0$, is chosen,[1]

$$\epsilon_\mu J^\mu = \xi_{t'}^\dagger \mathcal{F}\xi_t,$$
$$\mathcal{F} = i\boldsymbol{\tau}\cdot\boldsymbol{\epsilon}\mathcal{F}_1 + \boldsymbol{\tau}\cdot\hat{\mathbf{q}}\boldsymbol{\tau}\cdot(\hat{\mathbf{k}}\times\boldsymbol{\epsilon})\mathcal{F}_2 + i\boldsymbol{\tau}\cdot\hat{\mathbf{k}}\hat{\mathbf{q}}\cdot\boldsymbol{\epsilon}\mathcal{F}_3 + i\boldsymbol{\tau}\cdot\hat{\mathbf{q}}\hat{\mathbf{q}}\cdot\boldsymbol{\epsilon}\mathcal{F}_4 - i\boldsymbol{\tau}\cdot\hat{\mathbf{k}}\epsilon_0\mathcal{F}_5 - i\boldsymbol{\tau}\cdot\hat{\mathbf{q}}\epsilon_0\mathcal{F}_6. \tag{1}$$

The hat denotes unit vectors. The $\mathcal{F}_i$ are decomposed into electric, magnetic and scalar multipoles with the help of derivatives of the Legendre polynomials $P_l(z)$ [21],

$$\mathcal{F}_1 = \sum_{l=0}[lM_{l+} + E_{l+}]P'_{l+1}(z) + \big[(l+1)M_{l-} + E_{l-}\big]P'_{l-1}(z),$$
$$\mathcal{F}_2 = \sum_{l=1}\big[(l+1)M_{l+} + lM_{l-}\big]P'_l(z),$$
$$\mathcal{F}_3 = \sum_{l=1}[E_{l+} - M_{l+}]P''_{l+1}(z) + [E_{l-} + M_{l-}]P''_{l-1}(z),$$
$$\mathcal{F}_4 = \sum_{l=1}[M_{l+} - E_{l+} - M_{l-} - E_{l-}]P''_l(z),$$
$$\mathcal{F}_5 = \sum_{l=0}\big[lS_{l-} - (l+1)S_{l+}\big]P'_l(z),$$
$$\mathcal{F}_6 = \sum_{l=1}\big[(l+1)S_{l+}P'_{l+1}(z) - lS_{l-}P'_{l-1}(z)\big]. \tag{2}$$

The multipoles are complex valued functions of the center of mass energy and the photon virtuality. The discussion is restrained to the center of mass frame in the rest of the article.

3. We aim at a description of the multipoles close to the reaction threshold — where the momentum of the produced pion and of the nucleon is small. A nonrelativistic theory is the right tool for this task. The other kinematic variable, the photon virtuality k^2, has to be restricted to small values compared to $4M_\pi^2$. This allows for an expansion of the amplitudes in $k^2/(4M_\pi^2)$. The factor of four shows up because a power series in k^2 around the origin converges inside a circle in the complex plane up to the first branch point which lies at $k^2 = 4M_\pi^2$. A nonrelativistic treatment also offers the advantage that all the masses can be set to their physical value. Therefore, all the poles and branch points appear at the *correct place* in the Mandelstam plane. Moreover, the interaction of the nucleon and the pion is described by effective range parameters, which allows one to directly access the pion–nucleon scattering lengths, the main goal of the present analysis.

The covariant formulation of nonrelativistic field theories introduced and applied in Refs. [15–17,19] is used here. It incorporates the correct relativistic dispersion law for the particles. Note however that it is not mandatory to keep all the higher order terms, but merely a matter of convenience. The nonrelativistic proton, neutron and pion fields are denoted by ψ, χ and π_k, respectively. The kinetic part of the Lagrangian after minimal substitution takes the form (see Ref. [22])

$$\mathcal{L}_{kin} = \sum_\pm\big(i\pi_\pm^\dagger D_t\mathcal{W}_\pm\pi_\pm - i(D_t\mathcal{W}_\pm\pi_\pm)^\dagger\pi_\pm - 2\pi_\pm^\dagger\mathcal{W}_\pm^2\pi_\pm\big) + i\psi^\dagger D_t\mathcal{W}_p\psi - i(D_t\mathcal{W}_p\psi)^\dagger\psi - 2\psi^\dagger\mathcal{W}_p^2\psi + 2\chi^\dagger W_n(i\partial_t - W_n)\chi + 2\pi_0^\dagger W_0(i\partial_t - W_0)\pi_0, \tag{3}$$

with

$$W_0 = \sqrt{M_{\pi^0}^2 - \Delta}, \qquad W_n = \sqrt{m_n^2 - \Delta},$$
$$D_t\pi_\pm = (\partial_t \mp ieA_0)\pi_\pm, \qquad D_t\psi = (\partial_t - ieA_0)\psi,$$
$$\mathcal{W}_\pm = \sqrt{M_\pi^2 - \mathbf{D}^2}, \qquad \mathcal{W}_p = \sqrt{m_p^2 - \mathbf{D}^2},$$
$$\mathbf{D}\pi_\pm = (\nabla \pm ie\mathbf{A})\pi_\pm, \qquad \mathbf{D}\psi = (\nabla + ie\mathbf{A})\psi. \tag{4}$$

Since the photon is treated as an external field, its kinetic term is absent.

4. Close to threshold, the momenta of the outgoing pion and the outgoing proton, normalized with the pion mass, are small and therefore counted as a quantity of $O(\epsilon)$. The normalized momenta of the incoming proton and of the photon are counted as $O(1)$. All the masses are counted as $O(1)$. The mass differences of the charged and neutral pion, $\Delta_\pi/M_\pi^2 \equiv (M_\pi^2 - M_{\pi^0}^2)/M_\pi^2$ and of the proton and the neutron, $\Delta_N/M_\pi^2 \equiv (m_n^2 - m_p^2)/M_\pi^2$ are counted as $O(\epsilon^2)$. The power counting therefore is exactly the same as in the case of photoproduction, Ref. [19]. The only difference is that the off-shell photon introduces an additional scale into the problem, the virtuality k^2, which is taken to be small in comparison to $4M_\pi^2$ by assumption. Therefore, the quantity $k^2/(4M_\pi^2)$ is also counted as $O(\epsilon^2)$. This power counting is valid although derivatives on the incoming nucleon and photon fields generate terms of $O(1)$. Simply expand the large momenta in ϵ,

$$|\mathbf{k}| = \sum_{n,m}\alpha_{n,m}\mathbf{q}^{2n}k^{2m}, \qquad \alpha_{0,0} = \frac{M_{\pi^0}}{2}\frac{2+y}{1+y},$$
$$\alpha_{1,0} = \frac{y^2+2y+2}{4M_{\pi^0}(1+y)}, \qquad \alpha_{0,1} = -\frac{y^2+2y+2}{2M_{\pi^0}(1+y)(2+y)},$$
$$y = \frac{M_{\pi^0}}{m_p}, \tag{5}$$

and define the coupling of the leading order term such that it contains all the large terms with $\alpha_{0,0}$ (for more details, see Ref. [19]). The derivatives on the incoming fields are only needed to generate unit vectors in the direction of the incoming photon.

The rescattering of the pion on the nucleon introduces another expansion parameter $|\mathbf{q}|^n b$, where b denotes an arbitrary effective range parameter of πN scattering (at higher orders, b can also contain masses, see for instance Eq. (11)) with n the pertinent number to obtain a dimensionless quantity. In the following, I will indicate the order of a given quantity W as $a^m\epsilon^n$, where m simply counts the number of loops and n denotes the combined power of momenta $\mathbf{q}$ and k^2 present in W. It is straightforward to see how each of the terms divides up into the different dimensionless expansion parameters $|\mathbf{q}|^n b$ and ϵ. It was found in Ref. [19] that the expansion in $|\mathbf{q}|/M_\pi$ works well at least up to $|\mathbf{q}| \simeq 70$ MeV. The expansion in $|\mathbf{q}|^n b$ is expected to converge even faster, since the effective range parameters are much smaller than 1 in units of inverse M_π.

5. The Lagrangian needed for the calculation of the amplitudes for pion electroproduction reads $\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_\gamma + \mathcal{L}_{\pi N}$, where $\mathcal{L}_{kin}$ denotes the kinetic part, $\mathcal{L}_\gamma$ incorporates the interaction with the photon field and $\mathcal{L}_{\pi N}$ describes the pion–nucleon sector.

For the pion–nucleon sector, the Lagrangian was given before in Refs. [19,23]. For convenience, we write it down again. An equivalent description can be found in Ref. [24] (see Ref. [19] for an explicit comparison). For every channel n, we collect the charges of the outgoing and the incoming pions in the variables v and w, $(n; v, w)$: $(0; 0, 0)$, $(1; 0, +)$, $(2; +, +)$, $(3; 0, 0)$, $(4; -, 0)$, $(5; -, -)$, thereby assigning unique values to the variables v and w once n is given. The Lagrangian reads

[1] The $\mathcal{F}_{5,6}$ defined like that are sometimes also called $\mathcal{F}_{7,8}$ in the literature.

$$\mathcal{L}_{\pi N} = \begin{pmatrix}\psi^\dagger & \chi^\dagger\end{pmatrix}\begin{pmatrix}T_{\{0,5\}} & T_{\{1,4\}} \\ T_{\{1,4\}}^\dagger & T_{\{2,3\}}\end{pmatrix}\begin{pmatrix}\psi \\ \chi\end{pmatrix},$$
$$T_{\mathcal{C}} = \sum_{n\in\mathcal{C}}\big[C_n\pi_v^\dagger\pi_w + D_n^{(1)}\nabla^k\pi_v^\dagger\nabla^k\pi_w + D_n^{(2)}\pi_v^\dagger\overleftrightarrow{\Delta}\pi_w + iD_n^{(3)}\tau^k\epsilon^{ijk}\nabla^i\pi_v^\dagger\nabla^j\pi_w\big] \qquad (6)$$

with the abbreviation $f\overleftrightarrow{\Delta}g \equiv f\Delta g + (\Delta f)g$.

For $\mathcal{L}_\gamma$, the photon is treated as an external field. Gauge invariance requires that A^μ can only appear in covariant derivatives and through the Maxwell equations in the electric and magnetic fields $\mathbf{E} = -\nabla A^0 - \dot{\mathbf{A}}$ and $\mathbf{B} = \nabla\times\mathbf{A}$. All the terms are invariant under space rotations, parity and time reversal transformations. The upper index on the coupling constants counts the number of derivatives on the external vector field and is introduced for later convenience. The terms with coupling constants $G_i^{(k)}$ with $i = 0,\ldots,15$ were already given in Ref. [19] and contribute to the photoproduction amplitudes.

$$\begin{aligned}
\mathcal{L}_\gamma^{(0)} &= -iG_0^{(1)}\psi^\dagger\tau^k\psi E^k\pi_0^\dagger,\\
\mathcal{L}_\gamma^{(1)} &= -iG_1^{(2)}\psi^\dagger\tau^k\psi\nabla^jE^k\nabla^j\pi_0^\dagger + iG_2^{(1)}\psi^\dagger\tau^m\tau^l\psi B^l\nabla^m\pi_0^\dagger\\
&\quad - iG_3^{(2)}\psi^\dagger\tau^j\psi\nabla^jE^k\nabla^k\pi_0^\dagger - iG_{20}^{(2)}\psi^\dagger\tau^k\psi\nabla^jE^j\nabla^k\pi_0^\dagger,\\
\mathcal{L}_\gamma^{(2)} &= -iG_4^{(3)}\psi^\dagger\tau^k\psi\nabla^{jl}E^k\nabla^{jl}\pi_0^\dagger - iG_5^{(1)}\psi^\dagger\tau^k\psi E^k\Delta\pi_0^\dagger\\
&\quad + iG_6^{(2)}\psi^\dagger\tau^m\tau^l\psi\nabla^nB^l\nabla^{mn}\pi_0^\dagger - iG_7^{(3)}\psi^\dagger\tau^j\psi\nabla^{jl}E^k\nabla^{kl}\pi_0^\dagger\\
&\quad - iG_8^{(1)}\psi^\dagger\tau^j\psi E^k\nabla^{jk}\pi_0^\dagger - iG_{16}^{(3)}\psi^\dagger\tau^k\psi\mathcal{P}E^k\pi_0^\dagger\\
&\quad - iG_{21}^{(3)}\psi^\dagger\tau^k\psi\nabla^{mj}E^j\nabla^{mk}\pi_0^\dagger,\\
\mathcal{L}_\gamma^{(3)} &= -iG_9^{(2)}\psi^\dagger\tau^k\psi\nabla^jE^k\Delta\nabla^j\pi_0^\dagger - iG_{10}^{(4)}\psi^\dagger\tau^k\psi\nabla^{lmn}E^k\nabla^{lmn}\pi_0^\dagger\\
&\quad + iG_{11}^{(1)}\psi^\dagger\tau^m\tau^l\psi B^l\Delta\nabla^m\pi_0^\dagger\\
&\quad + iG_{12}^{(3)}\psi^\dagger\tau^m\tau^l\psi\nabla^{in}B^l\nabla^{min}\pi_0^\dagger\\
&\quad - iG_{13}^{(2)}\psi^\dagger\tau^j\psi\nabla^jE^k\Delta\nabla^k\pi_0^\dagger - iG_{14}^{(4)}\psi^\dagger\tau^j\psi\nabla^{jlm}E^k\nabla^{klm}\pi_0^\dagger\\
&\quad - iG_{15}^{(2)}\psi^\dagger\tau^j\psi\nabla^lE^k\nabla^{jkl}\pi_0^\dagger - iG_{17}^{(4)}\psi^\dagger\tau^k\psi\nabla^j\mathcal{P}E^k\nabla^j\pi_0^\dagger\\
&\quad - iG_{18}^{(4)}\psi^\dagger\tau^j\psi\nabla^j\mathcal{P}E^k\nabla^k\pi_0^\dagger + iG_{19}^{(3)}\psi^\dagger\tau^m\tau^l\psi\mathcal{P}B^l\nabla^m\pi_0^\dagger\\
&\quad - iG_{22}^{(2)}\psi^\dagger\tau^k\psi\nabla^jE^j\Delta\nabla^k\pi_0^\dagger - iG_{23}^{(4)}\psi^\dagger\tau^k\psi\nabla^j\mathcal{P}E^j\nabla^k\pi_0^\dagger.
\end{aligned} \qquad (7)$$

Here, the notation $\nabla^{i_1i_2\ldots i_k} \equiv \nabla^{i_1}\nabla^{i_2}\cdots\nabla^{i_k}$ and $\mathcal{P} \equiv -\partial_t^2 + \Delta$ is introduced. Since the structure of the Lagrangian for the remaining channels stays the same, one only has to replace the coupling constants and the field operators,

$$\begin{aligned}
(n+)&:\ \{\psi^\dagger, \pi_0^\dagger, G_i^{(n)}\} \to \{\chi^\dagger, \pi_+^\dagger, H_i^{(n)}\},\\
(n0)&:\ \{\psi, \psi^\dagger, G_i^{(n)}\} \to \{\chi, \chi^\dagger, L_i^{(n)}\},\\
(p-)&:\ \{\psi, \pi_0^\dagger, G_i^{(n)}\} \to \{\chi, \pi_-^\dagger, K_i^{(n)}\}.
\end{aligned} \qquad (8)$$

The full Lagrangian $\mathcal{L}_\gamma$ is given by adding the $\mathcal{L}_\gamma^{(i)}$ of all four channels.

6. In the pion–nucleon sector, the coupling constants of the nonrelativistic Lagrangian, C_i and $D_i^{(k)}$ can be expressed in terms of pion–nucleon scattering lengths of the S-wave and P-wave, a_{0+} and $a_{1\pm}$ and the effective range parameter b_{0+}, respectively. Adopting the notation of Ref. [25], in the isospin limit, the isospin decomposition of the πN scattering amplitudes reads

$$\begin{aligned}
T_{p\pi^0\to p\pi^0} &= T_{n\pi^0\to n\pi^0} = T^+,\\
T_{p\pi^0\to n\pi^+} &= T_{n\pi^0\to p\pi^-} = -\sqrt{2}T^-,\\
T_{n\pi^+\to n\pi^+} &= T_{p\pi^-\to p\pi^-} = T^+ + T^-.
\end{aligned} \qquad (9)$$

Defining $\mathcal{N} = 4\pi(m_p + M_\pi)$, one finds

$$\begin{aligned}
&C_0 = 2\mathcal{N}a_{0+}^+, && C_1 = 2\sqrt{2}\mathcal{N}a_{0+}^-, && C_2 = 2\mathcal{N}\big(a_{0+}^+ + a_{0+}^-\big),\\
&C_3 = C_0, && C_4 = C_1, && C_5 = C_2.
\end{aligned} \qquad (10)$$

The matching conditions for the $D_i^{(k)}$ are given in a generic form only. The isospin index of the threshold parameters can be inferred from Eq. (9).[2]

$$\begin{aligned}
&D_i^{(1)} = 2\mathcal{N}(2a_{1+} + a_{1-}), \qquad D_i^{(2)} = -\mathcal{N}\left(\frac{a_{0+}}{2m_pM_\pi} + b_{0+}\right),\\
&D_i^{(3)} = 2\mathcal{N}(a_{1-} - a_{1+}).
\end{aligned} \qquad (11)$$

Here, higher order terms in the threshold parameters have been dropped. The corrections to these relations which appear due to isospin breaking have to be calculated within the underlying relativistic theory. For the C_i, they can be found in Refs. [26–28]. In the isospin limit, the different couplings C_i are related according to Eq. (10). These relations do not hold anymore once isospin breaking corrections are taken into account.

The multipole coupling constants $G_i^{(n)}$, $H_i^{(n)}$, $K_i^{(n)}$ and $L_i^{(n)}$ on the other hand are related to the threshold parameters of the electric and magnetic multipoles of the pertinent channel. In the isospin limit, the expansion of the real part of the multipole $X_{l\pm}$ close to threshold is written in the form

$$\mathrm{Re}\,X_{l\pm}\big(s,k^2\big) = \sum_{k,m}\bar{X}_{l\pm,2k,2m}|\mathbf{q}|^{l+2k}k^{2m}, \qquad (12)$$

which defines the threshold parameters $\bar{X}_{l\pm,2k,2m}$. In the following, the relations of the coupling constants $G_i^{(n)}$ to these threshold parameters is given dropping terms of the order of the pion–nucleon threshold parameters. Since the nonrelativistic theory is not suited for the study of the dependence of the multipoles on $|\mathbf{k}|$, in this analysis, all vectors $\mathbf{k}$ are turned into unit vectors by the pertinent redefinition of the coupling constants,

$$G_i^{(n)} = G_i/\alpha_{0,0}^n. \qquad (13)$$

The higher order corrections due to factoring out $|\mathbf{k}|$ are taken care of in the matching relations. Again, these relations pick up isospin breaking corrections which have to be evaluated in the underlying relativistic theory.

Only the matching equations for the couplings of the Lagrangians $\mathcal{L}_\gamma^{(0)}$ and $\mathcal{L}_\gamma^{(1)}$ are indicated in the main text, the remaining relations are relegated to Appendix A. To ease notation, $\bar{X}_{i\pm} \equiv \bar{X}_{i\pm,0,0}$ is used.[3]

$$\begin{aligned}
&G_0 = \bar{E}_{0+}, && G_1 = 3(\bar{E}_{1+} + \bar{M}_{1+}),\\
&G_2 = -2\bar{M}_{1+} - \bar{M}_{1-}, && G_3 = 3(\bar{E}_{1+} - \bar{M}_{1+}).
\end{aligned} \qquad (14)$$

For the coupling constants H_i, K_i and L_i, the algebraic form of the relations is identical. However, the multipoles of the pertinent channels appear and the masses in Eq. (13) have to be adjusted. In the isospin limit, the leading multipole couplings fulfill

$$\sqrt{2}(G_0 - L_0) = H_0 + K_0. \qquad (15)$$

[2] The Condon–Shortley phase convention is used.

[3] The matching differs from the photoproduction case given in Ref. [19] because the amplitudes are normalized differently.

Since the effects of dynamical photons are not discussed here, all the coupling constants are taken to be real.[4]

7. In the following, we provide the expressions for the electric, magnetic and scalar multipoles E_{l+}, $S_{l\pm}$ and $M_{l\pm}$ for the channel $(p0)$.[5] The result is written in the form

$$X_{l,\pm}(s,k^2) = X_{l\pm}^{\text{tree}}(s,k^2) + X_{l\pm}^{\text{1Loop}}(s,k^2) + X_{l\pm}^{\text{2Loop}}(s,k^2)\cdots, \tag{16}$$

where $s=(p_1+k)^2$ and the ellipsis denote higher order terms in the perturbative expansion. The results for the other channels can be recovered by a simple replacement of the coupling constants which will be given later. Write

$$X_{l\pm}^{\text{tree}}(s,k^2) = \mathbf{q}^l\left[X_{l\pm}^t + X_{l\pm,q}^t\mathbf{q}^2 + X_{l\pm,k}^t\frac{k^2}{\alpha_{0,0}^2} + X_{l\pm,\eta}^t\eta + \cdots\right], \tag{17}$$

where $\eta = \frac{\alpha_{0,1}}{\alpha_{0,0}}k^2 + \frac{\alpha_{1,0}}{\alpha_{0,0}}\mathbf{q}^2$ collects the terms which are due to the expansion of k^0 and $\mathbf{k}$ in ϵ. One finds for the leading terms

$$\begin{aligned}
&E_{0+}^t = G_0, \qquad S_{0+}^t = G_0, \qquad 6E_{1+}^t = G_1 + G_3,\\
&3M_{1-}^t = G_3 - G_1 - 3G_2, \qquad 6M_{1+}^t = G_1 - G_3,\\
&6S_{1+}^t = G_1 + G_3, \qquad 3S_{1-}^t = G_1 + G_3 + 3G_{20},
\end{aligned} \tag{18}$$

for the coefficients of $\mathbf{q}^2$ and k^2

$$\begin{aligned}
3E_{0+,q}^t &= G_4 - 3G_5 + G_6 - G_8,\\
2E_{0+,k}^t &= G_0 + 2G_{16},\\
3S_{0+,q}^t &= G_4 - 3G_5 + G_7 - G_8 + G_{21},\\
S_{0+,k}^t &= G_{16},\\
30E_{1+,q}^t &= -5G_9 + 3G_{10} + 2G_{12} - 5G_{13} + G_{14} - 2G_{15},\\
12E_{1+,k}^t &= G_1 + G_3 + 2G_{17} + 2G_{18},\\
30M_{1+,q}^t &= -5G_9 + 3G_{10} + 2G_{12} + 5G_{13} - G_{14},\\
12M_{1+,k}^t &= G_1 - G_3 + 2G_{17} - 2G_{18},\\
15M_{1-,q}^t &= 5G_9 - 3G_{10} + 15G_{11} - 5G_{12} - 5G_{13} + G_{14},\\
6M_{1-,k}^t &= -G_1 + G_3 - 2G_{17} + 2G_{18} - 6G_{19},\\
30S_{1+,q}^t &= -5G_9 + 3G_{10} - 5G_{13} + 3G_{14} - 2G_{15},\\
6S_{1+,k}^t &= G_{17} + G_{18},\\
15S_{1-,q}^t &= -5G_9 + 3G_{10} - 5G_{13} + 3G_{14} - 5G_{15} - 15G_{22},\\
3S_{1-,k}^t &= G_{17} + G_{18} + 3G_{23},
\end{aligned} \tag{19}$$

and for the coefficients of η

$$\begin{aligned}
&E_{0+,\eta}^t = G_0, \qquad S_{0+,\eta}^t = G_0, \qquad 3E_{1+,\eta}^t = G_1 + G_3,\\
&3M_{1-,\eta}^t = -2G_1 - 3G_2 + 2G_3, \qquad 3M_{1+,\eta}^t = G_1 - G_3,\\
&3S_{1+,\eta}^t = G_1 + G_3, \qquad 3S_{1-,\eta}^t = 2G_1 + 2G_3 + 6G_{20}.
\end{aligned} \tag{20}$$

The coefficients $X_{l\pm}^t$ and $X_{l\pm,\eta}^t$ are not equal because the term with the coupling constant G_2 in the Lagrangian does not have a time derivative on the photon field.

8. All the one-loop contributions are proportional to the basic integral

$$\begin{aligned}
J_{ab}(P^2) &= \int\frac{d^Dl}{i(2\pi)^D}\\
&\quad\times\frac{1}{2\omega_a(\mathbf{l})2\omega_b(\mathbf{P}-\mathbf{l})}\frac{1}{(\omega_a(\mathbf{l})-l_0)(\omega_b(\mathbf{P}-\mathbf{l})-P_0+l_0)},\\
\omega_\pm(\mathbf{p}) &= \sqrt{M_\pi^2+\mathbf{p}^2}, \qquad \omega_i(\mathbf{p}) = \sqrt{m_i^2+\mathbf{p}^2}, \quad i=n,p\\
\omega_0(\mathbf{p}) &= \sqrt{M_{\pi^0}^2+\mathbf{p}^2}, \qquad P^2 = P_0^2 - \mathbf{P}^2.
\end{aligned} \tag{21}$$

In the limit $D\to 4$,

$$J_{ab}(P^2) = \frac{i}{16\pi s}\sqrt{\left(s-(m_a+M_{\pi^b})^2\right)\left(s-(m_a-M_{\pi^b})^2\right)}, \tag{22}$$

which is a quantity of order ϵ. The one-loop result for channel (c) up to and including order $O(a\epsilon^4)$ reads

$$\begin{pmatrix}
E_{0+}^{\text{1Loop}}(s,k^2)\\
\frac{1}{|\mathbf{q}|}M_{1+}^{\text{1Loop}}(s,k^2)\\
\frac{1}{|\mathbf{q}|}M_{1-}^{\text{1Loop}}(s,k^2)\\
\frac{1}{|\mathbf{q}|}E_{1+}^{\text{1Loop}}(s,k^2)\\
S_{0+}^{\text{1Loop}}(s,k^2)\\
\frac{1}{|\mathbf{q}|}S_{1+}^{\text{1Loop}}(s,k^2)\\
\frac{1}{|\mathbf{q}|}S_{1-}^{\text{1Loop}}(s,k^2)
\end{pmatrix} = \begin{pmatrix}
P_{11}^{(c)} & P_{12}^{(c)}\\
P_{21}^{(c)} & P_{22}^{(c)}\\
P_{31}^{(c)} & P_{32}^{(c)}\\
P_{41}^{(c)} & P_{42}^{(c)}\\
P_{51}^{(c)} & P_{52}^{(c)}\\
P_{61}^{(c)} & P_{62}^{(c)}\\
P_{71}^{(c)} & P_{72}^{(c)}
\end{pmatrix}\begin{pmatrix} J_{ab}(s)\\ J_{cd}(s)\end{pmatrix}, \tag{23}$$

where m_a, M_{π^b} denote the masses of the final state of the pertinent channel and m_c, M_{π^d} stands for the masses of the intermediate state that differ from the final state masses. This means for channel $(p0)$ $m_a = m_p, m_c = m_n, M_{\pi^b} = M_{\pi^0}$ and $M_{\pi^d} = M_\pi$. The elements $P_{ik}^{(c)}$ are functions of the pion momentum $\mathbf{q}$, the photon virtuality k^2 and the coupling constants of the Lagrangian. For the channel $(p0)$, one finds

$$\begin{aligned}
P_{11}^{(p0)} &= G_0C_0(1+\eta) + \mathbf{q}^2\left(C_0E_{0+,q}^t - 2D_0^{(2)}G_0\right)\\
&\quad + \frac{k^2}{\alpha_{0,0}^2}C_0E_{0+,k}^t,\\
P_{12}^{(p0)} &= C_1H_0(1+\eta) + h_{cd}^2\left(C_1E_{0+,q}^{t,(n+)} - D_1^{(2)}H_0\right) - \mathbf{q}^2D_1^{(2)}H_0\\
&\quad + \frac{k^2}{\alpha_{0,0}^2}C_1E_{0+,k}^{t,(n+)},\\
18P_{21}^{(p0)} &= \mathbf{q}^2\left(D_0^{(1)} - D_0^{(3)}\right)(G_1 - G_3),\\
18P_{22}^{(p0)} &= h_{cd}^2\left(D_1^{(1)} - D_1^{(3)}\right)(H_1 - H_3),\\
9P_{31}^{(p0)} &= \mathbf{q}^2\left(D_0^{(1)} + 2D_0^{(3)}\right)(G_3 - G_1 - 3G_2),\\
9P_{32}^{(p0)} &= h_{cd}^2\left(D_1^{(1)} + 2D_1^{(3)}\right)(H_3 - H_1 - 3H_2),\\
18P_{41}^{(p0)} &= \mathbf{q}^2\left(D_0^{(1)} - D_0^{(3)}\right)(G_1 + G_3),\\
18P_{42}^{(p0)} &= h_{cd}^2\left(D_1^{(1)} - D_1^{(3)}\right)(H_1 + H_3),\\
P_{51}^{(p0)} &= G_0C_0(1+\eta) + \mathbf{q}^2\left(C_0S_{0+,q}^t - 2D_0^{(2)}G_0\right)\\
&\quad + \frac{k^2}{\alpha_{0,0}^2}C_0S_{0+,k}^t,
\end{aligned}$$

[4] A more detailed discussion can be found in Refs. [19,28,29].

[5] We refrain from using an additional index on the multipoles to indicate the channel. Only when tree level coefficients of a channel different from $(p0)$ appear in the result, this will be indicated.

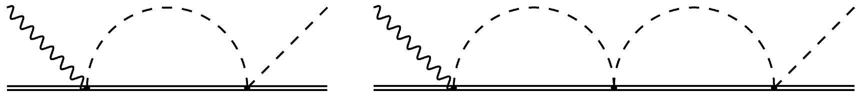

Fig. 1. One- and two-loop topologies needed to calculate the amplitude. The double line generically denotes a nucleon, the dashed line a pion and the wiggly line indicates the external electromagnetic field.

$$
\begin{aligned}
P_{52}^{(p0)} &= C_1 H_0(1+\eta) + h_{cd}^2\big(C_1 S_{0+,q}^{t,(n+)} - D_1^{(2)} H_0\big) - \mathbf{q}^2 D_1^{(2)} H_0 \\
&\quad + \frac{k^2}{\alpha_{0,0}^2} C_1 S_{0+,k}^{t,(n+)}, \\
18 P_{61}^{(p0)} &= \mathbf{q}^2\big(D_0^{(1)} - D_0^{(3)}\big)(G_1+G_3), \\
18 P_{62}^{(p0)} &= h_{cd}^2\big(D_1^{(1)} - D_1^{(3)}\big)(H_1+H_3), \\
9 P_{71}^{(p0)} &= \mathbf{q}^2\big(D_0^{(1)} + 2D_0^{(3)}\big)(G_1+G_3+3G_{20}), \\
9 P_{72}^{(p0)} &= h_{cd}^2\big(D_1^{(1)} + 2D_1^{(3)}\big)(H_1+H_3+3H_{20}),
\end{aligned} \tag{24}
$$

where $E_{0+,x}^{t,(c)}$ denotes the pertinent coefficient of the tree level result of channel (c), see Eq. (17), and h_{cd}^2 is given by

$$
h_{cd}^2 = \frac{(s-(m_c+M_{\pi^d})^2)(s-(m_c-M_{\pi^d})^2)}{4s}, \tag{25}
$$

which is a quantity of order ϵ^2. Up to the order considered here, the corrections in k^2 only contribute to $\mathcal{F}_1$ and $\mathcal{F}_6$ and they are independent of the scattering angle $\cos\Theta$. This is the reason why only the coefficients of the S-wave multipoles depend on k^2. Eqs. (23) and (24) clearly show the advantage of the nonrelativistic description: At leading order, the strength of the cusp in the channel $(p0)$ is parameterized in terms of the coupling constant C_1 (i.e. the scattering length a_{0+}^-) and the ratio H_0/G_0.

9. The two-loop corrections all have the topology shown in Fig. 1 and can therefore be cast into the form

$$
\begin{aligned}
E_{0+}^{\text{2Loop}}(s,k^2) &= \begin{pmatrix} J_{ab}(s) & J_{cd}(s)\end{pmatrix} \begin{pmatrix} T_{11}^{(c)} & T_{12}^{(c)} \\ T_{12}^{(c)} & T_{22}^{(c)} \end{pmatrix} \begin{pmatrix} J_{ab}(s) \\ J_{cd}(s)\end{pmatrix}, \\
S_{0+}^{\text{2Loop}}(s,k^2) &= \begin{pmatrix} J_{ab}(s) & J_{cd}(s)\end{pmatrix} \begin{pmatrix} V_{11}^{(c)} & V_{12}^{(c)} \\ V_{12}^{(c)} & V_{22}^{(c)} \end{pmatrix} \begin{pmatrix} J_{ab}(s) \\ J_{cd}(s)\end{pmatrix}.
\end{aligned} \tag{26}
$$

The coefficients $T_{ij}^{(c)}$ for the electric multipole in the channel $(p0)$ read

$$
\begin{aligned}
T_{11}^{(p0)} &= C_0^2 G_0(1+\eta) + C_0^2 E_{0+,q}^t \mathbf{q}^2 - 4C_0 G_0 D_0^{(2)} \mathbf{q}^2 \\
&\quad + C_0^2 E_{0+,k}^t \frac{k^2}{\alpha_{0,0}^2}, \\
T_{12}^{(p0)} &= \frac{1}{2}\big(C_1^2 G_0 + C_0 C_1 H_0\big)(1+\eta) + \frac{1}{2} C_1^2 E_{0+,q}^t \mathbf{q}^2 \\
&\quad - C_1 H_0 D_0^{(2)} \mathbf{q}^2 - C_1 G_0 D_1^{(2)} \mathbf{q}^2 \\
&\quad - \frac{1}{2} C_0 H_0 D_1^{(2)} \mathbf{q}^2 + \frac{1}{2} C_0 C_1 E_{0+,q}^{t,(n+)} h_{cd}^2 \\
&\quad - C_1 G_0 D_1^{(2)} h_{cd}^2 - \frac{1}{2} C_0 H_0 D_1^{(2)} h_{cd}^2 \\
&\quad + \frac{k^2}{2\alpha_{0,0}^2} C_1\big(C_1 E_{0+,k}^t + C_0 E_{0+,k}^{t,(n+)}\big), \\
T_{22}^{(p0)} &= C_1 C_2 H_0(1+\eta) - C_2 H_0 D_1^{(2)} \mathbf{q}^2 + C_1 C_2 E_{0+,q}^{t,(n+)} h_{cd}^2 \\
&\quad - C_2 H_0 D_1^{(2)} h_{cd}^2 - 2C_1 H_0 D_2^{(2)} h_{cd}^2 \\
&\quad + C_1 C_2 E_{0+,k}^{t,(n+)} \frac{k^2}{\alpha_{0,0}^2}.
\end{aligned} \tag{27}
$$

For the scalar multipole, the coefficients are

$$
\begin{aligned}
V_{11}^{(p0)} &= \frac{1}{3} C_0^2 G_7 \mathbf{q}^2 - \frac{1}{3} C_0^2 G_8 \mathbf{q}^2 + \frac{1}{3} C_0^2 G_{21} \mathbf{q}^2, \\
V_{12}^{(p0)} &= \frac{1}{2}\big(C_1^2 G_0 + C_0 C_1 H_0\big)(1+\eta) \\
&\quad + \frac{1}{2} C_1^2 S_{0+,q}^t \mathbf{q}^2 - C_1 H_0 D_0^{(2)} \mathbf{q}^2 \\
&\quad - C_1 G_0 D_1^{(2)} \mathbf{q}^2 - \frac{1}{2} C_0 H_0 D_1^{(2)} \mathbf{q}^2 + \frac{1}{2} C_0 C_1 S_{0+,q}^{t,(n+)} h_{cd}^2 \\
&\quad - C_1 G_0 D_1^{(2)} h_{cd}^2 - \frac{1}{2} C_0 H_0 D_1^{(2)} h_{cd}^2 \\
&\quad + \frac{k^2}{2\alpha_{0,0}^2} C_1\big(C_1 S_{0+,k}^t + C_0 S_{0+,k}^{t,(n+)}\big), \\
V_{22}^{(p0)} &= C_1 C_2 H_0(1+\eta) - C_2 H_0 D_1^{(2)} \mathbf{q}^2 + C_1 C_2 S_{0+,q}^{t,(n+)} h_{cd}^2 \\
&\quad - C_2 H_0 D_1^{(2)} h_{cd}^2 - 2C_1 H_0 D_2^{(2)} h_{cd}^2 \\
&\quad + C_1 C_2 S_{0+,k}^{t,(n+)} \frac{k^2}{\alpha_{0,0}^2}.
\end{aligned} \tag{28}
$$

The two-loop corrections up to and including order $O(a^2\epsilon^5)$ are independent of the scattering angle and therefore only contribute to the S-wave multipoles E_{0+} and S_{0+}.

10. Given the expressions of the various multipoles, one readily finds that the difference $E_{1+} - S_{1+}$ is, to a very high accuracy, free of loop corrections and therefore a polynomial in k^2 and $\mathbf{q}^2$,

$$
\begin{aligned}
\frac{1}{\mathbf{q}}(E_{1+} - S_{1+}) &= \frac{1}{2} E_{1+}^t \frac{k^2}{\alpha_{0,0}^2} + \frac{1}{15}(G_{12} - G_{14})\mathbf{q}^2 \\
&\quad + O\big(\epsilon^4, a\epsilon^5, a^2\epsilon^6, a^3\epsilon^3\big).
\end{aligned} \tag{29}
$$

Up to the corrections of higher order, this equation can be rewritten as a (very) low-energy theorem which relates the derivative of E_{1+} with respect to $|\mathbf{q}|$ evaluated at threshold to a combination of P- and E-waves,

$$
\begin{aligned}
& k^2 \frac{\mathrm{d}}{\mathrm{d}|\mathbf{q}|} E_{1+}\bigg|_{s=s_{\text{thr.}}} \\
&\quad = \frac{\alpha_{0,0}^2}{|\mathbf{q}|}(2E_{1+} - 2S_{1+} + 7E_{3+} - 3M_{3+} + 3M_{3-}).
\end{aligned} \tag{30}
$$

From Eq. (29), it is clear that this relation holds for all values of $\mathbf{q}$ and k^2 which lie in the region of validity of the effective theory, in particular for $\mathbf{q} = 0$ since each of the multipoles on the right-hand side of Eq. (30) is proportional to $|\mathbf{q}|$.

11. The result for the other channels are obtained from the tree level result of channel $(p0)$ and the coefficients $P_{ij}^{(p0)}$ and $T_{ij}^{(p0)}$ by the replacements

$$
\begin{aligned}
&(n+)\colon\ \{G_i, H_i, C_0, C_2\} \to \{H_i, G_i, C_2, C_0\}, \\
&(n0)\colon\ \{G_i, H_i, C_0, C_1, C_2\} \to \{L_i, K_i, C_3, C_4, C_5\}, \\
&(p-)\colon\ \{G_i, H_i, C_0, , C_1, C_2\} \to \{K_i, L_i, C_5, C_4, C_3\}.
\end{aligned} \tag{31}
$$

The replacement indicated for the C_x has to be done also for the corresponding $D_x^{(i)}$.

12. In this Letter, the electroproduction reaction of pions on the nucleon is studied using a nonrelativistic framework. The electric, scalar and magnetic multipoles E_{l+}, S_{l+} for $l = 0, 1$ and $M_{1\pm}$ are calculated in a systematic double expansion in the final state pion and nucleon momenta normalized with the pion mass (counted as a small quantity of order ϵ) and the photon virtuality k^2 as well as the threshold parameters of πN scattering. Explicit representations for the multipoles up to and including ϵ^3 at tree level, ϵ^4 at one loop and ϵ^5 at two loops are provided. The effective theory expansion shows a good convergence behavior in the low-energy region, at least up to a pion momentum of $|\mathbf{q}| \simeq 70$ MeV and for photon virtualities which satisfy $|k^2| \ll 4M_\pi^2$. It accurately describes the cusp structure and allows one to determine the pion–nucleon threshold parameters from experimental data. As an application, a new low-energy theorem relating the slope of E_{1+} at threshold to a combination of P- and E-wave multipoles is discussed.

A numerical analysis of all available experimental data of pion electro- *and* photoproduction in order to determine the pion–nucleon threshold parameters will be presented in a subsequent publication.

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Appendix A. Matching relations

The matching relations of the nonrelativistic couplings to the threshold parameters defined in Eq. (12) read

$$
\begin{aligned}
2G_4 &= 15(\bar{E}_{2+} + 2\bar{M}_{2+}),\\
2G_5 &= 2\zeta G_0 - 2\bar{E}_{2-} - 2\bar{E}_{0+,2,0} + 3(\bar{E}_{2+} - 2\bar{M}_{2-} + 6\bar{M}_{2+}),\\
G_6 &= -3(3\bar{M}_{2+} + 2\bar{M}_{2-}),\\
G_7 &= 15(\bar{E}_{2+} - \bar{M}_{2+}),\\
G_8 &= 3(\bar{E}_{2-} - \bar{M}_{2+} + \bar{M}_{2-} + \bar{E}_{2+}),\\
2G_9 &= 4G_1\zeta - 6\bar{E}_{3-} - 6\bar{E}_{1+,2,0} + 15\bar{E}_{3+}\\
&\quad - 24\bar{M}_{3-} - 6\bar{M}_{1+,2,0} + 45\bar{M}_{3+},\\
2G_{10} &= 35(\bar{E}_{3+} + 3\bar{M}_{3+}),\\
2G_{11} &= 2G_2\zeta + 2\bar{M}_{1-,2} - 9\bar{M}_{3-} + 4\bar{M}_{1+,2} - 12\bar{M}_{3+},\\
2G_{12} &= -15(3\bar{M}_{3-} + 4\bar{M}_{3+}),\\
2G_{13} &= 4G_3\zeta - 6\bar{E}_{3-} - 6\bar{E}_{1+,2,0} + 15\bar{E}_{3+}\\
&\quad - 6\bar{M}_{3-} + 6\bar{M}_{1+,2,0} - 15\bar{M}_{3+},\\
2G_{14} &= 105(\bar{E}_{3+} - \bar{M}_{3+}),\\
G_{15} &= 15(\bar{M}_{3-} + \bar{E}_{3-} + \bar{E}_{3+} - \bar{M}_{3+}),\\
2G_{16} &= 2\alpha_{0,0}^2\bar{E}_{0+,0,2} - G_0 - 2G_0\alpha_{0,0}\alpha_{0,1},\\
2G_{17} &= 6\alpha_{0,0}^2(\bar{E}_{1+,0,2} + \bar{M}_{1+,0,2}) - G_1 - 4G_1\alpha_{0,0}\alpha_{0,1},\\
2G_{18} &= 6\alpha_{0,0}^2(\bar{E}_{1+,0,2} - \bar{M}_{1+,0,2}) - G_3 + 4G_3\alpha_{0,0}\alpha_{0,1},\\
G_{19} &= -\alpha_{0,0}^2(\bar{M}_{1-,0,2} + 2\bar{M}_{1+,0,2}) - G_2\alpha_{0,0}\alpha_{0,1},\\
G_{20} &= \bar{S}_{1-} - 2\bar{S}_{1+},\\
G_{21} &= 6\bar{S}_{2-} - 9\bar{S}_{2+} + G_8,\\
2G_{22} &= 9\bar{S}_{3-} - 12\bar{S}_{3+} - 2\bar{S}_{1-,2,0} + 4\bar{S}_{1+,2,0} + 4\zeta G_{20},\\
G_{23} &= \alpha_{0,0}^2(\bar{S}_{1-,0,2} - 2\bar{S}_{1+,0,2}) - 2G_{20}\alpha_{0,0}\alpha_{0,1},
\end{aligned}
\tag{A.1}
$$

with $\zeta = \frac{\alpha_{1,0}}{\alpha_{0,0}}$.

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