Deterministic and probabilistic analysis of semi-elliptical cracks in austenitic steel

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Abstract

The aim of this study was deterministic and probabilistic analysis of fatigue life of the austenitic steel (1.4541) applied on industrial pipeline. Semi-elliptical cracks in 1.4541 steel were considered during studies and probabilistic analysis. Increases in the sizes of semi-elliptical cracks were determined experimentally and analytically for specimens made of 1.4541 steel used for the new structure of pipeline and 1.4541 steel after many years of operation. Crack propagation models of a semi-elliptical crack were developed for the purpose of describing crack length propagation in a case of random loading. Correctness of the proposed models was checked on the basis of the experimental results. Fatigue tests were performed in accordance with the block program of stresses.

1. Introduction

Fatigue crack propagation depends on many factors that have a random character, e.g. load changes, the component geometry or material properties. In such circumstances, it appears purposeful to use probabilistic models of crack propagation. Such an approach to the problem of fatigue life was presented by Holtam at al. (2010), Wang at al. (2012), Ahammed at al. (1997), Lu at al. (2010) and Ahammed (1997) among other authors.
Considering the complexity of the aforesaid issue, general relations that make use of the stress intensity factor or the J-integral must be modified properly while modeling fatigue crack growth in order to take the results of the experiment into account, Badena at al. (2012). Another problem is the randomness of many factors that affect the propagation of a crack which develops under operational conditions. It concerns mainly the random process of component loading, which causes the randomness of stress changes and, as a consequence, a random increase in crack length. For that reason, it seems purposeful to use the modifications of basic relations which describe the dynamics of cracking so that the obtained results can be used in the widest possible range of cases, Xu at al. (2009).

When reviewing publications it is crucial to pay attention to the review work Sobczyk and Spencer (1992). In the said work, the authors discussed, among other things, pre-developed random models of crack propagation by classifying them as probabilistic evolutionary models, cumulative discrete models and differential models. The authors' achievement was the use of cumulative discrete stochastic processes, including Markov diffusion processes and the birth process, to describe fatigue-related problems. The description of the possibilities of using the presented models in practice and a comparison of the results of theoretical analyses and experiments can be found at the end of the book. It is worth mentioning publication Kocanda at al. (1989), the authors of which presented a partial differential equation, which described crack development dynamics in a probabilistic sense, and which has been used by other authors until the present moment. Another advantage of the aforesaid work is that the authors developed a two-dimensional model, which takes into account both dimensions of an elliptic propagating crack.

In the present study, there has been proposed a generalization of the model that makes use of the difference equation, which describes crack growth dynamics in a probabilistic sense. Such an approach was presented by Kocanda and Jasztal (2012) and Tomaszek at al. (2012), among other publications. For the cracking case being considered, there was derived a differential equation that allowed for the dependencies of the deterministic model, the solution of which is a normal probability distribution of the crack length. Having at one's disposal the probability distribution with known parameters, for the assumed risk of exceeding the admissible crack length, there was estimated the fatigue life of model components that were cut out from industrial pipelines, which were made of austenitic steels.

2. Research methods

The industrial pipelines are often made of austenitic steels, because of their good mechanical and chemical properties. The material employed for the investigation is a steel called 1.4541 (austenitic stainless steel).

Experimental research were carried out on samples of 1.4541 steel with the chemical composition: 0.04% C, 0.80% Si, 0.40% Mo, 0.40% Ti, 17.88% Cr, 1.80% Mn, 8.89% Ni. Basic mechanical properties of this steel in the initial state are respectively: yield strength $R_{p0.2}=284$ MPa, ultimate tensile strength $R_m=600$ MPa. Tension tests on fatigue crack growth rate were performed for flat specimens. Samples of the shape and dimensions shown in Fig. 1 were subjected to axial tensile stress with a maximum amplitude $\sigma_{amax}=125, 137.5, 150, 162.5$ i $175$ MPa.

![Fig. 1. Specimen for tests on fatigue crack propagation under cyclic tension (zero-to-tension); specimen dimensions (a), dimensions of a semi-elliptic notch (b).](image)

Semi-elliptical initiators of cracking were made in specimens with the use of electro-erosion machine. The adopted ratio of the semi-minor axis of an ellipse, $a_0$, to the major axis, $2c_0$, was 0.2. Fatigue tests were performed in
accordance with the block program of stresses (see Fig. 2.). Selection of stress values were based on the results of the strength properties of steel and stress amplitude measurements in the wall of the industrial pipe.

Fig. 2. The block program of stresses with an irregular sequence of levels, which was used in fatigue tests.

Experimental research of fatigue life and semi-elliptical crack growth were carried out on samples cut from the industrial pipelines for transmission of 10.5 percent of nitric oxide after many years of operation (called "steel after many years of operation") and the new pipeline sections before installing them to an industrial installation (called "steel used for the new structure"). For the purpose of the realization of the present study, there was elaborated and verified a relevant methodology and there was built a measuring stand which made it possible to record crack length growths on the basis of the EPD method.

3. Experimental and analytical results.

3.1. Deterministic model of propagation of semi-elliptical fatigue cracks

The proposed description concerns the propagation of semi-elliptical type cracks of a shape that is illustrated in Fig. 3. It was assumed that cracking phenomenon can be modelled on the basis of the stress intensity factor, $K$.

In this study, it was assumed that the crack is growing steadily in both directions and also that the quotient of crack growth rates is constant and is equal to the quotient of material constants $C_{1}'$ and $C_{2}'$.

This allowed the arrange following system of equations:

$$
\begin{align*}
\frac{dc}{dN} &= C_{1}' \frac{2\sqrt{C_{1}'C_{2}'}\sqrt{ca}}{C_{1}'c + C_{1}'a} \left(\sqrt{\Delta K_{1}\Delta K_{2}}\right)^n \\
\frac{da}{dN} &= C_{2}' \frac{2\sqrt{C_{1}'C_{2}'}\sqrt{ca}}{C_{2}'c + C_{1}'a} \left(\sqrt{\Delta K_{1}\Delta K_{2}}\right)^n
\end{align*}
$$

where:

- $M_{k1}$ - the correction factor that allows for the finiteness of component dimensions for the $l_1$ direction,

- $M_{k2}$ - the correction factor that allows for the finiteness of component dimensions for the $l_2$ direction.

Having taken into account expressions for stress intensity factor ranges, from the system of equations (1), there was obtained the following:

$$
\begin{align*}
\frac{dc}{dN} &= C_{1}\Delta \sigma^{\frac{n}{2}} \frac{2\sqrt{C_{1}'C_{2}'}\sqrt{ca}}{C_{1}'c + C_{1}'a} \left(\Delta K_{1}^{*}\Delta K_{2}^{*}\right)^{n} \\
\frac{da}{dN} &= C_{2}\Delta \sigma^{\frac{n}{2}} \frac{2\sqrt{C_{1}'C_{2}'}\sqrt{ca}}{C_{2}'c + C_{1}'a} \left(\Delta K_{1}^{*}\Delta K_{2}^{*}\right)^{n}
\end{align*}
$$

where:

$$
C_{1} = C_{1}'\left(M_{k1}M_{k2}\right)^{\frac{n}{2}}; \quad C_{2} = C_{2}'\left(M_{k1}M_{k2}\right)^{\frac{n}{2}}
$$
On the base of the above equations, were obtained the relationship between the length and depth of the cracks with the number of load cycles. Initial conditions used in the calculations for N = 0: c(0) = c_0, a(0) = a_0.

\[
\begin{align*}
    c &= \frac{(C_2c_e - C_2c_0) + \sqrt{(C_2c_0 - C_2c_0)^2 + 4C_2C_1c_ea_e^2\Delta \sigma^2 \pi^2 N + (c_ea_e)^2}}{2C_2} ; \quad m = 2 \\
    a &= \frac{(C_1a_0 - C_1c_0) + \sqrt{(C_1a_0 - C_1c_0)^2 + 4C_1C_2c_ea_e^2\Delta \sigma^2 \pi^2 N + (c_ea_e)^2}}{2C_1} \\
    c &= \frac{(C_2c_e - C_2c_0) + \sqrt{(C_2c_0 - C_2c_e)^2 + 4C_2C_1c_ea_e^2\Delta \sigma^2 \pi^2 N + (c_ea_e)^2}}{2C_2} ; \quad m \neq 2 \\
    a &= \frac{(C_1a_0 - C_1c_0) + \sqrt{(C_1a_0 - C_1c_0)^2 + 4C_1C_2c_ea_e^2\Delta \sigma^2 \pi^2 N + (c_ea_e)^2}}{2C_1} 
\end{align*}
\]  

Crack surface area, depending on the number of the cycles of load changes, may be defined by the following relations:

\[
\begin{align*}
    S &= \frac{\pi c_ea_e^2\Delta \sigma^2 \pi^2 N + (c_ea_e)^2}{2} ; \quad m = 2 \\
    S &= \frac{\pi}{2} \left[ \frac{2 - m}{2} \sqrt{C_1C_2\Delta \sigma^2 \pi^2 N + (c_ea_e)^2} \right]^{\frac{1}{2-m}} ; \quad m \neq 2
\end{align*}
\]  

Calculation results are enable to predict the development of cracks in the whole range of their lengths. Selected curves of increases in both the crack surface area, S, and its basic dimensions, namely a and c, which were obtained during the experiment as well as from calculations that were carried out on the basis of the proposed model, in the function of the number of the cycles of load changes, N, are presented in Fig. 3 and Fig. 4.

![Fig. 3. Experimentally and analytically determined increases in the surface areas, S, in specimens which were made of 1.4541 steel used for the new structure and 1.4541 steel after many years of operation, and which were subject to tension under the conditions of a block program of stresses at \( \sigma_{\text{max}} = 137.5 \text{ MPa} \).](image-url)
Therefore, on the basis of relations (3) and (4), were calculated the required parameters semi-elliptical cracks. By introducing a designation that stands for the initial surface area of a crack, namely $S_0$, there are obtained the following expressions that make it possible to evaluate the fatigue lives of tested components with propagating semi-elliptical cracks:

a) for $m = 2$:

$$
N_{c} = \frac{\ln \left[ C_2 c_k - C_1 c_n + C_2 a_n \right] + \ln c_k - \ln (C_1 c_n a_n)}{2 \sqrt{C_1 C_2 \pi \Delta \sigma^2} \left( 2 a_c + 2 c \right)}
$$

$$
N_{s} = \frac{\ln \left[ C_2 a_k - C_1 a_n + C_2 a_n \right] + \ln a_k - \ln (C_2 c_k a_n)}{2 \sqrt{C_1 C_2 \pi \Delta \sigma^2} \left( 2 c_c + 2 c \right)}
$$

$$
N_{s} = \frac{\ln S_k - \ln S_0}{2 \sqrt{C_1 C_2 \Delta \sigma^2} \pi}
$$

(5)

b) for any value of exponent $m$ (excluding the special case where $m = 2$):

$$
N_{c} = \frac{\frac{2}{2 - m} \left[ \frac{C_2}{C_1} \left( c_k - c_n \right) + a_n \right]^{\frac{2 - m}{4}} c_k^{\frac{2 - m}{4}} - \left( c_n a_n \right)^{\frac{2 - m}{4}}}{\sqrt{C_1 C_2 \Delta \sigma^2} \pi^{\frac{n}{2}}}
$$

$$
N_{a} = \frac{\frac{2}{2 - m} \left[ \frac{C_1}{C_2} \left( a_k - a_n \right) + c_n \right]^{\frac{2 - m}{4}} a_k^{\frac{2 - m}{4}} - \left( c_n a_n \right)^{\frac{2 - m}{4}}}{\sqrt{C_1 C_2 \Delta \sigma^2} \pi^{\frac{n}{2}}}
$$

$$
N_{s} = \frac{\frac{2}{2 - m} \left( \frac{S_k}{\pi} \right)^{\frac{2 - m}{4}} - \left( \frac{S_0}{\pi} \right)^{\frac{2 - m}{4}}}{\sqrt{C_1 C_2 \Delta \sigma^2} \pi^{\frac{n}{2}}}
$$

(6)

As the fatigue life of a component, $N_t$, there should be adopted the lowest of the three values obtained respectively from relations (6) for particular computational cases:
It is necessary to bear in mind the assumptions that tests were made during experimental, namely that the fatigue life of specimens tested under the conditions of variable tension is determined by the number of the cycles of load changes until the appearance of a through crack. The calculated fatigue lives of tested specimens differ from the experimentally determined fatigue lives by 2.5-7%.

3.2. A probabilistic model of the propagation of semi-elliptical fatigue cracks

A probabilistic model of the propagation of a semi-elliptical crack was developed on the basis of the following assumptions:
- there exist fatigue crack lengths (in two mutually perpendicular directions), where in a certain range (or for a certain number of load cycles) the probability of the occurrence of a catastrophic fracture is zero,
- fatigue crack rates are described, in a deterministic sense, by relations that are presented in point 2 of the present study,
- load cycles, the duration of which is $\Delta t$ do not need to occur in a continuous manner - they can occur randomly with an intensity of $\lambda$, i.e. $\lambda \Delta t \leq 1$.

When assessing the fatigue life of a component it is crucial to allow both for the probability of exceeding critical lengths by the crack length and for the probability of exceeding the critical surface area of a crack. Assuming that the elliptic shape approximately reflects the actual crack shape, it becomes purposeful to formulate an equation that describes, in a probabilistic sense, the dynamics of crack area growth irrespective of the description of the dynamics of crack length growth.

Differential equations describing the dynamics of crack propagation in the probabilistic sense was derived on the base of the general description, proposed by Tomaszek at al. (2002).

$$\frac{\partial U(c,a,t)}{\partial t} = -\lambda \frac{\partial U(c,a,t)}{\partial c} \Delta c - \lambda \frac{\partial U(c,a,t)}{\partial a} \Delta a + \frac{\lambda^2}{2} \frac{\partial^2 U(c,a,t)}{\partial c^2} \Delta c^2 + \frac{\lambda^2}{2} \frac{\partial^2 U(c,a,t)}{\partial a^2} \Delta a^2 + \lambda \frac{\partial^2 U(c,a,t)}{\partial c \partial a} \Delta c \Delta a$$  (8)

$$\frac{\partial U^2(S,t)}{\partial t} = -\lambda \frac{\partial U^2(S,t)}{\partial S} \Delta S + \frac{1}{2} \lambda \frac{\partial^2 U^2(S,t)}{\partial S^2} \Delta S^2$$  (9)

where:
- $U(c,a,t)$ - the density function of crack length,
- $U^2(S,t)$ - the density function of crack surface area,
- $\Delta t, \Delta c, \Delta S$ - increments determined from appropriate depending of deterministic model.

The solution to the equations are two-dimensional normal distribution of crack length and the distribution of normal surface cracks. The parameters of distributions were estimated using the likelihood function.

If we base on the distributions of crack lengths and surface areas with known parameters, it is possible to determine the probability of not exceeding the critical crack length, $R_L(t)$, as well as the probability of not exceeding the critical value of the crack surface area, $R_S(t)$.

$$R_L(t) = P(c \leq c_t, a \leq a_t, t) = \int_{0}^{c_t} \int_{0}^{a_t} U(c,a,t) \, dc \, da$$  (10)

$$R_S(t) = P(S \leq S_t, t) = \int_{0}^{S_t} U^2(S,t) \, dS$$  (11)

In accordance with the assumptions of the probabilistic model, critical crack lengths, $c_t$ and $a_t$, as well as the critical crack surface area, $S_t$, should be determined in such a way that the risk of immediate component failure is small enough.

Equations (10) and (11) should be needed to calculate the probability. Total probability of not exceeding admissible values is determined from the relation:

$$R(t) = R_L(t)R_S(t)$$  (12)
Assuming that $R(t) \geq R_0$, where $R_0$ is the admissible minimum probability of not exceeding critical crack lengths, $c_k$ and $a_k$, and the critical value of the crack surface area, $S_k$, it is possible to determine the fatigue life of a component. In calculations, it was adopted that $R_0=0.99$.

In order to prove the correctness of the proposed probabilistic model, there were used the results of experimental tests on crack growth in specimens that were tested both under variable tension in accordance with the block program of stresses and under constant-amplitude variable bending.

A comparison of obtained results, which were developed statistically, is illustrated in the form of bar charts, which are presented in Fig. 5.

![Fig. 5. A comparison of experimentally determined and computational fatigue lives of components which were made of 1.4541 steel used for the new structure and 1.4541 steel after many years of operation, and which were subject to tension under the conditions of a block program of stresses.](image)

A compilation of statistically developed results of fatigue life tests, which were obtained from experimental tests and a probabilistic model, and which were calculated for the values of stress amplitudes $\sigma_{\text{max}}$ and $\sigma_{\text{gma}}$ adopted in the research program, is presented in the form of bar charts in Fig. 5. Calculated fatigue lives of specimens, which were expressed by the number of cycles until reaching the critical crack length, were lower than the experimentally determined fatigue lives and differed by 0.1-5.5% (depending on the stress amplitude value) in relation to the latter.

4. Conclusions

The results of conducted experimental and theoretical analyses of the fatigue lives of industrial pipelines, taking into account their operating times, made it possible to formulate the following conclusions:

- There was observed that the fatigue life of specimens that were cut out from pipeline sections that had operated for many years was shorter than the fatigue life of specimens made of steel that was used for the structure of a new pipeline. Under variable tension, in accordance to the block program of load changes, the observed decrease in fatigue life reached 15-26% for specimens without welds but with a semi-elliptical initiator of cracking.

- The obtained research results prove that it is possible to use the computational models which were presented in the present study both in the deterministic and in the probabilistic sense, for the prediction of the fatigue life margin of model components cut out from industrial pipelines with a propagating crack.
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References


