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Effect of Strain-Gradient Plasticity in Engineering Fracture Assessments

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Abstract

This study implements the conventional mechanism-based strain gradient plasticity (CMSG) in the engineering fracture assessment of structural steels, to estimate both the near-tip opening displacements and the probability of brittle fracture. The CMSG theory recognizes the dependence of the material hardening on both the strain and its gradient, for plastic deformations occurring at micron or sub-micron levels, through a material length scale. The CMSG presents a more realistic description of the stress, strain and displacement field in the immediate vicinity of the crack tip, than does the classical plasticity. This study therefore examines the near-tip opening displacement, commonly used in the assessment for ductile fracture in structural steels. This study also integrates the CMSG theory in calculating the microscopic crack driving force in a cleavage fracture assessment framework, namely the Weibull stress approach. The accuracy of the scalar Weibull stress relies significantly on the gradient-dependent, near-tip stress field, which subsequently impinges on the failure probability estimated using the Weibull stresses.

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Keywords: strain gradient plasticity; crack-tip opening displacement; material length scale; indentation.

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1. Introduction

The magnitude of the stresses and displacements at the immediate vicinity of the crack tip depends significantly on the accurate description of the material constitutive relationship. Recent research efforts (Gao et al. 1999) have discovered that the hardening of the materials depends not only on the strain values but their gradients when plastic deformation occurs at micron or sub-micron levels. Previous researchers (Chen et al. 1999) have demonstrated that the near-tip stresses computed from the mechanism-based gradient (MSG) plasticity remain much higher than the classical HRR solutions (Hutchinson 1968, Rice and Rosengren 1968). Huang et al. (2004) have further proposed a conventional mechanism-based strain gradient plasticity (CMSG) model to reduce the computational requirement in solving the higher-order stresses in the original MSG theory. Swaddiwudhipong et al. (2005) have subsequently implemented the CMSG theory in C^0 type elements to describe the gradient-based plastic deformation.

Nomenclature

E	elastic modulus
K_I	stress-intensity factor
N	hardening exponent
P_f	probability of fracture
R	radius
T	T-stress
l	material length scale
m	Weibull exponent
u,v	displacements in x- and y- directions
Θ	temperature
δ	near-tip displacement
ε^p	effective plastic strain
η^p	effective plastic strain gradient
θ	angle around the crack tip
σ_w	Weibull stress

Recent research efforts have extended the CMSG theory to a wide range of engineering applications, including the fatigue and fracture analysis of metals (Stamoulis and Giannakopoulos 2012), the assessment of nano-indentation tests (Haghshenas and Klassen 2013), thermo-mechanical assessment of metals (Voyiadjis and Faghihi 2013), and cleavage fracture analysis of ferritic steels (Qian et al. 2011). The strain gradient plasticity theory prescribes the material hardening as a function of the gradient of the strain over an intrinsic material length scale. This study reports effect of the strain gradient plasticity in computing the near-tip opening displacement and the Weibull stress assessment in the probabilistic fracture assessment of Euro-steels.

2. Conventional Mechanism-Based Strain Gradient Plasticity

In the conventional mechanism-based strain gradient plasticity theory, the flow stress, σ_f, depends on both the strain and strain gradient over a material length scale, as follows,

$$\sigma_f = \sigma_y \sqrt{f^2(\varepsilon^p) + l\eta^p} \tag{1}$$

where σ_y refers to the material yield strength and $f(\varepsilon^p)$ defines the classical strain hardening law,

$$f(\varepsilon^p) = \left(1+\frac{E\varepsilon^p}{\sigma_y}\right)^N \tag{2}$$

where N defines the material hardening exponent and E represents the material Young's modulus. The effective strain gradient η_p derives from the third-order strain-gradient tensor (Huang et al. 2004).

3. Near-Tip Opening Displacement

This study utilizes the typical structural steel material, S355 of which the material length scale for this material proves to be 7 μm under two different temperatures 20 °C and 300 °C (Qian et al. 2014). Figure 1a shows the uniaxial true-stress versus the true-strain curve for these two steels measured from standard tension specimens under two temperatures. Figure 1b compares the indentation test with the numerical simulation to determine the material length scale.

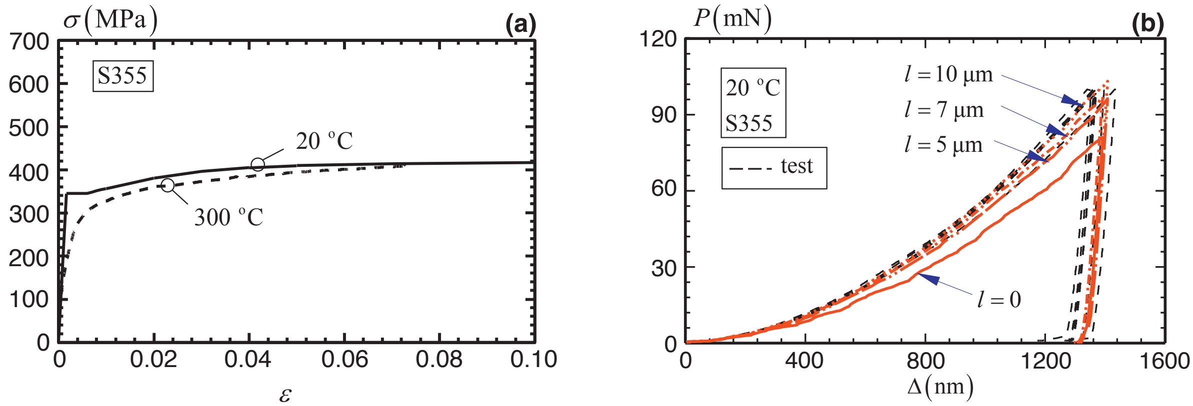

Fig. 1. Uniaxial true stress- true strain relationship for: (a) S355 steel; and (b) S690 steel; under two different temperatures.

Figure 2 illustrates the plane-strain modified boundary layer model investigated in this study. The small-scale yielding model shown in Fig. 2 contains an initial root radius, R_0, of 2.5 μm to facilitate the numerical convergence at large deformations. The smallest element size near the crack tip equals about 2 μm, following the convergence study reported by Qian et al. (2011). The numerical procedure imposes a displacement-controlled loading along the boundary of the semi-circle model as,

$$u(R,\theta) = \frac{K_I(1+\upsilon)}{E}\sqrt{\frac{R}{2\pi}}\cos\left(\frac{\theta}{2}\right)(3-4\upsilon-\cos\theta)+T\frac{1-\upsilon^2}{E}R\cos\theta \tag{3}$$

$$v(R,\theta) = \frac{K_I(1+\upsilon)}{E}\sqrt{\frac{R}{2\pi}}\sin\left(\frac{\theta}{2}\right)(3-4\upsilon-\cos\theta)-T\frac{\upsilon(1+\upsilon)}{E}R\cos\theta \tag{4}$$

There numerical procedure considers two loading conditions, namely the proportional loading and the non-proportional loading. The proportional loading applies the K_I and T-stress simultaneously to the modified boundary layer model, while the non-proportional loading applies the T-stress prior to the K_I loading. Figure 3 compares the near-tip opening displacement calculated at 2.5 μm behind the crack tip under the classical plasticity ($l = 0$) and the CMSG ($l = 7$ μm). The material properties follow that measured under room temperatures. The CMSG leads to a smaller near-opening displacement while sustaining the same K_I loading. The non-proportional loading does not create significant differences in the near-tip opening displacement compared to the proportional loading.

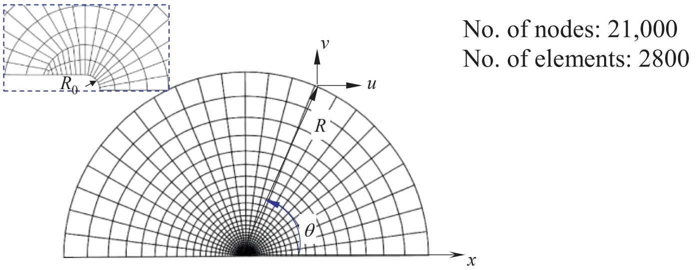

Fig. 2. A plane-strain modified boundary layer model.

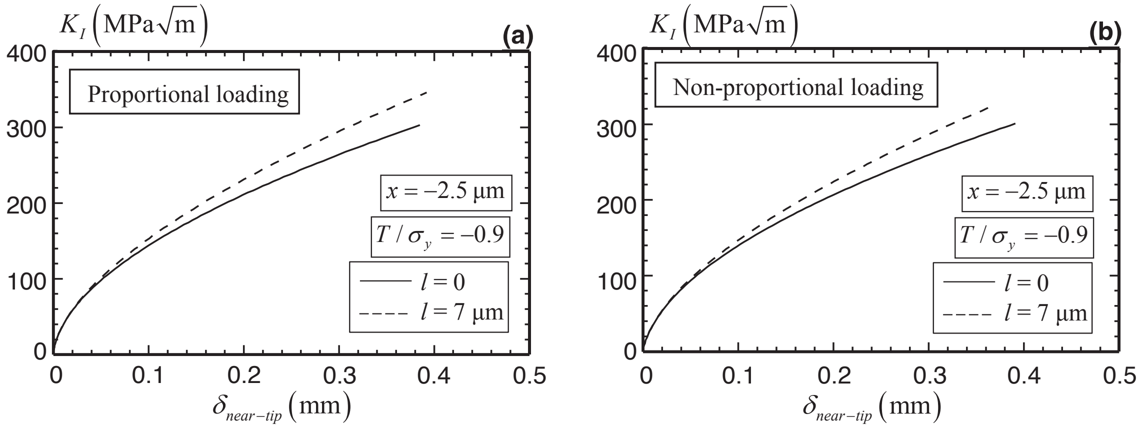

Fig. 3. Comparison of the near-tip opening displacement computed from the classical plasticity and the CMSG for the modified boundary layer model under: (a) proportional loading; and (b) non-proportional loading.

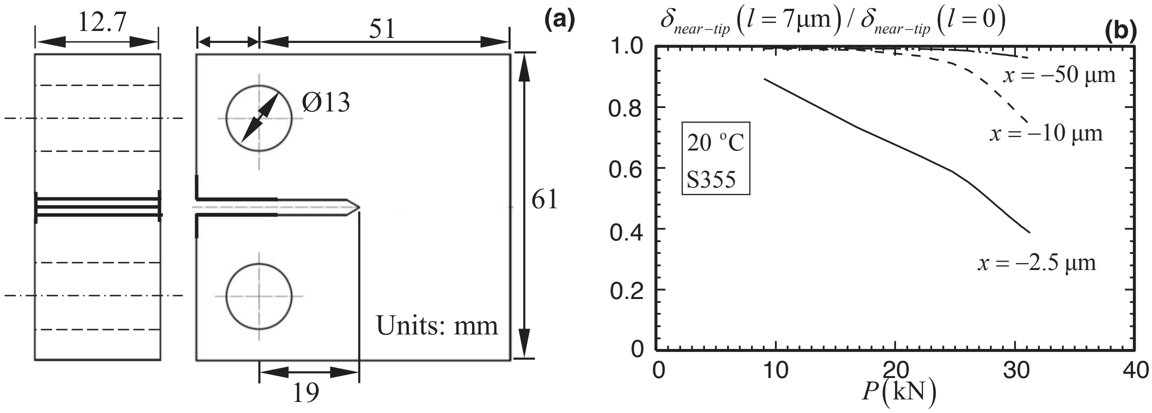

Fig. 4. (a) Configuration of a C(T) specimen; and (b) comparison of the near-tip opening displacement calculated at three different locations behind the crack tip.

This study also examines the effect of the CMSG in calculating the near-tip opening displacement for a realistic fracture specimen, namely the compact tension, C(T) specimen, as shown in Fig. 4a. The numerical procedure utilizes a 3-D model for the C(T) specimen with approximately 14,000 elements and 64,000 nodes. Figure 4b compares the near-tip opening displacement calculated at three different locations behind the crack tip from both the classical plasticity theory and the CMSG theory. The effect of the strain gradient plasticity on the calculated near-tip opening displacement dissipates for materials away from the crack tip. The effect of the strain gradient plasticity on the near-tip opening displacement becomes magnified as the plastic deformation propagates over a larger volume of material near the crack tip with increasing loads.

4. Weibull Stress Calculations

The Weibull stress framework has become a widely recognized procedure to determine the probability of cleavage fracture failure for ferritic steel materials (Wasiluk et al. 2006). Coupled with a weakest-link model with the microscopic fracture criterion, the probability of fracture depends on the scalar Weibull stress, σ_w,

$$P_f\left(\sigma_w\right)=1-\exp\left[-\left(\frac{\sigma_w^{m/4}-\sigma_{min}^{m/4}}{\sigma_u^{m/4}-\sigma_{min}^{m/4}}\right)^4\right] \tag{7}$$

where σ_u refers to the Weibull scale parameter, m stands for the Weibull shape parameter, which depends on the distribution of the microscopic flaws ahead of the crack tip, and σ_{min} denotes the threshold Weibull parameter. The microscopic crack driving force, σ_w, equals,

$$\sigma_w=\left[\frac{1}{V_0}\int_V \sigma_1^m dV\right]^{1/m} \tag{8}$$

where V refers to the volume of the fracture process zone and V_0 denotes a reference volume.

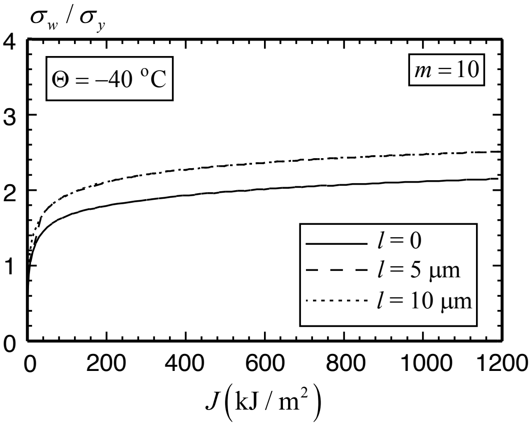

Fig. 5. Comparison of the Weibull stress values calculated based on the classical plasticity and the CMSG theory.

Figure 5 demonstrates the significant increases in the Weibull stress magnitudes calculated using the CMSG theory with two different material length scales (l = 5 μm and l = 10 μm) compared to those based on the classical plasticity (l = 0). Figure 6 compares the prediction of the brittle fracture failure using the Weibull stress calculated based on the classical plasticity and that using the CMSG for the experimental data for the Euro materials (Heerens and Hellmann 2002). The Weibull stress calculated using the CMSG theory predicts a closer estimation of the failure probability compared to that estimated using the classical plasticity for the toughness data measured at an ambient temperature of Θ = -20 °C.

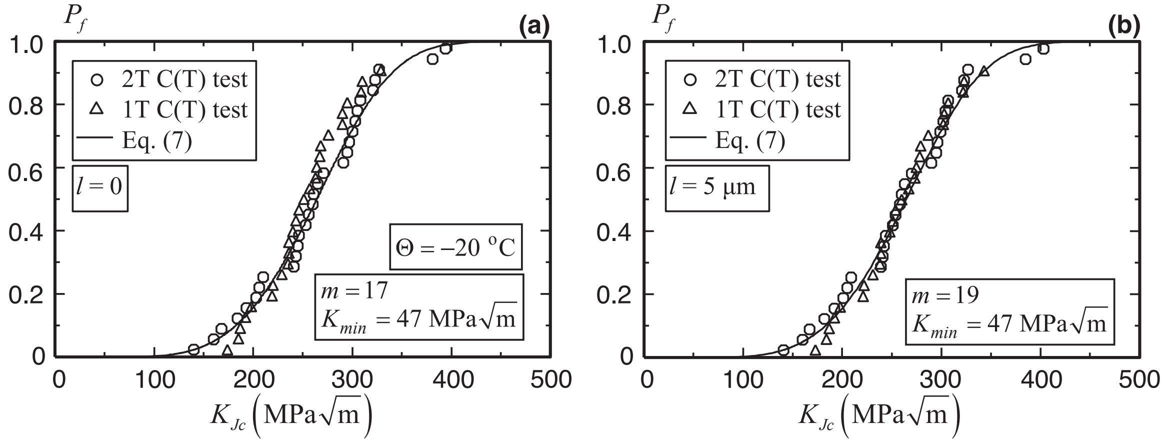

Fig. 6. Comparison of the Weibull statistical model with the experimental fracture toughness data using Weibull stress calculated based on: (a) the classical plasticity; and (b) the CMSG theory.

5. Conclusions

This study describes the engineering applications of the CMSG theory in fracture analysis. The strain gradient plasticity reduces significantly the near-tip opening displacement for materials within tens of micrometers behind the crack tip, compared to the opening displacement calculated using the classical plasticity. The Weibull stress calculated based on the CMSG theory elevates significantly of the magnitude of the Weibull stresses compared to the values calculated using the classical plasticity. The probability of brittle fracture calculated using the CMSG theory indicates a close agreement with the experimental data for the fracture data of Euro-steels at -20 °C.

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