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Type IIB flux compactifications on twistor bundles

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ABSTRACT

We construct a $U(1)$ bundle over $N(1,1)$, usually considered as an $SO(3)$ bundle on $\mathbf{CP}^2$, and show that type IIB supergravity can be consistently compactified over it. With the five form flux turned on, there is a solution for which the metric becomes Einstein. We further turn on 3-form fluxes and show that there is a one parameter family of solutions. In particular, there is a limiting solution of large 3-form fluxes for which two $U(1)$ fiber directions of the metric shrink to zero size. We also discuss compactifications over $N(1,1)$ to AdS_3. All solutions turn out to be non-supersymmetric.

1. Introduction

Compactifying solutions of supergravity theories provide a natural way of constructing consistent supergravity theories in lower dimensions. Moreover, some of the solutions turn out to be the near horizon geometry of M(D)-branes, and thus are of significance in AdS/CFT duality. Most of such solutions, however, preserve part of the supersymmetry and usually one needs to break it to construct more realistic models. Squashed and stretched solutions with fluxes in the compact direction, on the other hand, provide examples of supergravity solutions in which supersymmetry is spontaneously broken, and therefore, might be of interest in building the phenomenological models in the context of AdS/CFT duality [1].

Recently, we constructed new solutions of eleven-dimensional supergravity compactifying it to AdS_5 and $AdS_2 \times H^2$. We employed canonical forms on S^7 to write consistent ansätze for the 4-form field strength. The twistor space construction of $\mathbf{CP}^3$ was the key for identifying the new solutions [2], and as we will see in this Letter, this construction also proves useful in finding yet more solutions.

In this note, we extend the construction of [2] to the case of compact manifold $N(1,1)$, and use the twistor space language to describe it as $U(1)$ bundle over a base which itself is an S^2 bundle on $\mathbf{CP}^2$; the flag manifold. The first supergravity solutions of this kind were found in [3,4], and then explicit (squashed) metrics were constructed [5]. In Section 2, first we consider $N(1,1)$ as an $SO(3)$ bundle over $\mathbf{CP}^2$ and then rewrite the metric as a $U(1)$ bundle over the flag manifold. We then show that on this 7-dimensional manifold there exists a natural harmonic 2-form, the Kähler form of $\mathbf{CP}^2$, and use it to construct an 8-dimensional twistor bundle: a $U(1)$ bundle over $N(1,1)$. Interestingly, as the harmonic 2-form is anti-self-dual, the Ricci tensor of this 8-dimensional metric in a suitable basis is diagonal with constant components. On the other hand, in Section 3 we show that on this 8-dimensional manifold there exists a harmonic 3-form which we use to write down an ansatz for the 5-form field strength of type IIB supergravity. In this way, we are able to reduce the field equations to a set of algebraic equations. Among the three solutions we obtain one is Einstein. In Section 3.1, we generalize our solution by turning on 3-form fluxes, and show that there is a one parameter family of such solutions. In a limit of large 3-form fluxes two $U(1)$ fiber directions of the metric shrink to zero size. In Section 3.2, we discuss the supersymmetry of the solutions and show that they break supersymmetry. Section 4 is devoted to a discussion of compactification on $N(1,1)$ and the supersymmetry of the solution. Conclusions and the discussion are brought in Section 5.

2. $U(1)$ bundles over $N(1,1)$

$N(1,1)$ can be considered as an $SO(3)$ bundle over $\mathbf{CP}^2$ admitting two Einstein metrics, and hence providing Freund–Rubin type solutions of eleven-dimensional supergravity [6,1]. The bundle structure is very similar to that of S^7 where it is viewed as an $SU(2)$ bundle over S^4. However, $N(1,1)$ admits a 2-form, the Kähler form of $\mathbf{CP}^2$, which, as we will see, is anti-self-dual and harmonic. This allows us to construct a $U(1)$ bundle over $N(1,1)$ so that the Ricci tensor is diagonal and has constant coefficients. Therefore, with a suitable ansatz for the form fields we are able to reduce the field equations to some algebraic equations.

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2.1. $N(1,1)$ as an $SO(3)$ bundle over $\mathbf{CP}^2$

Let us start by taking the following 7-dimensional metric of $N(1,1)$ written as an $SO(3)$ bundle over $\mathbf{CP}^2$ [5,1]:

$$\begin{aligned} ds^2_{N(1,1)} &= d\mu^2 + \frac{1}{4}\sin^2\mu\big(\Sigma_1^2+\Sigma_2^2+\cos^2\mu\,\Sigma_3^2\big) \\ &\quad + \lambda^2\bigg((\sigma_1-\cos\mu\,\Sigma_1)^2+(\sigma_2-\cos\mu\,\Sigma_2)^2 \\ &\quad + \bigg(\sigma_3-\frac{1}{2}\big(1+\cos^2\mu\big)\Sigma_3\bigg)^2\bigg), \end{aligned} \tag{1}$$

where λ is the squashing parameter. Here $0\leqslant\mu\leqslant\pi/2$, and Σ_i's are a set of left-invariant one-forms on $SU(2)$:

$$\Sigma_1=\cos\gamma\,d\alpha+\sin\gamma\,\sin\alpha\,d\beta,$$
$$\Sigma_2=-\sin\gamma\,d\alpha+\cos\gamma\,\sin\alpha\,d\beta,$$
$$\Sigma_3=d\gamma+\cos\alpha\,d\beta,$$

with $0\leqslant\gamma\leqslant4\pi$, $0\leqslant\alpha\leqslant\pi$, $0\leqslant\beta\leqslant2\pi$. There is a similar expression for σ_i's:

$$\sigma_1=\sin\phi\,d\theta+\sin\theta\cos\phi\,d\tau,$$
$$\sigma_2=-\cos\phi\,d\theta+\sin\theta\sin\phi\,d\tau,$$
$$\sigma_3=-d\phi+\cos\theta\,d\tau,$$

where they now take value on $SO(3)$, i.e., $0\leqslant\tau\leqslant2\pi$, $0\leqslant\theta\leqslant\pi$, $0\leqslant\phi\leqslant2\pi$. They satisfy the $SU(2)$ algebra; $d\Sigma_i=-\frac{1}{2}\epsilon_{ijk}\Sigma_j\wedge\Sigma_k$, $d\sigma_i=-\frac{1}{2}\epsilon_{ijk}\sigma_j\wedge\sigma_k$, with $i,j,k,\ldots=1,2,3$.

As in the case of S^7, we can see that metric (1) can be rewritten as a $U(1)$ bundle over a base which itself is an S^2 bundle on $\mathbf{CP}^2$, the flag manifold, [7,8,2]:

$$\begin{aligned} ds^2_{N(1,1)} &= d\mu^2 + \frac{1}{4}\sin^2\mu\big(\Sigma_1^2+\Sigma_2^2+\cos^2\mu\,\Sigma_3^2\big) \\ &\quad + \lambda^2(d\theta-\sin\phi A_1+\cos\phi A_2)^2 \\ &\quad + \lambda^2\sin^2\theta\big(d\phi-\cot\theta(\cos\phi A_1+\sin\phi A_2)+A_3\big)^2 \\ &\quad + \lambda^2(d\tau-A)^2, \end{aligned} \tag{2}$$

where

$$A_1=\cos\mu\,\Sigma_1,\qquad A_2=\cos\mu\,\Sigma_2,$$
$$A_3=\frac{1}{2}\big(1+\cos^2\mu\big)\Sigma_3, \tag{3}$$

and

$$A=\cos\theta\,d\phi+\sin\theta(\cos\phi A_1+\sin\phi A_2)+\cos\theta A_3. \tag{4}$$

In the new form of the metric, (2), we can further rescale the $U(1)$ fibers to $\tilde\lambda$ so that the Ricci tensor, in a basis we introduce shortly, is still diagonal. So, let us take the metric to be

$$\begin{aligned} ds^2_{N(1,1)} &= d\mu^2 + \frac{1}{4}\sin^2\mu\big(\Sigma_1^2+\Sigma_2^2+\cos^2\mu\,\Sigma_3^2\big) \\ &\quad + \lambda^2(d\theta-\sin\phi A_1+\cos\phi A_2)^2 \\ &\quad + \lambda^2\sin^2\theta\big(d\phi-\cot\theta(\cos\phi A_1+\sin\phi A_2)+A_3\big)^2 \\ &\quad + \tilde\lambda^2(d\tau-A)^2, \end{aligned} \tag{5}$$

and choose the following basis

$$\begin{aligned} &e^0=d\mu,\qquad e^1=\frac{1}{2}\sin\mu\,\Sigma_1,\qquad e^2=\frac{1}{2}\sin\mu\,\Sigma_2, \\ &e^3=\frac{1}{2}\sin\mu\cos\mu\,\Sigma_3,\qquad e^5=\lambda(d\theta-\sin\phi A_1+\cos\phi A_2), \\ &e^6=\lambda\sin\theta\big(d\phi-\cot\theta(\cos\phi A_1+\sin\phi A_2)+A_3\big), \\ &e^7=\tilde\lambda(d\tau-A). \end{aligned} \tag{6}$$

In this basis the Ricci tensor is diagonal and reads

$$\begin{aligned} &R_{00}=R_{11}=R_{22}=R_{33}=6-4\lambda^2-2\tilde\lambda^2, \\ &R_{55}=R_{66}=4\lambda^2+1/\lambda^2-\tilde\lambda^2/2\lambda^4, \\ &R_{77}=4\tilde\lambda^2+\tilde\lambda^2/2\lambda^4. \end{aligned} \tag{7}$$

For $\lambda^2=\tilde\lambda^2=1/2$, and $\lambda^2=\tilde\lambda^2=1/10$ the metric becomes Einstein, and thus one can get a solution of the Freund–Rubin type [6]. One can also turn on the 4-form flux in the compact direction to get the Englert type solutions [3,5].

2.2. $U(1)$ bundles over $N(1,1)$

To start our discussion of constructing $U(1)$ bundles we need to borrow some preliminary results, adapted to the $N(1,1)$ case, from [2]. Let us first introduce the following three 2-forms

$$\begin{aligned} R_1 &= \sin\phi\big(e^{01}+e^{23}\big)-\cos\phi\big(e^{02}+e^{31}\big), \\ R_2 &= \cos\theta\cos\phi\big(e^{01}+e^{23}\big)+\cos\theta\sin\phi\big(e^{02}+e^{31}\big) \\ &\quad -\sin\theta\big(e^{03}+e^{12}\big), \\ K &= \sin\theta\cos\phi\big(e^{01}+e^{23}\big)+\sin\theta\sin\phi\big(e^{02}+e^{31}\big) \\ &\quad +\cos\theta\big(e^{03}+e^{12}\big), \end{aligned} \tag{8}$$

with the angles and basis given in the previous subsection. These three 2-forms are orthogonal to each other, i.e.,

$$R_1\wedge R_2=K\wedge R_1=K\wedge R_2=0. \tag{9}$$

With A in (4) rewritten as

$$A=\cot\theta\frac{e^6}{\lambda}+\frac{2\cot\mu}{\sin\theta}\big(\cos\phi e^1+\sin\phi e^2\big), \tag{10}$$

it is easy to prove that

$$de^5=-e^6\wedge A+2\lambda R_1,\qquad de^6=e^5\wedge A+2\lambda R_2. \tag{11}$$

Further, if we define

$$\operatorname{Re}\Omega=R_1\wedge e^5+R_2\wedge e^6,\qquad \operatorname{Im}\Omega=R_1\wedge e^6-R_2\wedge e^5, \tag{12}$$

using (11), we can see that

$$d\operatorname{Re}\Omega=8\lambda\omega_4-\frac{2}{\lambda}e^{56}\wedge K,\qquad d\operatorname{Im}\Omega=0, \tag{13}$$

with $\omega_4=e^0\wedge e^1\wedge e^2\wedge e^3$, the volume element of the base, which is closed; $d\omega_4=0$.

We can now look at an interesting feature of $N(1,1)$ as a bundle over $\mathbf{CP}^2$. The base manifold admits a closed 2-form, i.e., the Kähler form:

$$J=\frac{1}{4}da=\frac{1}{4}d\big(\sin^2\mu\,\Sigma_3\big)=e^{03}-e^{12}, \tag{14}$$

so that $dJ=0$. Moreover, we observe that on $N(1,1)$ with metric (5) J is also co-closed:

$$
\begin{aligned}
d*_7 J &= -d\big(J\wedge e^{567}\big)\\
&= -2\lambda J\wedge \operatorname{Im}\Omega\wedge e^7 - \tilde\lambda J\wedge e^{56}\wedge\big(2K + e^{56}/\lambda^2\big)\\
&= 0,
\end{aligned}
\tag{15}
$$

where we used

$$
\begin{aligned}
de^{56} &= 2\lambda \operatorname{Im}\Omega,\\
de^7 &= -\tilde\lambda F = -\tilde\lambda\, dA = \tilde\lambda\big(2K + e^{56}/\lambda^2\big),
\end{aligned}
\tag{16}
$$

and

$$
J\wedge K = J\wedge \operatorname{Im}\Omega = 0,
\tag{17}
$$

as K and $\operatorname{Im}\Omega$ are self-dual, whereas J is anti-self-dual on $\mathbf{CP}^2$. All this indicates that we can use the corresponding $U(1)$ connection of J to construct a $U(1)$ bundle over $N(1,1)$ so that its Ricci tensor is diagonal with constant coefficients. Therefore, for the metric of this 8-dimensional manifold, M, we take

$$
ds_8^2 = ds^2_{N(1,1)} + \hat\lambda^2(dz - a)^2,
\tag{18}
$$

with $\hat\lambda$ measuring the scale of the new $U(1)$ fiber. Adding

$$
e^8 = \hat\lambda(dz - a),
\tag{19}
$$

to the vielbein basis (6), the 8d Ricci tensor reads

$$
\begin{aligned}
&R_{00} = R_{11} = R_{22} = R_{33} = 6 - 4\lambda^2 - 2\tilde\lambda^2 - 8\hat\lambda^2,\\
&R_{55} = R_{66} = 4\lambda^2 + 1/\lambda^2 - \tilde\lambda^2/2\lambda^4,\\
&R_{77} = 4\tilde\lambda^2 + \tilde\lambda^2/2\lambda^4, \qquad R_{88} = 16\hat\lambda^2.
\end{aligned}
\tag{20}
$$

We see that as J is harmonic and anti-self-dual, we do not get mixed components and the Ricci tensor remains diagonal.

3. Type IIB compactifications to AdS_2

We now show that the eight dimensional metric constructed above admits a harmonic 3-form, and then use this 3-form to provide an ansatz for the five form field strength of type IIB supergravity. To begin with, we note that on this manifold there are generally three 4-forms which are closed and self-dual on $\mathbf{CP}^2$ [2]. On the other hand, since $de^8 = -4\hat\lambda J$ is anti-self-dual we can write down a 5-form which is also closed:

$$
*_8\,\omega_3 = \big(\alpha\omega_4 + \beta K\wedge e^{56} + \gamma e^7\wedge \operatorname{Im}\Omega\big)\wedge e^8 + \xi J\wedge e^{567},
\tag{21}
$$

with α, β, γ, and ξ being constant parameters. In fact, using (13), (16), and (17) we can see that $d*_8\,\omega_3 = 0$. Taking the Hodge dual (with $\epsilon_{01235678} = 1$), we have

$$
\omega_3 = -\alpha e^{567} - \beta K\wedge e^7 + \gamma \operatorname{Re}\Omega - \xi J\wedge e^8,
\tag{22}
$$

which we also require to be closed. Using (13) together with

$$
dK = -\operatorname{Im}\Omega/\lambda,
\tag{23}
$$

we see that ω_3 is closed if

$$
\beta = 2\alpha\lambda^2, \qquad \gamma = -2\alpha\lambda\tilde\lambda, \qquad \xi = -3\alpha\lambda^2\tilde\lambda/\hat\lambda.
\tag{24}
$$

Hence, on M there exists a harmonic 3-form; $d\omega_3 = d*_8\,\omega_3 = 0$.

To discuss type IIB supergravity, we take a direct product ansatz for the metric:

$$
ds_{10}^2 = ds_2^2 + ds_8^2,
\tag{25}
$$

together with the following ansatz for the self-dual 5-form:

$$
F_5 = \omega_3\wedge\epsilon_2 + *_8\,\omega_3,
\tag{26}
$$

which then satisfies the equation of motion, $d*F_5 = 0$, as ω_3 is harmonic.

Next, let us consider the Einstein equations. Taking the dilaton and axion to be constant, in the Einstein frame, they read

$$
\begin{aligned}
R_{MN} = {} & \frac{1}{4\cdot 4!}\Big(F_{MPQRS}F_N{}^{PQRS} - \frac{1}{10}F_{PQRSL}F^{PQRSL}g_{MN}\Big)\\
&+ \frac{e^{-\phi}}{4}\Big(H_{MPQ}H_N{}^{PQ} - \frac{1}{12}H_{PQR}H^{PQR}g_{MN}\Big)\\
&+ \frac{e^{\phi}}{4}\Big(F_{MPQ}F_N{}^{PQ} - \frac{1}{12}F_{PQR}F^{PQR}g_{MN}\Big).
\end{aligned}
\tag{27}
$$

Using (20) and ansatz (26), the Einstein equations reduce to the following algebraic equations:

$$
\begin{aligned}
&6 - 4\lambda^2 - 2\tilde\lambda^2 - 8\hat\lambda^2 = \alpha^2/4,\\
&4\lambda^2 + \frac{1}{\lambda^2} - \frac{\tilde\lambda^2}{2\lambda^4} = \big(2\beta^2 - \alpha^2 + 2\xi^2\big)/4,\\
&4\tilde\lambda^2 + \frac{\tilde\lambda^2}{2\lambda^4} = \big(4\gamma^2 - 2\beta^2 - \alpha^2 + 2\xi^2\big)/4,\\
&16\hat\lambda^2 = \big(4\gamma^2 + \alpha^2 + 2\beta^2 - 2\xi^2\big)/4.
\end{aligned}
\tag{28}
$$

First we note that there is a solution for which the metric is Einstein. Plugging (24) into the above equations, we get the following solution:

$$
\lambda^2 = \tilde\lambda^2 = 1/4, \qquad \hat\lambda^2 = 3/16, \qquad \alpha^2 = 12,
\tag{29}
$$

with the Ricci tensor along AdS_2:

$$
R_{\mu\nu} = -12 g_{\mu\nu}.
\tag{30}
$$

With the help of *Mathematica*, we have also found two more solutions of Eqs. (28) for which the metric is not Einstein:

$$
\lambda = \tilde\lambda \approx 0.4267, \qquad \hat\lambda \approx 0.2661, \qquad \alpha \approx 4.1667,
\tag{31}
$$

with $R_{\mu\nu} \approx -14.4583 g_{\mu\nu}$, and

$$
\begin{aligned}
&\lambda \approx 0.5609, \qquad \tilde\lambda \approx 0.4095, \qquad \hat\lambda \approx 0.4480,\\
&\alpha \approx 3.3464,
\end{aligned}
\tag{32}
$$

with $R_{\mu\nu} \approx -11.5538 g_{\mu\nu}$.

3.1. A one parameter family of solutions

Having found a solution for which the metric is Einstein, we are interested to see whether we can have solutions with H and F_3 fluxes turned on. For this we note that indeed there are two 3-forms which are closed:

$$
H = \zeta\, de^{78} = \zeta\tilde\lambda\big(2K + e^{56}/\lambda^2\big)\wedge e^8 + 4\zeta\hat\lambda e^7\wedge J,
\tag{33}
$$

and

$$
F_3 = \eta \operatorname{Im}\Omega = \eta\, de^{56}/2\lambda,
\tag{34}
$$

with ζ and η two constants. With the above ansätze for the 3-form fields, let us now turn to the type IIB equations of motion which, in the Einstein frame, read:

$$
\begin{aligned}
& d * d\phi = e^{2\phi}\, dc \wedge * dc - \frac{1}{2} e^{-\phi} H \wedge * H + \frac{1}{2} e^{\phi} \tilde{F}_3 \wedge * \tilde{F}_3, \\
& d\big(e^{2\phi} * dc\big) = -e^{\phi} H \wedge * \tilde{F}_3, \\
& d * \big(e^{-\phi} H - c e^{\phi} \tilde{F}_3\big) = F_3 \wedge F_5, \\
& d * \big(e^{\phi} \tilde{F}_3\big) = -H \wedge F_5, \\
& d * \tilde{F}_5 = H \wedge F_3,
\end{aligned} \tag{35}
$$

where,

$$
\begin{aligned}
& F_3 = dC_2, \qquad F_5 = dC_4, \qquad H_3 = dB, \\
& \tilde{F}_3 = F_3 - cH_3, \qquad \tilde{F}_5 = F_5 - C_2 \wedge H_3, \qquad * \tilde{F}_5 = \tilde{F}_5.
\end{aligned} \tag{36}
$$

First note that because of (9), (12), and (17) we have $H \wedge F_3 = 0$, and hence we can use the same F_5 as in the previous section, namely let

$$
\tilde{F}_5 = \omega_3 \wedge \epsilon_2 + *_8\, \omega_3, \tag{37}
$$

so that the last equation of (35) is satisfied. Taking ϕ to be constant, $c = 0$, and the form fields as in (33), (34), and (37) we can see that the rest of equations in (35) are also satisfied if

$$
e^{\phi} \eta = \gamma \zeta, \qquad \gamma^2 = 8\hat{\lambda}^2 + 2\tilde{\lambda}^2 + \tilde{\lambda}^2/4\lambda^4. \tag{38}
$$

This leaves us with four unknown coefficients to be fixed. However, when we plug these into Einstein equations (27) they collapse into 3 equations:

$$
\begin{aligned}
& 6 - 4\lambda^2 - 2\tilde{\lambda}^2 - 8\hat{\lambda}^2 = \frac{\gamma^2}{16\lambda^2\tilde{\lambda}^2} + 2b^2\big(\tilde{\lambda}^2 + 4\hat{\lambda}^2\big), \\
& 4\lambda^2 + \frac{1}{\lambda^2} - \frac{\tilde{\lambda}^2}{2\lambda^4} = \frac{\gamma^2}{2}\left(\frac{\lambda^2}{\tilde{\lambda}^2} - \frac{1}{8\lambda^2\tilde{\lambda}^2} + \frac{9\lambda^2}{4\hat{\lambda}^2}\right) + \frac{b^2\tilde{\lambda}^2}{2\lambda^4}, \\
& 4\tilde{\lambda}^2 + \frac{\tilde{\lambda}^2}{2\lambda^4} = \gamma^2\left(1 - \frac{1}{16\lambda^2\tilde{\lambda}^2} - \frac{\lambda^2}{2\tilde{\lambda}^2} + \frac{9\lambda^2}{8\hat{\lambda}^2}\right) \\
& \qquad + b^2\left(8\hat{\lambda}^2 - 2\tilde{\lambda}^2 - \frac{\tilde{\lambda}^2}{4\lambda^4}\right),
\end{aligned} \tag{39}
$$

with $b^2 = e^{-\phi}\zeta^2$. So, we get a free parameter, b, that is not determined by the equations of motion. Although we have not been able to find the most general solution of (39), by examining the pattern of numerical solutions generated by *Mathematica* we did derive a particular solution:

$$
\lambda^2 = \frac{1}{4}, \qquad \tilde{\lambda}^2 = \frac{1}{4(1+b^2)}, \qquad \hat{\lambda}^2 = \frac{3}{16(1+b^2)}, \tag{40}
$$

which can be checked by direct substitution in Eqs. (39). Note that for $b = 0$, we get the solution in the previous section where we had only F_5 turned on. In the extreme limit $b \to \infty$, $\tilde{\lambda}$ and $\hat{\lambda}$ go to zero and thus two $U(1)$ directions of metric (18) and (5) shrink to zero size. Surprisingly, the Ricci tensor of AdS_2 turns out to be independent of b,

$$
R_{\mu\nu} = -12 g_{\mu\nu}. \tag{41}
$$

3.2. Supersymmetry

In this section we show that the solution we found in Section 3, where we had turned on only the five-form flux with constant dilaton, breaks all supersymmetries. When the dilaton and axion are constant and there are no 3-form fluxes, the variation of the dilatino vanishes. However, we need to check whether the supersymmetry variation of the gravitino vanishes too, i.e.,

$$
\delta\psi_M = \nabla_M \varepsilon + \frac{i}{16 \cdot 5!} \Gamma^{NPQRS} \Gamma_M F_{NPQRS} \varepsilon = 0. \tag{42}
$$

To study the Killing equation, (42), on the direct product space $AdS_2 \times M$ of Section 3, let $\varepsilon = \epsilon \otimes \eta$, with ϵ and η the supersymmetry parameters along AdS_2 and M, respectively. We decompose the 10d Dirac matrices as

$$
\begin{aligned}
& \Gamma_\mu = \hat{\gamma}_\mu \otimes \gamma_9, \quad \mu = 0, 1, \\
& \Gamma_{m+1} = 1 \otimes \gamma_m, \quad m = 1, \ldots, 8,
\end{aligned}
$$

where $\hat{\gamma}_\mu$ and γ_m are the 2 and 8 dimensional Dirac matrices respectively, with $\hat{\gamma}_0 = i\sigma_2$, and $\hat{\gamma}_1 = \sigma_1$.

We can see that the supersymmetry is broken by looking at the Killing equation along AdS_2. First, note that

$$
F_{NPQRS} \Gamma^{NPQRS} = 10 F_{mnp\mu\nu} \Gamma^{mnp\mu\nu} (1 - \Gamma_{11}), \tag{43}
$$

so if we choose $\Gamma_{11}\varepsilon = (\sigma_3 \otimes \gamma_9)(\epsilon \otimes \eta) = \varepsilon$, with

$$
\Gamma_{11} = -\Gamma_{0123456789},
$$

then we have

$$
\begin{aligned}
& F_{NPQRS} \Gamma^{NPQRS} \Gamma_\mu \varepsilon \\
& \quad = 20 F_{mnp\rho\sigma} \Gamma^{mnp\rho\sigma} \Gamma_\mu \varepsilon \\
& \quad = 40 F_{mnp01} \big(1 \otimes \gamma^{mnp}\big)(\sigma_3 \otimes 1)(\hat{\gamma}_\mu \otimes \gamma_9)(\epsilon \otimes \eta) \\
& \quad = 40 F_{mnp01} \big(\hat{\gamma}_\mu \otimes \gamma^{mnp}\big)(\epsilon \otimes \eta).
\end{aligned} \tag{44}
$$

Therefore, to split Killing equation (42) along the AdS_2 and the compact direction, we need to require

$$
F_{mnp01} \gamma^{mnp} \eta = k\eta, \tag{45}
$$

for k a constant, so that along AdS_2 we have

$$
\nabla_\mu \epsilon + 40k \hat{\gamma}_\mu \epsilon = 0. \tag{46}
$$

But, since γ_{mnp} anticommutes with γ_9 and since η has a definite chirality, $\gamma_9 \eta = \eta$, Eq. (45) can only have a zero eigenvalue, i.e., we must have $k = 0$. On the other hand, if $k = 0$, then the integrability of Killing spinor equation $\nabla_\mu \epsilon = 0$ implies that the 2-dimensional Ricci tensor is vanishing which is not consistent with the AdS_2 factor that we obtained from solving the equations of motion. Therefore we conclude that the solution breaks supersymmetry. The above argument also applies to the solutions of Section 3.1.

4. Type IIB on $N(1,1)$

Now that we have discussed the compactification of type IIB on $U(1)$ bundles over $N(1,1)$, let us look at the related compactification of type IIB on $N(1,1)$ itself. We study solutions with only F_5 flux turned on. So, let us take a direct product ansatz for the metric:

$$
ds_{10}^2 = ds_3^2 + ds_{N(1,1)}^2, \tag{47}
$$

together with F_5 as

$$
F_5 = \alpha\big(J \wedge \epsilon_3 + J \wedge e^{567}\big), \tag{48}
$$

which is self-dual and closed because of (15). Using this ansatz and the Ricci components of $N(1,1)$ in (7), Einstein equations (27) reduce to

$$
\begin{aligned}
& 6 - 4\lambda^2 - 2\tilde{\lambda}^2 = 0, \\
& 4\lambda^2 + \frac{1}{\lambda^2} - \frac{\tilde{\lambda}^2}{2\lambda^4} = \alpha^2/2, \\
& 4\tilde{\lambda}^2 + \frac{\tilde{\lambda}^2}{2\lambda^4} = \alpha^2/2,
\end{aligned}
$$

which have just one solution;

$$\lambda^2 = \tilde{\lambda}^2 = 1, \qquad \alpha^2 = 9. \tag{49}$$

Note that in this solution the Ricci tensor along the base manifold, i.e., $\mathbf{CP}^2$, vanishes. The non-compact space is an AdS_3 with

$$R_{\mu\nu} = -9/2 g_{\mu\nu}. \tag{50}$$

Finally, let us discuss the supersymmetry of this solution. For $AdS_3 \times N(1,1)$ compactification, we take the 10d Dirac matrices as

$$\Gamma_\mu = \hat{\gamma}_\mu \otimes 1 \otimes \sigma_1, \quad \mu = 0, 1, 2,$$
$$\Gamma_{m+2} = 1 \otimes \gamma_m \otimes \sigma_2, \quad m = 1, \ldots, 7,$$

where $\hat{\gamma}_\mu$ and γ_m are 2 and 8 dimensional Dirac matrices respectively, with $\hat{\gamma}_0 = i\sigma_2$, $\hat{\gamma}_1 = \sigma_1$, and $\hat{\gamma}_2 = \sigma_3$. The supersymmetry parameter then decomposes

$$\varepsilon = \epsilon \otimes \eta \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \tag{51}$$

with $\Gamma_{11} = 1 \otimes 1 \otimes \sigma_3$. As in the previous section, let us first look at the Killing equation along the AdS_3 factor. Note that

$$\begin{aligned} &F_{NPQRS}\Gamma^{NPQRS}\Gamma_\mu\varepsilon \\ &\quad = 10F_{mn\nu\rho\sigma}\Gamma^{mn\nu\rho\sigma}\Gamma_\mu(1+\Gamma_{11})\varepsilon \\ &\quad = 5!F_{mn012}(\hat{\gamma}_\mu \otimes \gamma^{mn} \otimes 1)\left(\epsilon \otimes \eta \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right), \end{aligned} \tag{52}$$

so, to split the Killing equation along AdS_3 and $N(1,1)$ we must have

$$(\gamma^{03} - \gamma^{12})\eta = 2i\eta. \tag{53}$$

The integrability of the Killing equation along the base manifold $\mathbf{CP}^2$, on the other hand, requires

$$\left(-2\gamma^{01} + \gamma^{23} - \gamma^{\hat{2}\hat{3}} + \tilde{c}\gamma^{\hat{1}}(\gamma^{03} - \gamma^{12})\right)\eta = 0,$$
$$\left(-2\gamma^{23} + \gamma^{01} - \gamma^{\hat{2}\hat{3}} + \tilde{c}\gamma^{\hat{1}}(\gamma^{03} - \gamma^{12})\right)\eta = 0,$$

with $\tilde{c}$ a constant. The above equations imply

$$(\gamma^{03} - \gamma^{12})\eta = 0, \tag{54}$$

which is in conflict with Eq. (53). Therefore, we conclude that the solution breaks supersymmetry.

5. Conclusions

We constructed a $U(1)$ bundle over $N(1,1)$, and showed that type IIB supergravity can be consistently compactified over it. The twistor space formalism was crucial in deriving the solutions, specially when there were 3-form fluxes turned on. This approach has earlier been used in deriving new eleven-dimensional supergravity solutions [2], and also in [9] to study new solutions of massive IIA supergravity.

We noticed that $N(1,1)$ admits a harmonic 2-form and used it to write the twistor bundle eight dimensional metric. With this choice of the connection, the Ricci tensor turned out to be diagonal with constant components. Furthermore, we saw that this eight dimensional manifold allows a harmonic 3-form, which was then employed to write a consistent ansatz for the 5-form field strength of type IIB supergravity. In this way, we showed that the field equations could be reduced to a set of algebraic equations. Among the three solutions we found one was Einstein. The discussion became more interesting when we turned on 3-form fluxes and obtained a one parameter family of solutions. Amusingly, we observed that there was a limiting solution for which two fiber directions of the metric were shrinking to zero size, whereas the 2-dimensional cosmological constant turned out to be independent of the free parameter. At the end, we further studied the related compactification over $N(1,1)$ to AdS_3.

Since all the solutions we found in this Letter break supersymmetry it is interesting to see whether they are associated with some brane configurations. This would then allow us to study AdS/CFT in a non-supersymmetric setup.

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