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# Exit from inflation and a paradigm for vanishing cosmological constant in self-tuning models

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## Abstract

We propose a paradigm for the inflation and the vanishing cosmological constant in a unified way with the self-tuning solutions of the cosmological constant problem. Here, we consider a time-varying cosmological constant in self-tuning models of the cosmological constant. As a specific example, we demonstrate it with a 3-form field in 5D.

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## 1. Introduction

Ever since the inflationary idea has been proposed, how the universe settles after the inflationary period at the vacuum with the vanishing vacuum energy has been a dream to be solved but postponed until the solution of the cosmological constant problem is known. In 4-dimensional (4D) field theory models, it is known that there is no solution for the cosmological constant problem [1]. One must go beyond 4D to find a clue to the solution of the cosmological constant problem. In this sense, the Randall–Sundrum (RS) type models, in particular the RS-II type models [2]

are of great interest toward a clue toward a vanishing cosmological constant.

Indeed, a few years ago solutions of the cosmological constant problem have been tried under the name of self-tuning solutions [3–6]. In the late 1970's and early 1980's it was called the solutions with an undetermined integration constant(s). The old self-tuning solutions looked for flat space solutions whether or not it accompanies the de Sitter (dS) space and/or anti-de Sitter (AdS) space solutions. This kind of old self-tuning solutions is called *weak self-tuning solutions*. On the other hand, recently it has been tried to find a self-tuning solution without allowing the nearby dS and AdS space solutions [3]. This kind of new self-tuning solutions can be called *strong self-tuning solutions*. However, there seems no example for the strong self-tuning solution [4].

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The first example for the self-tuning solution is obtained in 5D with the three index antisymmetric tensor field  $A_{MNP}$ , with the  $1/H^2$  in the action where  $H_{MNPQ} = \epsilon^{MNPQ} \partial_M A_{NPQ}$  [5,7]. Certainly, this action has a few unsatisfactory features, but it renders an example for the existence of weak self-tuning solutions, and provides possible physics behind the self-tuning solutions. One such example is the existence of the region of parameter space where only the dS solutions are allowed, which can be used for the period of inflation [8]. From this example, we can envision a unification of the ideas of inflation in the early universe, presumably at the GUT era, and the solution of the cosmological constant problem.<sup>1</sup> Weak self-tuning solutions have been tried with a string-inspired Gauss–Bonnet action with some fine-tuning between bulk and/or brane parameters [10], and in models with brane gravity [11]. In view of the existence of a few weak self-tuning solutions, therefore, the time is ripe to consider a unified view on the inflation and vanishing cosmological constant now even though there has not appeared yet a universally accepted self-tuning solution.

In this vein, we try to find out time-dependent solutions of the cosmological constant for a simplified step function change (with respect to  $t$ ) of the cosmological constant. We have tried this kind of time-dependent step function for the brane tension before to show the existence of a flat space to another flat space solution in case the spontaneous symmetry breaking changes the vacuum energy of the observable sector [7]. Our motivation in this Letter is to see the time-dependent curvature change. However, the closed form dS solutions are difficult to find out. In fact, there has not appeared any closed form dS solution connected to a self-tuning solution. A time-dependent curvature solution is even more difficult to obtain. Therefore, in this Letter we just try to show the existence of such time dependent solutions of the curvature and put forward a paradigm how the cosmological constant can become zero after the inflationary era. We hope that the solution of the cosmological constant problem can be realized in this way if there appears a uni-

versally accepted weak self-tuning solution in the future.

There exist two examples of the closed form weak self-tuning solutions [5,6]. In this Letter, we try to show the paradigm with the weak self-tuning solution obtained with the  $1/H^2$  term by Kim, Kyae, and Lee (KKL) [5,7].

The unified view of the inflation and vanishing cosmological constant is realized in the following way. The universe starts with the parameters which allow only the dS space solutions [8]. Let us call this dS-only region *the D-region*. In this phase there results a sufficient inflation. The inflationary potential tried in 4D field theory models is the 4D potential at the brane located at  $y = 0$  in the RS-II models. The observable sector fields are localized at the  $y = 0$  brane. When the brane tension becomes sufficiently small, but not necessarily zero, the parameters enter into the region where the flat space, de Sitter space and anti-de Sitter space solutions are allowed [7]. Let us call this flat space allowing region *the F-region*. Note that the dynamics at the brane is also affected by gravity in the bulk but for our brane fields the effect is just changing the 4D Planck mass. Then the behavior of the fields living in the brane can be studied by the well-known 4D analysis. One proposal for the brane fields is the hybrid inflation [12] where parameters at the brane can change. In particular, the brane tension  $\Lambda_1$  can change so that the parameter range for the *D-region* is changed to that of the *F-region*. Then, we may consider an initial condition after exiting from the *D-region* is a dS space solution—since we anticipate no abrupt change of the curvature of the underlying space—in the *F-region*. Since the flat space, dS space and AdS space solutions are allowed in the F-region, we look for a time-dependent solution in the F-region. In particular, we look for the curvature changing solutions. If the effective 4D curvature tends to zero as  $t \rightarrow \infty$ ,

$$\bar{\Lambda}_{\text{eff}} \propto \frac{1}{t^p} \quad (1)$$

where  $p > 2$ , then we obtain a reasonable solution for the cosmological constant problem. It is necessary to require  $p > 2$  so that the radiation dominated phase of the standard Big Bang cosmology commences after inflation. The solution for the vanishing cosmological constant problem can be realized with the condi-

<sup>1</sup> The present tiny vacuum energy of order  $(0.003 \text{ eV})^4$  is expected to be understood by another independent mechanism such as by the existence of quintessence [9].

tion (1) satisfied with weak self-tuning models. If we cannot determine (1) classically as a solution of equations of motion, a quantum mechanical probabilistic determination can be given. In this sense, Baum and Hawking’s probabilistic interpretation [13] is clearly envisioned in this 5D example.

On the other hand, if we find a strong self-tuning solution, we must also show the existence of the D-region in that model also to accommodate the inflationary era. Otherwise, it is not cosmologically successful.

## 2. Time-dependent curvature in weak self-tuning model

As a prototype example of the weak self-tuning model, we consider the KKL model [5,7]. Here, a three-form field  $A_{MNP}$  ( $M, N, P = 0, 1, \dots, 4$ ) [5,7] is introduced. In this model, it has been shown [8] that there exists a band of brane tension  $\Lambda_1$ , allowing only the dS solutions,  $|\Lambda_1| > \sqrt{-6\Lambda_b}$  ( $\equiv$  D-region), where  $\Lambda_b$  is the bulk cosmological constant. On the other hand, both 4D flat and maximally curved ( $dS_4$  and  $AdS_4$  spaces) solutions are allowed for  $|\Lambda_1| < \sqrt{-6\Lambda_b}$  ( $\equiv$  F-region) [5,7]. Thus, the KKL model has the ingredient needed for inflation in the weak self-tuning model.<sup>2</sup>

In the KKL model, let us proceed to show a time-varying 4D cosmological constant.<sup>3</sup> Because of the difficulty of obtaining a closed form for the  $t$ -dependent solution, we consider just the instantaneous transition between two different  $dS_4$  curvature scales.

The 5D action considered in the KKL model is

$$S = \int d^4x \int dy \sqrt{-g} \left( \frac{1}{2} R - \Lambda_b + \frac{2 \cdot 4!}{H^2} - \frac{\sqrt{-g_4}}{\sqrt{-g}} \Lambda_1 \delta(y) \right), \quad (2)$$

where  $g, g_4$  are 5D and 4D metric determinants,  $H^2 = H_{MNPQ} H^{MNPQ}$ , and  $\Lambda_b, \Lambda_1$  are bulk and brane cosmological constants, respectively. Henceforth, we

use the dimensionless unit for the fundamental scale,  $M = 1$ . The fundamental unit  $M$  can be reintroduced when needed. Then, the ansatz for the  $dS_4$  solution is

$$ds^2 = \beta^2(y) (-dt^2 + e^{2\sqrt{\bar{\Lambda}}t} \delta_{ij} dx^i dx^j) + b^2 dy^2, \quad (3)$$

$$H_{\mu\nu\rho\sigma} = \sqrt{-g} \epsilon_{\mu\nu\rho\sigma} f(y), \quad H_{5ijk} = 0, \quad (4)$$

where  $\bar{\Lambda}$  is the  $dS_4$  curvature, and  $b$  is a constant, and  $f^2(y) = 2A/\beta^8(y)$  with an integration constant  $A$ . For this ansatz, the 4D curvature  $\bar{\Lambda}$  is constant. For a constant  $\bar{\Lambda}$ , one can easily obtain relevant equations assuming the Einstein space

$$R_{(\text{eff})\mu\nu} = 3\bar{\Lambda} g_{(\text{eff})\mu\nu}. \quad (5)$$

But, for a time-dependent  $\bar{\Lambda}$  the calculation is much more involved.

The (55) component Einstein equation in the KKL model gives the governing equation of  $\beta$  [7,8]

$$\frac{1}{b} \beta' = \pm \sqrt{\bar{k}^2 + k^2 \beta^2 - Q^2 \beta^{10}}, \quad (6)$$

where

$$k = \sqrt{-\frac{\Lambda_b}{6}}, \quad \bar{k}^2 = \bar{\Lambda}, \quad Q = \sqrt{\frac{1}{6A}}. \quad (7)$$

Moreover, the boundary condition for the warp factor at  $y = 0$  is given by

$$\left. \frac{\beta'}{\beta} \right|_{y=0^+} = -\frac{b}{6} \Lambda_1. \quad (8)$$

Then, if we take the negative sign on the RHS of Eq. (6) for a positive  $\Lambda_1$ , the warp factor becomes  $\beta(y) = \beta(b(-|y| + c); \bar{k}^2)$  with a positive integration constant  $c$ . Even if the exact form for  $\beta$  was not obtained, it has been shown numerically that there always exists a  $dS_4$  solution in the F-region [7,8]. Note that there is the repetition of bulk horizons among which only the first horizon at  $y = c$  is causally connected to the observer at  $y = 0$  and the length scale  $bc$  can be considered as the physical size of extra dimension.

Let us consider the instantaneous change of  $b(t)$  at  $t = t_0$  as<sup>4</sup>

$$b(t) = (b_f - b_i) \theta(t - t_0) + b_i. \quad (9)$$

<sup>2</sup> Other self-tuning solutions with different forms for the action of  $H_{MNPQ} = \partial_{[M} A_{NPQ]}$  have been also considered [6].

<sup>3</sup> This work has been reported at a recent conference [14].

<sup>4</sup> This form was also considered for maintaining the flat solution with a changing brane tension in Ref. [7].

Then, the warp factor has a form of  $\beta[|y|, t] = \beta[b(t)(-|y| + c); \bar{k}^2(t)]$  with time-dependent  $\bar{k}(t)$ . For a constant brane tension  $\Lambda_1$ , the boundary condition (8) at  $y = 0$  reads the time dependence of  $\bar{k}^2(t)$  as

$$\bar{k}^2(t)\beta^{-2}(0, t) + k^2 - Q^2\beta^8(0, t) = k_1^2, \quad (10)$$

where  $k_1 = \Lambda_1/6$ . Thus,  $\bar{k}^2(t)$  is a function of  $\theta(t - t_0)$  to be determined from knowing the exact form of  $\beta$ . Anyway,  $\bar{\Lambda}(t)$  has the initial value  $\bar{\Lambda}_i$  in terms of  $b_i c$  and the final value  $\bar{\Lambda}_f$  in terms of  $b_f c$  via Eq. (10).

In fact, the brane value of  $\beta$  and  $\bar{k}^2$  consistent with the boundary condition (10) are

$$\beta_i^2(0) > \beta_f^2(0), \quad \bar{k}_i^2 > \bar{k}_f^2, \quad (11)$$

or

$$\beta_i^2(0) < \beta_f^2(0), \quad \bar{k}_i^2 < \bar{k}_f^2, \quad (12)$$

where  $\beta_i(0) \equiv \beta(0, t < t_0)$ ,  $\beta_f(0) \equiv \beta(0, t > t_0)$ , and  $\bar{k}_i^2 \equiv \bar{k}^2(t < t_0)$ ,  $\bar{k}_f^2 \equiv \bar{k}^2(t > t_0)$ . On the other hand, by integrating Eq. (6) from  $y = 0$  to the first bulk horizon  $y_h$  where  $\beta = 0$ , we get the bulk horizon size as

$$b(t)c = \int_0^{\beta(0,t)} \frac{dx}{\sqrt{\bar{k}^2(t) + k^2x^2 - Q^2x^{10}}}. \quad (13)$$

Inserting  $\bar{k}^2(t)$  of Eq. (10) into Eq. (13) in terms of  $\beta(0, t)$  and making a change of integral variable with  $x' = x/\beta(0, t)$ , we can rewrite Eq. (13) as

$$b(t)c = \int_0^1 \frac{dx'}{\sqrt{k_1^2 - k^2(1 - x'^2) + \beta^8(0, t)Q^2(1 - x'^{10})}}. \quad (14)$$

Therefore, with the inequalities of Eqs. (11) and (12), we find that the change of  $b$  is  $b_i < b_f$  for  $\bar{k}_i^2 > \bar{k}_f^2$  and  $b_i > b_f$  for  $\bar{k}_i^2 < \bar{k}_f^2$ . In other words, a larger (smaller) 4D cosmological constant gives a smaller (larger) bulk horizon size.

From the ansatz for components of  $H$  as

$$H^{\mu\nu\rho\sigma} = \frac{1}{\sqrt{-g}} \epsilon^{\mu\nu\rho\sigma 5} \partial_5 \sigma, \quad (15)$$

$$H^{5ijk} = \frac{1}{\sqrt{-g}} \epsilon^{5ijk0} \partial_0 \sigma,$$

the solution for  $\sigma$  with time-dependent  $b(t)$  is given from the static  $b$  case as

$$\sigma(y, t) = b^2(t)\sqrt{2A} \times \int dy \beta^{-4}(b(t)(-|y| + c); \bar{k}^2(t)). \quad (16)$$

Therefore, we get the time derivative of  $\sigma$  as

$$\dot{\sigma} = \sqrt{2A} \left[ b\dot{b} \left( 2 \int dy \beta^{-4} + \beta^{-4} \right) + b^2 \dot{\bar{k}} \int dy \frac{\partial \beta^{-4}}{\partial \bar{k}} \right], \quad (17)$$

where there appear  $\delta(t - t_0)$  terms due to  $\dot{b}$  and  $\dot{\bar{k}}$ . Then, we find that the field equation for  $H$  is satisfied with this time-dependent  $\sigma$  even at  $t = t_0$  as has been done in the flat case [7].

Moreover, the time derivative terms of  $\beta$ ,  $b$  and  $\bar{k}$  in  $a \equiv e^{\bar{k}t}$  in the bulk Einstein equations are cancelled by the bulk matter fluctuation around the vacuum, with  $T_0^0 = -\rho$ ,  $T_i^i = p$ ,  $T_5^5 = p_5$  and  $T_0^5$  as nonvanishing components,

$$\rho = \frac{3}{\beta^2} \left( \left( \frac{\dot{a}}{a} + \frac{\dot{\beta}}{\beta} \right)^2 + \left( \frac{\dot{a}}{a} + \frac{\dot{\beta}}{\beta} \right) \frac{\dot{b}}{b} - \bar{k}^2 \right), \quad (18)$$

$$p = -\frac{1}{\beta^2} \left( 2 \left( \frac{\dot{a}}{a} + \frac{\dot{\beta}}{\beta} \right) \dot{\bar{k}} + 3 \left( \frac{\dot{a}}{a} + \frac{\dot{\beta}}{\beta} \right)^2 + \left( \frac{\dot{b}}{b} \right)^2 + \left( \frac{\dot{b}}{b} \right)^2 + 2 \left( \frac{\dot{a}}{a} + \frac{\dot{\beta}}{\beta} \right) \left( \frac{\dot{b}}{b} - \frac{\dot{\beta}}{\beta} \right) - \frac{\dot{b}}{b} \frac{\dot{\beta}}{\beta} - 3\bar{k}^2 \right), \quad (19)$$

$$p_5 = -\frac{3}{\beta^2} \left( \left( \frac{\dot{a}}{a} + \frac{\dot{\beta}}{\beta} \right) \dot{\bar{k}} + 2 \left( \frac{\dot{a}}{a} + \frac{\dot{\beta}}{\beta} \right)^2 - \left( \frac{\dot{a}}{a} + \frac{\dot{\beta}}{\beta} \right) \frac{\dot{\beta}}{\beta} - 2\bar{k}^2 \right), \quad (20)$$

$$T_0^5 = \frac{3}{b^2} \left( \frac{\beta' \dot{b}}{\beta b} - \left( \frac{\beta'}{\beta} \right) \dot{\bar{k}} \right). \quad (21)$$

Note that the bulk matter contributes only at  $t = t_0$  with terms proportional to  $\delta(t - t_0)$ ,  $\delta^2(t - t_0)$  and  $\dot{\delta}(t - t_0)$ . Then, we find that the 5D continuity equations for the bulk matter are automatically satisfied,

$$\dot{\rho} + 3 \left( \frac{\dot{a}}{a} + \frac{\dot{\beta}}{\beta} \right) (\rho + p) + \frac{\dot{b}}{b} (\rho + p_5) = (T_0^5)' + 4 \frac{\beta'}{\beta} T_0^5, \quad (22)$$

$$p'_5 + 3\frac{\beta'}{\beta}(p_5 - p) + \frac{\beta'}{\beta}(\rho + p_5) = 0. \quad (23)$$

Now let us calculate the 4D Planck mass and the 4D effective cosmological constant. Here we regard the extra dimension up to the first horizon at  $y_h = c$ . Then, by the integration of the 5D action gives

$$\begin{aligned} S &= \int d^4x \sqrt{-g_4} \int_{-c}^c b(t) dy \\ &\quad \times \beta^4 \left( \frac{1}{2} \beta^{-2} R_4 - \frac{4}{b^2} \frac{\beta''}{\beta} - \frac{6}{b^2} \left( \frac{\beta'}{\beta} \right)^2 \right. \\ &\quad \left. - \frac{1}{3} (-\rho + 3p + p_5) - \Lambda_b + \frac{2 \cdot 4!}{H^2} \right. \\ &\quad \left. - \frac{1}{b} \Lambda_1 \delta(y) + \mathcal{L}_m \right) + S_{\text{surface}} \\ &\equiv \int d^4x \sqrt{-g_4} \left( \frac{1}{2} M_{\text{P}}^2 R_4 - 3\Lambda \right), \end{aligned} \quad (24)$$

where  $\mathcal{L}_m = -\rho$  is the Lagrangian for the bulk perfect fluid which contributes only at  $t = t_0$ . Therefore, the 4D Planck mass is given by

$$M_{\text{P}}^2(t) = \int_{-c}^c b(t) dy \beta^2 \quad (25)$$

and the 4D cosmological constant is given by

$$\begin{aligned} \Lambda(t) &= \frac{1}{3} \int_{-c}^c b(t) dy \beta^4 \left[ \frac{1}{b^2} \left( 4 \frac{\beta''}{\beta} + 6 \left( \frac{\beta'}{\beta} \right)^2 \right) \right. \\ &\quad \left. + \frac{1}{3} (2\rho + 3p + p_5) \right. \\ &\quad \left. + \Lambda_b + \frac{3\beta^8}{A} + \frac{1}{b} \Lambda_1 \delta(y) \right]. \end{aligned} \quad (26)$$

Using the Einstein equations, we can rewrite  $\Lambda(t)$  as

$$\begin{aligned} \Lambda(t) &= \frac{1}{3b} \int_{-c}^c dy (\beta^3 \beta')' \\ &\quad + \int_{-c}^c b(t) dy \\ &\quad \times \beta^4 \left( \bar{\Lambda} \beta^{-2} + \frac{1}{9} (2\rho + 3p + p_5) \right). \end{aligned} \quad (27)$$

From the fact that  $\beta$  becomes zero at  $y_h = c$ , the resulting  $\Lambda(t)$  becomes

$$\Lambda(t) = \int_{-c}^c b(t) dy \left( \bar{\Lambda} \beta^2 + \frac{1}{9} (2\rho + 3p + p_5) \beta^4 \right). \quad (28)$$

Consequently, we get the ratio of the 4D cosmological constant to the 4D Planck mass which can be interpreted as the time-dependent effective curvature,

$$\begin{aligned} \bar{\Lambda}_{\text{eff}}(t) &= \frac{\Lambda(t)}{M_{\text{P}}^2(t)} \\ &= \bar{\Lambda}(t) + \left( \int_{-c}^c dy \beta^2 \right)^{-1} \\ &\quad \times \int_{-c}^c dy \frac{1}{9} (2\rho + 3p + p_5) \beta^4. \end{aligned} \quad (29)$$

Therefore, since the bulk matter contributes only at  $t = t_0$ , the difference of the effective c.c. for  $t > t_0$  from the one for  $t < t_0$  is given just from  $\bar{\Lambda}(t)$  which in our case is  $\bar{\Lambda}_f - \bar{\Lambda}_i$ . Note that any value of  $\bar{\Lambda}_f$  is possible. Thus, we cannot determine a classical path in our limited study. Any path is classically possible.

### 3. Vanishing curvature

In this section, we point out the importance of quantum mechanical correction [13] in case classical physics cannot determine the path. In our case, Hawking's scenario is physically clear: we start from a dS space in the F-region initially and look for probabilities ending in the flat or dS space solutions. The classical path is forbidden if the initial dS vacuum is a local minimum. However, in Eq. (29) the time-dependence of the curvature is obtained. It shows the existence of a classical solution for any instantaneous change of the curvature. Therefore, at this point we have not succeeded finding out a good action. It is our hope that some clever action is found such that the time-dependence of the classical path is determined in the framework of the self-tuning solution which satisfies the condition Eq. (1).

If we interpret our classical solution literally, any small change of  $\bar{\Lambda}$  is possible. In a sense, this case is

completely the opposite to the case with a potential barrier, forbidding a change of  $\bar{\Lambda}$ . We interpret this as any change  $\bar{\Lambda}$  has an equal probability. In this sense, classical physics does not determine the path. In this scenario, after the vacuum has changed to the one with an infinitesimally different  $\bar{\Lambda}$  from the initial one, we encounter the same question again, “What will be the next vacuum?”. In this way, we go to another vacuum and to another vacuum and hence we can wait for a long time in continuously different vacua to see the importance of quantum mechanical correction.

In this sense, if classical physics cannot determine the time-dependence, we can ask a quantum mechanical probability for the transition of the curvature. Here, we adopt Hawking’s Euclidian space integral for this probability function [13], from an initial curvature  $\bar{\Lambda}_i$  to the final curvature  $\bar{\Lambda}_f$ . Since we consider the 5D theory, we must integrate with respect to  $y$  also up to  $y_h$ . In our notation, the mass dimension of the curvature  $\bar{\Lambda}$  is 2, not 4. Moreover, the Planck mass comes from integration of extra dimension with the warp factor, so it has a dependence such as  $M_P = M_P(b_f, \bar{\Lambda}_f)$  which is finite for a vanishing  $\bar{\Lambda}_f$  and an infinite  $b_f$ . Thus, the probability is estimated to be proportional to

$$\exp\left(\alpha\left[\frac{M_P^2(b_f, \bar{\Lambda}_f)}{\bar{\Lambda}_f} - \frac{M_P^2(b_i, \bar{\Lambda}_i)}{\bar{\Lambda}_i}\right]\right), \quad (30)$$

where  $\alpha$  is a  $\mathcal{O}(1)$  positive numerical number and  $M_P = M_P(b, \bar{\Lambda})$  is finite. Since this probability function is infinitely larger for  $\bar{\Lambda}_f = 0^+$  compared to any other value of the final curvature, we obtain by  $\sim 100\%$  probability the vanishing final curvature. In our interpretation of Hawking’s probability, the underlying physics seems to be clear. We have a definite initial state with  $\bar{\Lambda}_i$  and ask for the probability of obtaining the final  $\bar{\Lambda}_f$ . This probabilistic determination applies when the classical path is not determined.

If we hope to obtain an exit from inflation by a classical argument only with a self-tuning cosmological constant, we need some run-away potential for a time-dependent  $g_{55} = b(t)$  such that an increasing  $b(t)$  gives a decreasing cosmological constant. But for a successful interpretation of cosmology the time-dependence must come with a sufficiently large power  $p$  in Eq. (1).

In passing, we remark that the strong self-tuning solution needs a parameter region allowing only the D-region to accommodate inflation.

## 4. Conclusion

We have considered the inflation and the vanishing cosmological constant in a unified way in 5D self-tuning models. In this framework, the inflation is designed to originate in the region where only the de Sitter space is allowed (D-region). Then, due to the parameter change at the brane the universe enters into the region where de Sitter, anti-de Sitter and flat spaces are allowed (F-region). Thus, the initial condition at the F-region is supposed to be a de Sitter space. We considered the time-dependence of curvature in the F-region and proposed that a solution of the cosmological constant problem by equations of motion is through the curvature change to zero as  $t \rightarrow \infty$ . If the classical equations does not determine the time-dependence of the curvature, a quantum mechanical probability a la Baum and Hawking is shown to determine the curvature in the F-region as  $0^+$ . The initial condition for this probabilistic solution is clearer in our set-up.

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