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Regular black holes in UV self-complete quantum gravity

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ABSTRACT

In this Letter we investigate the role of regular (curvature singularity-free) black holes in the framework of UV self-complete quantum gravity. The existence of a minimal length, shielding the trans-Planckian regime to any physical probe, is self-consistently included into the black hole probe itself. In this way we obtain to slightly shift the barrier below the Planck length, with the UV self-complete scenario self-consistently confirmed.

1. Introduction

The nature of space and time at the Planck scale is a long-standing argument of debate. Fluctuations in both geometry and topology are expected to become so violent to disrupt the very fabric of the spacetime manifold. The term "spacetime foam" is frequently used to portray this kind of gravitational quantum vacuum [1]. Any candidate theory of quantum gravity has to address this problem and provide some information about trans-Planckian physics, whatever it is. Even if String Theory is not yet a fully accomplished Unified Theory of Everything, it provides to day the most powerful framework to address quantum gravity problems. The price to pay for that is to dismiss the idea of "point-like" building blocks of matter in favor of one-dimensional, Planck size, fundamental objects. Unfortunately, the extended nature of (super) strings makes them unable to probe the trans-Planckian regime: as opposed to hypothetical point-like objects, increasing the energy is not enough to make them shorter and shorter; as more and more excitation modes are switched-on, the string elongates [2] bouncing back to a long-distance regime.[1] A quite different approach to the problem has been recently proposed by Dvali and collaborators in a series of papers [5,6], where String Theory is not explicitly involved. We shall comment this feature in the conclusions.

The general wisdom says that there is no self-consistent way to quantize gravity in the framework of "point-like" quantum field theory because in the foamy Planckian phase quantum fluctuations are out of control, and predictive power is lost even in supergravity models. Against this background, Dvali proposed a clever way to by-pass such a problem, by pointing-out the existence of a "black hole barrier" shielding the trans-Planckian regime to any physical probe. In a nutshell, gravity regularizes itself because of its unique ability to collapse high enough energy concentrations into black holes, with linear dimension increasing with energy, and not vice-versa. Thus, any point-like probe turns into a black hole when boosted to a "critical energy" $-s_* = \hbar c/2G_N$. Any further mass-energy increase reverses the Lorentz contraction in a sort of *Schwarzschild dilation* of the gravitational radius $R_s = 2G_N\sqrt{-s}/c^2$. The effect of gravity is to *shield* the deep-UV region behind the curtain of an event horizon (see Fig. 1).

The far reaching conclusion of this simple reasoning is that, contrary to any current wisdom, the quantum gravity trans-Planckian regime could be dominated by "classical", infra-red, field configurations. This result is reminiscent of T-Duality in String Theory, where a stringy probe cannot distinguish a length scale L from a length α'/L. Thus, $\sqrt{\alpha'}$ is the ultimate accessible distance to a stringy object. Keeping this in mind, a unique, and often overlooked, black hole property is to provide an ideal bridge between micro and macro physics [7–9]. Indeed, whatever the radius of the horizon, a black hole is always a "point-like" object, in the sense that the whole mass is packed inside an arbitrarily small region around the origin (classically the mass is collapsed into a single point). A black hole can be seen as a self-gravitating particle, and the infinite self-force the field applies to its own point-like source

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[1] For a different approach to a string induced minimal length see [3,4].

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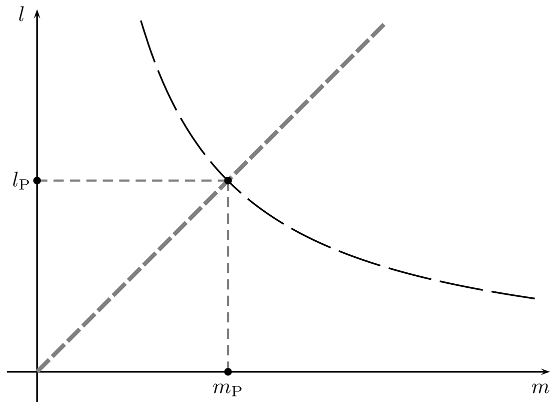

Fig. 1. The hyperbola represents the Compton wavelength of a "particle" of mass m. The straight line shows the linear increase of the Schwarzschild radius with respect to the mass of a black hole. The intersection between the two curves defines the Planck length, l_P, and the Planck mass, m_P.

translates into the presence of a "curvature singularity". From this point of view, a 1 gr mass black hole can be seen either as an infra-red field configuration of radius $10^5 l_P$, or a trans-Planckian test-particle with energy $10^5 E_P$.

Then, in the scenario briefly discussed above, the *minimal*, physically meaningful, length turns out to be the *Planck length* l_P, which is defined as the *cross-over* point between the Schwarzschild radius of a mass m black hole and the Compton wavelength of a particle with the same mass:

$$l_P \equiv (R_s)_{min} = \sqrt{\frac{2\hbar G_N}{c^3}}. \tag{1}$$

Any distance $d < l_P$ has no physical meaning being shielded by the horizon. Fig. 1 is a portrait of an elementary objects "*phase space*". Light objects with $m < m_P$ are what we colloquially call "particles". Their linear dimension is defined by the Compton wavelength encoding the quantum mechanical nature of a microscopic object. On the other hand, heavy objects with $m > m_P$ are gravity-dominated and they look like *classical* black holes of linear size R_s. The Planck scale represents the *critical point* where Quantum Mechanics intersects General Relativity and the Compton wavelength is "swallowed" by a "classical" black holes (the term "classical" means "solution of the Einstein equations", and is not referring to the actual size of the object).[2] The existence of a black hole barrier follows as a necessary consequence from the purely attractive character of gravity and is instrumental to the realization of the UV self-complete scenario. A possible critical remark to this scenario that has been raised in the literature, is that a Planckian black hole is highly unstable with respect to Hawking evaporation. Thus, it is conceivable that a Planckian probe will disintegrate, soon after its formation, into a burst of thermal radiation. While emitting Hawking radiation the black hole will shrink to smaller and smaller size. Then, in principle, a decaying black hole *can* probe distances smaller than the Planck length, at least during the final phase of its evaporation process. More precisely, one should say that the structure of the probe in these extreme conditions is unknown: maybe a transition to some excited string state could occur [12], and the whole self-completeness argument would require to be adapted to this different situation.

The purpose of this Letter is to provide an answer to this criticism. The root of the problem can be traced back to the fact that in the Schwarzschild geometry there is no lower bound to the radius of the black hole horizon during the evaporation process. This is, again, a consequence of the possibility to consider the source of the field concentrated into an arbitrary small volume. On the other hand, if a minimal distance exists point-like sources have no physical meaning. From standard quantum mechanics we know that "point-like", classical, particles can at most be represented by optimal localization, or minimal uncertainty, position states. In a recent series of papers [13–18] we introduced this idea in General Relativity and found black hole solutions generated by a minimal width Gaussian distribution of matter. For the reader's convenience, we list below the main features of these objects.

i) They are curvature singularity free. This is a straightforward consequence of spreading the source over a finite volume. The arbitrary large curvature region close to the origin is turned into a de Sitter vacuum core with finite curvature.
ii) They admit an extremal, degenerate, configuration even in the neutral, non-rotating case. The presence of both an inner (Cauchy) horizon and an outer (Killing) horizon is a characteristic feature of this regular solutions.[3]
iii) The Hawking temperature is bounded from above and vanishes for the extremal configuration. The heat capacity is positive in the small black hole phase, making these solutions thermodynamically stable.
iv) A detailed investigation of the quantum properties of these objects, in relation to production and decay at LHC can be found in [19–21].

In what follows we will see how it is possible to make *self-consistent* the UV self-completeness proposal by taking into account the existence of a minimal length in the black hole probe itself. The advantage of this approach is that the minimal size black hole is a zero Hawking temperature, stable, extremal configuration, which will evade the above mentioned criticism.

2. Regular Schwarzschild black hole

Black hole type solutions of the Einstein field equations are plagued by the presence of curvature singularities, where tidal forces arbitrarily blow up. From a physical point of view, no measurable quantity can become infinite. Indeed, the presence of a singularity cannot be seen as a "physical" effect, rather it sounds like a warning that we are pushing a classical theory, i.e. General Relativity, where it stops to be effective and loses its predictive power. A possible cure to the "singularity sickness" is suggested by non-commutative geometry, where manifold fluctuations make it impossible to measure lengths shorter than a minimal length $\sqrt{\theta}$. The parameter θ is a measure of how much non-commuting coordinates deviate from their classical, commuting, counterparts. In a series of papers we introduced a phenomenological approach where the key feature of non-commuting geometry, i.e. the existence of a minimal length, is encoded into Einstein equations by re-modelling matter sources in terms of minimal width Gaussian distributions. For more details we refer the reader to the original

[2] To appreciate the specific meaning of "classical" in this framework, it may be useful to recall an analogy with Yang–Mills "instantons". Also in this case one talks of "classical solutions", even if such field configurations are confined to microscopic scale. With this analogy in mind, one can say that instantons play an essential role in non-perturbative Yang–Mills theory, and black holes control gravity in the trans-Planckian regime. In both cases, the dynamics of the theory is described in terms of classical field configurations instead of particle-like excitations [10,11].

[3] The stability of the Cauchy horizon is an open issue which is currently under investigation [31,32]. In any case, this discussion is not relevant to the problem we are discussing in this work.

papers [22–25]. We would only like to comment about the sensitivity of the solution with respect to the choice of the source. Regular black holes can be obtained both by coupling gravity to non-linear electrodynamics [26–28], and by engineering appropriate sources, e.g. [29,30] (for a general review about this topic, see [22]). In our case, the Gaussian form of the matter distribution is not a choice but an exact result recovered from the underlying non-commutative geometry. Strictly speaking, we could extend the Gaussian distribution to a Maxwell-like form, i.e. $\rho_G(r) \to r^n \rho_G(r)$, $n \geqslant 0$ integer, without spoiling the regularity of the black hole solution. The physical difference is clear, we replace a massive droplet source with an hollow shell of matter. From the geometrical side, the inner de Sitter core will be replaced by a flat Minkowski central region. All the other appealing features of the black hole solution are preserved.

The simplest solution of the modified Einstein equations is the so-called "non-commutative" Schwarzschild metric

$$\begin{aligned} ds^2 &= -f(r)\,dt^2 + f^{-1}(r)\,dr^2 + r^2\,d\Omega_2^2, \\ d\Omega_2^2 &\equiv d\theta^2 + \sin^2\theta\,d\phi^2 \quad (c=1,\ G_N=1), \\ f(r) &= 1 - \frac{4M}{\sqrt{\pi}r}\gamma\left(\frac{3}{2},\frac{r^2}{4\theta}\right), \\ \gamma\left(\frac{3}{2},\frac{r^2}{4\theta}\right) &= \int_0^{r^2/4\theta} dt\, t^{1/2} e^{-t}, \end{aligned} \tag{2}$$

where, $\sqrt{\theta}$ is the width of the Gaussian mass-energy distribution of the source. Expanding $f(r)$ near the origin, one sees that the central curvature singularity is replaced by a de Sitter vacuum core characterized by an effective cosmological constant $\Lambda_{\mathrm{eff.}} = M/(\sqrt{\pi}\theta^{3/2})$. The line element (2) smoothly interpolates between the de Sitter geometry at short distance, i.e. $r \ll \sqrt{\theta}$, and the Schwarzschild metric at large distance $r \gg \sqrt{\theta}$. Some cautionary remark about the short distance limit is due. This is the range where our effective description breaks down and the very concept of smooth spacetime loses its meaning. However, through the looking glass of gravity a non-commutative fluctuating manifold is filtered into a non-trivial "vacuum" of de Sitter type.

In the intermediate distance range non-standard black hole configurations can be realized above a certain mass threshold. Let us consider the zeros of the metric function, $f(r_H) = 0$, and plot the total mass-energy M as a function of the Schwarzschild radius r_H (see Fig. 2)

$$M = \frac{\sqrt{\pi}}{4}\frac{r_H}{\gamma(3/2, r_H^2/4\theta)}. \tag{3}$$

Even a neutral, non-spinning, object of mass $M_1 > M_0$ is a black hole with an outer (Schwarzschild) horizon of radius r_1^+ and an inner (Cauchy) horizon of radius r_1^-. As the mass decreases towards M_0 the two horizons merge into a single, degenerate, null surface, with $r_H = r_0$. This is an *extremal* black hole. For lower masses there are no more horizons and the object is a regular, particle-like, lump of matter. The presence of two horizons and the existence of an extremal configuration make the thermodynamic behavior of this uncharged, non-spinning, black hole quite similar to the thermal evolution of a standard, charged, Reissner–Nordstrom black hole. The Hawking temperature is bounded from above and vanishing as the extremal configuration is reached. This is the so-called "scram-phase" [23] leading to a stable massive remnant in the form of a degenerate extremal black hole. This endpoint configuration is the most relevant one in the framework of self-complete quantum gravity as it provides us the smallest probe we can think of. Let us give a closer look to the extremal black hole represented by the minimum of the curve in Fig. 2. The minimum is characterized by

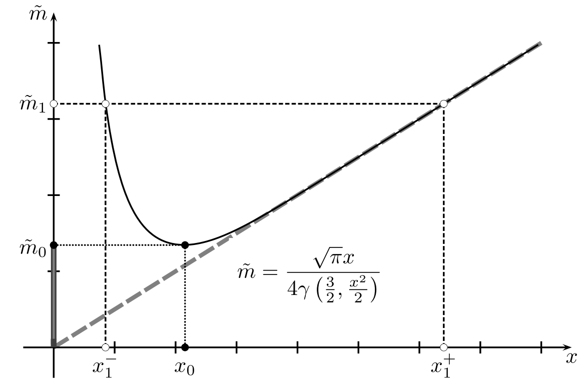

Fig. 2. This is the plot of Eq. (3) in terms of rescaled variables $x \equiv r_H/\sqrt{2\theta}$, $\tilde{m} \equiv MG_N/\sqrt{2\theta}$. $\tilde{m}_0$, x_0 are the mass and radius of the "extremal" black hole configuration. If $\tilde{m} > \tilde{m}_0$ we have a non-degenerate black hole with event horizon of radius x_+ and inner Cauchy horizon of radius x_-. For $\tilde{m} < \tilde{m}_0$ we have a particle-like object with no horizons.

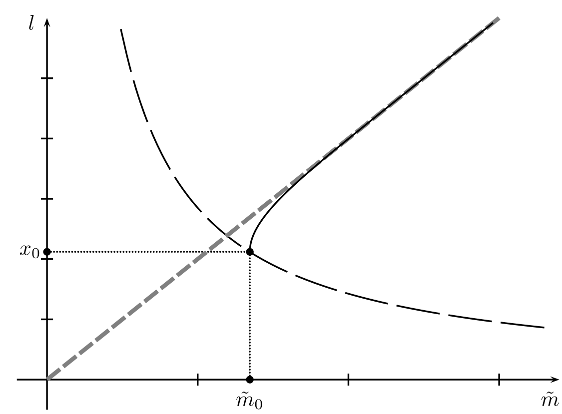

Fig. 3. The dashed curve is the rescaled Compton hyperbola for the critical value $\sqrt{\theta} \simeq l_{\mathrm{P}}/3.393$. The continuous curve is the outer horizon branch of (3).

$$f(r_0) = 0, \tag{4}$$

$$\left(\frac{dM}{dr_H}\right)_{r_H=r_0} = 0 \quad\Rightarrow\quad r_0^3 = 4\theta^{3/2}\gamma\left(\frac{3}{2},\frac{r_0^2}{4\theta}\right)e^{r_0^2/4\theta}. \tag{5}$$

Notice that in terms of the length unit $\sqrt{\theta}$ the horizon curve becomes θ-independent while the Compton hyperbola can be shifted by varying the value of θ:

$$\frac{l_C}{\sqrt{2\theta}} = \frac{\hbar}{2\theta(m/\sqrt{2\theta})} = \frac{l_{\mathrm{P}}^2}{4\theta}\cdot\frac{1}{m/\sqrt{2\theta}} = \frac{l_{\mathrm{P}}^2}{4\theta}\cdot\frac{1}{\tilde{m}}.$$

Thus, having the rescaled horizon curve fixed and the rescaled Compton wavelength freely adjustable, it is consistent to look for the value of θ allowing an intersection point between the two curves for the values of mass and event horizon radius that define an extremal configuration. This peculiar crossing point is obtained for $\sqrt{\theta} \simeq l_{\mathrm{P}}/3.393$ (Fig. 3). A relationship between the minimal length r_0 and the Planck length not involving M_0 can then be obtained combining (4), (5) and $r_0 = l_{\mathrm{P}}^2/(2M_0)$

$$r_0^3 = \frac{\sqrt{\pi}r_0}{M_0}\theta^{3/2}e^{r_0^2/(4\theta)} = 2\sqrt{\pi}\left(\frac{\sqrt{\theta}}{l_{\mathrm{P}}}\right)^3 r_0^2\, e^{r_0^2/(4\theta)} l_{\mathrm{P}} \tag{6}$$

so that

$$L_* \stackrel{\text{def.}}{=} r_0 \simeq 2\sqrt{\pi}\,\frac{9.8138}{(3.393)^3} l_{\text{P}} \simeq 0.891 l_{\text{P}} \quad \text{and} \quad M_* \stackrel{\text{def.}}{=} M_0 = \frac{\hbar}{L_*}$$

are the new values for the "Planck" length and mass. Thus, the black hole barrier is just slightly shifted below the Planck scale and the UV self-completeness scenario is self-consistently preserved.

3. Conclusions

We conclude this Letter by pointing out some interesting connections among self-complete quantum gravity, string theory, non-commutative geometry, regular back holes and un-particles.

The very concept of point-particle is only a low energy approximation for a one-dimensional string, and the naive idea that shorter and shorter length scales can be probed by injecting more and more energy into the probe breaks down at the string scale $l_s = \sqrt{\alpha'}$. To make contact with the UV self-complete scenario we recall the Correspondence Principle for Black holes and Strings [12]. In [33] Susskind suggested that there exists a one-to-one correspondence between Schwarzschild black holes and fundamental string states. The argument follows from the fact that in the strong coupling regime the size of an highly excited string is less than its Schwarzschild radius. On the other hand, the interest for non-commutative geometry was boosted in the high energy physics community by the recognition that spacetime coordinates turn into non-commuting objects as an effect of string–D-brane coupling in the presence of a Neveu–Schwartz background field [34,35]. Uncertainty in the localization of any physical event, near and beyond a certain length scale $l_{NC} = \sqrt{\theta}$, becomes an unavoidable feature of any physical theory. We encoded this intrinsic limit into our regular black hole solution by smearing the central curvature singularity, or mass-energy density, into a minimal width Gaussian distribution.

Finally, self-complete quantum gravity provides a different view of the minimal distance which can be probed in a *gendanken* high energy experiment as the radius of a thermodynamically stable, extremal, regular black hole. Our self-consistent approach allows to push the black hole barrier slightly below the Planck length, but it is still there. Is this the end of the story?

A couple of years ago Georgi introduced a possible new sector of the elementary particle Standard model, where scale invariance is realized in the form of a continuous mass spectrum [36,37]. The new objects have been called *un-particles* to distinguish them from ordinary matter. The interactions between particles and un-particles introduce an entire new phenomenology to be, hopefully, tested at LHC. As far as gravity is concerned, un-gravitons[4] lead to deviations from the Newton law [40] which turn, at the non-perturbative level, into modifications of the Schwarzschild geometry [41–44]. The un-graviton modified metric results to be formally equivalent to the line element in the presence of *fractal* extra dimensions. The non-trivial way in which scale invariance is realized in the un-particle sector seems to be the key to access a new *fractal phase* of spacetime geometry [45–49]. A recent analysis of high energy un-matter diffusion provided a new interpretation of $\sqrt{\theta}$ as the *critical temperature* marking the transition from a smooth geometry to a trans-Planckian "spacetime steam" [50]. This new scenario and its connection with the UV self-complete quantum gravity model are currently under investigation.

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References

[1] J.A. Wheeler, Ann. Phys. (NY) 2 (1957) 604;
J.A. Wheeler, K. Ford, Geons, Black Holes, and Quantum Foam: A Life in Physics, Norton, New York, USA, 1998, 380 pp.
[2] D. Amati, M. Ciafaloni, G. Veneziano, Phys. Lett. B 216 (1989) 41.
[3] M. Fontanini, E. Spallucci, T. Padmanabhan, Phys. Lett. B 633 (2006) 627.
[4] E. Spallucci, M. Fontanini, arXiv:gr-qc/0508076.
[5] G. Dvali, C. Gomez, Self-completeness of Einstein gravity, arXiv:1005.3497 [hep-th].
[6] G. Dvali, S. Folkerts, C. Germani, Physics of trans-Planckian gravity, arXiv: 1006.0984 [hep-th].
[7] G. Chapline, E. Hohlfeld, R.B. Laughlin, D.I. Santiago, Int. J. Mod. Phys. A 18 (2003) 3587.
[8] R.B. Laughlin, Int. J. Mod. Phys. A 18 (2003) 831.
[9] B.P. Kosyakov, Found. Phys. 38 (2008) 678.
[10] G. Dvali, G.F. Giudice, C. Gomez, A. Kehagias, UV-completion by classicalization, arXiv:1010.1415 [hep-ph].
[11] G. Dvali, D. Pirtskhalava, Dynamics of unitarization by classicalization, arXiv: 1011.0114 [hep-ph].
[12] G.T. Horowitz, J. Polchinski, Phys. Rev. D 55 (1997) 6189.
[13] P. Nicolini, A. Smailagic, E. Spallucci, Phys. Lett. B 632 (2006) 547.
[14] P. Nicolini, A. Smailagic, E. Spallucci, ESA Spec. Publ. 637 (2006) 11.1, arXiv: hep-th/0507226.
[15] S. Ansoldi, P. Nicolini, A. Smailagic, E. Spallucci, Phys. Lett. B 645 (2007) 261.
[16] E. Spallucci, A. Smailagic, P. Nicolini, Phys. Lett. B 670 (2009) 449.
[17] P. Nicolini, E. Spallucci, Class. Quant. Grav. 27 (2010) 015010.
[18] A. Smailagic, E. Spallucci, Phys. Lett. B 688 (2010) 82.
[19] T.G. Rizzo, JHEP 0609 (2006) 021.
[20] R. Casadio, P. Nicolini, JHEP 0811 (2008) 072.
[21] D.M. Gingrich, JHEP 1005 (2010) 022.
[22] S. Ansoldi, Spherical black holes with regular center: a review of existing models including a recent realization with Gaussian sources, in: Proceedings of Conference on Black Holes and Naked Singularities, Milan, Italy, 10–12 May, 2007, arXiv:0802.0330 [gr-qc].
[23] P. Nicolini, Int. J. Mod. Phys. A 24 (2009) 1229.
[24] R. Banerjee, B. Chakraborty, S. Ghosh, P. Mukherjee, S. Samanta, Found. Phys. 39 (2009) 1297.
[25] L. Modesto, J.W. Moffat, P. Nicolini, Phys. Lett. B 695 (2011) 397.
[26] E. Ayon-Beato, A. Garcia, Phys. Rev. Lett. 80 (1998) 5056.
[27] E. Ayon-Beato, A. Garcia, Gen. Rel. Grav. 31 (1999) 629.
[28] E. Ayon-Beato, A. Garcia, Phys. Lett. B 464 (1999) 25.
[29] S.A. Hayward, Phys. Rev. Lett. 96 (2006) 031103.
[30] S. Shankaranarayanan, N. Dadhich, Int. J. Mod. Phys. D 13 (2004) 1095.
[31] D. Batic, P. Nicolini, Phys. Lett. B 692 (2010) 32.
[32] E. Brown, R.B. Mann, Phys. Lett. B 694 (2010) 440.
[33] L. Susskind, arXiv:hep-th/9309145.
[34] E. Witten, Nucl. Phys. B 268 (1986) 253.
[35] N. Seiberg, E. Witten, JHEP 9909 (1999) 032.
[36] H. Georgi, Phys. Rev. Lett. 98 (2007) 221601.
[37] H. Georgi, Phys. Lett. B 650 (2007) 275.
[38] P. Gaete, E. Spallucci, Phys. Lett. B 661 (2008) 319.
[39] P. Gaete, E. Spallucci, Phys. Lett. B 668 (2008) 336.
[40] H. Goldberg, P. Nath, Phys. Rev. Lett. 100 (2008) 031803.
[41] J.R. Mureika, Phys. Lett. B 660 (2008) 561.
[42] J.R. Mureika, Phys. Rev. D 79 (2009) 056003.
[43] J.R. Mureika, E. Spallucci, Phys. Lett. B 693 (2010) 129.
[44] P. Gaete, J.A. Helayel-Neto, E. Spallucci, Phys. Lett. B 693 (2010) 155.
[45] G. Calcagni, JHEP 1003 (2010) 120.
[46] G. Calcagni, Phys. Rev. Lett. 104 (2010) 251301.
[47] L. Modesto, P. Nicolini, Phys. Rev. D 81 (2010) 104040.
[48] P. Nicolini, B. Niedner, arXiv:1009.3267 [gr-qc].
[49] G. Calcagni, arXiv:1012.1244 [hep-th].
[50] P. Nicolini, E. Spallucci, Phys. Lett. B 695 (2011) 290.

[4] The effective actions for various unparticle fields have been discussed in [38,39].