SOURCES

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VOLTERRA ARCHIVE AT THE ACCADEMIA NAZIONALE DEI LINCEI

By Giorgio Israel
Istituto Matematico "G. Castelnuovo,"
Università degli Studi,
Città Universitaria, 00100 Rome, Italy

The Accademia Nazionale dei Lincei recently acquired all the correspondence and manuscripts of Vito Volterra. They constitute a vast archive and throw light on every aspect of the figure and work of the famous Italian mathematician. This note contains a brief biography of Volterra and a description of the archive.

As Guido Castelnuovo observed, "Vito Volterra was one of the greatest mathematicians Italy has ever had. He was famous throughout the world for his writings, his lectures and his talks in various cities of Europe and America." Furthermore, "during the first twenty-five years of the century he was the inspirer and organizer of Italian science. We may say that no important scientific institution arose in our country, during that period, to which he had not given the original impulse" [Castelnuovo 1947].

Vito Volterra was born in Ancona on May 3, 1860, the only son of Angelica Almagià and Abramo Volterra, a cloth merchant. His life was difficult from the start: his father died when he was two years old and, owing to economic problems, he and his mother were obliged to live in the home of his uncle Alfonso Almagià, an official of the Banca d'Italia. He lived first in Turin and then chiefly in Florence, where he attended two excellent schools, the Dante Alighieri Technical School and the

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Galileo Galilei Technical Institute. Here his master was the physicist Antonio Roiti, who played an important part in his education. By appointing the young man assistant in his own Institute, Roiti prevented him from having to give up his studies and become a bank clerk, under the weight of financial difficulties and family pressure. Thus young Vito was able to enroll in the Faculty of Science at Florence University (in 1878) and win a competition to become an internal student in the Scuola Normale Superiore di Pisa where he completed his studies.

From his early youth, two tendencies emerge which, linked together, constitute the fundamental characteristic of Volterra's thought. On the one hand we find a tendency to reduce natural problems to mathematical schemata. Almost all his biographers recall his attempt--inspired by J. Verne's novel, "De la terre à la lune"--to calculate the trajectory of a projectile fired from the earth toward the moon; an attempt, that is, at solving a particular version of the three-body problem. And so

> *at an early age he conceived the idea of studying a phenomenon by dividing the time in which it is produced into partial intervals, and studying the phenomenon during each interval, while considering the causes that produce it to be unvarying. This same idea is fundamental to the infinitesimal calculus and forms the basis for other concepts and their application by Volterra to linear substitutions, linear differential equations, functions of lines and functions depending on an infinite number of variables.* [Pérès 1954].

On the other hand Volterra always showed a decided interest in questions arising from the study of natural phenomena, avoiding problems which were too abstract or of a purely formal nature. Thus, although he was strongly influenced by having such a great master as Ulisse Dini, he soon abandoned the abstract approach of analysis in favor of Enrico Betti's point of view, the latter being more inclined to intermingle analytical questions with physico-mathematical themes. So it was not by chance that Volterra, after graduating in Physics in 1882, became assistant in Rational Mechanics for which Betti held the Chair.

These tendencies are clearly reflected in the first group of works of his youth (from 1881 to 1885) which, after two notes on discontinuous dotted functions and on the principles of integral calculus (inspired by Dini's research), were definitely oriented toward themes of a decidedly physical or physico-mathematical character, (electrochemistry, potential theory, elasticity theory).

In Volterra's view, mathematical research is always inspired by concrete problems:

Lengthy experience teaches us that particular problems arising from the examination of natural phenomena are the ones which lead to the creation of the most fruitful analytical methods, susceptible to further extension in the course of their development, while general questions that are artificially established a priori are often less profitable." [V. Volterra 1929]

This point of view may seem paradoxical in the light of contemporary axiomatic conceptions, but it was typical of the 19th-century movement in favor of "rigor": that is, of the idea that the greatest contribution toward the extension of concepts in mathematics should come from "physical and natural theories" [V. Volterra 1912]. Thus Volterra's introduction, in 1887, of the key concept of functional analysis (or "abstract analysis")-the concept of *functional* or "function of lines" as he called it--was inspired by a concrete problem: the study of "hereditary" phenomena, that is, of those phenomena in which the evolution of a system does not depend on its initial state alone, but on the whole of its past.

Volterra's predominant interest in the application of theory distracted him from the more precisely formal aspects; his definitions (particularly that of the derivative of a function of lines) were exposed to criticism on the part of Hadamard [1902]. Volterra explained his estrangement from research into the foundations of functional analysis in this way:

I had been unable to conclude this critical research because my attention had immediately been attracted in other directions: in fact, the application of theoretical principles to new problems that arose, and the possibility of solving ancient problems that had remained unsolved, excited my curiosity and my interest above all; this naturally resulted in a tendency not to proceed immediately with a profound examination of the more abstract aspects, but to leave this study till later. [V. Volterra 1929]

The period between 1890 and 1908 was highly productive: Volterra's researches (contained in more than 60 scientific notes) range from subjects including mechanics of the earth, rational mechanics, and the elasticity theory to the study of the equations of electrodynamics, and to subjects of a distinctly mathematical character within the sphere of the theory of linear differential equations or concerning the problem of the inversion of integrals.

Volterra's introduction of the concept of function of lines originated in an idea that took shape during the last years of this period: that of developing "hereditary mechanics" or, more generally, a mathematical theory of "hereditary" phenomena. In fact, between 1909 and 1914, Volterra perfected the mathematical instrument most adequate for developing this idea--the theory of integral and integro-differential equations. Consistent with his "classical" outlook, he saw in this theory an extension of the domain of intervention of the infinitesimal calculus and, therefore, of the principle of determinism (duly extended) to describe a vast class of phenomena hitherto not dealt with by physics and mathematics. This was a contribution for which Volterra is justly famous and which, in a certain sense, closes one of the two great cycles that characterize his scientific production.

In the meantime Volterra had become an important personage, not only for his research, but also on an institutional level. In 1887 he was appointed full Professor at the University of Pisa; in 1900 he succeeded Eugenio Beltrami in the Chair of Mathematical Physics at the University of Rome. He became Corresponding Member of the Accademia Nazionale dei Lincei in 1887 and National Associate in 1899. Since the end of the 19th century, Volterra had shown a lively interest in the problems of the reorganization of scientific institutions. His approach to these problems reflected his awareness of living in a period of transition to one dominated by technology. Here science would no longer be a separate branch of knowledge, but would increasingly become an instrument of active intervention in the realm of nature. This eruption into practical activities on the part of scientific research involved considerable modifications in its internal structure and in its institutional organization. In particular, two facts were emerging:

> *a closer relationship between scientists and the general public, due to a common state of mind engendered by the scientific sentiment which dominates the present world; and the great crisis which today shakes so many branches of knowledge.* [V. Volterra 1907]

It was therefore necessary to promote the foundation of scientific associations to organize research; of organisms for directing and stimulating its application; and of cultural associations, capable of developing contacts between the scientific and the outside world. Thus Volterra contributed to the foundation in 1897 of the *Società Fisica Italiana;* he founded the *Comitato di Ricerche,* which later became the *Consiglio Nazionale delle Ricerche;* he revived the *Associazione Italiana per il Progresso delle Scienze,* whose main purpose was to bring together scientists and cultured laymen. It would take too long to relate the innumerable institutional enterprises which were promoted by Volterra at a legislative level while he was a Senator of the Realm (from 1905 onward). We will cite only a few: the foundation of the Thalassographic Committee and his initiative in favor of oceanographic research; his activity with regard to the reorganization of the Schools of Engineering; his legislative intervention in favor of the control of radioactivity and a creation of a national telegraphic and telephonic network; his role as President of the *Bureau International des Poids et Mesures;* and his manifold activities concerning the application of scientific research to warfare during the First World War.

Volterra's democratic ideas and his vision of science as an international concern soon clashed with the autarchic and nationalistic conception of science inherent in Fascism. He was a strong opponent of Fascism, not only in the political field but also on a cultural level, and he opposed its objectives on restructuring scientific institutions. When he was already President of the Accademia Nazionale dei Lincei (an office which he held from 1923 to 1926, after having been Vice-President from 1920 to 1923), he signed the "Intellectual's Declaration" against Fascism and tried to resist the educational reform law (promoted by Giovanni Gentile) by forming within the Academy a commission which had the task of examining the law thoroughly and criticizing its contents. In 1931 he was one of the very few university professors who refused to swear the oath of loyalty to the Fascist regime; for this reason he lost his Chair and was discharged from all the Italian Academies and cultural institutes.

In this period, during which Volterra became progressively detached from every public activity and was increasingly obliged to look abroad for his scientific contacts, the second great phase of his scientific production began. Since the early years of the century Volterra had been interested in the possibility of applying mathematics to biology and economics [V. Volterra 1901] by introducing into these fields the analytic methods of classical determinism. This objective, consistent with his point of view, began to take shape in 1926 when Volterra, stimulated by a problem posed by his son-in-law Umberto D'Ancona, produced a model of the dynamics of two populations--one of predators and the other of preys--by means of a system of two nonlinear ordinary differential equations. He later generalized this model to increasingly complex cases [V. Volterra 1931]. In these researches Volterra pursued a dual objective: that of applying all the methods which he had elaborated, almost in a synthesis of his entire activity, and in particular the use of integro-differential equations for the description of "hereditary" phenomena; and that of stating laws for the dynamics of populations corresponding to those of physics. The latter culminated in his announcement and demonstration, in two papers, of a principle of minimum vital action analogous to that of mechanics.

Intent on these researches, but now very ill, Volterra died on October 11, 1940, in his villa at Ariccia (near Rome), in complete isolation, by will of the regime which he had opposed.

Volterra was a man of culture in the widest sense of the term. G. Castelnuovo recalls the extremely important contributions which Volterra made toward the enrichment and reorganization of the Library of the Accademia Nazionale dei Lincei [Castelnuovo 1947]. In private, too, he was a passionate bibliophile and, over the years, he had collected in his villa at Ariccia an imposing library, which comprised ancient and modern volumes, some of great value. With the same care he conserved all his papers, notes, and correspondence. And so, through the recent donation by his son and daughter, Professor Edoardo Volterra and Luisa d'Ancona Volterra, of all this material to the Accademia Nazionale dei Lincei, the Academy itself has taken possession of an imposing archive, which is of great interest to historians of science, in particular of mathematics. The archive has been consigned to the Academy in a satisfactory state of order; there is a catalogue of the manuscripts, correspondence, and other materials to aid the scholar. Here and there a few problems exist with regard to the date of some manuscripts and letters and their arrangement in chronological order, but these are not difficult to solve.

The papers conserved in the archive cover, with surprising completeness, Volterra's activities from the earliest years of his youth onward. Even his activities at school and university can be found in the 59 notebooks (which also contain notes related to later periods).

One of the most important parts of the archive is represented by the manuscripts which have been collected in 37 boxes and comprise thousands of pages. They reflect the whole of Volterra's scientific activity, in the real sense of the word, and date back to 1877 (except for a few undated manuscripts, probably written slightly earlier). The collection ends with his most recent researches in the period 1938-1940. All the preparatory work for most of Volterra's publications can be found in these pages: preliminary calculations, early rough drafts, various successive drafts, and final copies of his works. A summary list of the themes treated in his manuscripts, arranged in order of box and period, is presented below:

(1) Undated manuscripts on various subjects (probably prior to 1877). Notes from which the concept of function of lines (i.e., functional) emerges (1877 and following years).

(2) Functions of a complex variable. Potential theory. Electrochemistry. Electrostatics (1880-1883).

(3) Mechanics. Linear differential equations (1883-1886).
(4) Functions of a complex variable. Fuchsian functions. Rough draft of a scientific note of 1887 on the concept of function of lines. Linear differential equations (1886-1890).
(5) Functions of a complex variable. Functions of lines. Elasticity theory. Optics. Calculus of variations (1890-1892).
(6) Linear differential equations. Partial differential equations. Electromagnetism. Mechanics (1892-1895).
(7) Dynamics of the earth. Inversion of integrals (1895-1896).
(8) Subjects similar to those contained in (7) (1896-1898).
(9) Foundations of the theory of partial differential equations. Celestial mechanics (1899-1902).
(10) Partial differential equations. Celestial mechanics. Principle of Riemann-Dirichlet (1899-1902).
(11) Celestial mechanics. Exercises on the functions of lines. Parabolic equations. Equation of waves (1901-1905).
(12) Theory of elasticity (1905-1906).
(13) Functionals. Hyperbolic equations. Celestial mechanics (1906-1908).
(14) Mathematical physics. Magnetic rays (1908-1909).
(15) Integro-differential equations with applications. Hydrodynamics (1909-1910).
(16) Integro-differential equations (1910-1911).
(17) Integral and integro-differential equations (1910-1913).
(18) Integro-differential equations. Conference on the concept of heredity (1911-1912).
(19) Lectures on functionals. Elastic hysteresis. Partial differential equations. Integral and integro-differential equations (1912).
(20) Commemorations (Arzelà, Pasteur, Poincaré, Darboux). Integro-differential equations. Hereditary mechanics. Texts of speeches (1912-1914).
(21) Functional derivative equations. Problems depending on function of lines. Integro-differential equations. Texts speeches (1914-1919).
(22) Problems of mathematical physics and applications (dirigible). Texts of speeches. Commemoration of Dini (1915-1918).
(23) Speeches in wartime. Permutable functions. Lectures held in Texas (1918-1920).
(24) Biographical notes on mathematicians (1920-1921).
(25) Speeches held at the Accademia Nazionale dei Lincei (1921-1923).
(26) Lectures on permutable functions (1924).
(27) Speeches and commemorations. Inauguration of the Institut Henri Poincaré (1924-1926).

(28) Manuscripts and calculations on the fluctuations of animal species that live together (1926).
(29) Mathematical theory of the struggle for existence. Biological fluctuations. Biological hereditary phenomena (1926-1928).
(30) Speeches made at the Institut Henri Poincaré. Schrödinger's equation and integral equations (1928-1929).
(31) Functionals and integro-differential equations. Biological associations (1930-1932).
(32) Praise of Painlevé. Lectures on the partial derivative equations held at the Institute Henri Poincaré (1932-1935).
(33) Lectures on the biological associations at the Institute Henri Poincaré. Principle of minimum vital action (1935-1936).
(34) Functionals. Biological fluctuations (1936-1938).
(35) Biological fluctuations and relative experimental data (1938-1940).
(36) Functionals.
(37) Letters addressed to J. Pérès. Other manuscripts.

A second group of materials comprises 17 files containing (in synthesis) the following:

I-III. Reports, judgments on examination candidates, degree theses, etc. (1896-1929).
IV. Prefaces and reviews of other authors (1882-1936).
V. Materials concerning the Nobel Prize.
VI. Materials concerning the Office of Inventions and Research, the Air Force, problems of helium and radium, etc.
VII-XI. Bureau International des Poids et Mesures (1918-1940).
XII. Zoological station. International Commission for the Mediterranean. Istituto Oceanografico e Talassografico.
XIII-XIV. International Congresses of Mathematicians. Journeys to scientific institutions abroad. Accademia Nazionale dei Lincei. Accademia Pontificia, Società Italiana per il Progresso delle Scienze, Società Italiana di Fisica. Unione Matematica Italiana, etc.
XV. Questions concerning books. International Bibliographical Catalogue, etc.
XVI. Turin Polytechnic. Astronomical Observatory in Rome.
XVII. Certificates. Documents on political discrimination.

Besides these, there are 178 diplomas, certificates, and decrees of appointment, 27 decorations and medals, and an interesting collection of 290 prints and photographs of mathematicians. Finally, a large volume contains many newspaper cuttings concerning Volterra's activities.

The other very interesting section of the archive is represented by the correspondence; it is a very large collection, contained in 34 boxes comprising the letters of about 1500 correspondents (according to an approximate calculation). Almost all the great names in the scientific world of that time appear there; therefore it would be impossible to enter into detail here. [1] It should be pointed out that although all the correspondence is of interest from an institutional point of view, it is not equally so from the point of view of scientific content. It is significant to note that the scientific content in the correspondence relating to the years of the 19th century is greatly superior to that of the correspondence relating to the years of the 20th century (and diminishes in the course of time). This is undoubtedly due to a progressive abandonment of the custom of discussing scientific questions by letter and to an increasing diffusion of Congresses which permitted direct exchanges. This change is part of a profound modification in the organization of scientific research, and Volterra himself described it with great efficacy in the course of his commemoration of Henri Poincaré:

> *The scientist who, a few years ago, remained shut up and almost hidden in his study or in his laboratory, today comes into contact continually with other scholars and with the general public; he is assailed with requests from all sides, and unfortunately he is solicited to answer before an exact reply has matured in his brain. Congresses and scientific meetings have multiplied and popular talks have been added to academic lectures (...). The hurry and tumult of modern life have invaded the peaceful refuges of learned men. This movement has created a particular state of mind in scholars; it has transformed their lives, their manner of working and even of thinking.* [V. Volterra 1913]

More detailed information may be obtained by writing to the author.

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NOTE

1. However, I will prepare, in the course of my research, a complete catalogue of correspondents (with a brief summary of the content of correspondences). I also plan to publish a selection of the most significant correspondences.

REFERENCES

Castelnuovo, G. 1947. Vito Volterra. *Rendiconti delle Adunanze Solenni della Accademia Nazionale dei Lincei* 5, Roma, 5-9.
Hadamard, J. 1902. Sur les dérivées des fonctions de lignes. *Bulletin de la Société Mathématique de France* 30, 40-43.
Pérès, J. 1954. Cenni biografici. In *Vito Volterra, Opere matematiche. Memorie e note,* Accademia Nazionale dei Lincei, Rome: 1954-1962.
Volterra, E. 1967. Vito Volterra, *Dictionary of scientific biography* 10, 85-88.
Volterra, V. 1901. Sui tentativi di applicazione delle Matematiche alle scienze biologiche e sociali. *Annuario della Regia Università di Roma,* 3-28.
——— 1907. Il momento scientifico presente e la nuova Società Italiana per il Progresso delle Scienze. *Rivista di Scienza* 2, No. 1, 225-237.
——— 1912. L'evoluzione delle idee fondamentali del calcolo infinitesimale. *La Revue du Mois,* Paris.
——— 1913. L'applicazione del calcolo ai fenomeni di eredità. *La Revue du Mois,* Paris.
——— 1929. La théorie des fonctionnelles appliquée aux phénomènes héréditaires, *Revue générale des sciences pures et appliquées.* 41, 197-206.
——— 1931. *Leçons sur la théorie mathématique de la lutte pour la vie,* M. Brelot, ed., Vols. I-VI, pp. 1-214. Paris: Gauthier-Villars.
——— 1954-1962. *Opere matematiche. Memorie e note.* 5 vols. Rome: Accademia Nazionale dei Lincei.