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The history, logic and uses of the Equivalent Initial Flaw Size approach to total fatigue life prediction

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Abstract

Total fatigue life is traditionally composed of the time to crack initiation plus the time for the initiated crack to grow to a critical crack size. Fracture mechanics does reasonably well in predicting the growth portion but there is still a lot of uncertainty about the definition of an initiated crack and scatter associated with the number of cycles to “initiation”. This paper will review some of the history, logic and uses of the Equivalent Initial Flaw Size (EIFS) approach to total life prediction. In short, this is a method where found cracks are analytically grown backwards to time equal zero (time or cycles) to determine an initial flaw, referred to as an EIFS. By growing a number of found cracks back to time equal zero a distribution of EIFS can be established. Example of establishing this distribution are given for the C-130 aircraft with a 7075 aluminum structure and for gas powered turbine blades made of directional solidified super-alloys.

1. Introduction

The purpose of this paper is to briefly review the Equivalent Initial Flaw Size (EIFS) approach to total fatigue life predictions. First the EIFS will be discussed and defined. Then some examples of the determination of the EIFS will be given. The EIFS will be compared to some studies on the “small-crack” behavior.

As one knows, the total fatigue life of a part is typically made up of the time to crack initiation plus the time for that initiated crack to grow to failure in the particular structural part. The use of fracture mechanics to predict crack growth is pretty mature and does a reasonable job. The weaker part of this total life prediction is the estimation of time to crack initiation. Initiation is very much a function of material (type, quality, heat treatment, etc.), machining (residual stresses, machine marks, etc.), environment (temperature, aggressive, etc.), local stress concentrations (such as holes, notches, $K_I$’s) and, of course, loading history. Fig 1 is a schematic of two approaches to determine the EIFS. One approach assumes that there is a “small-crack” present from day one and it grows from day one. So that assumed small crack would be the EIFS. The other approach is a little more traditional where one assumes a

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time to crack initiation. “Initiation” here would be the detectable size for a given nondestructive inspection technique (NDI). The EIFS would be determined by extrapolated back to time equal zero from the actual measured crack growth. The extrapolation would be done using the best information available (loadings, geometry and da/dN vs ΔK). If there is actual fatigue growth data from inspection of actual structure that has been in operation, then the realistic material, machining, environment, stress concentrations and loadings are included in the determination of the EIFS.

![Fig. 1. Schematic of two approaches to determine the EIFS.](image)

The idea is that this distribution of EIFS can then be used as a starting point for life predictions on other similar structures. Since the EIFS is a distribution it lends itself to doing probabilistic analysis of life. A schematic of this concept is shown in Fig 2. Notice that distributions of crack sizes can be determined at different times or a distribution of times to critical crack sizes can be estimated.

![Fig. 2. The EIFS distribution can serve as a starting point for probabilistic life predictions.](image)
2. Examples of the determination of EIFS distributions from actual in-service structures

2.1. C-130 wing box

For his Master’s thesis the author worked in 1974 on a NASA Langley Research Center sponsored project [1,2]. The stated objective of the project was to reduce C-130 service-flight inspection data to determine times to crack initiation. But during the course of the investigation the author came up with the idea of using fracture mechanics to extrapolate the found cracks back to time equal zero in order to estimate the “initial flaw sizes”. Around this same time period Rudd and Gray used an EIFS approach on the McDonnell Douglas F-4C/D aircraft [3].

The early C-130 wing boxes experienced significant fatigue cracking. These wing boxes were made of 7075-T6 aluminum alloy. By the early 1970s the C-130 (first launched in 1954) had considerable service hours. All of the C-130 inspection data examined refer to rivet holes in the center wing box section of the aircraft (Fig 3). The center wing box was divided into nearly 100 inspection locations which were symmetric about the centerline of the aircraft. The airplane was periodically inspected and the sizes of the cracks growing from the rivet hole in the skin were recorded. Small cracks were permitted to grow through several inspections. Eventually cracks were repaired or, in some cases, the whole center wing box was replaced. The original inspection data was received from Warner-Robbins Air Logistic center (ALC/ACDCJ), on a magnetic nine track tape. The following information was available for each inspected point: aircraft serial number, total flying hours at time of inspection, date of inspection, military command, military base, facility where inspected, number of inspection, crack location as marked in Fig 3, crack size and number (there could be more than one crack at a location).

![Fig. 3. The location of the wing-boxes on the C-130 aircraft and the location of the holes that exhibited the most fatigue cracks. These holes most likely to crack were all located at the corners of the wing box.](image-url)

The simple Paris relation was used to grow the cracks backwards.

\[
da/dN = C(\Delta K)^n \tag{1}\]
This equation was expanded and converted to a time, t, base to go with the recorded flight hours.

\[ \frac{da}{dt} = C(\Delta\sigma(a)1/2f_{th})^n \frac{dN}{dt} = C(\Delta\sigma(a)1/2f_{th})^n \frac{dN}{dt} a^{n/2} \]  

(2)

Letting \( C^* = C(\Delta\sigma(a)1/2f_{th})^n \frac{dN}{dt} \) and \( n^* = n/2 \)

(3)

where \( C^* \) is in units of \([\text{in./in.}^{n/2}/\text{flight hours}]\), Eq. 2 becomes

\[ \frac{da}{dt} = C^* a^{n^*} \]  

(4)

Since the values found in \( C^* \) are not well known (the exact stresses at the holes, flight cycles per flight hour, the geometric correction factor for the complex rivet holes on the wing box) it was decided to group all of those together and let it fall out of the fit to the inspected crack growth behavior. It was decided to set \( n^* \) equal to 1.5 since the Paris exponent for 7075 aluminum alloy is essentially 3. So, for each location where the same crack was found to grow in subsequent inspections, the \( da/dt \) versus the crack length was used to determine \( C^* \) for that location. Rivet hole locations in the corners of the wing box covers were the ones most prone to cracking. On one cover this corresponded to locations 73, 74, 75 and 76. On the symmetric cover this corresponded to locations 89, 90, 91 and 92 as shown in Fig 3. Since the wing box were identical about the plane’s centerline we grouped the data: (73, 89), (74, 90), (75, 91), and (76, 92).

The initial attempt at reducing the data used a \( \Delta K_{th} \) (threshold stress intensity range) of 3 MPa-m\(^{1/2}\). This gave a limit as to how far one could extrapolate the crack backwards. A skewed distribution resulted with a definite limit to how small the initial flaw size could be. Since this did not make much sense, it was decided not to use a threshold value. This was consistent with the “small crack effect” growth below threshold reported by many at a later time.

Table 1 presents the data from the four locations. Notice that the aircraft were in two operations groups: TAC (Tactical Air Command) had 263 airplanes inspected and PACAF (Pacific Air Command) had 223 airplanes inspected. The mean initial flaw sizes for each grouping ranged from 0.06 to 0.14 mm assuming a normal distribution. This is very consistent considering that the data is from different hole locations and from different aircraft that may have been used very differently to fly different missions in different environments. Also notice that the \( C^* \) values are also reasonably consistent. It is interesting to note that the two highest values are from the PACAF that had many flights in the Vietnam War. Since \( C^* \) reflects applied stress and usage, one might expect some hard landings and maneuvers in the war usage.

Table 1. Summary of Initial Flaw Data

<table>
<thead>
<tr>
<th>Hole Locations</th>
<th>Command</th>
<th>Normal Distribution</th>
<th>C* mean</th>
<th>No. of Data Pts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( a_0, \text{mean (mm)} )</td>
<td>Std. Dev.</td>
<td></td>
</tr>
<tr>
<td>76 92</td>
<td>TAC</td>
<td>0.13</td>
<td>0.10</td>
<td>0.103</td>
</tr>
<tr>
<td>76 92</td>
<td>PACAF</td>
<td>0.06</td>
<td>0.04</td>
<td>0.151</td>
</tr>
<tr>
<td>75 91</td>
<td>TAC</td>
<td>0.12</td>
<td>0.11</td>
<td>0.119</td>
</tr>
<tr>
<td>75 91</td>
<td>PACAF</td>
<td>0.13</td>
<td>0.08</td>
<td>0.100</td>
</tr>
<tr>
<td>74 90</td>
<td>TAC</td>
<td>0.14</td>
<td>0.06</td>
<td>0.090</td>
</tr>
<tr>
<td>74 90</td>
<td>PACAF</td>
<td>0.07</td>
<td>0.03</td>
<td>0.126</td>
</tr>
<tr>
<td>73 89</td>
<td>TAC</td>
<td>0.12</td>
<td>0.07</td>
<td>0.107</td>
</tr>
<tr>
<td>73 89</td>
<td>PACAF</td>
<td>0.12</td>
<td>0.06</td>
<td>0.093</td>
</tr>
</tbody>
</table>

Shown in Fig 4 is a typical distribution of the initial flaw sizes. Notice that there are some rather large flaws present. It was found that the Johnson \( S_n \) distribution [4] fit the flaw size distribution the best. Much more data and information about the approach and statistics can be found in references 1 and 2.
2.2. Gas turbine combustor blades

Research was conducted for GE Power Generation Division to look at the determination of EIFS distributions from blade inspections and to incorporate that information into a probabilistic lifing approach [5,6,7] as indicated in Fig 2. Starting with the EIFS distribution, crack length as a function of usage time could be determined accounting for scatter in usage and material properties (da/dN data). Inspections were carried out on several types of gas turbines that had different types of blades. In particular, some combustor blades had round holes and another type of blade had elliptical holes as shown in Fig 5.

The inspection data consisted of blade unique serial number, crack size, crack growing from one side of the hole or both sides, service history (number of trips, starts, and operational hours) at time of inspection, and turbine type. In addition to the differences in blade geometry, the inspected turbines tended to fall into two classes of operational groupings: those that were turned on and allowed to run for very long periods of time (many days or weeks at a time) and those that were turned on and off on almost a daily basis. This difference in service history is shown to make a big difference in the size of cracks found as a function of operational hours as shown in Fig 6. Engines 7 and 10 were those that were turned on and off on almost a daily basis.
Fig. 5. Two blade types that contain different geometry cooling holes. The round hole was drilled and the elliptical hole was cast.

Fig. 6. Shows that more cracks and bigger cracks appeared after a shorter number of hours for those units that were cut on and off on almost a daily basis.
As mentioned earlier, the turbine essentially experiences three difference types of loading that tend to drive a crack: Starts (and the ensuing shut down), Trips (the emergency shut down) and Time running (a sustained load at high temperatures that could cause time dependent creep-like crack growth). The simple crack growth rate (CGR) equation was developed to account for these events by combining three Paris Law type equations. Equation (5), for trips, assumes that the bucket experiences stresses similar to those of a startup/shutdown cycle but assumes temperature differences from the startup/shutdown cycle. The starts are approached as a low frequency cycle with a very low or even negative $R$ ratio that is representative of the large stress ranges a blade experiences and is represented by Equation (6). It is assumed that the Paris exponent, $m$, for the starts and trips equations are similar values because of the similarity of the trip and start phenomena. The Paris $C$ constants for each equation remain different because of the temperature and loading rates may vary between the Start event and the Trip event. These assumptions are made because $da/dN$ vs. $\Delta K$ curves shift with temperature and frequency but generally have similar slopes on log-log plots [7].

Equation (7) contribution, for hours, is characteristic of the stresses experienced by the blade while it is running. This stress has a high frequency and $R$ ratio and is considered similar to a sustained load with superimposed vibrations. Because of this, the Paris equation for hours is modified from a stress range to a mean stress and is now akin to a creep crack growth phenomenon. The authors realize that there are more complex and accepted models for creep crack growth, such as $C^*$, but we are taking this approach for simplicity and consistency. The cracks will grow at different rates under the different loading events and are expressed by the equations below.

$$\frac{da}{dN_{trip}} = C_{trip} \cdot (f(g) \cdot \Delta \sigma_{trip} \sqrt{\pi a})^{m_{trip}}$$ (5)

$$\frac{da}{dN_{start}} = C_{start} \cdot (f(g) \cdot \Delta \sigma_{start} \sqrt{\pi a})^{m_{start}}$$ (6)

$$\frac{da}{dN_{hour}} = \frac{da}{dt} = C_{hour} \cdot (f(g) \cdot \sigma_{hour} \sqrt{\pi a})^{m_{hour}}$$ (7)

The $f(g)$ is the stress intensity factor geometric correction factor that is dependent on the cooling hole geometry, the crack length and whether there is only one crack on one side or a crack on both sides. The $C_{trip}$, $C_{start}$ and $C_{hour}$ are considered to be “influence” coefficients. These will be determined from the data to provide a best fit. These coefficients will reflect the influence that each of these events has on growing the crack.

It is necessary to combine all these crack growth equations to have an accurate physically based model. The stresses from starts and trips are assumed to be similar, thus the same value for each is used for this model. It is also required that a frequency factor, $f$, be included to convert the start and trip crack growth rate equations into units of crack growth per time. The frequency factor will be unique to each turbine because each turbine sees a different usage history.

$$f_{trip} = \frac{trips}{hours}$$ (8)

$$f_{start} = \frac{starts}{hours}$$ (9)

These frequency factors change the units of the growth rate equations from growth increment per event to growth increment per time for the Starts and Trips. The service history of each blade is now included in the crack growth rate (CGR) equation so all equations can be combined to give the final crack growth rate equation shown below.
The assumed stress values for this work are $\Delta \sigma_t = \Delta \sigma_s = 565 \text{ MPa (82 ksi)}$ and $\sigma_h = 537 \text{ MPa (78 ksi)}$. The stress values for the trips and starts are assumed to be similar. The exponent $m_{\text{start}}=4.3$ is from Highsmith and Johnson [7]. No testing was performed for conditions similar to those of a trip, so the $m_{\text{trip}}$ value was set equal to that of the $m_{\text{start}}$ coefficient. Since the geometric factors were mentioned earlier, the resulting unknowns for this equation are now $C_t$, $C_s$, $C_h$, $m_{\text{hour}}$, and $a_i$. The initial crack size, $a_i$, becomes a variable because an initial value must be entered into the equation to perform coefficient optimization calculations.

It should be pointed out that the $C_s$ and maybe the $C_t$, could be estimated from the laboratory test data in [7], but that would be risky. The real crack growth data experienced by the blade is at the same temperatures as tested in [7] but the environment is combustion by-products. These by-products of hydrogen sulfides would be expected to accelerate the crack growth rate over that of lab air. Thus the $C$’s are backed out of the inspection data.

To determine the unknowns, the CGR equation (10) is used to calculate the crack growth forward through time with multiple combinations of variables. The equation is converted from its current form into a form where the increment of crack growth can be calculated for a set increment of time, Eqn.11.

$$\frac{da}{dt} = C_t \cdot f_t \cdot [f(g) \cdot \Delta \sigma_t \sqrt{\pi a}]^{m_{\text{trip}}} + C_s \cdot f_s \cdot [f(g) \cdot \Delta \sigma_s \sqrt{\pi a}]^{m_{\text{start}}} + C_h \cdot [f(g) \cdot \sigma_h \sqrt{\pi a}]^{m_{\text{hour}}}$$

(10)

Since some assumed crack size is required to calculate stress intensity and begin the crack growth simulation, an initial “guess” for $a_i$ is required for the forward calculations. This initial guess is one of the variables, along with the influence coefficients, that must be optimized. From this guess $a_i$, Eqn. 11 is numerically integrated forward in time until it reaches the hours at which the inspection data for each engine was recorded (or until the integration is aborted at a critical crack size). Once all variable combinations have been calculated forward to either a critical crack size or the inspection time, the optimized set of coefficients selected are those which best predicted the inspection crack data for all of the engines being analyzed, as determined by a minimum sum squares error in crack size. The algorithm developed uses an incremental approach to select the optimized coefficients. For each variable, an upper and lower bound are specified before program execution, as is the number of increments into which that interval is to be divided. Then the forward crack growth calculation is performed using each of the values that define these subinterval boundaries, using all permutations of the multiple values for all variables. The best set of variables is chosen based on the lowest sum squares error in predicting the inspection data. This process is repeated with the new upper and lower bound defined by the previous subintervals on either side of the value chosen in the previous step, and then this new interval is divided into the specified number of increments in a “divide and conquer” type approach. (If, in the initial run, the best value selected is the upper or lower limit of the overall interval, the limits are expanded and the process restarts.) These iterations are repeated until the change in error falls below some user-specified percentage. An example of the best-fit forward calculations is shown in Fig 7. The X data points are holes with one crack and O data points are the holes containing two cracks.
Fig. 7. Shows an example of a forward calculation where the parameters were optimized to fit real measured crack lengths. All cracks were grown forward from a best fit initial crack length, \( a_i \).

Once the optimized coefficients are determined, the crack growth rate equation is used to grow each of the found inspected cracks backwards to time equals zero. Similar to the forward calculation, Eqn. 11 is numerically integrated in one hour time steps from the inspection time down to zero, only in this calculation each crack increment \( \Delta a \) is subtracted rather than added. Note that the “initial” crack size in this integration is the actual measured crack size from inspection data, not the optimized \( a_i \) from the forward calculations. That \( a_i \) was used as a single starting point to optimize the \( C_f, C_s, C_h, \) and \( m_{hour} \); in the backward calculations, each and every measured crack is regressed to its own \( a_i \) (crack size at time zero). It should also be noted that no threshold stress intensity factors are used as crack growth cut offs, since the EIFS is only a hypothetical initial crack size calculated using only the linear Paris equation formulation of crack growth. (This implies concurrently that forward calculations for life based on these EIFS should also not use any crack growth threshold.) Performing this reverse calculation from all of the crack sizes found at inspection for all engines generates a set of \( a_i \) values at time zero, and these are the EIFS distribution. Since a deterministic crack growth equation is used in the forward and backward calculations (i.e., the Paris equation coefficients and exponents are constants and not random variables), all variability in the material quality as manifested in fatigue cracks is incorporated into this EIFS distribution characterization. Finally, whether or not the resultant EIFS distribution converges on the initial guess \( a_i \) is an important criterion in optimizing the regression.

Fig 8 is an example of the backward growth from the numerous found cracks. These were for both the turbines that were allowed to run for long periods of time and those that were essentially daily use. Included in these were holes that contain only one crack and those that contained two cracks. Notice that these extreme use cases all grew back to essentially the same distribution. This adds credibility to the approach.
Fig. 8. Shows the back extrapolation from found cracks. The “daily” usage and the “long-term” usage grow back to essentially the same distribution of EIFS.

The EIFS distribution shown in Fig 9 includes data from the daily and long-term units and for those holes (41) containing one crack and those holes (127) containing two cracks. The fact that all of these variables fall within the same distribution adds more credibility to the given approach. Many more details can be found in reference 5.

Fig. 9. Distribution of all cracks, both cyclic and long-term usage and holes containing 1 or 2 cracks.
3. Other approaches to EIFS

Newman and colleagues have conducted several studies on pre-existing defects in materials and have shown that these defects appear to begin growing very early in fatigue life. See references [8, 9] as examples of this work. In these papers they use microscopy to measure the size of the initial defects and crack growth analysis that includes the “small crack growth effects” to show that indeed these defects may be growing cracks from the very beginning of fatigue cycling. In [8] they conclude that for 7075 aluminum alloys the initial flaw size ranges from 6 to 9 μm.

The initial flaw sizes suggested by Newman, et al., are significantly smaller than found from the C-130 inspection data by the author over 30 years ago. The average of the initial flaws shown in Table 1 is about 100 μm. There are several reasons for these results. First and foremost, the specimens used by Newman, et al. were finely polished (minimized residual stresses), open semi-circular surface, and tested in a laboratory environment. The actual C-130 holes that fatigued were factory drills holes (potential for residual stress and surface roughness), fastener loaded (may have had some fretting), and operational environments (grime, salt air, etc.). The EIFS from the real operational aircraft may have had the mentioned accelerating fatigue factors that were not present in the sterile testing environment of Newman’s data. Further, Newman used the latest knowledge on the small crack growth effects in aluminum alloys to account for accelerated crack growth at lower delta K’s. In some way the author did something like that by ignoring the $\Delta K_{th}$ in the backward extrapolation.

Another series of EIFS investigations was conducted on 2024-T3 aluminum alloy by Fawaz and colleagues [10, 11]. In these investigations tests were conducted on realistic fuselage splice joint designs that had been used on commercial aircraft for many years. Four types were chosen, two longitudinal lap-splice joints and two butt splice joints. These were tested in laboratory conditions at the Air Force Research Laboratory’s wide panel test facility at Wright-Patterson Air Force Base. In this case the investigators used both FASTRAN and AFGROW crack growth predictions programs to determine the EIFS. In this investigation the range of EIFS found ranged from 4 to about 30 μm [11]. In [10] Fawaz, et al found mean EIFS ranging from 7 to 56 μm. These values are getting closer to those shown in Table 1. The Fawaz EIFS values do have the realistic load bearing/fretting that would be found in bolt bearing and by-pass conditions that would tend to deliver larger EIFS that the sterile coupons that Newman tested. However, the effects of actual operational environments were not included. It is well known that high humidity, salty air, grime, etc can effect fatigue initiation and therefore the EIFS distribution.

4. Some observations

Laboratory tests like those conducted by Newman and Fawaz are very useful in defining microstructural defects that initiate cracks and tuning/verifying crack growth models to the small crack growth and predicting over all life. Newman makes a very good case that defects in aluminum alloys are growing from the first applied cycle. However, the use of laboratory tests conducted by Newman do not reflect the realistic conditions of an operational aircraft (material quality, manufacturing type and quality, actual loading histories and environmental effects). All of these can have significant effects on crack initiation and early growth thus profoundly affecting the EIFS distributions, most likely making the EIFS larger.

Since inspections are becoming routine for almost all aircraft and many engine types, why not use this data early on to establish an EIFS data base that can be used to project fleet reliability into the future and to establish durability limits. It is suggested that an EIFS distribution established on one earlier aircraft type could be transferred to a newer aircraft type as long as the material, manufacturing procedures and operational envelopes were about the same.

Depending on the amount of information available will dictate how one goes about determining the EIFS distribution. In the C-130 case, there were numerous cracks that were left in service such that we knew their growth as a function of flights and could back out some information on the crack driving stresses. For the gas turbine blade example, we only had the final found crack size. The blades were removed from service if cracked. However we had a record of the usage in terms of starts, trips and number of operational hours, so we could find a relationship for the crack driving terms for the number of starts, trips and hours. In each of the cases presented the author used actual $da/dN$ vs $\Delta K$ data for the appropriate material but ignored the $\Delta K_{th}$ limit. Newman did use more of the small crack growth understanding in that area but modeled threshold behavior using lower region of the crack-growth-rate.
The bottom line here is what ever methods and assumptions one uses to develop the EIFS distribution, those same methods and assumptions should be used to grow the EIFS’s back out to predict life.

5. Summary

The Equivalent Initial Flaw Size (EIFS) approach to life prediction has been summarized. Two examples of EIFS distribution determination from actual operations platforms have been presented: one from over 35 years ago (C-130 cargo plane) and one rather recent (gas turbine engine blades). Work of other researchers in this area was noted and differences in approaches and applicability pointed out. Finally some concluding remarks were made about the EIFS approach and application.

References


