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How universe evolves with cosmological and gravitational constants

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Abstract

With a basic varying space–time cutoff $\tilde{\ell}$, we study a regularized and quantized Einstein–Cartan gravitational field theory and its domains of ultraviolet-unstable fixed point $g_{\text{ir}} \gtrsim 0$ and ultraviolet-stable fixed point $g_{\text{uv}} \approx 4/3$ of the gravitational gauge coupling $g = (4/3)G/G_{\text{Newton}}$. Because the fundamental operators of quantum gravitational field theory are dimension-2 area operators, the cosmological constant is inversely proportional to the squared correlation length $\Lambda \propto \xi^{-2}$. The correlation length ξ characterizes an infrared size of a causally correlate patch of the universe. The cosmological constant Λ and the gravitational constant G are related by a generalized Bianchi identity. As the basic space–time cutoff $\tilde{\ell}$ decreases and approaches to the Planck length ℓ_{pl} , the universe undergoes inflation in the domain of the ultraviolet-unstable fixed point g_{ir} , then evolves to the low-redshift universe in the domain of ultraviolet-stable fixed point g_{uv} . We give the quantitative description of the low-redshift universe in the scaling-invariant domain of the ultraviolet-stable fixed point g_{uv} , and its deviation from the Λ CDM can be examined by low-redshift ($z \lesssim 1$) cosmological observations, such as supernova Type Ia.

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1. Introduction

As one of fundamental theories for interactions in Nature, the classical Einstein theory of gravity, which plays an essential role in the standard model of modern cosmology (Λ CDM), should be realized in the scaling-invariant domain of a fixed point of its quantum field theory, analogously to other renormalizable gauge field theories in the standard model of particle physics. It was suggested [1] that the quantum field theory of gravity regularized at an ultraviolet (UV) cutoff might have a non-trivial UV-stable fixed point and asymptotic safety, namely the renormalization group (RG) flows are attracted into the UV-stable fixed point with a finite number of physically renormalizable operators for the gravitational field. The evidence of such UV-stable fixed point has been found in the different short-distance regularization frameworks of dimensional continuation [2], the large N (the number of matter fields) approximation [3], lattice methods [4], the truncated exact renormalization group [5–7], and perturbation theory [8], as well as our approach to the quantum gravity [9,10] that will be discussed below.

The regularized and quantized gravitational field theory, see for example the bare action \mathcal{A}_{EC} of Eq. (4) below, must be well-defined in a space–time and gauge independent regularization (cutoff) scheme for controlling quantum field fluctuating modes at short distances. In the scaling-invariant domain of the UV-stable fixed point, the operators of \mathcal{A}_{EC} obey RG-equations, the irrelevant operators are suppressed by the cutoff, whereas the relevant operators become renormalizable operators at long distances. It is expected that the classical Einstein theory would be realized as an effective theory with two relevant and renormalizable operators

$$\mathcal{A}_{\text{EC}}^{\text{eff}} = \int \frac{d^4x}{2\kappa} \det(e)(R - 2\Lambda) + \dots, \quad \kappa \equiv 8\pi G, \quad (1)$$

where R is the Ricci scalar, Λ and G are the cosmological and gravitational constant, and “ \dots ” stands for matter fields and possible high-dimensional operators (see Ref. [11]).

It is a nature and important issue to study the asymptotic safety of quantum gravitational field and their applications to the universe evolution. Some attempts have been made [12,13]. We present our preliminary study of this issue in this article.

2. The quantum gauge theory of gravitational field and its correlation length

By the analogy of quantization of non-abelian gauge field theories, we adopted [9] the diffeomorphism and local (Lorentz) gauge-invariant regularization scheme, which is a background-independent simplicial complex with a unique space–time running cutoff $\tilde{\ell} = \pi/\tilde{\Lambda}_{\text{cutoff}}$, to regularize and quantize the Einstein–Cartan field theory for gravitational field, massless fermion and gauge fields.¹

The basic gravitational variables in the Einstein–Cartan theory constitute a pair of tetrad and spin-connection fields $[e_\mu^a(x), \omega_\mu^{ab}(x)]$, whose Dirac-matrix values are $e_\mu(x) = e_\mu^a(x)\gamma_a$ and $\omega_\mu(x) = \omega_\mu^{ab}(x)\sigma_{ab}$ with Dirac matrices γ_a , σ_{ab} and γ_5 . Analogously to the Wilson-loop defined in non-abelian gauge theories, we introduce the vertex field

$$v_{\mu\nu}(x) \equiv \gamma_5 \sigma_{ab} (e_\mu^a e_\nu^b - e_\nu^a e_\mu^b) / 2, \quad (2)$$

and define the diffeomorphism and *local* Lorentz gauge-invariant holonomy field [9]

¹ In this article, we do not explicitly show the matter section of fermion and gauge fields.

$$X_C(e, \omega) = \mathcal{P}_C \text{tr exp} \left\{ ig \oint_C v_{\mu\nu}(x) \omega^\mu(x) dx^\nu \right\}, \tag{3}$$

along the loop C on the four-dimensional Euclidean manifold, where g is the gravitational gauge coupling. We introduced the $SO(4)$ group-valued spin-connection field $U_\mu(x) = e^{ig\tilde{\ell}\omega_\mu(x)}$, and the smallest holonomy field $X_h(v, U)$ along the closed triangle path of the 2-simplex $h(x)$ in the simplicial complex with the basic space–time cutoff $\tilde{\ell}$. We regularize and quantize the Euclidean Einstein–Cartan theory (see Eqs. (120), (124) and (134) in Ref. [9]),

$$\mathcal{A}_{\text{EC}}(v, U) = \frac{1}{8g^2} \sum_{h \in \mathcal{M}} X_h(v, U) + \text{H.c.}, \tag{4}$$

where $\sum_{h \in \mathcal{M}}$ is the sum over all 2-simplices $h(x)$ of the simplicial complex, and the partition function

$$\mathcal{Z}_{\text{EC}}[\tilde{\ell}, g(\tilde{\ell})] = \int \mathcal{D}e \mathcal{D}U \exp -\mathcal{A}_{\text{EC}}. \tag{5}$$

Henceforth, we will use “the theory” as the abbreviation of the regularized and quantized the Euclidean Einstein–Cartan field theory of Eqs. (3), (4) and (5). In this theory there are only two fundamental fields: the connection field-strength $g\omega_\mu$ and dimension-2 area-operator $\tilde{\ell}^2 (e_\mu \wedge e_\nu)/2$ with two fundamental parameters: the gravitational coupling g and unique dimensional scale $\tilde{\ell}$. In the naive continuum limit of $\tilde{\ell} g \omega_\mu \ll 1$ or $\tilde{\ell} \rightarrow 0$, Eq. (4) formally reduces to Eq. (1) with the gravitational gauge coupling

$$g = (4/3)G/G_0, \tag{6}$$

the cosmological constant $\Lambda = 0$ and high-dimensional operators are naively suppressed by $\mathcal{O}[(\tilde{\ell} g \omega_\mu)^4]$. The G is the running gravitational “constant” and G_0 is its present value, i.e., the Newton constant $G_{\text{Newton}} = \ell_{\text{pl}}^2$.

Instead of taking the naive continuum limit ($\tilde{\ell} \rightarrow 0$), one should first find the critical points of phase transitions. In the neighborhood of these critical points, one should then adopt the Kadanoff–Wilson approach to find an effective action at the scale of the correlation length $\tilde{\xi}$ by integrating over short-distance modes at the space–time cutoff $\tilde{\ell}$. Namely, in the simplicial complex of the cutoff $\tilde{\ell}$, one integrates the partition function $\mathcal{Z}_{\text{EC}}[\tilde{\ell}, g(\tilde{\ell})]$ of Eq. (5) over short-distance modes at the cutoff $\tilde{\ell}$ to obtain the effective partition $\mathcal{Z}'_{\text{EC}}[\tilde{\ell}', g(\tilde{\ell}')] in the simplicial complex of the cutoff $\tilde{\ell}'$, where $\xi > \ell' > \ell$. By comparing $\mathcal{Z}'_{\text{EC}}[\tilde{\ell}', g(\tilde{\ell}')] to $\mathcal{Z}_{\text{EC}}[\tilde{\ell}, g(\tilde{\ell})]$, one could possibly obtain the running gravitational gauge coupling $g(\tilde{\ell})$, i.e., $\beta(g)$ -function, and an effective quantum field theory in the continuum space and time.$$

We integrate the partition function $\mathcal{Z}_{\text{EC}}[\tilde{\ell}, g(\tilde{\ell})]$ of Eq. (5) over the short-distance quantum degrees of freedom (e, U) of small loops of the length $\mathcal{C}_{\tilde{\ell}} \sim \tilde{\ell}$ and area $A(\mathcal{C}_{\tilde{\ell}}) \sim \tilde{\ell}^2$ so as to obtain an effective partition function $\mathcal{Z}_{\text{EC}}[\tilde{\xi}, g(\tilde{\xi})]$ in terms of larger loops of the correlation length $\mathcal{C}_{\tilde{\xi}} \sim \tilde{\xi}$ and area $A(\mathcal{C}_{\tilde{\xi}}) \sim \tilde{\xi}^2$ ($\tilde{\xi} > \tilde{\ell}$). For the reason that the holonomy fields $X_h(v, U)$ are dimension-2 area-operators in Eq. (4), the leading contribution to the effective action $\mathcal{A}_{\text{EC}}[\tilde{\xi}, g(\tilde{\xi})]$ contains a new volume term, i.e., the cosmological term

$$A(\mathcal{C}_{\tilde{\xi}})/\tilde{\xi}^2 \propto \sum_{\mathcal{C}_{\tilde{\ell}}} A(\mathcal{C}_{\tilde{\ell}})/\tilde{\xi}^2 \propto \sum_x \tilde{\ell}^4 / (\tilde{\ell}^2 \tilde{\xi}^2) \propto \sum_x \tilde{\ell}^4 \tilde{\rho}_{\tilde{\Lambda}}, \tag{7}$$

where the energy density reads

$$\tilde{\rho}_\Lambda \propto 1/(\tilde{\ell}^2 \tilde{\xi}^2), \tag{8}$$

and the related cosmological constant is given by

$$\tilde{\Lambda} \propto \tilde{\xi}^{-2}. \tag{9}$$

Note that this preliminary discussion does not demand $\tilde{\xi} \gg \tilde{\ell}$ and Eq. (9) can be $\tilde{\ell}$ -dependent.

We have to confess that the result (9) bases on the dimensional analysis. The rigorous calculations by both analytical and numerical approaches are necessarily required to obtain the cosmological term (7). This is a difficult task of the theory. Nevertheless we argue that because the fundamental operators in the theory \mathcal{A}_{EC} of Eq. (4) are the dimension-2 area operators, rather than dimension-4 density operators, the dependence of the cosmological constant on the correlation length has the form of Eq. (9).

3. The UV-unstable fixed point and its domain

In this section, we discuss the UV-unstable fixed point g_{ir} and its domain (g_{ir} -domain) of the theory of Eqs. (4) and (5). The regularized action (4) at the space–time cutoff $\tilde{\ell}$ is actually the ratio of activation energy per the fundamental area operator X_h and squared gravitational gauge coupling g^2 , the latter acts as if it is a “temperature”.

In the weak coupling limit $g \approx 0$, all quantum fields of gravity and matter are asymptotically decoupled. Moreover, quantum fields $\{e, U\}$ are frozen to the configurations of completely randomly fluctuating and uncorrelated fields at the cutoff scale $\tilde{\ell}$ and the fundamental operators X_h have activation energies at the cutoff scale $\tilde{\ell}^{-1}$. This “random” configuration is a maximal-entropy configuration. It corresponds to the $\langle X_h \rangle = 0$, where the expectational value $\langle \cdot \cdot \cdot \rangle$ is evaluated with respect to the partition function (5). The reasons are that the partition function (5), i.e., the amplitude of configurations $\{e, U\}$, is dominated by the “random” configuration $\langle X_h \rangle = 0$, and other configurations with $\langle X_h \rangle \neq 0$ are exponentially suppressed by their weight $\exp -\langle X_h \rangle / g^2$ for $g \approx 0$. This implies [10] that (i) there is a disorder phase $\langle X_h \rangle = 0$ at $g \approx 0$, where the fundamental operators X_h are suppressed by their activation energies $\sim \tilde{\ell}^{-1}$, and (ii) the correlation between two fundamental operators X_h is limited in their neighborhood and the correlation length $\tilde{\xi} \gtrsim \tilde{\ell}$.

As the coupling g deviates from $g \approx 0$ and increases for $g \gtrsim 0$, the more and more fundamental area operators X_h start to activate and correlate each other. As a result the long-ranged order (e, U) -configurations of $\langle X_h \rangle \gtrsim 0$ are excited by the probability $\sim \exp -\langle X_h \rangle / g^2$. This means that in a length scale larger than the cutoff $\tilde{\ell}$, more and more degrees of freedom of quantum fields $\{e, U\}$ are correlated and the activation energy (per X_h) decreases. This implies the occurrence of a phase transition at the critical coupling

$$g_{ir} = (4/3)(G_{ir}/G_0) \gtrsim 0, \quad G_{ir} \gtrsim 0. \tag{10}$$

On the other hand, as the coupling g increases and moves away from the critical coupling g_{ir} , the ratio $(\tilde{\xi}/\tilde{\ell})$ of the correlation length $\tilde{\xi}$ and UV-cutoff $\tilde{\ell}$ increases, indicating that the critical coupling g_{ir} should be a UV-unstable fixed point. This feature can also be seen from the perturbation result [8] of positive one-loop β -function $\beta(g_N)$, where g_N is a small dimensionless Newton constant in the effective theory of gravitational field with the UV-cutoff.

However, we know neither the exact g_{ir} -value ($g_{ir} \approx 0$), nor the measure and properties of the g_{ir} -domain. Instead of being a scaling-invariant domain, the part of g_{ir} -domain is expected

to be a crossover domain where the correlation length $\tilde{\xi}$ is not much larger than the UV-cutoff $\tilde{\ell}$ so that $\tilde{\xi}$ does not decouple from $\tilde{\ell}$ variation. Therefore, in addition to operators in the effective Einstein action (1), there are unsuppressed high-dimension operators of the regularized action (4) following the RG flows in the crossover domain. In this article, we postulate that the measure (g -range) of the crossover in the g_{ir} -domain is small, namely, the ratio $\tilde{\xi}/\tilde{\ell}$ rapidly increases for a small g -increase, corresponding to the positive β -function $\beta(g) \equiv -\tilde{\ell} \partial g(\tilde{\ell})/\partial \tilde{\ell} \approx 0^+$. This postulation implies that for some small g -values in the g_{ir} -domain, we would be allowed to make some approximate calculations by an expansion in the powers of $(\tilde{\ell}/\tilde{\xi}) < 1$. We will come back to this point in the last section. Moreover, at the critical coupling g_{ir} , the phase transition possibly occurs from the Planck phase ($g \lesssim g_{\text{ir}}$), where all fundamental interactions including gravity are unified, to the phase ($g \gtrsim g_{\text{ir}}$) of grand unification theory of gauge interactions to matter-fields.

4. The UV-stable fixed point and its domain

We briefly recall the UV-stable fixed point g_{uv} and its scaling-invariant domain (g_{uv} -domain) discussed in Ref. [10]. As the running space–time cutoff $\tilde{\ell}$ decreases, the increasing coupling g approaches to g_{uv} ($g \nearrow g_{\text{uv}}$). In the g_{uv} -domain, the large-loop holonomy fields $\langle X_{\mathcal{C}} \rangle$ of Eq. (11) are not suppressed because the smallest loops X_h undergo condensation by jointing together side by side to form surfaces whose boundaries appears as large loops $X_{\mathcal{C}}$. The correlation length ξ characterizes the size of dominantly non-vanishing holonomy fields

$$\langle X_{\mathcal{C}} \rangle = \int \mathcal{D}e \mathcal{D}U X_{\mathcal{C}}(e, U) \exp -\mathcal{A}_{\text{EC}}(e, U) \tag{11}$$

$$\sim \int \mathcal{D}A(\mathcal{C}) \exp -A(\mathcal{C})/\xi^2, \tag{12}$$

where $\int \mathcal{D}A(\mathcal{C})$ is the functional measure of all possible surface-area $A(\mathcal{C})$ bound by the large loop \mathcal{C} . As a result, the system undergoes a second-order phase transition at a critical coupling g_{uv} . As the cutoff $\tilde{\ell}$ decreases, the increasing coupling approaches to the UV-stable fixed point g_{uv} ($g \nearrow g_{\text{uv}}$). In its domain of scaling invariance, the surface-area of large loops $X_{\mathcal{C}}$ proliferates and becomes macroscopically large with the scaling invariant correlation length $\xi \gg \tilde{\ell}$ and area $\xi^2 \gg \tilde{\ell}^2$. We approximately obtained the scaling law [10]

$$\xi^2 \propto \tilde{\ell}^2 g_{\text{uv}}^2 / (g_{\text{uv}}^2 - g^2), \quad \xi \propto \tilde{\ell} / (g_{\text{uv}} - g)^{\nu/2}, \tag{13}$$

the critical coupling $g_{\text{uv}} = (4/3)G_{\text{uv}}/G_0 \sim \mathcal{O}(1)$ and critical exponent $\nu = 1$ of the UV-stable fixed point g_{uv} . Assuming that the coupling g approaches the fixed point g_{uv} , as the cutoff $\tilde{\ell}$ approaches the Planck length ($\tilde{\ell} \rightarrow \ell_{\text{pl}}$), we obtained $g_{\text{uv}} \approx (4/3)$ or $G \approx G_0$ [see Eq. (6)]. The area law (12) represents the leading contribution to the partition function (5) after integrating over vacuum–vacuum quantum fluctuations of small loops $\mathcal{C}_{\tilde{\ell}}$ at short distances $\tilde{\ell}$. The volume term $A(\mathcal{C})/\xi^2$ in Eq. (12) has the same form of the cosmological term in Eq. (7).

In this scaling-invariant domain of the UV-stable fixed point

$$g_{\text{uv}} = (4/3)(G_{\text{uv}}/G_0) \approx 4/3, \quad G_{\text{uv}} \approx G_0, \tag{14}$$

the Kadanoff–Wilson approach leads to the RG equations for physically relevant and renormalizable operators of the effective Einstein theory (1) at long distances. Applying the effective Einstein theory (1) to the present universe, we have (i) the gravitational coupling $G \approx G_0$ and the correlation length ξ being the order of the size of the present universe horizon, (ii) the operator of Ricci scalar $R \propto \xi^{-2}$ and (iii) the cosmological term

$$\Lambda \propto \xi^{-2}, \quad (15)$$

and the corresponding energy density

$$\rho_\Lambda \propto (\tilde{\ell}\xi)^{-2}, \quad (16)$$

instead of $\rho_\Lambda \propto (\tilde{\ell})^{-4}$.²

In the g_{uv} -domain, the irrelevant operators of the theory (4) are suppressed by the powers of $(\tilde{\ell}/\xi)$, and the UV-cutoff $\tilde{\ell}$ is “removed” because of $\tilde{\ell} \ll \xi$. In the continuous spacetime, as a result, the Euclidean quantum or classical Einstein–Cartan field theory (1) with finite numbers of relevant operators (e.g. Λ and R terms) is realized as an effectively renormalizable theory in the sense of asymptotic safety. Such an effective Euclidean field theory is expected to become an effective 3 + 1 field theory (1) after the Wick rotation [9,10]. The scaling-invariant correlation length ξ , equivalently the lowest lying mass scale $m = \xi^{-1}$ of quantum or classical gravitational fields, characterizes the infrared size ($L \propto \xi$) of a causally correlate patch of the universe.

Note that the g_{uv} -domain we chose the running space–time cutoff $\tilde{\ell}$ to approach the Planck length ℓ_{pl} ($\tilde{\ell} \rightarrow \ell_{\text{pl}}$), rather than take the limit $\tilde{\ell} \rightarrow 0$ to remove the cutoff for the following reasons. The first, in a renormalizable field theory in the scaling-invariant domain of a non-trivial fixed point, the removal of the cutoff $\tilde{\ell}$ means that renormalized physical operators at the scale ξ do not depend on quantum field fluctuations at the cutoff $\tilde{\ell} \ll \xi$, the irrelevant operators are suppressed by the powers of $(\tilde{\ell}/\xi)$. The second, the physical correlation length ξ and coupling g have to be determined by comparing final results with observations and experiments carried out at the known scales.

Since fixed points and their scaling-invariant domains are universal, i.e., independent of different regularization schemes at the UV-cutoff, we expect that the non-trivial UV-stable fixed point g_{uv} of gravitational gauge coupling should relate to the non-Gaussian fixed point (g_N^*, λ^*) of $g_N = \tilde{k}^2 G(\tilde{k})$, $\lambda = \Lambda(\tilde{k})/\tilde{k}^2$ for energy $\tilde{k} \rightarrow \infty$ obtained in Refs. [5–8]. However, in this article the cosmological constant appears as $\tilde{\Lambda} \propto \tilde{\xi}^{-2}$ of Eq. (9) or $\Lambda \propto \xi^{-2}$ of Eq. (15), relating to the scaling invariant correlation length of the theory (4), rather than a primary dimensional parameter following the RG equation in theory space of operators.

5. The correlation length and scaling factor of FRW universe

The correlation lengths $\tilde{\xi}$ and ξ are defined by the two-point correlation function of the theory (4) with the UV-cutoff $\tilde{\ell}$. They can be either the spatial length defined by the equal-time correlation function of two spatial points, or the time elapsing defined by the correlation function in time. In the domains of fixed points g_{ir} and g_{uv} , an effective action of the Euclidean field theory of gravitation is obtained by integrating over a time-constant slice up to the size $\tilde{\xi}$ or ξ , and integrating over an imaginary time variable τ is up to the causally-correlate time $\tilde{\xi}$ or ξ . Therefore, after the Wick rotating from the effective Euclidean field theory to the effective 3 + 1 field theory for describing the universe evolution, the correlation lengths $\tilde{\xi}$ and ξ are not only proportional to the infrared size L , but also proportional to the time elapsing t of the expanding universe. We generally denote $\tilde{\xi} = \tilde{\xi}(t)$ and $\xi = \xi(t)$ monotonically increasing their values in time. The UV-cutoff $\tilde{\ell} = \tilde{\ell}(t)$ acts as a Lagrangian variable monotonically decreasing its value in time. In

² This is reminiscent of the “vacuum-energy” density $\rho_\Lambda^{\text{vac}} \approx 1/(\ell_{\text{pl}}^2 \xi^2)$ rather than $\rho_\Lambda^{\text{vac}} \approx 1/(\ell_{\text{pl}}^4)$ as a candidate for the cosmological term, proposed by V.G. Gurzadyan and S.-S. Xue more than 14 years ago, IJMPA 18 (2003) 561–568, see astro-ph/0105245 and astro-ph/0510495.

fact, as a unique and arbitrary scale of the theory (4), the running UV-cutoff $\tilde{\ell}$ plays the roles of not only regularizing high-energy modes at short distances, but also providing a fundamental length unit measuring the correlation length $\tilde{\xi}/\tilde{\ell}$ or $\xi/\tilde{\ell}$. The $\tilde{\xi}$ or ξ turns out to be the physical unit for dimensional operators. The dynamics of the theory determine the coupling $g = g(t)$ and β -function $\beta[g(t)]$ as functions of time.

In the flat ($k = 0$) spatial section of the Robertson–Walker geometry with rotation and translation symmetries,

$$ds^2 = L^2(t)d\mathbf{x}^2 = a^2(t)dx^2, \tag{17}$$

where the spatial size $L(t)$ is the dimensional scaling factor, describing the length scale of the causally correlated universe at each time-constant slice in the four-dimensional space time. The dimensionless scaling factor is then given by $a(t) = L(t)/\tilde{\ell}(t)$, since the UV-cutoff $\tilde{\ell}(t)$ is a unique and basic scale of the theory (4). As discussed the dimensional scaling factor $L(t) \propto \tilde{\xi}(t)$ or $L(t) \propto \xi(t)$, and the dimensionless scaling factor of RW symmetry is given by

$$\tilde{a}(t) = L(t)/\tilde{\ell}(t) \propto \tilde{\xi}(t)/\tilde{\ell}(t), \tag{18}$$

$$a(t) = L(t)/\tilde{\ell}(t) \propto \xi(t)/\tilde{\ell}(t), \tag{19}$$

which is monotonically increasing in time, describes and measures the stretching of spatial manifold of space time in the universe expansion. Note that in Eq. (18) we consider the postulation that the ratio $\tilde{\xi}/\tilde{\ell}$ becomes sufficiently large in the part of g_{ir} -domain so that we approximately adopt the effective Einstein theory (1) and RW-geometry (17). We shall come back to this point in the last section.

These discussions do not preclude the possibility that the correlation length $\xi(t)$ of the theory discussed in this article describes only a part of the entirely causally correlated universe. In this case, instead of a unique scaling factor $a(t)$ for the homogeneous and isotropic RW symmetry, we will probably be led to an effective cosmological term, which is contributed from the universe inhomogeneity described by different correlation lengths at different parts of the universe.

Moreover, conventional 3 + 1 local quantum field theories with the IR-cutoff L and the UV-cutoff Λ_{cutoff} , whose entropy and energy scales extensively $S \sim L^3 \Lambda_{\text{cutoff}}^3$ and $E \sim L^3 \Lambda_{\text{cutoff}}^4$, vastly overcount degrees of freedom for a very large IR-cutoff. The reason is that these theories are described in terms of Lagrangian volume-density operators, they have extensivity of the entropy built in. Therefore, it is required to have a constrain on the IR- and UV-cutoffs, given by the up bound energy [14]

$$E \sim L^3 \Lambda_{\text{cutoff}}^4 \lesssim LM_{\text{pl}}^2, \tag{20}$$

where the maximum energy density $\rho_{\text{max}} \approx \tilde{\Lambda}_{\text{cutoff}}^4$ and the Planck mass $M_{\text{pl}} \propto 1/\ell_{\text{pl}}$. When Eq. (20) is near saturation, the maximum entropy is

$$S_{\text{max}} \approx (\pi L^2 M_{\text{pl}}^2)^{3/4}. \tag{21}$$

As discussed in Sections 3 and 4, the infrared cutoff $\tilde{\xi}$ and energy density $\tilde{\rho}_{\tilde{\Lambda}}$ (8) [or ξ and ρ_{Λ} (16)] satisfy the constrain (20)

$$\tilde{E}_{\tilde{\Lambda}} \approx \tilde{\xi}^3 \tilde{\rho}_{\tilde{\Lambda}} \propto \tilde{\xi} \tilde{\ell}^{-2} \lesssim \tilde{\xi} \ell_{\text{pl}}^{-2}, \tag{22}$$

$$E_{\Lambda} \approx \xi^3 \rho_{\Lambda} \propto \xi \tilde{\ell}^{-2} \lesssim \xi \ell_{\text{pl}}^{-2}. \tag{23}$$

They reach the saturation, when $\tilde{\ell} \rightarrow \ell_{\text{pl}}$, corresponding the maximal entropy. The reason is that the regularized Euclidean quantum gravity (4) is described in terms of area-density operators, rather than volume-density operators. In accordance with the energy density $\tilde{\rho}_{\tilde{\Lambda}}$ (8) or ρ_{Λ} (16), the entropy density $\tilde{s}_{\tilde{\Lambda}} \approx \tilde{\rho}_{\tilde{\Lambda}} \tilde{\ell} \propto 1/(\tilde{\xi}^2 \tilde{\ell})$ or $s_{\Lambda} \approx \rho_{\Lambda} \tilde{\ell} \propto 1/(\xi^2 \tilde{\ell})$. The up bound entropy is

$$\tilde{S}_{\tilde{\Lambda}} \simeq \tilde{\xi}^3 \tilde{s}_{\tilde{\Lambda}} \propto \tilde{\xi} \tilde{\ell}^{-1} \lesssim \tilde{\xi} \ell_{\text{pl}}^{-1}, \tag{24}$$

$$S_{\Lambda} \simeq \xi^3 s_{\Lambda} \propto \xi \tilde{\ell}^{-1} \lesssim \xi \ell_{\text{pl}}^{-1} \tag{25}$$

which increases as $\tilde{\xi}$ or ξ increasing and $\tilde{\ell}$ decreasing. This entropy is much smaller than the extensive entropy $\xi^3 \tilde{\ell}^{-3}$ built by Lagrangian volume-density operators. These discussions show that the energy and entropy densities representing the cosmological term in Eq. (1) are contributed from the ensemble of quantum fluctuating degrees of freedom of the space time, which is described by the simplicial complex whose fundamental element is a 2-simplex, i.e., a triangle area operator at short distances $\tilde{\ell}$.

As $\tilde{\xi}(t)$ or $\xi(t)$ increases and $\tilde{\ell}(t)$ decreases in time, the space–time entropy (24) or (25)

$$\tilde{S}_{\tilde{\Lambda}}(t) \propto \tilde{\xi}(t) \tilde{\ell}^{-1}(t) \propto \tilde{a}(t) \text{ or } S_{\Lambda}(t) \propto \xi(t) \tilde{\ell}(t)^{-1} \propto a(t) \tag{26}$$

increases in time. This shows that $\tilde{\xi}(t)$ or $\xi(t)$ increasing and $\tilde{\ell}(t)$ decreasing in time are in accordance with the entropy of expanding universe increasing in time. As will be discussed, the proliferation and increase of space–time entropy $\tilde{S}_{\tilde{\Lambda}}(t)$ and $S_{\Lambda}(t)$ drive the universe inflation and acceleration. Whereas, the space–time energy Eq. (22) or (23)

$$\tilde{E}_{\tilde{\Lambda}}(t) \propto \tilde{\xi}(t) \tilde{\ell}^{-2}(t) \propto \tilde{a}(t) \tilde{\ell}^{-1}(t) \text{ or } E_{\Lambda}(t) \propto \xi(t) \tilde{\ell}^{-2}(t) \propto a(t) \tilde{\ell}^{-1}(t), \tag{27}$$

increases in time, as $\tilde{\xi}(t)$ or $\xi(t)$ increases and $\tilde{\ell}(t)$ decreases in time. This indicates that the matter-field sector (including dark matter) and the space–time sector (space–time originated cosmological term) should interact each other, resulting in the energy exchange between the matter-field sector and space–time sector [15].

We expect that these two sectors must interact each other via microscopic particle–antiparticle (including dark matter particles) creation and annihilation. It is expected that the interaction of two sectors should be strong in the earlier universe evolution, and weak in the latter universe evolution. These issues will be studied in future. In this article, we will only use the generalized Bianchi identity (40) below for total energy–momentum conservation within the framework of classical Einstein equation for the universe evolution.

In the following sections, we shall discuss the possible scenario that due to the proliferation and increase of the space–time entropy, the universe evolves in time. At the initial time $t = t_0$ and UV-cutoff $\tilde{\ell}(t_0) = \tilde{\ell}_0$, the evolution starts with the inflation in the domain of the UV-unstable fixed point g_{ir} for the early universe. The universe evolution continues with acceleration in the scaling-invariant domain of the UV-stable fixed point g_{uv} for the present and future universe, as the basic space–time UV-cutoff $\tilde{\ell}(t)$ decreases and approaches the Planck length ℓ_{pl} . We mainly consider that the evolution of low-redshift universe in the scaling-invariant domain of the UV-stable fixed point g_{uv} . Using the scaling law (13), we solve the Einstein equation for the cosmic scaling factor $a = a[\xi, g(\tilde{\ell})]$ by taking into account the generalized Bianchi identity for the relation between the cosmological and gravitational constants $\Lambda = \Lambda[\xi, g(\tilde{\ell})]$. In the last section, we give a brief and preliminary discussion on the inflation in the domain of the UV-unstable fixed point g_{ir} for the early universe. This scenario is different from that discussed in Refs. [12], where the universe evolution follows the RG flow going away from the non-Gaussian fixed point

(g^*, λ^*) of high energies ($\tilde{k} \rightarrow \infty$) to the RG-branch of low energies ($\tilde{k} \rightarrow 0$) for the present universe.

6. The domain of UV-stable fixed point for the low-redshift universe

In the previous section, we mentioned that the $\xi(t)$ increasing and $\tilde{\ell}(t)$ decreasing in time consistently describe the universe expansion with entropy increasing in time. We further assume that the low-redshift universe has already been in the scaling-invariant domain of UV-stable fixed point g_{uv} . Namely, $\tilde{\ell}(t) \searrow \ell_{pl}$ and $g(t) \nearrow g_{uv}$, physical quantities $m(g, \tilde{\ell})$ are scaling-invariant and satisfy the renormalization-group invariant equation, i.e., $\tilde{\ell}(dm/d\tilde{\ell}) = 0$,

$$\tilde{\ell} \frac{\partial m}{\partial \tilde{\ell}} - \beta(g) \frac{\partial m}{\partial g} = 0, \tag{28}$$

where the β -function is

$$\beta(g) \equiv -\tilde{\ell} \frac{\partial g(\tilde{\ell})}{\partial \tilde{\ell}} = \mu \frac{\partial g(\mu)}{\partial \mu}, \tag{29}$$

and the UV-cutoff $\mu \equiv \pi/\tilde{\ell} = \tilde{\Lambda}_{cutoff}$. In the neighborhood of the UV-stable fixed point g_{uv} , where $\xi \gg \tilde{\ell} > \ell_{pl}$, the coupling $g(\tilde{\ell})$ and β -function can be generally expanded as a series

$$g(\tilde{\ell}) = g_{uv} + c_0(\tilde{\ell}/\xi)^{1/\nu} + \mathcal{O}[(\tilde{\ell}/\xi)^{2/\nu}], \tag{30}$$

$$\beta(g) = \beta(g_{uv}) + \beta'(g_{uv})(g - g_{uv}) + \mathcal{O}[(g - g_{uv})^2] > 0, \tag{31}$$

where $\beta(g_{uv}) = 0$, $\beta'(g_{uv}) = -1/\nu$, the coefficient $c_0 > 0$ and the critical exponent $\nu > 0$. In the neighborhood of the fixed point g_{uv} , the behavior of the β -function

$$\beta'(g_{uv})(g - g_{uv}) > 0, \quad \text{for } g < g_{uv} \tag{32}$$

indicates the fixed point g_{uv} is UV-stable, as $\tilde{\ell}(t) \searrow \ell_{pl}$ and $g(t) \nearrow g_{uv}$.

Selecting the scaling-invariant physical quantity to be the correlation length $m = \xi^{-1}$, as the solution to Eq. (28), we obtain that the correlation length ξ follows the scaling law [cf. Eq. (13)]

$$\begin{aligned} \xi(t) &= \tilde{\ell}(t) \exp + \int_{g_{uv}}^{g(t)} \frac{dg'}{\beta(g')}, \quad \text{for } g(t) < g_{uv}, \\ &= \tilde{\ell}(t) \left[\frac{c_0}{g_{uv} - g(t)} \right]^{v/2}, \end{aligned} \tag{33}$$

which represents an intrinsic scale of the theory in the scaling-invariant domain. The dimensionless scaling factor

$$a(t) \propto \xi(t)/\tilde{\ell}(t) = \left[\frac{c_0}{g_{uv} - g(t)} \right]^{v/2}, \quad \text{for } g(t) < g_{uv} \tag{34}$$

for the low-redshift universe. Using the scaling factor $a \equiv a(t)$ and gravitational coupling $g \equiv g(t)$, we will omit the time variable henceforth.

We first introduce the scaling factor and gravitational coupling values at the present time t_0 : a_0 and $g_0 = g(a_0) \approx 4/3$ [cf. Eq. (6)] for $G = G_0$. The relation (6) between gravitational couplings g and G gives $(g/g_0) = (G/G_0)$. We consider the following evolution of universe in the scaling-invariant domain:

- (i) $g \lesssim g_0 \lesssim g_{uv}$ and $a \lesssim a_0 < a_c$ in the past;
- (ii) $g_0 \lesssim g \lesssim g_{uv}$ and $a_0 < a < a_c$ in the future,

where a_c and $g_{uv} = g(a_c)$ are the scaling factor and gravitational coupling values at the future time when the fixed point g_{uv} is approached $g(a_c) = g_{uv}$.

For two different values a and a_0 of the scaling factor, the scaling law (33) yields

$$a^2 \propto \frac{(c_0)^\nu}{(g_{uv} - g)^\nu}; \quad a_0^2 \propto \frac{(c_0)^\nu}{(g_{uv} - g_0)^\nu}, \tag{35}$$

and the ratio of two equations leads to the running gravitational coupling as a function of the scaling factor

$$\left(\frac{g}{g_0}\right) = \left(\frac{g_{uv}}{g_0}\right) + \left(1 - \frac{g_{uv}}{g_0}\right) \left(\frac{a}{a_0}\right)^{-2/\nu}, \tag{36}$$

$$\approx 1 + \delta_G \ln(a/a_0) \approx \left(\frac{a}{a_0}\right)^{\delta_G}, \tag{37}$$

where in the second line, $a \lesssim a_0$, $\ln(a_0/a) \ll 1$, and the parameter $\delta_G \equiv (g_{uv}/g_0 - 1)2/\nu > 0$ is assumed to be small enough. Here, we treat g_{uv} and $\nu/2$ as parameters, hence the parameter δ_G , to be fixed by observations. They can be in principle obtained by non-perturbative calculations of β -function, including both gravitational and matter fields. The critical index $\nu/2$ actually relates to the anomalous dimension of the gravitational coupling g .

Eq. (37) shows that the low-redshift universe in the past $a(t) \lesssim a_0$, was approaching the present universe, $a(t) \nearrow a_0$ and $g(t) \nearrow g_0$. Eq. (36) shows that the present universe is evolving into the universe in the remote future $a(t)/a_0 \gg 1$ and $g(t)/g_0 > 1$. Eq. (36) also shows when the coupling $g(t)$ will be approaching the UV-fixed point, $g(t) \nearrow g_{uv}$, the universe will be approaching its “infinite” size $a_c/a_0 \rightarrow \infty$ in the “infinity” time $t_c/t_0 \rightarrow \infty$.

7. Einstein equation and generalized Bianchi identity

In the following sections, we will use the “scaling” relation (35), (36), (37) of scaling factor $a(t)$ and gravitational coupling $g(t)$ together with the classical Einstein equations and total energy–momentum conservation to study the evolution of low-redshift universe. Based on the observational facts and symmetry principle, at long distances the Einstein tensor \mathcal{G}_{ab} and the classical Einstein equation coupling to the total energy–momentum tensor T_{ab} of matter fields can be in general written as (see for example [16] page 153)

$$\mathcal{G}_{ab} = -8\pi G T_{ab}; \quad \mathcal{G}_{ab} = R_{ab} - (1/2)g_{ab}R - \Lambda g_{ab}. \tag{38}$$

The cosmological term Λg_{ab} in LHS of Einstein equation shows its gravitational origin. The covariant differentiation of Eq. (38) and the Bianchi identity

$$[\mathcal{G}_a^b]_{;b} = -8\pi [G T_a^b]_{;b}, \quad [R_a^b - (1/2)\delta_a^b R]_{;b} \equiv 0, \tag{39}$$

lead us to the generalized Bianchi identity: the conservation law of the energy–momentum of matter-field sector and space–time sector (the cosmological term)

$$(\Lambda)_{;b} g_a^b = 8\pi (G)_{;b} T_a^b + 8\pi G (T_a^b)_{;b}, \tag{40}$$

where the cosmological and gravitational “constants” are no longer constant, i.e., $(\Lambda)_{;b} = (\Lambda)_{,b}$ and $(G)_{;b} = (G)_{,b}$. Using the result of Eqs. (8) and (9) and $\tilde{a} \propto \tilde{\xi}/\tilde{\ell}$ for the early universe, or using the result of Eqs. (15) and (16) and $a \propto \xi/\tilde{\ell}$ for the low-redshift universe, we in principle completely determine the relation of cosmological constant and gravitational coupling $\tilde{\Lambda} = \tilde{\Lambda}[\tilde{a}, g(\tilde{\ell})]$ or $\Lambda = \Lambda[a, g(\tilde{\ell})]$ by the generalized Bianchi identity (40).

Suppose that the cosmological term Λg^{ab} does not exchange any mass-energy with matter fields for two possibly approximate cases: (i) the energy-density ρ_Λ is too small to energetically create many pairs of particles and antiparticles in the low-redshift universe; (ii) the energy density ρ_M of particles and antiparticles is high and approximately comparable with $\tilde{\rho}_\Lambda$ in the early universe. In these two cases, the conservation law of Eq. (40) reduces to the energy–momentum conservation of matter fields $(T_a^b)_{;b} = 0$ and relation

$$(\Lambda)_{,b} = 8\pi(G)_{,b} T^b_a = 8\pi(G)_{,b} T^{bc} g_{ca}, \quad (41)$$

which relates the variations of Λ and G in the presence of matter fields. We practically study these two cases in the rest of this article.

8. Equation of state

Matter fields in RHS of Einstein equation (38) are usually described by a perfect fluid with the energy–momentum tensor

$$T^{ab} = p_M g^{ab} + (p_M + \rho_M) U^a U^b, \quad (42)$$

and the equation of state

$$p_M = (\Gamma_M - 1)\rho_M \equiv \omega_M \rho_M, \quad (43)$$

where p_M and ρ_M are the pressure and energy-density of matter field fluid, whose four-velocity U^μ obeys the condition $g_{\mu\nu} U^\mu U^\nu = -1$. The thermal index $\Gamma_M > 1$, i.e., $\omega_M > 0$ is due to the facts that (i) the entropy of matter fields conserves, i.e., the number of particles conserves in the universe expansion; (ii) the decreasing internal energy E_M (excluding mass-energy) of matter fields, i.e., $\delta E_M = \delta(\rho_M V) \leq 0$, paying for the universe expanding its volume $\delta V > 0$. As a consequence of the particle-number and energy-conservation laws $(n_{\text{particle}} U^b)_{;b} = 0$ and $U_a (T_M^{ab})_{;b} = 0$ along a flow line in the universe expansion, we have

$$(\rho_M U^b)_{;b} + p_M U^b_{;b} = 0. \quad (44)$$

Recalling that $dV/dt = V U^b_{;b}$, where V and t are the comoving volume and time, we have along each flow line

$$p_M \delta V + \delta E_M = 0, \quad \Rightarrow \quad p_M \geq 0. \quad (45)$$

Eq. (43) and $\rho_M > 0$ lead to $\Gamma_M - 1 = \omega_M \geq 0$. The ω_M value varies from $\omega_M = 1/3$ for ultra-relativistic matter fields to $\omega_M = 0$ for extremely non-relativistic matter fields.

In order to understand the observational effects possibly due to the cosmological term Λg^{ab} in LHS of Einstein equation (38), one moves it to the RHS of Eq. (38) and substitutes it by an exotic “dark energy” fields.³ By analogy with a perfect fluid of matter fields, one in general proposes the energy–momentum tensor of the exotic “dark energy” fields to be

³ Some confusions between the cosmological term and vacuum-energy were pointed out, see Section II of Ref. [17].

$$T_D^{ab} \equiv p_D g^{ab} + (p_D + \rho_D) U^a U^b \tag{46}$$

and the equation of state $p_D = \omega_D \rho_D$. Moreover, for the case of $G \equiv G_0$ being constant the conservation law $(T_D^{ab})_{;b} = 0$ is demanded, independently of the conservation law $(T_M^{ab})_{;b} = 0$ of matter fields [18]. Here we purposely use the subscript “ D ” to indicate that as a kind of exotic matter field, this “dark energy” (46) does not necessarily relate to the cosmological term Λg_{ab} in LHS of Einstein equation (38).

In fact, the cosmological term Λg^{ab} in LHS of Einstein equation (38) clearly has its gravitational origin as discussed above. If we move the cosmological term Λg^{ab} from LHS to RHS of Einstein equation (38), and rewrite it in the form

$$\Lambda g^{ab} \equiv -8\pi G T_\Lambda^{ab}, \quad T_\Lambda^{ab} \equiv p_\Lambda g^{ab} + (p_\Lambda + \rho_\Lambda) U^a U^b. \tag{47}$$

As a result, the equation of state relating the pressure p_Λ and energy density $\rho_\Lambda = \Lambda/(8\pi G)$: $p_\Lambda = \omega_\Lambda \rho_\Lambda$, $\omega_\Lambda \equiv -1$. The conservation law $(T_\Lambda^{ab})_{;b} = 0$ follows for both $\Lambda \equiv \Lambda_0$ and $G \equiv G_0$ being constants. In the case of both G and Λ dynamically varying, the generalized conservation law Eq. (40) or (41) is fulfilled with the equation of state $p_\Lambda = \omega_\Lambda \rho_\Lambda$ and $\omega_\Lambda \equiv -1$.

Actually, our results of cosmological term Λg^{ab} and its properties presented in Sections 2 and 5 are in agreement with the equation of state $p_\Lambda = \omega_\Lambda \rho_\Lambda$ and $\omega_\Lambda \equiv -1$. The negative pressure $p_\Lambda < 0$ or $\omega_M < 0$ is due to the facts that in the universe expansion $\delta V > 0$, (i) the space–time entropy (26) increases, (ii) the space–time energy E_Λ (27) increases, i.e., $\delta E_\Lambda > 0$. This shows that the entropy and energy of stretching space–time manifold with a decreasing fundamental UV-cutoff $\tilde{\ell}$ are completely different from the entropy and internal (kinetic) energy of particles moving in the manifold.⁴

In the universe expansion, the space–time manifold in the form of the simplicial complex is stretched as the correlation length ξ increases. On the other hand, the fundamental element of the simplicial complex, i.e., a 2-simplex (a triangle area) of size $\tilde{\ell}$, becomes smaller. As a result the total entropy S_Λ (26) increases. This is an entropic repulsive force resulting from the entire universe’s statistical tendency to increase its entropy. Against the gravitational attractive force, this entropic force acts in the universe and leads to its inflation and acceleration. This entropic force is totally different from a particular underlying microscopic force of particles.

9. The universe evolution with varying G and Λ

In this section, for the case of varying gravitational and cosmological “constants”, G and Λ , we study the Einstein equation (38) with the Robertson–Walker metric (17) for the evolution of low-redshift universe. We implement the relation of the scaling factor $a(t) \propto \xi/\tilde{\ell}(t)$ of Eq. (19) and the scaling law of Eqs. (34)–(37) in the scaling-invariant domain of the UV-stable fixed point g_{uv} .

Using the energy–momentum tensor (42) with $U^0 = 1$, $U^i = 0$, $T^{00} = \rho_M$ and $T^{ii} = p_M g^{ii}$, the time–time and space–space components of the Einstein equation (38) in the Robertson–Walker metric are given by,

$$3\ddot{a} = -[4\pi G(\rho_M + 3p_M) - \Lambda]a, \tag{48}$$

$$a\ddot{a} + 2\dot{a}^2 + 2k = [4\pi G(\rho_M - p_M) + \Lambda]a^2. \tag{49}$$

⁴ A physical interpretation was given in the last section of Ref. [17].

As usually, the Hubble rate $H = \dot{a}/a$, the “time”-variable $x = a/a_0 = 1/(1+z)$ and $d(\dots)/dt = (Hx)d(\dots)/dx$. The values $z_0 = 0$ and $a/a_0 = 1$ represent the present time of the universe. The values $z > 0$ and $a/a_0 < 1$ represent the time in the past; the values $-1 < z < 0$ and $a/a_0 > 1$ represent the time in the future. Indicating the values of cosmological variables at the present time by subscript or superscript “0”, we rewrite Eqs. (48) and (49) as

$$H^2 = H_0^2 \left(\frac{G}{G_0} \right) \left(\Omega_M + \Omega_\Lambda + \Omega_k \right), \quad (50)$$

$$x \frac{dH^2}{dx} + 2H^2 = H_0^2 \left(\frac{G}{G_0} \right) \left[2\Omega_\Lambda - (1 + 3\omega_M)\Omega_M \right], \quad (51)$$

and the deceleration parameter

$$q \equiv -\frac{(\ddot{a}a)}{\dot{a}^2} = \frac{1}{2} \frac{(G/G_0)}{(H^2/H_0^2)} \left[\Omega_M(1 + 3\omega_M) - 2\Omega_\Lambda \right], \quad (52)$$

where the conventional definitions of the critical density ρ_c^0 ,

$$\rho_c^0 = 3H_0^2/(8\pi G_0), \quad \Omega_{M,\Lambda,k} = \rho_{M,\Lambda,k}/\rho_c^0, \quad (53)$$

and the curvature density $\rho_k = -k/(8\pi G a^2)$. If the gravitational and cosmological “constants” are set to equal to their values at the present time, $G = G_0$ and $\Lambda = \Lambda_0$, the above equations become usual equations in the Λ CDM. Moreover, at the present time,

$$\Omega_M^0 + \Omega_\Lambda^0 + \Omega_k^0 = 1, \quad \Omega_{M,\Lambda,k}^0 = \rho_{M,\Lambda,k}^0/\rho_c^0, \quad (54)$$

and $\Omega_k^0 = -k/(H_0^2 a_0^2)$ with the present values of energy densities $\rho_M^0, \rho_\Lambda^0 = \Lambda_0/(8\pi G_0), \rho_k^0 = -k/(8\pi G_0 a_0^2)$.

In addition, the generalized Bianchi identity (40) becomes,

$$\frac{d\Lambda}{dt} = -8\pi \left[\rho_M \frac{dG}{dt} + G \frac{d\rho_M}{dt} + G \frac{3\dot{a}}{a} (p_M + \rho_M) \right], \quad (55)$$

or

$$x \frac{d}{dx} (g\Omega_\Lambda + g\Omega_M) = -3g(1 + \omega_M)\Omega_M, \quad (56)$$

where $g = G/G_0$, $g_0 = 1$ and Eq. (54) for $x_0 = 1$. In Eq. (56), the coupling $g = g(x)$ is given by the scaling law, for example Eq. (36) if we know the β -function. However, we still need another independent equation relating Ω_Λ and Ω_M to find the solution $\Omega_\Lambda(x)$ and $\Omega_M(x)$ to Eq. (56). The relation between Ω_Λ and Ω_M should be determined by the interaction of the space–time and matter-field sectors, as already discussed after Eq. (27) in Section 5. If this relation is known, we will be possibly able to find a resolution to the coincidence problem why Ω_Λ^0 and Ω_M^0 are the same order of magnitude in the present universe.

Analogously to the $\beta(g)$ -function (29), using $(d/dt) = \dot{\tilde{\ell}}(d/d\tilde{\ell}) = \dot{\mu}(d/d\mu)$ and $\dot{\tilde{\ell}} \neq 0, \dot{\mu} \neq 0$, we define the β -functions

$$\beta_\Lambda \equiv \mu \partial \Lambda(\mu) / \partial \mu, \quad \beta_M \equiv \mu \partial \rho_M / \partial \mu, \quad \beta_a \equiv \mu \partial \ln a / \partial \mu. \quad (57)$$

Eq. (55) can be written as

$$\beta_\Lambda = -8\pi G_0 [\rho_M \beta(g) + g\beta_M + 3g\beta_a (p_M + \rho_M)]. \quad (58)$$

In the scaling-invariant domain of the UV-stable fixed point g_{uv} , as $\tilde{\ell}(t) \searrow \ell_{pl}$ and $a(t) \nearrow a_0$, $\beta_a > 0$, $\beta_M < 0$ and $\beta(g) > 0$, see Eq. (29).

We turn to study the low-redshift universe, and suppose that the cosmological term and matter fields completely decouple from each other in this epoch. The energy–momentum conservation of matter fields $(T_a^b)_{;b} = 0$, i.e.,

$$d(\rho_M a^3)/da = -3p_M a^2, \Rightarrow \Omega_M = \Omega_M^0 (a_0/a)^{3(1+\omega_M)}. \tag{59}$$

The generalized Bianchi identity (40) reduces to (41). Eq. (55) becomes

$$\frac{d\Lambda}{dt} = -8\pi\rho_M \frac{dG}{dt}, \text{ or } \frac{d(G\rho_\Lambda)}{dx} = -\rho_M \frac{dG}{dx}, \tag{60}$$

and Eq. (58) becomes

$$\beta_\Lambda = -8\pi G_0 \rho_M \beta(g) < 0. \tag{61}$$

This indicates that the cosmological constant $\Lambda(t)$ decreases, i.e., $\Lambda(t) \searrow \Lambda_0$, as $a(t) \nearrow a_0$ and $\tilde{\ell}(t) \searrow \ell_{pl}$ in the scaling-invariant domain of the UV-stable fixed point g_{uv} .

Using Eqs. (36) and (37) in the scaling-invariant domain, with the boundary condition $x_0 = 1$, $a = a_0$ at the present time, we integrate the “time”-variable $x = a/a_0$ in Eq. (60) over the region $x \leq 1$ for the past or the region $x \geq 1$ for the future, and obtain the evolution of cosmological constant

$$\frac{\Lambda}{\Lambda_0} = \frac{G\Omega_\Lambda}{G_0\Omega_\Lambda^0} = 1 - \left(\frac{\delta_\Lambda}{\kappa}\right) \left[1 - \left(\frac{a_0}{a}\right)^\kappa\right] \tag{62}$$

$$= 1 - \delta_\Lambda \ln x \sum_{n=0}^{\infty} \frac{(\kappa \ln x)^n}{(n+1)!} \approx \left(\frac{a}{a_0}\right)^{-\delta_\Lambda}, \tag{63}$$

in the second line, $\ln(a_0/a) = \ln(1+z) \ll 1$ for the low-redshift universe. The parameter $\kappa = 3(1+\omega_M) - \delta_G > 0$. The parameter δ_Λ is related to the parameter δ_G in the scaling law (37)

$$\delta_\Lambda = \delta_G (\Omega_M^0 / \Omega_\Lambda^0) > 0, \quad \delta_\Lambda < \delta_G \ll 1 \tag{64}$$

due to the generalized Bianchi identity (60). The small parameters δ_G and δ_Λ should be determined and their relation (64) should be checked by observational data of low-redshift universe. Eq. (63) shows that in the past $a \lesssim a_0$, $a \nearrow a_0$ and $\Lambda \searrow \Lambda_0$.

For the case of low redshifts $z \lesssim \mathcal{O}(1)$, $\ln(a_0/a) = \ln(1+z) \ll 1$, and the indexes $\delta_G < \delta_\Lambda \ll 1$, Eqs. (37) and (63) yield

$$(G/G_0) \approx (1+z)^{-\delta_G}, \quad (\Lambda/\Lambda_0) \approx (1+z)^{\delta_\Lambda}, \tag{65}$$

and $(\Omega_\Lambda/\Omega_\Lambda^0) \approx (1+z)^{\delta_G/\Omega_\Lambda^0}$. The correlation of gravitational and cosmological constants is

$$(\Lambda/\Lambda_0) \approx (G/G_0)^{-(\Omega_M^0/\Omega_\Lambda^0)}; \quad (\Omega_\Lambda/\Omega_\Lambda^0) \approx (G/G_0)^{-1/\Omega_\Lambda^0}, \tag{66}$$

depending on the values Ω_Λ^0 and Ω_M^0 . The ratio of Ω_M and Ω_Λ is

$$(\Omega_M/\Omega_\Lambda) \approx (\Omega_M^0/\Omega_\Lambda^0)(1+z)^{3(1+\omega_M)-\delta_G/\Omega_\Lambda^0}, \tag{67}$$

consistently with Eq. (60). However, the interaction of space–time and matter–field sectors has been neglected.

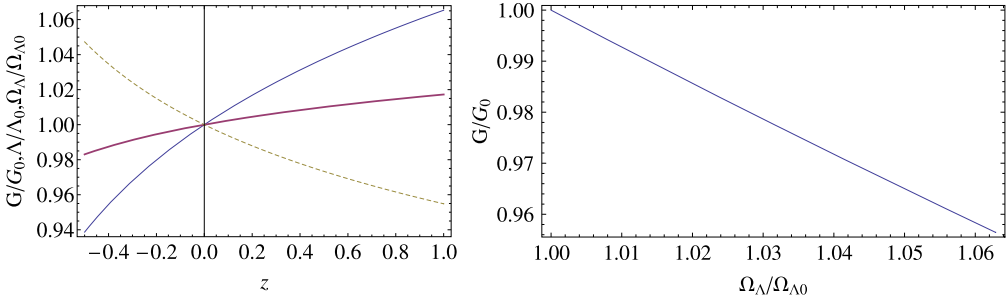


Fig. 1. As functions of the redshift z , we illustrate (i) G/G_0 (dashed line), Λ/Λ_0 (thick solid line) and $\Omega_\Lambda/\Omega_\Lambda^0$ (thin solid line) of Eq. (65); (ii) the correlation (66) of the gravitational and cosmological constants.

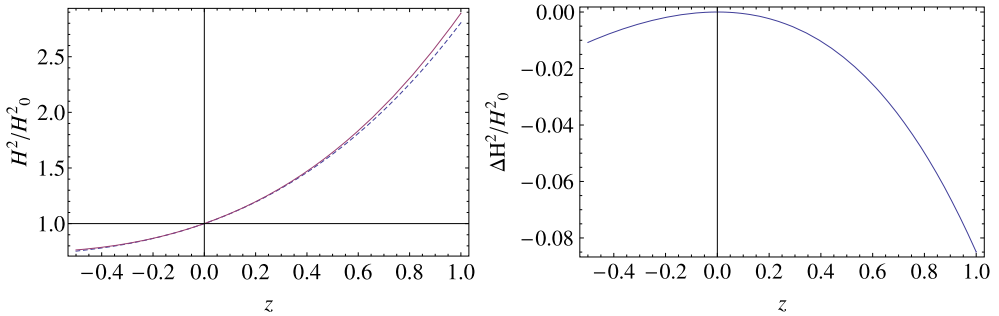


Fig. 2. We illustrate the Hubble function H^2/H_0^2 (dashed line) of Eq. (68) in contrast with its Λ CDM counterpart (solid line), and their discrepancy $\Delta H^2 \equiv H^2(\delta_G) - H^2(0)$ in terms of the redshift z .

Correspondingly, Eq. (50) is approximately replaced by

$$H^2 \approx H_0^2 \left[\Omega_M x^{+\delta_G} + \Omega_\Lambda^0 x^{-\delta_\Lambda} + \Omega_k^0 x^{-2} \right], \tag{68}$$

in contrast with the Λ CDM equations ($\delta_G = \delta_\Lambda = 0$), the same discussions apply for the deceleration parameter (52) and the luminosity distance

$$d_H(z) = \int_0^{a_0/(1+z)} dz' / (1+z') H(z'), \tag{69}$$

where $H(z)$ is given by Eq. (68). The relations (64) and (68) can be examined [19] by using the measurements of low-redshift ($z \lesssim 1$) cosmological observations, e.g. Type Ia supernovae.

In this scenario, the time evolutions of gravitational constant G/G_0 and cosmological constant Λ/Λ_0 depend on one parameter δ_G in Eq. (37). In order to gain some physical insight into these time evolutions, and their impacts on the cosmological parameters, we chose as an example $\omega_M \approx 0$, $\delta_G \approx 0.06$ and $k = 0$ for illustrations. In Fig. 1, G/G_0 , Λ/Λ_0 and $(\Omega_\Lambda/\Omega_\Lambda^0)$ are plotted in terms of the red shift $z \in [-0.5, 1]$. It is shown that G/G_0 and Λ/Λ_0 (or $\Omega_\Lambda/\Omega_\Lambda^0$) are slightly increasing and decreasing as the redshift z decreasing, and they are correlated. In Fig. 2, we plot the Hubble function of Eq. (68) to compare and contrast with its Λ CDM counterpart, and show their discrepancy increasing with the redshift z , i.e., $[H^2(z), \Delta H^2(z)]$. In Fig. 3, we plot the deceleration parameter (52) in contrast with its Λ CDM counterpart, and show their discrepancy

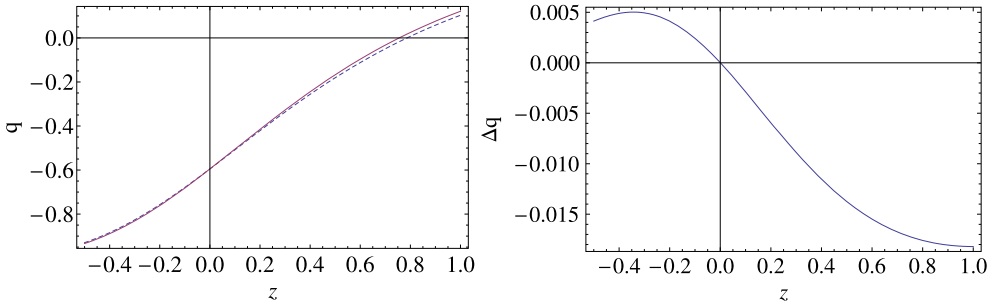


Fig. 3. We illustrate the deceleration parameter q (dashed line) of Eq. (52) in contrast with its Λ CDM counterpart (solid line), and their discrepancy (right) in terms of the redshift z .

increasing with the redshift z , i.e., $[q(z), \Delta q(z)]$. The same calculations can be done for the luminosity distance $[d_H(z), \Delta d_H(z)]$.

To end this section, it is interesting to see that in the remote future $a/a_0 \gg 1$, $z \rightarrow -1 + 0^+$, Eqs. (36) and (62) show that $G/G_0 \rightarrow G_{uv}/G_0$ and $\Lambda/\Lambda_0 \rightarrow \Lambda_c/\Lambda_0 = (1 - \delta_\Lambda/\kappa) \approx 1$ towards the UV-stable fixed point ($g \rightarrow g_{uv}$, $\tilde{\ell} \rightarrow \ell_{pl}$). Eq. (50) implies that the universe possibly undergo a very slow “inflation”

$$a \simeq a_0 \exp(H_0^2 \Omega_\Lambda^0)^{1/2} t \approx a_0 \exp(t/10^{18} \text{ s}), \tag{70}$$

approaching to $a_c/a_0 = \infty$ in a constant acceleration $q \rightarrow -1 + 0^+$.

10. The domain of UV-unstable fixed point for the universe inflation

In the last section, we give a preliminary discussion on the possibility whether the inflation can occur in the domain of UV-unstable fixed point $g_{ir} \gtrsim 0$ of Eq. (10). As discussed in Section 3, this g_{ir} -domain is a crossover domain rather than the scaling-invariant one, like g_{uv} -domain. In order to gain some physical insight into this domain, we assume that $\tilde{\xi} > \tilde{\ell}$ at least for some part of the g_{ir} -domain, and expand the coupling $g(\tilde{\ell})$ and β -function (29) as a series

$$g(\tilde{\ell}) = g_{ir} + \tilde{c}_0 (\tilde{\ell}/\tilde{\xi})^{1/\tilde{\nu}} + \mathcal{O}[(\tilde{\ell}/\tilde{\xi})^{2/\tilde{\nu}}], \quad (\tilde{\xi} > \tilde{\ell}), \tag{71}$$

$$\beta(g) = \beta(g_{ir}) + \beta'(g_{ir})(g - g_{ir}) + \mathcal{O}[(g - g_{ir})^2] > 0, \tag{72}$$

where the coefficient $\tilde{c}_0 > 0$, $\beta(g_{ir}) = 0$ and $\beta'(g_{ir}) = 1/\tilde{\nu} > 0$. In the g_{ir} -domain, the behavior of the β -function is

$$\beta'(g_{ir})(g - g_{ir}) > 0, \quad \text{for } g > g_{ir} \gtrsim 0 \tag{73}$$

indicating the fixed point g_{ir} is UV-unstable, the coupling $g(t)$ moves away from g_{ir} , as the UV-cutoff $\tilde{\ell}(t)$ decreases.

In this g_{ir} -domain, the scaling law for $\tilde{\xi} \gg \tilde{\ell}$ is not completely valid and the action is more complicate than the Einstein action (1). Nevertheless, as a preliminary study and for the reasons discussed in Section 3, we approximately use Eq. (28) for $m = \tilde{\xi}^{-1}$ and obtain

$$\begin{aligned} \tilde{\xi}(t) &\approx \tilde{\ell}(t) \exp + \int_{g_{ir}}^{g(t)} \frac{dg'}{\beta(g')}, \quad \text{for } g(t) > g_{ir}, \\ &= \tilde{\ell}(t) \left[\tilde{c}_0 [g(t) - g_{ir}] \right]^{\tilde{\nu}/2}, \end{aligned} \tag{74}$$

and the dimensionless scaling factor

$$\tilde{a}(t) \propto \tilde{\xi}(t)/\tilde{\ell}(t) = \left[\tilde{c}_0 [g(t) - g_{\text{ir}}] \right]^{\tilde{\nu}/2}, \quad \text{for } g(t) > g_{\text{ir}}. \tag{75}$$

Suppose that the initial scaling factor \tilde{a}_0 and gravitational coupling $\tilde{g}_0 = g(\tilde{a}_0) \gtrsim g_{\text{ir}}$, where a smaller subscript or superscript “0” is used to indicate quantities’ values at the initial time t_0 , differently from the normal subscript or superscript “0” indicating quantities’ values at the present time t_0 . The initial energy densities are

$$\tilde{\rho}_\Lambda^0 = \tilde{\Lambda}_0 / (8\pi \tilde{G}_0), \quad \tilde{\rho}_M^0 \approx 1/\tilde{\xi}_0^4, \quad \tilde{\rho}_k^0 = -k / (8\pi \tilde{G}_0 \tilde{\xi}_0^2), \tag{76}$$

where $\tilde{\Lambda}_0 \propto \tilde{\xi}_0^{-2}$ of Eq. (9) and $\tilde{G}_0 = \tilde{g}_0 G_0 = \tilde{g}_0 \ell_{\text{pl}}^2 \gtrsim 0$. The space–time entropy $\tilde{S}_\Lambda^0 \propto \tilde{a}_0$ and energy $\tilde{E}_\Lambda^0 \propto \tilde{a}_0 \tilde{\ell}^{-1}(t_0)$ are given by Eqs. (26) and (27). Analogously to Eq. (54), we define

$$\tilde{\rho}_c^0 = 3\tilde{H}_0^2 / (8\pi \tilde{G}_0), \quad \tilde{\Omega}_{M, \tilde{\Lambda}, k}^0 = \tilde{\rho}_{M, \tilde{\Lambda}, k}^0 / \tilde{\rho}_c^0, \tag{77}$$

where $\tilde{\rho}_c^0$ is the critical density, the Hubble rate $\tilde{H} = \dot{\tilde{a}}(t)/\tilde{a}(t)$ and $\tilde{H}_0 = \dot{\tilde{a}}(t_0)/\tilde{a}(t_0)$.

In the g_{ir} -domain, we consider the universe evolution in $g \gtrsim \tilde{g}_0 \gtrsim g_{\text{ir}}$ and $\tilde{a} > \tilde{a}_0 > \tilde{a}_{\text{ir}}$ where \tilde{a}_{ir} is the scaling factor for $g_{\text{ir}} = g(\tilde{a}_{\text{ir}})$. For two different values \tilde{a} and \tilde{a}_0 of the scaling factor, Eq. (75) yields

$$\tilde{a}^2 \propto (\tilde{c}_0)^{\tilde{\nu}} (g - g_{\text{ir}})^{\tilde{\nu}}; \quad \tilde{a}_0^2 \propto (\tilde{c}_0)^{\tilde{\nu}} (\tilde{g}_0 - g_{\text{ir}})^{\tilde{\nu}}, \tag{78}$$

and the ratio of these two equations leads to the running gravitational coupling as a function of the scaling factor

$$\left(\frac{g}{\tilde{g}_0} \right) = \left(\frac{g_{\text{ir}}}{\tilde{g}_0} \right) + \left(1 - \frac{g_{\text{ir}}}{\tilde{g}_0} \right) \left(\frac{\tilde{a}}{\tilde{a}_0} \right)^{2/\tilde{\nu}} \tag{79}$$

$$\approx \left(\frac{\tilde{a}}{\tilde{a}_0} \right)^{2/\tilde{\nu}}, \quad g_{\text{ir}} \gtrsim 0 \tag{80}$$

where $[G(t)/\tilde{G}_0] = [g(t)/\tilde{g}_0]$.

In order to gain a physical insight into the possibility whether or not the inflation can take place in the g_{ir} -domain, we assume the decoupling of the cosmological term and matter fields, and approximately adopt Eq. (41) or (60). We write Eq. (60) as

$$\frac{d\tilde{\Lambda}}{dt} = -8\pi\rho_M \frac{dG}{dt}, \quad \text{or} \quad \frac{d(G\tilde{\rho}_\Lambda)}{d\tilde{x}} = -\tilde{\rho}_M \frac{dG}{d\tilde{x}}, \tag{81}$$

where $\tilde{x}(t) = \tilde{a}(t)/\tilde{a}_0(t_0)$ and $\tilde{x}_0(t_0) = 1$. Then integrating Eq. (79) over “time”-variable $\tilde{x} \equiv \tilde{a}(t)/\tilde{a}_0$ in the region $\tilde{x} > 1$ with the boundary condition $\tilde{x}(t_0) = 1$, we obtain

$$\begin{aligned} (\tilde{\Lambda}/\tilde{\Lambda}_0) &= (G\tilde{\Omega}_\Lambda^0) / (\tilde{G}_0\tilde{\Omega}_\Lambda^0) \\ &= 1 - (\tilde{\delta}_\Lambda/\tilde{\kappa}) \left[1 - \left(\frac{\tilde{a}_0(t_0)}{\tilde{a}(t)} \right)^{\tilde{\kappa}} \right], \end{aligned} \tag{82}$$

where the parameters are

$$\tilde{\kappa} = 3(1 + \omega_M) - 1/\tilde{\nu} > 0, \quad \tilde{\delta}_\Lambda = (1/\tilde{\nu})(\tilde{\Omega}_M^0/\tilde{\Omega}_\Lambda^0) > 0, \tag{83}$$

and $\omega_M \approx 1/3$ for ultra relativistic particles.

In the initial condition, it is compellingly reasonable to assume that the matter–field energy-density $\tilde{\rho}_M^0$ is much smaller than the space–time energy-density $\tilde{\rho}_\Lambda^0$, see Eq. (76), so that $(\tilde{\Omega}_M^0/\tilde{\Omega}_\Lambda^0) = (\tilde{\rho}_M^0/\tilde{\rho}_\Lambda^0) \ll 1$. Suppose that $\tilde{\nu} \gg 1$ and $\beta'(g_{\text{ir}}) = 1/\tilde{\nu} \ll 1$, namely, the β -function is very flat in the g_{ir} -domain, see also the discussion presented in the last paragraph of Section 3. As a result, $\tilde{\delta}_\Lambda/\kappa \ll 1$ in Eq. (82), $\tilde{\Lambda} \approx \tilde{\Lambda}_0$ slowly varies. Eq. (50) becomes

$$\tilde{H}^2 = \tilde{H}_0^2(G/\tilde{G}_0)(\tilde{\Omega}_M + \tilde{\Omega}_\Lambda + \tilde{\Omega}_k) \approx (\tilde{\Lambda} - k\tilde{a}^{-2})/3, \quad (84)$$

which shows an inflationary de Sitter solution, $\tilde{\Lambda}_0 \approx 3\tilde{H}_0^2$,

$$\tilde{a}(t) \approx \tilde{a}(t_0) \exp(\tilde{\Lambda}_0/3)^{1/2}(t - t_0). \quad (85)$$

Equation (79) gives the coupling

$$\tilde{g}(t) \approx \tilde{g}(t_0) \exp(2/\tilde{\nu})(\tilde{\Lambda}_0/3)^{1/2}(t - t_0), \quad (86)$$

which demands $\beta'(g_{\text{ir}}) = 1/\tilde{\nu} \ll 1$ to be consistent with small variation of the coupling g (79). The space–time entropy $\tilde{S}_\Lambda \propto \tilde{a}(t)$ and energy $\tilde{E}_\Lambda \propto \tilde{a}(t)\ell^{-1}(t)$ of Eqs. (26) and (27) increase.

The proliferation and increase of space–time entropy \tilde{S}_Λ drive the universe inflation (85). Moreover, the entropy of particles and antiparticles increases by converting the space–time energy-density $\tilde{\rho}_\Lambda$ to the matter–field energy-density $\tilde{\rho}_M$. In the initial inflation epoch the space–time energy-density $\tilde{\rho}_\Lambda$ is much more larger and energetic for the production of particles and antiparticles.

Similarly to the relation (54) and $\rho_{\text{total}}^0 = \rho_c^0$ in the present time t_0 , Eqs. (77) and (84) give $(\tilde{\Omega}_M^0 + \tilde{\Omega}_\Lambda^0 + \tilde{\Omega}_k^0) = 1$, indicating the total energy-density $\tilde{\rho}_{\text{total}}^0 = \tilde{\rho}_c^0$ in the initial time t_0 , although the initial values $\tilde{a}_0, \tilde{\xi}_0, \tilde{g}_0$ and densities (76) are unknown. Needless to say, in order to understand the quantitative properties of entire inflationary process, for example, whether the inflation ends with the e -folding factor $\ln[\tilde{a}(t)/\tilde{a}_0(t_0)] \approx 60$, we have to study the rate of the energy-conversion and particle-creation from the space–time energy-density $\tilde{\rho}_\Lambda$ to the matter–field energy-density $\tilde{\rho}_M$, and adopt the generalized Bianchi identity (40) or (55). The initial values $\tilde{a}_0, \tilde{\xi}_0, \tilde{g}_0$ and densities (76), as well as the value $1/\tilde{\nu}$ could be determined by either theories or observations.

In addition to the problem previously discussed, there are many other open questions. We mention a few more examples. Actually, the correlation length $\xi(t)$ is much larger than the UV-cutoff $\tilde{\ell}(t)$ in the entire universe evolution from the g_{ir} -domain to the g_{uv} -domain. After the crossover domain, we speculate only one scaling law $a(t) \propto [\xi(t)/\tilde{\ell}(t)] = \xi[g(t)]$, provided the full β -function $\beta(g)$ is known in the entire range of the coupling $g(t) \in (g_{\text{ir}}, g_{\text{uv}}]$. Moreover, induced by very massive torsion fields, the four-fermion interactions depend on the gravitational gauge coupling g in the Einstein–Cartan theory [20]. How these self-interactions of matter fields affect on the universe evolution. These unsolved issues deserve studies in future.

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