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Ab initio approach to the non-perturbative scalar Yukawa model

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ABSTRACT

We report on the first non-perturbative calculation of the scalar Yukawa model in the single-nucleon sector up to four-body Fock sector truncation (one "scalar nucleon" and three "scalar pions"). The light-front Hamiltonian approach with a systematic non-perturbative renormalization is applied. We study the n-body norms and the electromagnetic form factor. We find that the one- and two-body contributions dominate up to coupling $\alpha \approx 1.7$. As we approach the coupling $\alpha \approx 2.2$, we discover that the four-body contribution rises rapidly and overtakes the two- and three-body contributions. By comparing with lower sector truncations, we show that the form factor converges with respect to the Fock sector expansion.

1. Introduction

Solving quantum field theories in the non-perturbative regime is not only a theoretical challenge but also essential to understand the structure of hadrons from first principles. The light-front (LF) Hamiltonian quantum field theory approach provides a natural framework to tackle this issue [1,2]. A great advantage of this approach is that it provides direct access to the hadronic observables. In the LF dynamics, the system is defined at a fixed LF time $x^+ \equiv t + z$. The physical states are obtained by diagonalizing the LF Hamiltonian operator. The vacuum in LF quantization is trivial. As a result, it is particularly convenient to expand the physical states in the Fock space. For example, a physical pion state can be written in terms of quarks (q), antiquarks ($\bar{q}$) and gluons (g) as $|\pi\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + |q\bar{q}gg\rangle + \cdots$.

In order to do practical calculations, the Fock space has to be truncated. A natural choice, taking advantage of the LF dynamics, is the Fock sector truncation, also known as the light-front Tamm–Dancoff (LFTD) method [2]. A number of non-perturbative renormalization schemes have been developed based on the LFTD [3–6]. Thus we arrive at a few-body problem and predictions can be systematically improved by including more Fock sectors. The LFTD method is a non-perturbative approach in Minkowski space, which can be compared with other non-perturbative methods, e.g., Lattice quantum field theory in Euclidean space. Of course, this approach only works if the Fock sector expansion converges in the non-perturbative region. In practice, one can compare successive Fock sector truncations and check numerically whether the relevant physical observables converge. We will see that good convergence is achieved for the scalar Yukawa model in a non-perturbative regime with a four-body Fock sector truncation. Similar results, though by a different method, were found in Refs. [7,8] for the Wick–Cutkosky model [9].

We apply this approach to a scalar version of the Yukawa model that describes the pion-mediated nucleon–nucleon interaction. The Lagrangian density of the model reads

$$\mathscr{L} = \partial_\mu N^\dagger \partial^\mu N - m^2|N|^2 + \tfrac{1}{2}\partial_\mu \pi \partial^\mu \pi - \tfrac{1}{2}\mu_0^2 \pi^2 + g_0|N|^2\pi + \delta m^2 |N|^2, \tag{1}$$

where g_0 is the bare coupling, δm^2 is the mass counterterm of the field $N(x)$. It is convenient to introduce a dimensionless coupling constant

$$\alpha = \frac{g^2}{16\pi m^2}.$$

For the sake of brevity, we refer to the fundamental degrees-of-freedoms (d.o.f.'s) $N(x)$ and $\pi(x)$ as "scalar nucleon" and "scalar pion" field respectively. We also introduce a Pauli–Villars (PV) scalar pion (with mass μ_1) to regularize the ultraviolet (UV) divergence [10]. Then, a sector dependent method known as the Fock sector dependent renormalization (FSDR) developed in Ref. [6] is used to renormalize the theory. FSDR is a systematic non-perturbative renormalization scheme based on the covariant light-front dynamics (CLFD, see Ref. [11] for a review) and Fock sector

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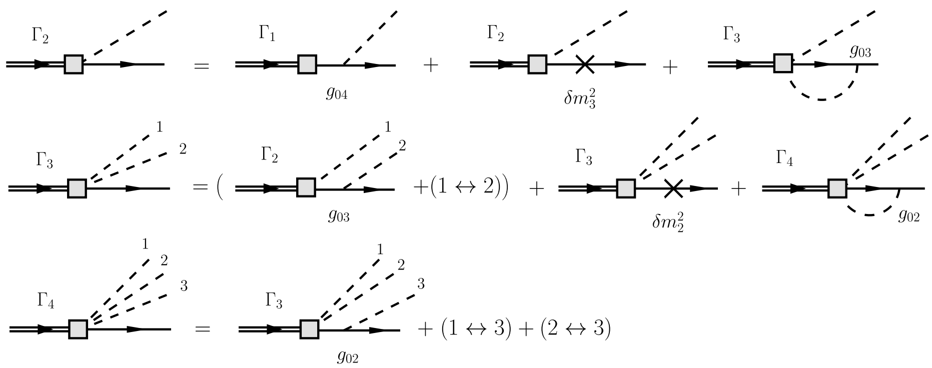

Fig. 1. The diagrammatic representation of the system of equations in the four-body truncation.

expansion. It has shown great promise in the application to the Yukawa model and QED [12,13].

The scalar Yukawa model is known to exhibit a vacuum instability [14]. It can be stabilized by either adding the quartic terms $\frac{1}{4!}\pi^4$, $\frac{1}{2}|N|^4$ and $\frac{1}{2}|N|^2\pi^2$ to the Lagrangian, or restricting the nucleon–antinucleon d.o.f. [15]. The latter leads to the exclusion of the pion self-energy correction, sometimes referred to as the "quenched approximation". For the sake of simplicity, here we study this restricted version of the theory. Then the bare mass of the scalar pion becomes the physical mass, $\mu_0 = \mu$. It should be emphasized, though, that our formalism is capable of dealing with the (scalar) antinucleon d.o.f. The scalar nucleon and scalar pion d.o.f.'s generate non-perturbative dynamics at large coupling sufficient for our purposes.

Previously, this model has been solved in the same approach up to three-body truncation (one scalar nucleon, two scalar pions) [6]. The results from the two- and three-body truncations agree at small couplings; yet they deviate in the large coupling region. Therefore, it is crucial to extend the non-perturbative calculation to higher Fock sectors. In this paper, we present the calculation of the four-body truncation (one scalar nucleon, three scalar pions). By comparing successive truncations, we can examine the convergence of the Fock sector expansion. We presented a preliminary version of this work in Ref. [16].

We first introduce our formalism in the next section. The LF Hamiltonian field theory will be briefly mentioned and the non-perturbative renormalization procedure will be explained. Then a set of coupled integral equations will be derived for the four-body truncation. In Section 3, we present the numerical results, including the calculation of the electromagnetic form factor. We conclude in Section 4.

2. Light-front Hamiltonian field theory

The LF Hamiltonian for the scalar Yukawa model is

$$\hat{P}^- = \int \mathrm{d}^3x \Big[\boldsymbol{\partial}_\perp N^\dagger \cdot \boldsymbol{\partial}_\perp N + m^2|N|^2 + \tfrac{1}{2}\boldsymbol{\partial}_\perp \pi \cdot \boldsymbol{\partial}_\perp \pi + \tfrac{1}{2}\mu_0^2\pi^2 - g_0|N|^2\pi - \delta m^2|N|^2\Big]_{x^+=0}. \tag{2}$$

The physical states can be obtained by solving the time-independent Schrödinger equation

$$\hat{P}^- |p\rangle = \frac{\boldsymbol{p}_\perp^2 + M^2}{p^+}|p\rangle, \tag{3}$$

where $\boldsymbol{p}_\perp$ and p^+ are the transverse and longitudinal momentum, respectively. Thanks to boost invariance in the LF dynamics, we can take $\boldsymbol{p}_\perp = 0$ without loss of generality.

The system is solved in the single-nucleon sector. The state vector admits a Fock space expansion,

$$|p\rangle = \sum_n \int D_n\, \psi_n(\boldsymbol{k}_{1\perp}, x_1, \ldots, \boldsymbol{k}_{n\perp}, x_n; p^2) \times |\boldsymbol{k}_{1\perp}, x_1, \ldots, \boldsymbol{k}_{n\perp}, x_n\rangle, \tag{4}$$

where $x_i \equiv \frac{k_i^+}{p^+}$, and

$$D_n = 2(2\pi)^3\delta^{(2)}(\boldsymbol{k}_{1\perp} + \cdots + \boldsymbol{k}_{n\perp})\delta(x_1 + \cdots + x_n - 1) \times \prod_{i=1}^{n} \frac{\mathrm{d}^2k_{i\perp}\mathrm{d}x_i}{(2\pi)^3 2x_i}.$$

The n-body Fock state $|\boldsymbol{k}_{1\perp}, x_1, \ldots, \boldsymbol{k}_{n\perp}, x_n\rangle$ consists $(n-1)$ scalar pions and 1 scalar nucleon. We use the last pair $(\boldsymbol{k}_{n\perp}, x_n)$ to denote the momentum of the scalar nucleon. ψ_n, known as the LF wave function (LFWF), is a boost invariant. The LFWFs are normalized to unity, $\sum_n I_n = 1$, where

$$I_n = \frac{1}{(n-1)!}\int D_n \Big|\psi_n(\boldsymbol{k}_{1\perp}, x_1, \ldots, \boldsymbol{k}_{n\perp}, x_n; p^2)\Big|^2 \tag{5}$$

is the probability that the system appears in the n-body Fock sector. In the scalar Yukawa model, these quantities are regulator independent, in contrast to more realistic theories such as Yukawa and QED. Note that $\psi_1 = \sqrt{I_1}$ is a constant.

It is convenient to introduce the n-body vertex functions,

$$\Gamma_n(\boldsymbol{k}_{1\perp}, x_1, \ldots, \boldsymbol{k}_{n-1\perp}, x_{n-1}; p^2) = (s_{1,\ldots,n-1} - p^2)\psi_n(\boldsymbol{k}_{1\perp}, x_1, \ldots, \boldsymbol{k}_{n\perp}, x_n; p^2) \tag{6}$$

for $n > 1$ and $\Gamma_1 = (m^2 - p^2)\psi_1$, where

$$s_{i_1,\ldots,i_{n-1}} \equiv (k_{i_1} + \cdots + k_{i_{n-1}} + k_n)^2 = \sum_{i=i_1}^{i_{n-1}} \frac{\boldsymbol{k}_{i\perp}^2 + \mu_{j_i}^2}{x_i} + \frac{\boldsymbol{k}_{n\perp}^2 + m^2}{x_n}$$

is the invariant mass squared of the Fock state, and μ_{j_i} $(j_i = 0, 1)$ is the mass of the i-th scalar pion. We have suppressed $\boldsymbol{k}_{n\perp}$ and x_n in Γ_n, by virtue of the momentum conservations $\boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp} + \cdots + \boldsymbol{k}_{n\perp} = 0$, $x_1 + x_2 + \cdots + x_n = 1$. For simplicity we will also omit the dependence on p^2 in Γ_n for the ground state $p^2 = m^2$.

Written in terms of the vertex functions Γ, Eq. (3) can be represented diagrammatically using the LF graphical rules [17,18] (see Ref. [11] for a review). Fig. 1 shows the diagrams for the four-body truncation.

The two-body vertex function Γ_2 plays a particular role in renormalization. It comprises all radiative corrections allowed

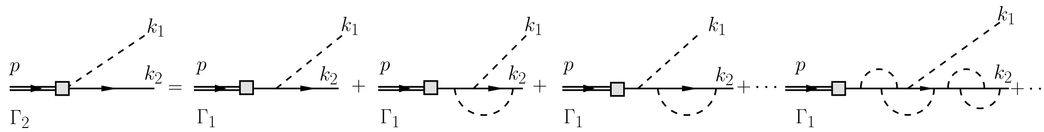

Fig. 2. The perturbative expansion of the two-body vertex function. The solid lines represent the scalar nucleons; the dashed lines represent the scalar pions; the double lines represent the dressed nucleons.

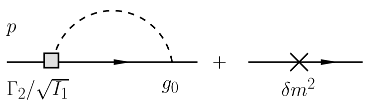

Fig. 3. The self-energy correction, loop correction Σ plus mass counterterm δm^2, expressed in terms of the two-body vertex function Γ_2. Note the external lines are amputated.

by the Fock sector truncation, including the amputated vertex $V_3(k_1, k_2, p)$ and the self-energy $\Sigma((p-k_1)^2)$ (see Fig. 2):

$$\Gamma_2(\boldsymbol{k}_{1\perp}, x_1; p^2) = Z((p-k_1)^2)V_3(k_1,k_2,p)\sqrt{I_1}, \tag{7}$$

where the function $Z(q^2) = \big(1 - \frac{\Sigma(q^2)-\Sigma(m^2)}{q^2-m^2}\big)^{-1}$ is a generalization of the field strength renormalization constant $Z = \big(1 - \frac{\partial}{\partial q^2}\Sigma(q^2)\big)^{-1}_{q^2=m^2} = I_1$. Note the presence of the scalar pion spectator, which means that in the n-body truncation, the self-energy correction in the expression for Γ_2 is the $(n-1)$-body self-energy.

The dependence of renormalization constants on the Fock sector is a general feature of the Fock sector expansion. We use g_{0n} and δm_n^2 to denote the bare coupling and the mass counterterm from the n-body truncation, respectively. According to the LSZ reduction formula, the physical coupling $g = \mathcal{T}_{fi} = \sqrt{Z}V_3^\star(k_1,k_2,p)\sqrt{I_1}$. Here "$\star$" means that V_3 is evaluated at the renormalization point, the physical mass shell $s_1 = m^2 \Rightarrow \boldsymbol{k}_{1\perp}^2 = -(1-x_1)\mu^2 - x_1^2m^2 \equiv \boldsymbol{k}_{1\perp}^{\star 2}$. These relations provide the on-shell renormalization condition [5,6,12],

$$\Gamma_2^{(n)}(\boldsymbol{k}^\star_{1\perp}, x_1; p^2 = m^2) = g\sqrt{Z^{(n-1)}}. \tag{8}$$

Here the Fock sector dependence is shown explicitly. For example, $\Gamma_2^{(n)}$ represents the two-body vertex function found in the n-body truncation. Note that $\boldsymbol{k}_{1\perp}^{\star 2}$ is negative, which means Eq. (8) has to be imposed through analytic continuation.

The two-body vertex function Γ_2 also provides a non-perturbative means to calculate the self-energy correction (see Fig. 3). Following the LF graphical rules, the self-energy in the n-body truncation is

$$\Sigma^{(n)}(p^2) = -\left(I_1^{(n)}\right)^{-\frac{1}{2}} \int \frac{\mathrm{d}^2k_{1\perp}}{(2\pi)^3} \int_0^1 \frac{\mathrm{d}x_1\, g_{0n}}{2x_1(1-x_1)} \times \frac{\Gamma_2^{(n)}(\boldsymbol{k}_{1\perp}, x_1; p^2)}{s_1 - p^2}. \tag{9}$$

Note that in our formalism the state vector in Eq. (4), rather than its one-body component, is normalized to unity. So according to the definition of the self-energy, the one-body LFWF $\psi_1 = \sqrt{I_1}$ is excluded from Γ_2 in the above expression. Then the mass renormalization condition in the on-shell scheme implies $\delta m_n^2 = \Sigma^{(n)}(m^2)$.

As mentioned, the system of equations for $\Gamma_{2\text{–}4}$ resulted from truncating Eq. (3) to at most four-body (one scalar nucleon and three scalar pions) are shown in Fig. 1. After substituting Γ_4 into the second equation and applying the renormalization condition Eq. (8), the system of equations becomes

$$\begin{aligned}\Gamma_2^{j_1}(\boldsymbol{k}_{1\perp}, x_1) = {} & g/\sqrt{I_1^{(3)}} + \frac{\delta m_3^2\, \Gamma_2^{j_1}(\boldsymbol{k}_{1\perp}, x_1)}{(1-x_1)(s_1-m^2)} \\ & + \sum_{j_2=0}^{1}(-1)^{j_2}\int\frac{\mathrm{d}^2k_{2\perp}}{(2\pi)^3}\int_0^{1-x_1}\frac{\mathrm{d}x_2\, g_{03}(\xi_{21})}{2x_2(1-x_1-x_2)} \\ & \times\left(\frac{\Gamma_3^{j_1j_2}(\boldsymbol{k}_{1\perp}, x_1, \boldsymbol{k}_{2\perp}, x_2)}{s_{12}-m^2}\right. \\ & \left. - \frac{\widetilde{\Gamma}_3^{0j_2}(\boldsymbol{k}^\star_{1\perp}, x_1, \boldsymbol{k}_{2\perp}, x_2)}{s^\star_{12}-m^2}\right),\end{aligned} \tag{10}$$

$$\begin{aligned}&\Gamma_3^{j_1j_2}(\boldsymbol{k}_{1\perp}, x_1, \boldsymbol{k}_{2\perp}, x_2) \\ &\quad = Z^{(2)}(q_{12}^2)\left[\frac{g_{03}(\xi_{21})\,\Gamma_2^{j_1}(\boldsymbol{k}_{1\perp}, x_1)}{(1-x_1)(s_1-m^2)} + g_{02}^2\sum_{j_3=0}^{1}(-1)^{j_3}\right. \\ &\quad \times\int\frac{\mathrm{d}^2k_{3\perp}}{(2\pi)^3}\int_0^{1-x_1-x_2}\frac{\mathrm{d}x_3}{2x_3(1-x_1-x_3)(1-x_1-x_2-x_3)} \\ &\quad \left.\times\frac{1}{s_{123}-m^2}\frac{\Gamma_3^{j_1j_3}(\boldsymbol{k}_{1\perp}, x_1, \boldsymbol{k}_{3\perp}, x_3)}{(s_{13}-m^2)}\right] + (1\leftrightarrow 2)\end{aligned} \tag{11}$$

where $\xi_{21} = x_2/(1-x_1)$, $s^\star_{12} = \frac{\boldsymbol{k}^{\star 2}_{1\perp}+\mu_0^2}{x_1} + \frac{\boldsymbol{k}^2_{2\perp}+\mu_{j_2}^2}{x_2} + \frac{(\boldsymbol{k}^\star_{1\perp}+\boldsymbol{k}_{2\perp})^2+m^2}{1-x_1-x_2}$, $q_{12}^2 = m^2 - (1-x_1-x_2)(s_{12}-m^2)$, and $Z^{(2)}$ comes from combining the two-body self-energy corrections. We have included the PV scalar pions ($j=1$) in the equations along with the "physical" pions ($j=0$). As mentioned, g_{02}, δm_2^2, g_{03}, δm_3^2 are sector dependent renormalization "constants" obtained from the two- and three-body truncations [6]. In fact, g_{03} depends on the momentum fraction x, which is a manifestation of the violation of the Lorentz symmetry by the Fock sector truncation [12]. $\widetilde{\Gamma}_3$ is an auxiliary function that satisfies the integral equation,

$$\begin{aligned}&\widetilde{\Gamma}_3^{0j_2}(\boldsymbol{k}^\star_{1\perp}, x_1, \boldsymbol{k}_{2\perp}, x_2) \\ &\quad = Z^{(2)}(q_{12}^{\star 2})\left[\frac{g_{03}(\xi_{12})\Gamma_2^{j_2}(\boldsymbol{k}_{2\perp}, x_2)}{(1-x_2)(s_2-m^2)} + g_{02}^2\sum_{j_3=0}^{1}(-1)^{j_3}\right. \\ &\quad \times\int\frac{\mathrm{d}^2k_{3\perp}}{(2\pi)^3}\int_0^{1-x_1-x_2}\frac{\mathrm{d}x_3}{2x_3(1-x_1-x_3)(1-x_1-x_2-x_3)} \\ &\quad \times\frac{1}{s^\star_{123}-m^2}\left(\frac{\widetilde{\Gamma}_3^{0j_3}(\boldsymbol{k}^\star_{1\perp}, x_1, \boldsymbol{k}_{3\perp}, x_3)}{s^\star_{13}-m^2}\right. \\ &\quad \left.\left. + \frac{\Gamma_3^{j_2j_3}(\boldsymbol{k}_{2\perp}, x_2, \boldsymbol{k}_{3\perp}, x_3)}{s_{23}-m^2}\right)\right],\end{aligned} \tag{12}$$

where $\xi_{12} = x_1/(1-x_2)$, $q_{12}^{\star 2} = m^2 - (1-x_1-x_2)(s^\star_{12}-m^2)$, $s^\star_{123} = \frac{\boldsymbol{k}^{\star 2}_{1\perp}+\mu_0^2}{x_1} + \frac{\boldsymbol{k}^2_{2\perp}+\mu_{j_2}^2}{x_2} + \frac{\boldsymbol{k}^2_{3\perp}+\mu_{j_3}^2}{x_3} + \frac{(\boldsymbol{k}^\star_{1\perp}+\boldsymbol{k}_{2\perp}+\boldsymbol{k}_{3\perp})^2+m^2}{1-x_1-x_2-x_3}$. Note that Eq. (10) can be eliminated by substituting Γ_2 into Eq. (11) and Eq. (12).

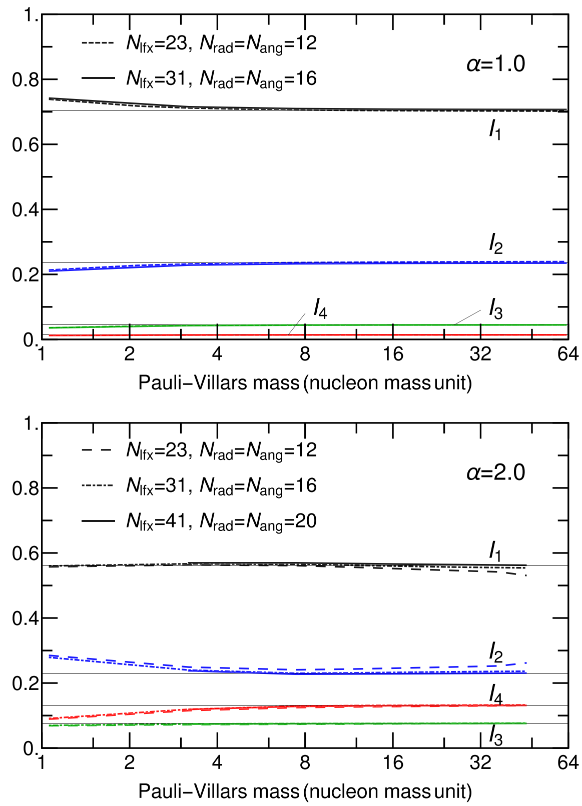

Fig. 4. The Fock sector norms I_{1-4} as a function of the PV mass μ_1 for $\alpha = 1.0$ (top), $\alpha = 2.0$ (bottom). Results evaluated on different grids are shown.

3. Numerical results

We employ an iterative procedure to solve Eqs. (10)–(12). The momenta are discretized on chosen grids in the transverse radial and angular coordinates as well as in the longitudinal coordinate, where the grid sizes are controlled by the number of abscissas, $N_{\rm rad}$, $N_{\rm ang}$, and $N_{\rm lfx}$. Then the integrals are approximated by the Gauss–Legendre quadrature. We start with an initial guess of the vertex functions and update them iteratively, until reaching a pointwise absolute tolerance $\max\{|\Delta\Gamma|\} < 10^{-4}$. We solved the system at $m = 0.94$ GeV, $\mu_0 = 0.14$ GeV. The numerical results are obtained using Cray XE6 Hopper at NERSC.

Fig. 4 plots the Fock sector normalization factors I_n (see Eq. (5)) as a function of the PV mass μ_1 for two selected coupling constants. It shows that for sufficiently large grids, I_n converge as μ_1 increases. However, for a fixed grid, increasing μ_1 would increase the numerical error while decreasing the systematic error introduced by the finite regulator, as larger μ_1 requires more coverage in the UV hence larger grid size. A PV mass $\mu_1 = 15$ GeV suffices for our purposes here.

There exist two critical couplings at $\alpha_c \approx 2.6$ and $\alpha'_c \approx 2.2$. In the two-body truncation, one finds the bare coupling,

$$\frac{1}{g^2} - \frac{1}{g_{02}^2} = \frac{1}{16\pi^2 m^2}\left[f\left(\frac{\mu_0}{m}\right) - f\left(\frac{\mu_1}{m}\right)\right],$$

where $f(\lambda) = \int_0^1 dx\, x(1-x)/((1-x)\lambda^2 + x^2)$ and $f(\lambda \to \infty) = 0$. If the physical coupling constant $\alpha > \alpha_c \equiv \pi/f(\mu_0/m)$, the two-body bare coupling g_{02} diverges at some finite PV mass. Such a singularity in g_{02} (known as the "Landau pole" in a similar case in QED) propagates from the two-body truncation to the four-body truncation via g_{02} used in the FSDR. At $\alpha = \alpha'_c$, the determinant of the Hamiltonian in the three-body truncation crosses zero. Similarly, this singularity propagates from the three-body truncation to the four-body truncation and the iterative procedure in the four-body truncation diverges at $\alpha \gtrsim \alpha'_c$.

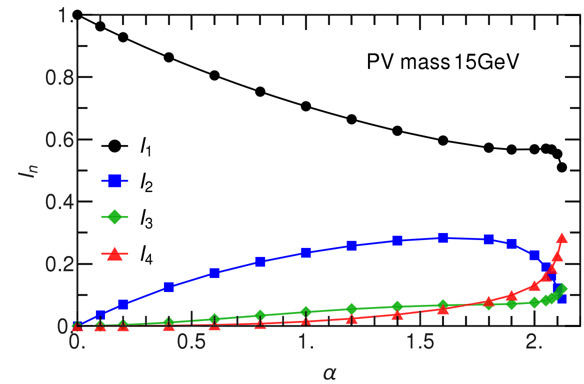

Fig. 5. The Fock sector norms I_{1-4} as a function of the coupling constant α. Results are evaluated on the grid $N_{\rm lfx} = 41$, $N_{\rm rad} = N_{\rm ang} = 20$, with a PV mass $\mu_1 = 15$ GeV.

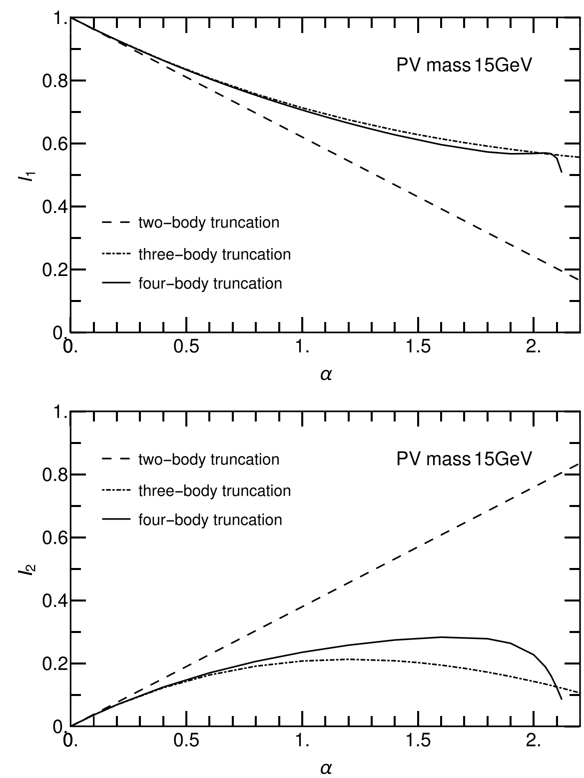

Fig. 6. Comparison of the Fock sector norms I_1 (top) and I_2 (bottom) from successive two-, three- and four-body truncations.

Fig. 5 shows the contribution of each Fock sector in the four-body truncation for couplings up to $\alpha = 2.12$. A natural Fock sector hierarchy $I_1 > I_2 > I_3 > I_4$ can be observed, up to $\alpha \approx 1.7$. Beyond $\alpha \approx 1.7$, I_4 exceeds I_3 and begins a steep climb with increasing α. Meanwhile, I_2 turns over and starts to fall. The net effect is that I_4 exceeds I_2 and I_3 at about $\alpha \approx 2.1$. Clearly, as we approach α'_c, dramatic changes in the I_n's are emerging and it appears that the Fock space expansion breaks down. Nevertheless, the lowest sectors $|N\rangle + |N\pi\rangle$ are observed to dominate the Fock space up to $\alpha \approx 2.0$, where these two sectors constitute 80% of the full norm.

Fig. 6 compares the Fock sector norms from the four-body truncation with their counterparts from the two- and three-body truncations. The result suggests a convergence as the number of

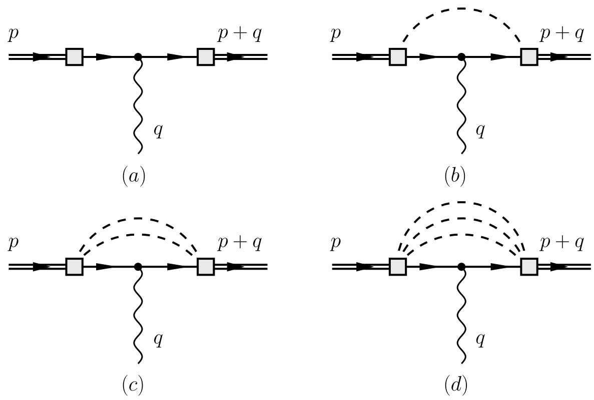

Fig. 7. The various Fock sector contributions to the electromagnetic form factor for the scalar nucleon (solid lines). The dashed lines represent the internal scalar pions. The wavy lines represent the external photons.

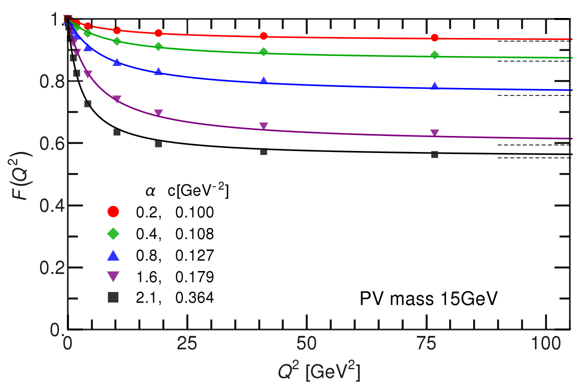

Fig. 8. The elastic electromagnetic form factor $F(Q^2)$ in the four-body truncation for couplings $\alpha = 0.2, 0.4, 0.8, 1.6,$ and 2.1. The numerical results (symbols) are fitted by Eq. (15) (lines).

constituent bosons increases, especially for the coupling below $\alpha \approx 1.0$. Note that the one-body norm I_1 changes little from the three-body truncation to the four-body truncation, even around $\alpha \approx 1.7$.

The obtained LFWFs are now available for computing physical observables. Here we consider the elastic electromagnetic form factor for photon coupling to the scalar nucleon, which is obtained from the matrix element of the "+" component of the current (see Fig. 7),

$$\langle p+q|J^+(0)|p\rangle = 2p^+ F(Q^2), \tag{13}$$

where $q^+ = 0$, $Q^2 = -q^2 = \boldsymbol{q}_\perp^2 > 0$. In LF dynamics, the form factor obtains the form [19]:

$$F(Q^2) = \sum_n \frac{1}{(n-1)!} \int D_n \psi_n^*(\boldsymbol{k}'_{1\perp}, x_1, \dots, \boldsymbol{k}'_{n\perp}, x_n) \times \psi_n(\boldsymbol{k}_{1\perp}, x_1, \dots, \boldsymbol{k}_{n\perp}, x_n) \tag{14}$$

where $\boldsymbol{k}'_{i\perp} = \boldsymbol{k}_{i\perp} - x_i \boldsymbol{q}_\perp$ $(i = 1, 2, \dots, n-1)$, for the spectators and $\boldsymbol{k}'_{n\perp} = \boldsymbol{k}_{n\perp} + (1 - x_n)\boldsymbol{q}_\perp$ for the struck parton. Fig. 8 shows the form factor for some selected couplings. In the limit of $Q^2 \to 0$, $F(0) = 1$, consistent with the charge conservation; in the limit of $Q^2 \to \infty$, $F(\infty) = I_1$, representing a point-like charge. The form factors can be approximated by

$$F(Q^2) \approx \frac{1 + c I_1 Q^2}{1 + c Q^2}. \tag{15}$$

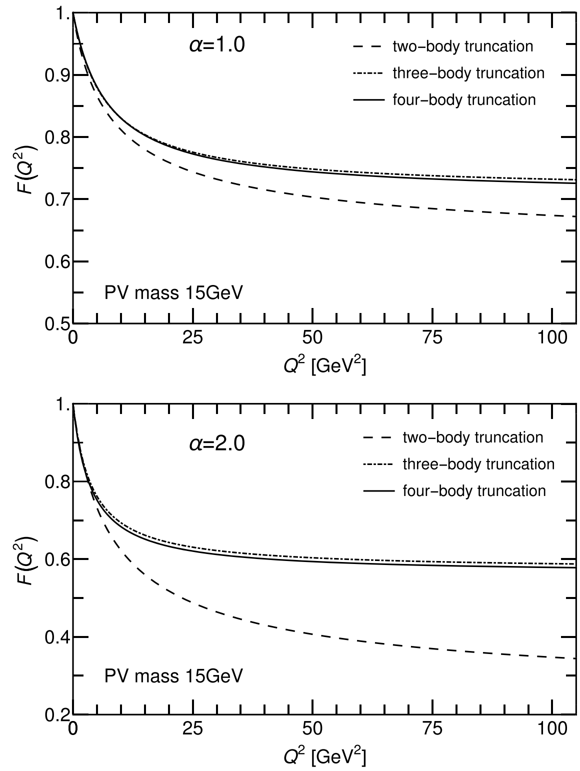

Fig. 9. Comparison of the form factors calculated in the two-, three- and four-body truncations at $\alpha = 1.0$ (top) and $\alpha = 2.0$ (bottom). The three- and four-body form factors are fitted by Eq. (15).

Fig. 9 compares the form factors obtained from the two-, three- and four-body truncations for two selected couplings. The three- and four-body truncation results show good agreement even at the non-perturbative couplings, suggesting a reasonable convergence with respect to the Fock sector expansion.

4. Discussion and conclusions

We solve the single-nucleon sector of the scalar Yukawa model in light-front dynamics within a four-body (up to one scalar nucleon and three scalar pions) Fock sector truncation. Fock sector dependent renormalization is implemented. The coupled system of linear integral equations is derived and solved numerically. The numerical study of the Fock sector norms suggests that up to $\alpha \approx 1.7$ the system is dominated by the lowest Fock sectors. By comparing the form factors from successive Fock sector truncations (two-, three- and four-body), we find that the Fock space expansion of the form factor for the scalar nucleon converges as the number of scalar pions increases even in the non-perturbative region.

Solving the one-nucleon sector is also the first step for the study of the two-nucleon sector – a bound-state problem, which has been extensively studied in various approaches (see, e.g., [20] and the references therein). However not all these approaches are from first principles. In our approach, the two-nucleon sector obeys similar integral equations. The bare couplings and the mass counterterms, according to FSDR, are already provided by the one-nucleon sector (up to three dressing pions). Therefore, our approach allows a systematic study of the theory with a non-perturbative renormalization.

This calculation demonstrates that the light-front Tamm–Dancoff method, equipped with the Fock sector dependent renormalization, is a general *ab initio* non-perturbative approach to quantum field theories. While the solution of the scalar Yukawa model may be useful for, e.g., chiral effective field theory studies, this approach has also been applied to more realistic field theories, including the Yukawa model (truncation up to one spinor and two scalars) [13] and QED (truncation up to one electron and two photons) [5]. In these theories, the vertex functions also diverge, in contrast to the scalar Yukawa model. However, after the renormalization, the physical observables converge as expected. Nevertheless, the study of the higher Fock sector expansion in these models is in principle similar to the current one, which indicates the potential of this approach as an alternative to other first-principle methods, e.g. the lattice gauge theory, especially in the study of hadron structures.

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