Switching Between Formation in a Moving Shape for Multi-Robots via Synchronization Approach

Ibrahim M.H. Sanhoury*; Shamsudin H.M. Amin; Abdul Rashid Husain
Centre of Artificial Intelligence and Robotics, Materials and Manufacturing Research Alliance, Universiti Teknologi Malaysia, 81310 Johor Bahru, Malaysia

Abstract

This paper extends the synchronization approach for formation control of multiple mobile robots. In this work, the formation shape is moving in a straight line while it is changing with time. Each robot is controlled to track its desired trajectory while synchronizing its movement with the two neighboring robots to maintain a time-varying desired formation. The proposed synchronous controller guarantees the asymptotic stability of both position errors and synchronization errors. Simulation results show the effectiveness of the proposed synchronous controller in navigation.

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Keywords: formation control; synchronization approach; dynamic model; trajectory tracking.

1. Introduction

The importance of the research in cooperative control of multiple mobile robots systems has been increased rapidly since its initiation at the start of 1980s. This is motivated due to the rapid advances in the technology of computing, sensing, and communication. Cooperative control of multiple mobile robots has been widely studied during the last decades. There are several challenging problems in cooperative control, among these problems formation control has been considered as an important cooperative task due to its several applications such as surveillance, search and rescue, transportation, and formation.

Several approaches have been reported in the literature to solve the formation control task. Among these approaches, behavior-based, virtual structure and the leader-following are the most common methods used for formation control. In the behavior-based approach [1-4] several desired behaviors are advocated for each robot in the group, and the effective formation control is resulted from a weighted summation of each behavioral output. This method is suitable for generating the control input in the occurrence of multiple competing objectives, and there is explicit feedback through the communication channel between the adjacent robots. However, there is a difficulty to describe the group behavior obviously; further, this method is complicated to describe the group dynamics, and there is a difficulty to evaluate the stability of the whole system. In the virtual structure approach [5-10] the entire formation is controlled as a single entity, where the desired position for every robot is given to the virtual structure that tracks the trajectories for each robot in the formation. This technique is straightforward for describing the formation strategy; moreover, the stability of the system is guaranteed, and it is more robust to formation via utilizing the group dynamics. However, it is difficult to control multiple mobile robot formation in a decentralized manner; furthermore, it is unsuitable for time-varying formation. In the leader-

* Corresponding author. Tel.: +6173148703.
E-mail address: sanhoury124@yahoo.com
follower approach [11-16], each follower robot was controlled to track one leader with \( l - \phi \) controller, or two leaders with \( l - l \) controller. This method is easy to implement by two controllers; also, it has simple structure, and it depends on the information from its local sensors only. However, there is no feedback error from the followers to the leader robot; furthermore, the leader robot is a single entity for failure of the whole formation, and it is difficult to consider the performance abilities of different robots.

Other approaches to formation are also available like artificial potential [17], graph theory [18], and recently synchronization approach [19]. In the synchronization method, the control goal is derived according to the desired formation, which based on the position synchronization error defined as the differential position errors between each pair of the two adjacent robots. Cross-coupling control established by Koren [20] is an effective method to achieve this approach that can stabilize the formation errors efficiently. In this method, the motion control for each robot is divided into two parts: the first part is to force each robot to move along the desired path to achieve the tracking control goal. The second part is to synchronize the motion of each robot in the group with the two nearby robots. In this way, all the robots in the group will be synchronized, and the system complexity will be reduced in order to achieve the control goal. Synchronization (formation) errors were introduced to measure how multi-robots achieve the desired formation. In the synchronization approach, the controller can be designed in a decentralized way; besides, the controller design and the computational power are reduced, and the controller is scalable. However, this method still needs further research to be conducted in order for different type of mobile robot’s dynamics to perform formation effectively.

This paper is an extension to our former work [21], where the center of the formation shape will move in a straight line, while each robot in the group will track it is desired trajectory in the shape, and simultaneously synchronizing its motion with the two nearby robots to maintain the desired time-varying formation. This work differed from [21], where the robots only switching between different formation shapes in a time-varying manner with the same formation shape center. Through achieving these results, the multi-robot can be further navigated while maintain time-varying formation via this approach.

2. Formation Control of Multi-Robot via Synchronization

Fig. 1 illustrates a wheeled mobile robot, where \( q = [x, y, \phi]^T \) denotes the position coordinate of the robot in the x-y plane, and \( \phi \) denotes the heading angle measured from the x-axis. The dynamic model of the \( i \)th wheeled mobile robot WMR is given as follows:

\[
\begin{align*}
M_i \ddot{q}_i + V_i(q, \dot{q}) &= \tau_i
\end{align*}
\]

\[
I_{dab} \ddot{\phi}_i = \tau_{\phi_i}
\]

where

\[
M_i = \begin{bmatrix} M_{ab} & 0 \\ 0 & M_{ab} \end{bmatrix} , \quad V_i(q, \dot{q}) = \begin{bmatrix} 2m_{wi}d_i I_{dab} \phi_i^2 \cos \phi_i \\ 2m_{wi}d_i I_{dab} \phi_i^2 \sin \phi_i \end{bmatrix} , \quad \tau_{\phi_i} = \begin{bmatrix} \tau_{\phi \phi_i} \\ \tau_{\phi \phi_i} \end{bmatrix} , \quad \ddot{q}_i = \begin{bmatrix} \dot{\phi}_i^2 \\ \dot{\phi}_i^2 \end{bmatrix} , \quad I_{dab} = I_m - 4m_{wi}d_i^2 ,
\]

\( M_{ab} = m_1 I_{dab} + m_2 m_{wi} + 2m_{wi} I_i = I_{dab} + 2m_{wi} b_i^2 + 2I_m \), \( b_i \) is the distance between each driving wheel and the axis of symmetry, \( m_{wi} \) the mass of the WMR without the driving wheels and the rotor of the motors, \( m_i \) the mass of each driving wheel with the rotor, \( I_i \) the moment of inertia of the WMR without the driving wheels and the rotor of the motors about the vertical axis through \( P_{ci} \), \( I_m \) the moment of inertia of each wheel and the rotor of the motor about the wheel diameter, \( P_e \) the intersection of the axis of the symmetry with the driving wheel axis, \( P_c \) the center of the mass of the WMR, \( d_i \) is the distance from \( P_c \) to \( P_i \) along the positive X-axis, X-Y denoted to the robot frame, and x-y refer to the world coordinate.

A time-varying desired shape is introduced for each robot in a similar manner to [19], represented as \( S(p,t) \), where \( p \) denotes 2-D position vector and \( t \) the time. The target location \( q^d_i \) for the \( i \)th robot should be positioned in the curve as \( S(q^d_i, t) = 0 \). The aim is to design the control inputs for the dynamics (1) and (2), such that the robot converges to its desired position \( q^d_i \) while maintain its location in the desired shape \( S(p,t) \). The desired heading \( \phi^d_i \) of the \( i \)th robot is defined such that the robot heading is always oriented towards the robot desired location \( q^d_i \).

The position and heading errors of the \( i \)th robot are defined as: \( e_i = q^d_i - q_i \), and \( \Delta \phi = \phi^d_i - \phi_i \), respectively. The robot is required to accomplish a translational control aim of \( e_i \rightarrow 0 \) and \( \Delta \phi \rightarrow 0 \) as \( t \rightarrow \infty \), as well as to achieve a formation control aim for sustaining the robots on the desired curve.

The following example shows how the synchronization control goal is determined based on the formation goal \( S(q, t) = 0 \).

Example: Consider that \( n \) robots are required to maintain in an ellipse curve during the motions, while the center of the ellipse moving in straight line. The coordinates \( q_i \) of the \( i \)th robot are required to meet the following restrictions:

\[
\begin{align*}
\dot{q}_i &= \begin{bmatrix} x_i \\ y_i \end{bmatrix} \\
\dot{\phi}_i &= \begin{bmatrix} \dot{\phi}_i^1 \\ \dot{\phi}_i^2 \end{bmatrix} \\
\tau_{\phi_i} &= \begin{bmatrix} \tau_{\phi \phi_i^1} \\ \tau_{\phi \phi_i^2} \end{bmatrix} \\
\end{align*}
\]
\[
q_i(t) = \begin{bmatrix}
 x_i(t) \\
y_i(t)
\end{bmatrix} = \begin{bmatrix}
 \cos \varphi_i(t) \\
 \sin \varphi_i(t)
\end{bmatrix} \begin{bmatrix}
 a(t) \\
b(t)
\end{bmatrix} + \begin{bmatrix}
 x_{cen}(t) \\
y_{cen}(t)
\end{bmatrix} = A_i(t) \begin{bmatrix}
 a(t) \\
b(t)
\end{bmatrix} + \begin{bmatrix}
 x_{cen}(t) \\
y_{cen}(t)
\end{bmatrix}
\]

(3)

where \(a\) and \(b\) denote the longest and the shortest radii of the ellipse, respectively, \(\varphi_i = \tan h(\frac{b \sin \alpha_i}{a \cos \alpha_i})\), with \(\alpha_i = \tan h(\frac{y_i}{x_i})\), denotes the angle of the robot lying on the ellipse with respect to the center of the ellipse, and \((x_{cen}(t), y_{cen}(t))\) denotes the center of the ellipse with respect to the time. Assume that the robots are not located in the longest or the shortest axis of the ellipse such that the inverse of \(A_i\) exists. The synchronization constrains to \(q_i\) are derived as:

\[
A_i^{-1}(q_i - \text{Center}(t)) = A_2^{-1}(q_2 - \text{Center}(t)) = \cdots = A_n^{-1}(q_n - \text{Center}(t)) = \begin{bmatrix}
 a \\
b
\end{bmatrix}^T
\]

(4)

where \(\text{Center}(t) = [x_{cen}(t) \ y_{cen}(t)]^T\). From this example, the synchronization constraints can be generally represented as follows:

\[
c_1(q_1 - \text{Center}(t)) = c_2(q_2 - \text{Center}(t)) = \cdots = c_n(q_n - \text{Center}(t))
\]

(5)

where \(c_i\) denotes the coupling parameter for the \(i\)th robot, and its inverse exits based on (4). Furthermore, (5) can be hold for all the desired coordinate \(q_i^d\),

\[
c_1(q_1^d - \text{Center}(t)) = c_2(q_2^d - \text{Center}(t)) = \cdots = c_n(q_n^d - \text{Center}(t))
\]

(6)

Subtracting (5) from (6) yields the control goal as follows;

\[
c_1e_1 = c_2e_2 = \cdots = c_ne_n
\]

(7)

The control goal represented by (7) implicitly, and it can be divided into \(n\) sub goals of \(c_i e_i = c_{i+1} e_{i+1}\). Note that, when \(i = n\), \(n + 1\) is donated as 1. Then, the synchronization errors can be defined as a subset of all possible pairs of two neighboring robots as follows;

\[
\varepsilon_1 = c_1e_1 - c_2e_2 \\
\varepsilon_2 = c_2e_2 - c_3e_3 \\
\vdots \\
\varepsilon_n = c_ne_n - c_1e_1
\]
where $e_i$ denotes the synchronization error of the $i$th robot. Notice that, if the synchronization error $e_i = 0$ for all the robots $i = 1, \ldots, n$, the control goal (7) can be automatically accomplished. The necessary condition for the formation shape is that the shape must be represented mathematically, such that the synchronization constraints (5) can be achieved.

3. Synchronous Formation Control Law

In order for both position errors and synchronization errors to converge to zero, a coupled position errors $E_i$ that links these errors is introduced as follows:

$$E_i = c_i e_i + \beta \int_0^t (e_i - e_{i-1}) d \zeta$$

(9)

where $\beta$ is a diagonal positive gain matrix. Notice from (8) and (9) that the coupled position errors for the $i$th robot feeds back the information from the two nearby robots $i-1$ and $i+1$.

Differentiating (9) with respect to time, yields:

$$\dot{E}_i = \dot{c}_i e_i + c_i \dot{e}_i + \beta (e_i - e_{i-1})$$

(10)

To achieve $\dot{E}_i \to 0$ and $E_i \to 0$, a control vector $u_i$ that leads to a combined position and velocity error is defined as follows:

$$u_i = c_i \dot{q}_i + \dot{c}_i e_i + \beta (e_i - e_{i-1}) + \Lambda E_i$$

(11)

where $\Lambda$ is a diagonal positive gain matrix. The definition of $u_i$ lead to the following position/velocity vector as:

$$r_i = u_i - c_i \dot{q}_i = c_i \dot{e}_i + \dot{c}_i e_i + \beta (e_i - e_{i-1}) + \Lambda E_i = \dot{E}_i + \Lambda E_i$$

(12)

Then, the translational controller is designed to drives $r_i$ to zero, such that the coupled errors $E_i$ and $\dot{E}_i$ tend to zero as well.

An input torque that controlling the robot translation (1) is designed as follows:

$$\tau = M_i c_i^{-1} (\dot{u}_i - \dot{c}_i \dot{q}_i) + K_n c_i^{-1} r_i + c_i^T K_x (e_i - e_{i-1}) + V (q, \dot{q})$$

(13)

where $k_n$ and $k_x$ are positive feedback control gains. The last term in (13) is to compensate for the centripetal and Coriolis effect of the WMR.

Substituting the proposed synchronous controller (13) into the translational dynamic model of the WMR (1), yields the closed-loop dynamics of the system as follows:

$$M_i c_i^{-1} \tau_i + K_n c_i^{-1} r_i + c_i^T K_x (e_i - e_{i-1}) = 0$$

(14)

In order to proof the asymptotic stability for the closed-loop system (14) readers can refer to our previous work [21].

To control the robot’s heading, a general computed torque scheme is used as follows:

$$\tau_{\phi} = I_i \left( \dot{\phi}_{\gamma} + k_{\phi} \Delta \dot{\phi} + k_{\mu} \Delta \phi \right)$$

(15)

where $k_{\phi}$ and $k_{\mu}$ are computed torque control gains. The desired orientation $\phi_{\gamma}$ is defined such that the $i$th robot is always oriented towards its desired position.

Substituting (15) into the robot rotational dynamic (2), yields the closed-loop dynamic system as follows:

$$\Delta \dot{\phi} + k_{\phi} \Delta \dot{\phi} + k_{\mu} \Delta \phi = 0$$

(16)

It is directly achieves $\Delta \phi = 0$ and $\Delta \dot{\phi} = 0$ as time $t \to \infty$, and therefore, the stability of the robot rotation is guaranteed.
4. Simulation Results

Simulations are carried out to verify the effectiveness of the proposed synchronous control law. All the desired formation shapes are assumed to be regular, closed, smooth, and simple planar curves. In this study, a generalized super ellipse with varying parameters that its center is moving in straight line was selected to represent different types of formation curves

\[
\begin{align*}
    x_i &= \pm a \cos^m \varphi_i + x_{\text{cent}} \\
    y_i &= \pm b \sin^m \varphi_i + y_{\text{cent}}
\end{align*}
\]

where \( m \) represents the exponent index, \( a, b, \) and \( \varphi_i \) are as defined in (3), and \( (x_{\text{cent}}, y_{\text{cent}}) \) denotes the center of the formation shape. Throughout this simulation, the value of \( \varphi_i \) is fixed with respect to the center of the shape. The exponent \( m, a, \) and \( b \) can be time-varying parameters.

In this simulation, four WMR are located in an ellipse curve as shown in Fig 2, where the parameters for each robot are given in [21]. During the simulation, the center of the formation shape will move in a straight line, while the formation shape is switched from an ellipse curve \( m_0 = 1 \) to a rounded rectangle shape \( m_f = 1/8 \). In this case, \( a \) and \( b \) are considered to be fixed value. The robots are required to maintain a desired time-varying hyper ellipse curves during the switching between the ellipse to a rounded rectangle, while the center of the shape is moving in a straight line. The exponent index is changed as:

\[
m(t) = m_0 + (m_f - m_0) \left(1 - e^{-t}\right)
\]

The center of formation shapes is given with respect to the time as follows;

\[
\begin{align*}
    x_{\text{cent}}(t) &= 2t \\
    y_{\text{cent}}(t) &= 2t
\end{align*}
\]

The desired trajectory for each robot is given as follows:

\[
q^d_i(t) = \begin{bmatrix} x_i^d(t) \\ y_i^d(t) \end{bmatrix} = \begin{bmatrix} \cos^{m(i)} \varphi_i \\ \sin^{m(i)} \varphi_i \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} x_{\text{cent}} \\ y_{\text{cent}} \end{bmatrix} = A_i(t) \begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} x_{\text{cent}} \\ y_{\text{cent}} \end{bmatrix}
\]

The coupled parameter matrix is determined as follows:

\[
c_i(t) = A_i^{-1}(t) = \begin{bmatrix} \cos^{m(i)} \varphi_i \\ \sin^{m(i)} \varphi_i \end{bmatrix}^{-1}
\]

The simulation sampling period was set to 0.005 second. The synchronous controller parameters were selected for each robot as: \( \beta = \text{diag} \{1.4, 3\}, \Lambda = \text{diag} \{0.6, 5\}, \kappa = \text{diag} \{2, 3\}, \kappa_\psi = \text{diag} \{10000, 10000\}, k_p = 10, \) and \( k_\psi = 15 \). The initial heading for the four robots are zero degree.

Fig. 3.a shows the position errors in the \( x \)- and \( y \)-directions, while Fig. 3.b shows the synchronization errors in the same directions. Notice from the figures that the values of both position errors’ and synchronization errors’ increases to finite amount and consequently, converged to zero until reaching the final desired shape and position. From the simulation, both position errors and synchronization errors are minimized to zero, where a better formation is achieved.

5. Conclusion

The proposed synchronous controller guarantees asymptotic stability of both position errors and synchronization errors. The dynamic model of the WMR is divided into a translational and rotational dynamic model. The rotational control law allows each robot to be always oriented towards its desired position. The simulation results show the effectiveness of the synchronization method to control the formation shape to follow a straight line while the shape is changing with time. Our future work will focus on integrating the synchronization method with path planning and navigation to achieve better performance in real applications.
Fig. 2. Switching from an ellipse to a rounded rectangle while shape is moving in straight line

Fig. 3. (a) position errors; (b) synchronization errors.
Acknowledgements

The authors would like to thank MOHE and UTM for the Research University Grant (RUG) Tier 1, under the vot no. Q.J130000.2501.02H72 on multi-robotics formation control.

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