which is dreadfully misleading.) But this isomorphism $\phi$ (which Prather does not make explicit) itself does not immediately yield a recipe; i.e., a residue $[x]_n$ such that $\phi[x]_n = ([a_1]_{n_1}, [a_2]_{n_2}, \ldots, [a_k]_{n_k})$ is not a solution of the simultaneous congruences! Such confusion is most unfortunate, for it serves to convince the intelligent layman that "higher mathematics" is beyond his (or her) grasp, a prejudice to which he (or she) is all too prone.

But let the last word on this very fine and worthy enterprise not be one of carping criticism. The authors have done a very difficult and very worthwhile job well; and the major virtues of this text massively outweigh its minor vices.


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If the 14th century witnessed significant advances in logic and saw a considerable interest in what may be properly called the philosophy of mathematics wherein problems about the infinitely large and small exerted a special fascination, there can be no doubt that the first half of the 13th century represents the high point of medieval achievements in theoretical mathematics, namely, in the fields of arithmetic, geometry, and algebra. These achievements were largely the work of two mathematicians, Leonardo of Pisa (Fibonacci) and Jordanus de Nemore. Although Leonardo is better known than Jordanus, the latter, whose work is the subject of this review, may well have been his equal. Because of the publication of two of his major treatises on statics in 1952 (by Marshall Clagett and E. A. Moody), Jordanus is at present more renowned for his achievements in applied, rather than theoretical, mathematics. But of the three mathematical fields mentioned above, Jordanus made major contributions to each: the *De triangulis* in geometry, the *Arithmetica* in arithmetic, and the *De numeris datis* (the treatise under review here) in algebra.

"The *De numeris datis* of Jordanus de Nemore is recognized as the first advanced algebra composed in western Europe."

With these opening words (p. 1), Barnabas Hughes begins a 53-page introduction which includes an account of the life and
works of Jordanus; a claim that Jordanus used analysis in the De numeris datis; an examination both of the possible sources that Jordanus may have used and of the status of algebra in the early 13th century as embodied in the works of al-Khwarizmi; abu-Kamil, and Fibonacci; a description of the fifteen manuscripts used for the present edition and the familial relationships between them: a description of a number of digests and excerpts of the De numeris datis; and, finally, the methodology used in establishing the Latin text and translation. In his translation Hughes has chosen to follow the Cartesian convention and use letters from the beginning of the alphabet for constants and from the end to represent unknowns. Since Jordanus used no such convention, we find, as a typical example, that when Hughes translates (in Bk. 1, prop. 1) "Given is the number \( a \) which is divided into \( w, x, y, \) and \( z \)," Jordanus has really said "Given is the number \( a \) which is divided into \( b, c, d, \) and \( e \)." Perhaps this explains why the Latin text appears first and then the translation. If text and translation faced each other, which would be more convenient, the reader would constantly, and perhaps annoyingly, be confronted with \( a, b, c, \) and \( d \) translated on the opposite page as \( w, x, y, \) and \( z \).

Besides a very useful bibliography (pp. 197-204), an index of Latin terms (pp. 205-207), and a general index (pp. 209-212), Hughes has provided a helpful "Symbolic Translation" (pp. 189-196), where in four successive columns he provides for each proposition in the De numeris datis (see pp. 46-47 for a description) "the book and theorem number ... the hypothesis in the appropriate number of equations ... the reduction of the hypothesis to the canonical form reached by Jordanus, which exposes either the unknown(s) or the penultimate step," and finally, in the last column, a reference to "the pertinent proposition(s) whereby the unknown(s) is ultimately found."

Historians of mathematics and science should welcome this admirable scholarly contribution by Barnabas Hughes. A proper edition was long overdue. Of fifteen manuscripts collated, Hughes identifies three as superior with two more proving useful in the text of the fourth and final book (pp. 45-46). The translation has been deliberately rendered freely rather than literally (see p. 125, where Hughes provides a sample translation in literal and liberal modes). In making this wise decision, Hughes has provided a highly readable, but nonetheless faithful, rendition of the Latin text.

According to Hughes, the significance of the De numeris datis lies in its use of mathematical analysis. In his Introduction to the Analytical Art of 1591, Francois Viète employed three essential steps for algebraic analysis that were presumably analogous to geometric analysis as described by Pappus of Alexandria in the early fourth century A.D. As Hughes explains it, following the explication of Jacob Klein [Klein 1968], the
The first step represents the construction of the equation; the second presents the steps leading to the canonical form, which is the solution in the form of an indeterminate equation; and the final stage is the determinate solution where specific numerical values provide an example that verifies the canonical solution. Although Viète consciously applied the Greek method of analysis to algebra, calling the latter "the analytical art," Jordanus, without any mention, or apparent awareness, of the historic role of analysis, followed virtually the same procedure some 350 years earlier.

The brevity of Hughes' discussion of alleged analysis in the *De numeris datis* is unfortunate because a number of difficulties are apparent. Hughes quotes (p. 5) the following from Pappus: "Now analysis is the passage from the thing sought, as if it were admitted, through the things which follow in order (from it), to something admitted as a result of synthesis.... If the thing admitted is possible and obtainable, what they call in mathematics given, the (problem) set forth will also be possible, and again the proof will be the reverse of analysis." In some manner, Viète transformed this and other conceptions into the threefold algebraic procedure detected by Klein and thus transformed algebra into the "analytical art." But Hughes holds that even these details of Viète's achievements were anticipated by Jordanus centuries earlier.

Further reflection on Jordanus' treatise, however, raises doubts about such a neat and tidy application of the threefold subdivision of the analytic art to Jordanus. To illustrate, let me reproduce Hughes' formulation of a typical problem in the *De numeris datis* (Bk. IV, prop. 6):

If the ratio of two numbers together with the sum of their squares is known, then each of them is known.

Let the ratio of \( x \) and \( y \) be given. Let \( d \) be the square of \( x \) and \( c \) the square of \( y \): and let \( d + c \) be known.

Now the ratio of \( d \) to \( c \) is the square of the ratio of \( x \) and \( y \). Hence, the former is known. Consequently, \( d \) and \( c \) are known.

\[
x:y = a, \quad x^2 + y^2 = b \quad (1)
\]
\[
x:y = a, \quad x^2 = d, \quad y^2 = c
\]
\[
d + c = b \quad (2)
\]
\[
d:c = x^2:y^2 = a^2 \quad (3)
\]
\[
(d/c + 1)y^2 = b \quad (4)
\]
\[
y = \left[ b/(a^2 + 1) \right]^{1/2} \quad (5)
\]
For example, let the ratio of two numbers be 2 and the sum of their squares be 500.

\[ y = \left(\frac{500}{4} + 1\right)^{1/2} \]  \hspace{1cm} (6)

Now, since the square of one number is 4 times the square of the other, it follows that 500 is 5 times the square of the other, which makes it 100. The root of this is 10 for the smaller number, and for the larger, 20.

Where in this theorem is the "something admitted as a result of synthesis," which Pappus' sense of analysis requires in the passage quoted above? When we turn to the symbolic translation of this problem (p. 194), we find column four empty (as it is for most propositions). But column four is the place which, according to Hughes, "refers the reader to the pertinent proposition(s) whereby the unknown(s) is ultimately found" (p. 47). Thus, even with Hughes' explanations, most of Joudanus' proofs do not seem to fit Pappus' conception of analysis.

They do, however, appear to accord well with the first two steps elucidated by Jacob Klein and cited above. The third step, namely, the "computation of unequivocally determinate numbers that fulfill the conditions set for the problem" (p. 6), is, however, quite problematic. The numerical example appended to the end of every theorem is treated as if it were part of a self-conscious analytic methodology developed by Jordanus himself.

But a less elegant and perhaps more plausible explanation of this practice may be found in the difference between the mathematical status of geometry and algebra during the Middle Ages. Where a Q.E.D. was sufficient to terminate a geometric proof without resort to a concrete example, an abstract algebraic theorem needed something more to convince the reader of its truth. Hence numerical examples usually accompanied algebraic theorems, as one can see in al-Khwarizmi's algebra, which could have served as a model for Jordanus. Indeed al-Khwarizmi may have inadvertently revealed the less certain status of algebra when, after discussing and exemplifying six types of algebraic equations, he explained that "now, however, it is necessary that we should demonstrate geometrically the truth of the same problems which we have explained in numbers" (translated by Louis C. Karpinski and reprinted in [Grant 1974, 110]). Thus the practice of furnishing particular examples for algebraic theorems, which probably began from a sense of the inferiority of algebra with respect to geometry, may have
become standard procedure by the time Jordanus wrote. In any event, there seems little reason to believe it formed part of any methodology of analysis. We ought not to invoke a more complicated explanation when a simpler one is readily available.

Although a more detailed consideration of these problems would have been most welcome, we must not lose sight of the splendid contribution which Professor Hughes has made to the history of medieval mathematics. Without his careful scholarship and devotion, we would still lack a major mathematical treatise of the Middle Ages.

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In a play on a name, Augustus De Morgan once remarked (in essence): "I can't Kant." Sir William Rowan Hamilton, on the other hand, could handle Kant and even incorporated some of the German philosopher's ideas into his mathematical writings. In his major statement on the foundations of algebra ("Essay on Algebra as the Science of Pure Time"), for example, Hamilton claimed to construct algebra from the Kantian intuition of pure time.

Despite Hamilton's acknowledgment of a significant debt to Kant, his use of Kantian metaphysics in science and mathematics remained an unexplored historical curiosity for a century and a quarter. Many early historians simply ignored it; a few even deplored what they say as the Kantian intrusion into science, arguing that it merely obscured some of Hamilton's major insights. Then, in the 1960s, "Hamiltonitis" struck: there was a resurgence of interest in Hamilton's mathematics and science, and in their possible Kantian roots. In one of his early articles on the influence of Naturphilosophie (particu-