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ORIGINAL ARTICLE

Reduced differential transform method to solve two and three dimensional second order hyperbolic telegraph equations

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Abstract In this article, an analytical solution procedure is described for solving two and three dimensional second order hyperbolic telegraph equation using a reliable semi-analytic method so called the reduced differential transform method (RDTM) subject to the appropriate initial condition. Using this method, it is possible to find an exact solution or a closed approximate solution of a differential equation. Various numerical examples are carried out to check the accuracy, efficiency, and convergence of the described method. The method is a powerful mathematical tool for solving a wide range of problems arising in engineering and sciences.

1. Introduction

In the recent development, the communication system plays a key role in the worldwide society. The radio frequency (RF) and microwave communication (MW) systems generate high frequency communication which plays a significant role in various industrial applications.

The above systems use the transmission media for transferring the information carrying signal from one point to another point. This transmission media can be categorized into two groups, namely, guided and unguided. In a guided medium, the signal is transferred through the coaxial cable or transmission line and therefore, the guided media are capable of transporting the high frequency voltage and current waves. In case of unguided media, the electromagnetic waves carry the signal over part of or the entire communication path through RF and MW channels. These electromagnetic waves are transmitted and received through antenna.

In a guided transmission media, especially cable transmission medium is investigated to address the problem of efficient telegraph transmission. A cable transmission medium classified as a guided transmission medium represents a physical system that directly propagates the information between two or more locations. In order to optimize the guided communication system it is necessary to determine or project power and signal losses in the system, because all the systems have such losses.

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To determine these losses and eventually ensure a maximum output, it is necessary to formulate some kind of equation to calculate these losses.

Assume that $u(x,y,t)$ and $i(x,y,t)$ be the electric voltage and the current in a double conductor, satisfying the telegraph equations in two dimensions (2D):

$$\left.\begin{array}{l}\frac{\partial^2 u}{\partial t^2}+2\alpha\frac{\partial u}{\partial t}+\beta^2 u=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+f_1(x,y,t),\\ \frac{\partial^2 i}{\partial t^2}+2\alpha\frac{\partial i}{\partial t}+\beta^2 i=\frac{\partial^2 i}{\partial x^2}+\frac{\partial^2 i}{\partial y^2}+f_2(x,y,t),\end{array}\right\},(x,y,t)\in\Omega;\alpha>0,\beta>0 \tag{1}$$

where $\Omega=[a,b]\times[c,d]\times[t>0]$ .The initial conditions are assumed to be

$$\left.\begin{array}{l}u(x,y,0)=g_1(x,y),\\ u_t(x,y,0)=g_2(x,y),\\ i(x,y,0)=h_1(x,y),\\ i_t(x,y,0)=h_2(x,y),\end{array}\right\},(x,y)\in\Omega \tag{2}$$

while the boundary conditions are expressed as follows

$$\left.\begin{array}{l}u(x,y,t)=\xi_1(x,y,t),(x,y)\in\Gamma_p,t\geqslant 0,\\ \frac{\partial u}{\partial \eta}(x,y,0)=\xi_2(x,y,t),(x,y)\in\Gamma_q,t\geqslant 0,\\ i(x,y,t)=\psi_1(x,y,t),(x,y)\in\Gamma_p,t\geqslant 0,\\ \frac{\partial i}{\partial \eta}(x,y,0)=\psi_2(x,y,t),(x,y)\in\Gamma_q,t\geqslant 0,\end{array}\right\} \tag{3}$$

where Γ_p and Γ_q are non-intersecting curves such that $\Gamma_p\cup\Gamma_q=\Gamma$, Γ is the closed curve bounding the domain Ω and η is the unit outward vector to Γ.

Similarly, the three dimensional (3D) telegraph equation is expressed as follows:

$$\left.\begin{array}{l}\frac{\partial^2 u}{\partial t^2}+2\alpha\frac{\partial u}{\partial t}+\beta^2 u=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}+f_1(x,y,z,t),\\ \frac{\partial^2 i}{\partial t^2}+2\alpha\frac{\partial i}{\partial t}+\beta^2 i=\frac{\partial^2 i}{\partial x^2}+\frac{\partial^2 i}{\partial y^2}+\frac{\partial^2 i}{\partial z^2}+f_2(x,y,z,t),\end{array}\right\}, \quad (x,y,z,t)\in\Omega;\alpha>0,\beta>0 \tag{4}$$

where $\Omega=[a,b]\times[c,d]\times[e,f]\times[t>0]$, with initial conditions

$$\left.\begin{array}{l}u(x,y,z,0)=g_1(x,y,z),\\ u_t(x,y,z,0)=g_2(x,y,z),\\ i(x,y,z,0)=h_1(x,y,z),\\ i_t(x,y,z,0)=h_2(x,y,z)\end{array}\right\},(x,y,z)\in\Omega \tag{5}$$

and the boundary conditions consist of

$$\left.\begin{array}{l}u(x,y,z,t)=\xi_1(x,y,z,t),(x,y,z)\in\Gamma_p,t\geqslant 0,\\ \frac{\partial u}{\partial \eta}(x,y,z,0)=\xi_2(x,y,z,t),(x,y,z)\in\Gamma_q,t\geqslant 0,\\ i(x,y,z,t)=\psi_1(x,y,z,t),(x,y,z)\in\Gamma_p,t\geqslant 0,\\ \frac{\partial i}{\partial \eta}(x,y,z,0)=\psi_2(x,y,z,t),(x,y,z)\in\Gamma_q,t\geqslant 0,\end{array}\right\} \tag{6}$$

For $\alpha>0,\beta=0$, Eqs. (1) and (4) represent damped wave equations in two and three dimensions respectively.

Through the literature survey it can be seen that telegraph equation is much more appropriate than ordinary diffusion equation for modeling the reaction diffusion. The hyperbolic partial differential equations model the vibrations of structures (e.g. machines, buildings and beams) and they are the basis for fundamental equations of atomic physics. The telegraph equation is described as an important equation for modeling various problems arising in engineering and science fields to name a few, wave propagation (Weston and He, 1993), random walk theory (Banasiak and Mika, 1998), signal analysis (Jordan and Puri, 1999) etc. Recently, it can be observed that there has been given much concentration to the development of exact and numerical computational methods for one dimensional and two dimensional telegraph equations (Mohanty and Jain, 2001; Mohanty et al., 2002; Mohanty, 2004; 2005; 2009; Dehghan and Shokri, 2008; Saadatmandi and Dehghan, 2010; Dehghan and Ghesmati, 2010; Dehghan et al., 2011; Lakestani and Saray, 2010; Jiwari et al. 2012; Momani, 2005; Chen et al. 2008; Raftari and Yildirim, 2012; Das et al., 2011; Srivastava et al., 2013a,b; Ahmad and Hassan, 2013; Keskin and Oturanc, 2009).

The present paper describes an analytical scheme, the reduced differential transform method to provide approximate analytical results of the two and three dimensional telegraph equations. The accuracy and efficiency of the proposed method are demonstrated by several test examples. The biggest benefit of the described method is that it finds the solution of telegraph equation directly without using any transformation, linearization, discretization or any other restrictive conditions. Further, the method can be easily implemented in multidimensional problems arising in many areas of science and engineering.

2. Reduced differential transform method (RDTM)

In this section, the basic definitions of the reduced differential transform method are described.

Let us consider a function of four variables$w(x,y,z,t)$, and assume that it can be represented as a product $w(x,y,z,t)=F(x,y,z)G(t)$. On extending the basis of the properties of the one-dimensional differential transformation (Abazari and Ganji, 2011), the function $w(x,y,z,t)$ can be represented as follows

$$\begin{aligned} w(x,y,z,t)&=\sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\sum_{i_3=0}^{\infty}F(i_1,i_2,i_3)x^{i_1}y^{i_2}z^{i_3}\sum_{j=0}^{\infty}G(j)t^j\\ &=\sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\sum_{i_3=0}^{\infty}\sum_{j=0}^{\infty}W(i_1,i_2,i_3)x^{i_1}y^{i_2}z^{i_3}t^j,\end{aligned} \tag{7}$$

where $W(i_1,i_2,i_3)=F(i_1,i_2,i_3)G(j)$ is called the spectrum of $w(x,y,z,t)$.

Assume that R_D denotes the reduced differential transform operator and R_D^{-1} indicates the inverse reduced differential transform operator, then the basic definition and operation of the RDTM are described below.

Definition 2.1. If $w(x,y,z,t)$ is analytic and continuously differentiable with respect to space variables x,y and time variable t in the domain of interest, then the spectrum function (Abazari and Ganji, 2011; Abazari and Abazari, 2012)

$$R_D[w(x,y,z,t)]\approx W_k(x,y,z)=\frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}w(x,y,z,t)\right]_{t=t_0} \tag{8}$$

is the reduced transformed function of $w(x,y,z,t)$.

In this article, the lowercase $w(x,y,z,t)$ stands for the original function while the uppercase $W_k(x,y,z)$ represents the reduced transformed function. The differential inverse reduced transform of $W_k(x,y,z)$ is defined by

$$R_D^{-1}[W_k(x,y,z)]\approx w(x,y,z,t)=\sum_{k=0}^{\infty}W_k(x,y,z)(t-t_0)^k \tag{9}$$

Now, combining the Eqs. (8) and (9), we get

$$w(x,y,z,t) = \sum_{k=0}^{\infty} \frac{1}{k!} \left[\frac{\partial^k}{\partial t^k} w(x,y,z,t) \right]_{t=t_0} (t - t_0)^k \tag{10}$$

From the Eq. (10), it is obvious that the reduced differential transform is derived from the function's power series expansion.

Definition 2.2. Assume $u(x,y,z,t) = R_D^{-1}[U_k(x,y,z)]$, $v(x,y,z,t) = R_D^{-1}[V_k(x,y,z)]$, and let convolution $\otimes$ denotes the reduced differential transform version of the multiplication, then the basic operations of the reduced differential transform are given in Table 1.

3. RDTM for two dimensional telegraph equation

Implementing the RDTM to the two dimensional telegraph Eq. (1), we have the following expression

$$\left.\begin{array}{c} (k+1)(k+2)U_{k+2}(x,y) + 2\alpha(k+1)U_{k+1}(x,y) \\ +\beta^2 U_k(x,y) = \frac{\partial^2}{\partial x^2} U_k(x,y) + \frac{\partial^2}{\partial y^2} U_k(x,y) + R_D[f_1(x,y,t)], \\ (k+1)(k+2)I_{k+2}(x,y) + 2\alpha(k+1)I_{k+1}(x,y) \\ +\beta^2 I_k(x,y) = \frac{\partial^2}{\partial x^2} I_k(x,y) + \frac{\partial^2}{\partial y^2} I_k(x,y) + R_D[f_2(x,y,t)], \end{array}\right\}, \quad (x,y,t) \in \Omega; \alpha > 0, \beta > 0.$$

Now implementing the aforesaid method to the initial conditions (2), we get

$$\left.\begin{array}{l} U_0(x,y) = g_1(x,y), \\ U_1(x,y) = g_2(x,y), \\ I_0(x,y) = h_1(x,y), \\ I_1(x,y) = h_2(x,y), \end{array}\right\}, (x,y) \in \Omega.$$

From above two equations we get the values of $U_k(x,y)$, $I_k(x,y), k = 2,3,4,\ldots$ etc. Applying the differential inverse reduced transform of $U_k(x,y); I_k(x,y), k = 0,1,2,3,\ldots$, one can obtain the approximate solution for $u(x,y,t)$ and $i(x,y,t)$ given by

$$u(x,y,t) = \sum_{k=0}^{\infty} U_k(x,y)t^k = U_0(x,y) + U_1(x,y)t + U_2(x,y)t^2 + U_3(x,y)t^3 + \ldots$$

$$i(x,y,t) = \sum_{k=0}^{\infty} I_k(x,y)t^k = I_0(x,y) + I_1(x,y)t + I_2(x,y)t^2 + I_3(x,y)t^3 + \ldots$$

4. RDTM for three dimensional telegraph equation

Implementing the aforesaid method to the three dimensional telegraph Eq. (4), we get the following equation

$$\left.\begin{array}{l} (k+1)(k+2)U_{k+2}(x,y,z) + 2\alpha(k+1)U_{k+1}(x,y,z) + \beta^2 U_k(x,y,z) \\ = \frac{\partial^2}{\partial x^2} U_k(x,y,z) + \frac{\partial^2}{\partial y^2} U_k(x,y,z) + \frac{\partial^2}{\partial z^2} U_k(x,y,z) + R_D[f_1(x,y,z,t)], \\ (k+1)(k+2)I_{k+2}(x,y,z) + 2\alpha(k+1)I_{k+1}(x,y,z) + \beta^2 I_k(x,y,z) \\ = \frac{\partial^2}{\partial x^2} I_k(x,y,z) + \frac{\partial^2}{\partial y^2} I_k(x,y,z) + \frac{\partial^2}{\partial z^2} I_k(x,y,z) + R_D[f_2(x,y,z,t)], \end{array}\right\}, \quad (x,y,z,t) \in \Omega; \alpha > 0, \beta > 0.$$

Now applying the method to the initial conditions (5), we have

$$\left.\begin{array}{l} U_0(x,y,z) = g_1(x,y,z), \\ U_1(x,y,z) = g_2(x,y,z), \\ I_0(x,y,z) = h_1(x,y,z), \\ I_1(x,y,z) = h_2(x,y,z), \end{array}\right\}, (x,y,z) \in \Omega.$$

From above two equations we get the values of $U_k(x,y,z)$, $I_k(x,y,z), k = 2,3,\ldots$ etc. Using the differential inverse reduced transform of $U_k(x,y,z); I_k(x,y,z), k = 0,1,2,\ldots$, we have the approximate solution for $u(x,y,z,t)$ and $i(x,y,z,t)$ as follows

$$u(x,y,z,t) = \sum_{k=0}^{\infty} U_k(x,y,z)t^k = U_0(x,y,z) + U_1(x,y,z)t + U_2(x,y,z)t^2 + \ldots$$

$$i(x,y,z,t) = \sum_{k=0}^{\infty} I_k(x,y,z)t^k = I_0(x,y,z) + I_1(x,y,z)t + I_2(x,y,z)t^2 + \ldots$$

5. Computational illustrations

In this section, the method explained in Section 2 is described by taking several examples of both linear and nonlinear 2D and 3D telegraph equations to validate the efficiency and reliability of the aforesaid technique.

Example 5.1. Consider the 2D linear Telegraph equation (Jiwari et al., 2012)

$$\frac{\partial^2 u}{\partial t^2} + 2\frac{\partial u}{\partial t} + u = \frac{1}{2}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{11}$$

Table 1 Basic operations of the reduced differential transform method.

Original function	Reduced differential transformed function
$R_D[u(x,y,z,t)v(x,y,z,t)]$	$U_k(x,y,z) \otimes V_k(x,y,z) = \sum_{r=0}^{k} U_r(x,y,z)V_{k-r}(x,y,z)$
$R_D[\alpha u(x,y,z,t) \pm \beta v(x,y,z,t)]$	$\alpha U_k(x,y,z) \pm \beta V_k(x,y,z)$
$R_D[\frac{\partial^n}{\partial t^n} u(x,y,z,t)]$	$(k+1)(k+2)\ldots\ldots(k+n)U_{k+n}(x,y,z)$
$R_D[\frac{\partial^{m+n+p+s}}{\partial x^m \partial y^n \partial z^p \partial t^s} u(x,y,z,t)]$	$\frac{(k+s)!}{k!} \frac{\partial^{m+n+p}}{\partial x^m \partial y^n \partial z^p} U_{k+s}(x,y,z)$
$R_D[x^m y^n z^p t^q]$	$\begin{cases} x^m y^n z^p, k = q \\ 0, \text{otherwise} \end{cases}$
$R_D[e^{\lambda t}]$	$\frac{\lambda^k}{k!}$
$R_D[\sin(\alpha x + \beta y + \gamma z + \omega t)]$	$\frac{w^k}{k!} \sin(\frac{\pi k}{2!} + \alpha x + \beta y + \gamma z)$
$R_D[\cos(\alpha x + \beta y + \gamma z + \omega t)]$	$\frac{w^k}{k!} \cos(\frac{\pi k}{2!} + \alpha x + \beta y + \gamma z)$

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subject to the initial conditions (the solution is periodic in x and y)

$$\left.\begin{array}{l} u(x,y,0)=\sinh(x)\sinh(y), \\ u_t(x,y,0)=-2\sinh(x)\sinh(y), \end{array}\right\}. \quad (12)$$

Implementing the RDTM to Eq. (11), we get the following relation

$$(k+1)(k+2)U_{k+2}(x,y)+2(k+1)U_{k+1}(x,y) = \frac{1}{2}\left(\frac{\partial^2}{\partial x^2}U_k(x,y)+\frac{\partial^2}{\partial y^2}U_k(x,y)\right)-U_k(x,y). \quad (13)$$

Using the aforesaid method to the initial conditions (12), we have

$$U_0(x,y)=\sinh(x)\sinh(\mathrm{y});\ U_1(x,y)=-2\sinh(x)\sinh(\mathrm{y}). \quad (14)$$

Using Eq. (14) in Eq. (13), we have the following $U_k(x,y)$ values successively as

$$U_2(x,y)=2\sinh(x)\sinh(y)=\tfrac{(-2)^2}{2!}\sinh(x)\sinh(y); \quad U_3(x,y)=\tfrac{(-2)^3}{3!}\sinh(x)\sinh(y); \quad \ldots;\ U_k(x,y)=\tfrac{(-2)^k}{k!}\sinh(x)\sinh(y). \quad (15)$$

Using the differential inverse reduced transform of $U_k(\mathrm{x},\mathrm{y})$, we obtain the expression

$$\begin{aligned} u(x,y,t) &= \sum_{k=0}^{\infty}U_k(x,y)t^k \\ &= U_0(x,y)+U_1(x,y)t+U_2(x,y)t^2+U_3(x,y)t^3+\ldots \\ &= \sinh(x)\sinh(y)\left(1+(-2)t+\frac{(-2)^2}{2!}t^2+\frac{(-2)^3}{3!}t^3+\ldots+\frac{(-2)^k}{k!}t^k+\ldots\right). \end{aligned} \quad (16)$$

The solution (16), in closed form, is expressed as follows

$$\mathrm{u}(x,y,t)=e^{-2t}\sinh(x)\sinh(y). \quad (17)$$

Example 5.2. Consider the 3D linear Telegraph equation (Weston and He, 1993)

$$\frac{\partial^2 u}{\partial t^2}+2\frac{\partial u}{\partial t}+u=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2} \quad (18)$$

subject to initial conditions (the solution is periodic in x, y and z)

$$\left.\begin{array}{l} u(x,y,z,0)=\sinh(x)\sinh(y)\sinh(z), \\ u_t(x,y,z,0)=-\sinh(x)\sinh(y)\sinh(z), \end{array}\right\}. \quad (19)$$

Applying the aforesaid technique to Eq. (18), we obtain the following recurrence formula

$$(k+1)(k+2)U_{k+2}(x,y,z)+2(k+1)U_{k+1}(x,y,z) = \tfrac{\partial^2}{\partial x^2}U_k(x,y,z)+\tfrac{\partial^2}{\partial y^2}U_k(x,y,z)+\tfrac{\partial^2}{\partial z^2}U_k(x,y,z)-U_k(x,y,z). \quad (20)$$

Using the described method to the initial conditions (19), we get

$$U_0(x,y,z)=\sinh(x)\sinh(y)\sinh(z);\ \ U_1(x,y,z) =-\sinh(x)\sinh(y)\sinh(z). \quad (21)$$

Using Eq. (21) in Eq. (20), one can get the following $U_k(\mathrm{x},\mathrm{y},z)$ values successively as

$$U_2(x,y,z)=\tfrac{(-1)^2}{2!}\sinh(x)\sinh(y)\sinh(z); \quad U_3(x,y,z)=\tfrac{(-1)^3}{3!}\sinh(x)\sinh(y)\sinh(z); \quad \ldots;\ U_k(x,y,z)=\tfrac{(-1)^k}{k!}\sinh(x)\sinh(y)\sinh(z). \quad (22)$$

Using the differential inverse reduced transform of $U_k(x,y,z)$, we have

$$\begin{aligned} u(x,y,z,t) &= \sum_{k=0}^{\infty}U_k(x,y,z)t^k \\ &= \sinh(x)\sinh(y)\sinh(z) \\ &\quad \times\left(1+(-1)t+\tfrac{(-1)^2}{2!}t^2+\tfrac{(-1)^3}{3!}t^3+\ldots\ldots+\tfrac{(-1)^k}{k!}t^k+\ldots\ldots\right) \end{aligned} \quad (23)$$

The solution (23), in closed form, can be given by

$$\mathrm{u}(x,y,z,t)=e^{-t}\sinh(x)\sinh(y)\sinh(z). \quad (24)$$

Example 5.3. Consider the following 2D nonlinear Telegraph equation (Dehghan and Ghesmati, 2010)

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{\partial^2 u}{\partial t^2}+2\frac{\partial u}{\partial t}+u^2-e^{2(x+y)-4t}+e^{(x+y)-2t} \quad (25)$$

under the initial conditions (the solution grows exponentially in x and y)

$$\left.\begin{array}{l} u(x,y,0)=e^{x+y}, \\ u_t(x,y,0)=-2e^{x+y}, \end{array}\right\}. \quad (26)$$

Applying the aforesaid technique to Eq. (25), we obtain the following iterative expression:

$$\begin{aligned} &(k+1)(k+2)U_{k+2}(x,y)+2(k+1)U_{k+1}(x,y) \\ &= \frac{\partial^2}{\partial x^2}U_k(x,y)+\frac{\partial^2}{\partial y^2}U_k(x,y) \\ &\quad -\sum_{r=0}^{k}U_r(x,y)U_{k-r}(x,y)+e^{2(x+y)}\left(\frac{(-4)^k}{k!}\right)-e^{(x+y)}\left(\frac{(-2)^k}{k!}\right). \end{aligned} \quad (27)$$

Applying the RDTM to the initial conditions (40), we obtain

$$U_0(x)=e^{x+y};\ U_1(x)=-2e^{x+y}. \quad (28)$$

Using Eq. (28) in Eq. (27), we get the following $U_k(\mathrm{x},\mathrm{y})$ values successively as

$$U_2(x,y)=2e^{x+y}=\tfrac{(-2)^2}{2!}e^{x+y};\ U_3(x,y)=\tfrac{(-2)^3}{3!}e^{x+y};\ U_4(x,y)=\tfrac{(-2)^4}{4!}e^{x+y}; \quad \ldots U_k(x,y)=\tfrac{(-2)^k}{k!}e^{x+y}. \quad (29)$$

Using the differential inverse reduced transform of $U_k(\mathrm{x},\mathrm{y})$, one can get

$$\begin{aligned} u(x,y,t) &= \sum_{k=0}^{\infty}U_k(x,y)t^k=U_0(x,y)+U_1(x,y)t+U_2(x,y)t^2+U_3(x,y)t^3+\ldots \\ &= e^{x+y}\left(1+(-2)t+\tfrac{(-2)^2}{2!}t^2+\tfrac{(-2)^3}{3!}t^3+\ldots\ldots+\tfrac{(-2)^k}{k!}t^k+\ldots\ldots\right). \end{aligned} \quad (30)$$

The solution (30), in closed form, is given by

$$\mathrm{u}(x,y,t)=e^{(x+y)-2t}. \quad (31)$$

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Example 5.4. Consider the following 3D nonlinear Telegraph equation (Weston and He, 1993)

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=\frac{\partial^2 u}{\partial t^2}+2\frac{\partial u}{\partial t}+u^2-e^{2(x+y+z)-4t}+e^{(x+y+z)-2t} \tag{32}$$

under the initial conditions (the solution grows exponentially in x, y and z)

$$\left.\begin{aligned} u(x,y,z,0)&=e^{x+y+z},\\ u_t(x,y,z,0)&=-e^{x+y+z},\end{aligned}\right\}. \tag{33}$$

Implementing the aforesaid technique to Eq. (32), we obtain the following iterative expression:

$$\begin{aligned}&(k+1)(k+2)U_{k+2}(x,y,z)+2(k+1)U_{k+1}(x,y,z)\\&=\tfrac{\partial^2}{\partial x^2}U_k(x,y,z)+\tfrac{\partial^2}{\partial y^2}U_k(x,y,z)+\tfrac{\partial^2}{\partial z^2}U_k(x,y,z)\\&-\sum_{r=0}^{k}U_r(x,y,z)U_{k-r}(x,y,z)+e^{2(x+y+z)}\left(\tfrac{(-4)^k}{k!}\right)\\&-e^{(x+y+z)}\left(\tfrac{(-2)^k}{k!}\right).\end{aligned} \tag{34}$$

Using the aforesaid scheme to the initial conditions (33), we have

$$U_0(x,y,z)=e^{x+y+z};\, U_1(x,y,z)=-e^{x+y+z}. \tag{35}$$

Using Eq. (35) in Eq. (34), we obtain $U_k(\mathrm{x,y,z})$ values successively as

$$\begin{aligned}U_2(x,y,z)&=\tfrac{(-1)^2}{2!}e^{x+y+z};\, U_3(x,y,z)=\tfrac{(-1)^3}{3!}e^{x+y+z};\\ U_4(x,y,z)&=\tfrac{(-1)^4}{4!}e^{x+y+z};;.....;\, U_k(x,y,z)=\tfrac{(-1)^k}{k!}e^{x+y+z}.\end{aligned} \tag{36}$$

Using the differential inverse reduced transform of $U_k(\mathrm{x,y,z})$, we have

$$\begin{aligned}u(x,y,z,t)&=\sum_{k=0}^{\infty}U_k(\mathrm{x,y,z})t^k\\&=U_0(\mathrm{x,y,z})+U_1(\mathrm{x,y,z})t+U_2(\mathrm{x,y,z})t^2+\\&=e^{x+y+z}\left(1+(-1)t+\tfrac{(-1)^2}{2!}t^2+\tfrac{(-1)^3}{3!}t^3+......+\tfrac{(-1)^k}{k!}t^k+......\right).\end{aligned} \tag{37}$$

The solution (37), in closed form, is given as follows

$$\mathrm{u}(x,y,z,t)=e^{(x+y+z)-t}. \tag{38}$$

6. Conclusions

In this article, the reduced differential transform method is described to find the analytical solution of two and three dimensional hyperbolic linear and nonlinear telegraph equations. The method is applied in a direct way without using transformation, linearization, discretization or any other restrictive conditions. The effectiveness of the method is shown from the computational solutions, which shows that the RDTM rapidly converges, highly accurate, and is an easily implementable mathematical method for the multidimensional problems emerging in various domains of science and engineering.

References

Abazari, R., Abazari, M., 2012. Numerical simulation of generalized Hirota-Satsuma coupled KdV equation by RDTM and comparison with DTM. Commun. Nonlinear Sci. Numer. Simul. 17, 619–629.

Abazari, R., Ganji, M., 2011. Extended two-dimensional DTM and its application on nonlinear PDEs with proportional delay. Int. J. Comput. Math. 88 (8), 1749–1762.

Ahmad, Z.F., Hassan, I., 2013. Analytical solution for a generalized space-time fractional telegraph equation. Math. Methods Appl. Sci. 36 (14), 1813–1824.

Banasiak, J., Mika, J.R., 1998. Singularly perturbed telegraph equations with applications in the random walk theory. J. Appl. Math. Stoch. Anal. 11 (1), 9–28.

Chen, J., Liu, F., Anh, V., 2008. Analytical solution for the time-fractional telegraph equation by the method of separable variables. J. Math. Anal. Appl. 338, 1364–1377.

Das, S., Vishal, K., Gupta, P.K., Yildirim, A., 2011. An approximate analytical solution of time-fractional telegraph equation. Appl. Math. Comput. 217 (18), 7405–7411.

Dehghan, M., Ghesmati, A., 2010. Combination of mesh less local weak and strong (MLWS) forms to solve the two dimensional hyperbolic telegraph equation. Eng. Anal. Boundary Elem. 34, 324–336.

Dehghan, M., Shokri, A., 2008. A numerical method for solving the hyperbolic telegraph equation. Numer. Methods Partial Differ. Equ. 24, 1080–1093.

Dehghan, M., Yousefi, S.A., Lotfi, A., 2011. The use of He's variational iteration method for solving the telegraph and fractional telegraph equations. Int. J. Numer. Methods Biomed. Eng. 27, 219–231.

Jiwari, R., Pandit, S., Mittal, R.C., 2012. A differential quadrature algorithm to solve the two dimensional linear hyperbolic telegraph equation with Dirichlet and Neumann boundary conditions. Appl. Math. Comput. 218, 7279–7294.

Jordan, P.M., Puri, A., 1999. Digital signal propagation in dispersive media. J. Appl. Phys. 85 (3), 1273–1282.

Keskin, Y., Oturanc, G., 2009. Reduced differential transform method for partial differential equations. Int. J. Nonlinear Sci. Numer. Simul. 10 (6), 741–749.

Lakestani, M., Saray, B.N., 2010. Numerical solution of telegraph equation using interpolating scaling functions. Comput. Math. Appl. 60 (7), 1964–1972.

Mohanty, R.K., 2004. An unconditionally stable difference scheme for the one-space dimensional linear hyperbolic equation. Appl. Math. Lett. 17, 101–105.

Mohanty, R.K., 2005. An unconditionally stable difference formula for a linear second order one space dimensional hyperbolic equation with variable coefficients. Appl. Math. Comput. 165, 229–236.

Mohanty, R.K., 2009. A new unconditionally stable difference schemes for the solution of multi-dimensional telegraphic equations. Int. J. Comput. Math. 86 (12), 2061–2071.

Mohanty, R.K., Jain, M.K., 2001. An unconditionally stable alternating direction implicit scheme for the two space dimensional linear hyperbolic equation. Numer. Methods Partial Differ. Equ. 7, 684–688.

Mohanty, R.K., Jain, M.K., Arora, U., 2002. An unconditionally stable ADI method for the linear hyperbolic equation in three space dimensional. Int. J. Comput. Math. 79, 133–142.

Momani, S., 2005. Analytical and approximate solutions of the space- and time fractional telegraph equations. Appl. Math. Comput. 170 (2), 1126–1134.

Raftari, B., Yildirim, A., 2012. Analytical solution of second-order hyperbolic telegraph equation by variation iteration and homotopy perturbation methods. Results Math. 61 (1–2), 13–28.

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Saadatmandi, A., Dehghan, M., 2010. Numerical solution of hyperbolic telegraph equation using the Chebyshev Tau method. Numer. Methods Partial Differ. Equ. 26, 239–252.

Srivastava, V.K., Awasthi, M.K., Tamsir, M., 2013a. RDTM solution of Caputo time fractional-order hyperbolic telegraph equation. AIP Adv. 3, 032142.

Srivastava, V.K., Awasthi, M.K., Chaurasia, R.K., Tamsir, M., 2013b. The telegraph equation and its solution by reduced differential transform method. Modell. Simul. Eng. Article ID 746351.

Weston, V.H., He, S., 1993. Wave splitting of the telegraph equation in R3 and it application to inverse scattering. Inverse Prob. 9, 789–812.