French, but also for anybody who intends to write a professional historical paper in English in which he has to quote from Cauchy's book.

This book is useful and well done.

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The History and Development of Nomography
By H.A. Evesham. (Docent Press). 2011 (original copyright 1982). 267 pp., paperback

Nomography is loosely defined as the theory and practice by which the results of geometry are used to facilitate numerical calculation through graphical representations of formulae. It has had a remarkable history. Having its origins in various graphical attempts to ease practical calculations such as the conversion to metric measurements in France in the late 18th century, it grew to a subject in its own right through the work many mathematicians and engineers. Papers began to appear in the 1840s on the effect of deformation on such graphical representations, referred to as nomograms, with the aim of making them more readable. In the following decades material on analytical criteria for representation by graphical means was produced. The first publication [d'Ocagne, 1884] by Maurice d'Ocagne, the man responsible for naming and organizing the discipline, came out in 1884; it described a type of nomogram used to this day. From that year until the early 1930s the subject experienced its greatest progress. This was a period which saw the arrival of three systematic works by d'Ocagne and an expanding literature of both a pure and applied nature. The nomograms themselves found widespread use, first with the expansion of the French railways in the 1840s, and later, among other projects, in irrigation efforts in Egypt in the early 20th century. The theoretical side experienced a lull for several decades following the 1930s but interest was renewed in the late 1950s and early 1960s, when Russian mathematicians in particular paid increasing attention to the mathematics of nomogram construction. The introduction of electronic calculators in the 1970s and more sophisticated electronic computing devices in later years relegated nomography to lesser importance, although nomograms have been in constant use from the early 20th century to the present. In very recent years there has been a resurgence in interest in the subject due in part to the ability to create nomograms using present-day computers. For this aspect one can consult, for example, the beautiful and sometimes fanciful constructions of Doerfler [2006].

The relationship between mathematics and nomography is a fascinating one. In the broadest sense, of course, the subject is mathematical in that geometry is a branch of mathematics as are the analytical relations nomography attempts to render calculable.

However, as a glance at any textbook of the subject shows, its exposition uses a set of mathematical ideas which includes determinants and projective transformations in its more elementary stages and sophisticated algebraic ideas and partial differential equations in its more theoretical aspects. But even this casual survey does not do justice either to the broad range of mathematics which can be invoked in discussions of nomographic problems or the role that pure mathematics played in its development. This material is for the most part available only in original papers which take some effort to find.

It is thus with gratitude that we can welcome the issue by Docent Press of *The History and Development of Nomography* by H.A. Evesham, the most extensive historical treatment of the subject available. (The quotes and pagination for this paragraph are taken from the "Introduction" of Evesham's doctoral thesis of 1982. Docent Press has reproduced the thesis in the volume under review; the reproduction omits this introductory section, however. In the paragraphs after this one, the pagination from the new edition is used.) Evesham's perspective on the subject is that of the mathematician, and he notes early in his work that, although nomography is known primarily through its engineering role, "It [nomography] is unusual in the sense that it lies outside the experience of many present day [1982] mathematicians and it is interesting because difficult mathematical problems arise from simple ideas" [Evesham, 2011, p. 2]. He has chosen to "trace the theoretical development of the subject viewed as a branch of mathematics" [p. 6]. Of particular interest to the author is the "development of a practical idea into a theoretical structure .... This development was necessary because the improvements of the practical idea, necessary to make its use more effective, gave rise to theoretical problems of some complexity" [pp. 6–7]. He adds that "To those who believe that much of Pure Mathematics originates as the refinement of practical ideas, the history of nomography makes and interesting case study" [p. 7]. He believes that the subject is of interest to general historians of science, an assertion which has found confirmation in some recent papers [Hankins, 1999; Tournes, 2000, in press].

Throughout the book Evesham details the development of two main mathematical ideas relevant to nomography, mentioned in the introductory paragraph above. The first is the problem of anamorphosis, roughly, the mathematics involved in any graphical transformation of a curve into a straight line. This problem arises due to the need to render nomograms more easily constructible and readable. The second is the determination of whether an analytical relationship among variables can be represented by a nomogram, a problem of obvious interest for the applicability of nomography. Using these two ideas he shows how in different eras the development of nomography runs parallel to and is grounded in the pure mathematics of the day. He also demonstrates the interdependence of this mode of computation with pure mathematics: a computational issue can give rise to purely mathematical problems and concepts, and solutions of these problems as well as new mathematical considerations can be used to improve the construction of the diagrams, especially by decreasing the effort involved.

The volume begins with a brief chapter on the "Origins of Geometric Computation", in which some antecedents of nomography are considered. One prominent ancestor was L. Pouchet's 1795 work on graphical scales for conversion of weights in the new metric system. This publication included graphical methods of computation for the four elementary arithmetic operations. The graphs produced would eventually become known as intersection nomograms, that is, graphs in which a function of two variables is evaluated by locating each domain variable on its own curve and finding the intersection of these curves; this point runs through a third curve on which the value satisfying the relation is read. The second chapter covers the development of nomography as a distinct discipline through the end of the 19th century. This begins with consideration of the 1843 work of L. Lalanne, a civil engineer working on the French railway system. Lalanne was the first to to note that anamorphosis of an intersection nomogram will not change the relative positions and intersections of the curves, and hence the computed values remain intact under such a transformation. Those transforms which carry curves into straight lines are highly desirable, and Lalanne gives illustrations of such transforms. This was followed by an 1867 paper of P. de Saint-Robert, who was the first to arrive a sufficient condition for the expression of an equation $f(x, y, z) = 0$ in the form $Z(z) = X(x) + Y(y)$, a form which allows the relation to be expressed as a nomogram. The condition uses partial differential equations. Evesham notes the significance of the result: it "gives an effective procedure for solving the problem, if it can be solved, which leaves nothing to insight or intuition" [p. 22]. This tension between the intuitive methods for nomogram construction, exemplified by Lalanne's work, and the need for exact mathematical criteria such as Saint-Robert's is a theme in the history of nomography which is amply developed in the book.

The chapter continues with the work of the J. Massau, a Belgian railway engineer, who was concerned with the large amount of effort required to produce nomograms; his mathematical work attempts to address this issue. Although it is noted that his signal contribution was the introduction of determinants into the discipline, it is a mark of Evesham's thorough efforts that he digs out many more aspects of Massau's work, including ideas which have not "stood the test of time" [p. 41], but which are of mathematical significance. Next comes the first contribution of Maurice d'Ocagne, appearing in the same year as Massau's paper (1884), when the former was a 22-year-old student engineer. With it d'Ocagne introduced what is now known as the alignment nomogram: a relationship $f(x, y, z) = 0$ is evaluated by the introduction of a scale for each variable. Two variable values are chosen on their respective scales, a straight line connects the two points, and the intersection of the line with the third scale occurs at the value of that variable which, together with the first two, satisfies the relation. d'Ocagne's first alignment nomogram was one for the solution of the depressed cubic, earlier treated as an intersection nomograpm by Lalanne. To read modern treatments of the construction of this type of graph is to miss the rich context in which it was created, and again it is a strong feature of Evesham's account that it emphasizes the tradition of co-ordinate systems of the day in which it arose, among them that of the German mathematician and physicist J. Plucker. The 1883 hexagonal nomograms of C. Lallemand are described, as are more methods to develop criteria, similar to that of Saint-Robert, for the simplification of the form of an analytic expression for nomographic purposes.

There follows an appraisal of the first book to summarize the theory and practice of nomography: d'Ocagne's *Nomographie-Les Calculs Usuels effectués au moyen des Abaques*, which appeared in 1891 [d'Ocagne, 1891]. Evesham discusses its contents, importantly noting that d'Ocagne, although an engineer, meant the book to be a mathematical one, not just a collection of techniques for construction of the graphs. d'Ocagne himself provides an analogy: he compares his newly named Nomography with Descriptive Geometry, noting the latter's reliance on Pure Geometry while his creation rests on Analytic Geometry. Among the contributions in this volume is the use of homographic transformations, another mathematical tool to change a nomogram into a more easily readable form. The contents of d'Ocagne's paper at the 1893 World's Columbian Exposition in Chicago is noted, along with some debatable conjectures as to his status as a mathematician. Evesham uses nomography as a springboard for an eight-page discussion of the spread of graphical ideas to Great Britain at the end of the 19th century, a discussion in which opinions are offered on the relative scientific and technological education of various groups of engineers in that country at that time. The chapter concludes with a consideration of the main theoretical problem of alignment nomograms, that of determining necessary and sufficient conditions under which a relation of the form $f(x, y, z) = 0$ can be expressed in such a nomogram, and its first attempted solution in 1898 by E. Duporcq. Again, the evaluation of the work is important: Evesham finds it of little practical value but suggests that it focused attention on the problem; this resulted in an extensive line of research which continued well into the next century. Capsule summaries of the lives of Lalanne, Lallemand and d'Ocagne are given; though not stated as such at this point, this is a clear opportunity for present day historians to expand upon the stories of some key historical figures in nomography, not the least d'Ocagne himself. The end of the 19th century saw the publication of the first standard text on the subject, d'Ocagne's *Traité de Nomographie* [d'Ocagne, 1899], at which point nomography could legitimately be considered a distinct subject in its own right.

It is indicative of the increased activity in the field and Evesham's comprehensive investigation of the work of that era that Chapter 3, though roughly 80 pages in length, covers only the first decade of the 20th century. First analyzed are d'Ocagne's *Traité* and the work of R. Soreau. The latter's efforts offered new mathematical concepts, such as a slightly more general definition of linear dependence of functions, with which to analyze and classify nomograms. The review of the *Traité* includes d'Ocagne's analysis of thirteen types of nomographic forms by their algebraic equations. Accompanying this list is an analysis of his attempt to form nomograms of such equations, assuming only rational functions appear in the form. This is a problem in pure algebra, the reduction of an algebraic form to one of several canonical types up to homographic transformation, and the solution is presented by Evesham. This type of problem, in his opinion, was included by d'Ocagne to emphasize the depth of the mathematics involved in his discipline. The *Traité* itself is rightly described as having been the model for most subsequent texts on nomography. Evesham's discusses Hilbert's 13th problem, which concerned the possibility of the solution of the general seventh degree equation. In his statement of this problem Hilbert referenced nomography; thus his problem list brought the discipline to the attention of pure mathematicians who otherwise might not have been aware of it. A careful explanation of the role that nomography played in Hilbert's thinking on this problem is given. d'Ocagne's swift reply, in which he described a special nomogram with a movable element to solve the equation, is also analyzed.

A section of Chapter 3 is devoted to the spread of nomography from France as a result of the publication of the *Traité*, with the most attention paid to its reception in Great Britain. Curiously, little is mentioned of its migration to America, where it in fact received much attention in the 1910s. Speculation is given for nomography's growth in other countries, especially Poland and Russia in later decades. In the latter case, the interest in the 1950s, for example, is accounted for by a need to catch up with the West's rapid development of electronic calculation; nomography is seen as an "interim measure to narrow the gap" [p. 121].

Roughly 35 pages of this chapter describe the contributions of one J. Clark, a Professor of Mathematics at L'École Polytechnique in Cairo, about whom little is known even at the present time. In it one sees again the depth of Evesham's scholarship, for in no other reference of which this reviewer is aware is any comparable treatment of this material available. It would be folly to attempt a summary here, but one result of Clark's work was the production of alignment nomograms whose scales are cubic equations such as the Folium of Descartes. These have achieved a measure of exposure in the present day [Doerfler, 2006]. Evesham compares Clark's approach, which de-emphasized the application of anamorphosis to produce straight line nomograms, to the "abandonment of the parallel principle in Geometry, and which led in turn to the discovery of non-Euclidean geometries" [p. 123]. Three canonical forms for Clark's cubic nomograms are given. Although the point hardly needs to be made, it is a measure of the progress of the subject that the depth of Clark's mathematics and the aesthetic appeal of his nomograms stand in contrast to the everyday nomographic needs of the engineer. Publications by d'Ocagne which comment on aspects of Clark's work are examined, and in the process Evesham expends a great deal of effort on clarifying issues of priority for certain nomographic ideas. The chapter ends with a note that, after this hectic decade, the pace of new developments became more "leisurely" [p. 172].

Chapter 4, covering the period from the 1910s through the 1950s, considers four substantial mathematical contributions to the problem of anamorphosis and nomogram construction, each different in spirit. In 1912 T.H. Gronwall provided a necessary and sufficient condition for the equation $f(x, y, z) = 0$ to be reduced to alignment form; the condition involves two lengthy and complicated partial differential equations. (Again Evesham's thoroughness is such that he spotted two typographical errors in the equations.) This was quickly followed in 1915 by an alternative criterion provided by O.D. Kellogg, in which the concept of linear independence of functions of several variables is key. Kellogg and Gronwall were mathematicians working in the United States at this time; and although Gronwall's background included a stint as a civil engineer during the 1900s, neither paper was intended to be of great practical use. The only reference in English of which this reviewer is aware which treats either of these papers in any comparable detail is [Epstein, 1958), which in fact devotes a chapter to the application of some of Kellogg's ideas, as well as some space to Clark's earlier work. Evesham devotes 10 pages to development of these ideas. The scene then shifts to 1959 and Poland and Russia. From the former came the work of M. Warmus, whose direction was algebraic and led to a "Scheme of Computation", showing how a given form can be reduced to a determinant form (and hence usable for nomogram construction) via a lengthy classification process. In the development, an interesting theorem on the linear independence of a collection of functions of one variable is proved. A final effort, due to G.E. Dzems-Levy in Russia, involved "approximate nomograms". Evesham notes how characteristic of the times this type of treatment was, "times in which the development of computational aids was making approximation techniques more readily acceptable" [p. 191]. The final page of this chapter contrasts the four efforts in terms of practicality and philosophical position inherent in the solutions.

The book's penultimate chapter is devoted to "Later Developments", largely an exploration of volume 4 of the 1959 issue of the Russian journal *Vychislitelnaya Matematika*, an issue devoted entirely to nomography. Evesham, in the midst of an exposition of these mathematical papers, pauses for a paragraph to note that in one entry the question of calculating generalized Laguerre polynomials arises. He speculates on a possible connection between these calculations and the contemporary use of the polynomials in guided missile weaponry. Another paper discusses the use of nomograms to study the behavior of functions, rather than merely to aid in their computation. Again the theoretical development of nomography is seen to draw on such sophisticated and modern mathematics as W. Blaschke's book on the theory of nets [Blaschke and Gerrit, 1938], which is used in a contribution from the collection, and it is pointed out that Blaschke himself wrote a paper on nomography. At the end of the chapter the 1970s work of one last mathematician, R.C. Buck of the University of Wisconsin, is examined, and in the process the anticipation of one of his results by an earlier Russian worker is revealed.

The concluding chapter briefly considers the question of the future of nomography, especially in the light of the the availability of electronic computational devices, and raises the possibilities for future historical work. Evesham in 1982 was not sanguine about the future of his subject for several reasons, and in a prescient statement he offers the following observation: "This is not to say that whenever a [electronic] calculator is used it will produce a result which in some way is better than that which would have been produced by a nomogram, but that the former has some kind of psychological advantage over the latter. An answer is displayed in lights and therefore has some veracity although the calculation which led to it may not have been at all appropriate" [p. 245]. This remark presages the concern with the "computer as oracle" which has become so prevalent as the use of the computer in the classroom and the workplace progressed in the years following the publication of this book. Evesham offers as subjects for future investigation geometric computation before 1840, the use of nomograms in certain fields, biographical works on some of the principals in the story of nomography such as Clark, Gronwall, Kellogg, Warmus, and Dzems-Levy, and further exposition of Hilbert's 13th problem.

In a sense one can summarize Evesham's accomplishment by saying that he has done for nomography and the nomogram what several historians of the same era were beginning to do for the electronic digital computer. In Phillips [1987] one finds essays by Martin Davis [pp. 137–165] and William Aspray [pp. 166–194] on the latter subject. The first of these sets the development of the computer in the context of the abstract theories of computation developed by mathematical logicians such as Leibniz, Gödel, and Turing; the second discusses the use of the computer to study certain theoretical problems at the Institute for Advanced Study at Princeton. In the cases of both digital and analog computation the devices can be used with no knowledge of the mathematics related to them, but their connection to deep mathematics is evident on closer examination. The omnipresence of digital computers tends to obscure the history and mathematics of analog methods. This volume is among other things a corrective to this imbalance; it is unique in its scope and depth, and likely will not be superceded by a similar study. It should be of interest to anyone curious about the parallel development of significant mathematics with routine computational work and desirous of understanding how one force drove the other through a roughly century-and-a-half long story.

References

Blaschke, W., Gerrit, Bol, 1938. Geometrie der Gewebe. Springer-Verlag.

d'Ocagne, M., 1884. Procède Nouveau de calcul Graphique. Annales des Ponts et Chaussées 8, 531–540.

d'Ocagne, M., 1891. Les Calcules Usuels effectués au moyens des Abaques. Gauthier-Villars, Paris.

d'Ocagne, M., 1899. Traité de Nomographie. Gauthier-Villars, Paris.

Doerfler, R., 2006. Dead Reckonings: Lost Art in the Mathematical Sciences. Available from: <http://www.myreckonings.com/wordpress/>.

Epstein, L.I., 1958. Nomography. Interscience Publishers, Inc., New York.

Evesham, H.A., 2011. The History and Development of Nomography. Docent Press, original publication 1982.

Hankins, T.L., 1999. Blood, dirt, and nomograms, a particular history of graphs. Isis 90, 50–90.

Phillips, E.R. (Ed.), 1987. Studies in the History of Mathematics. MAA Studies in Mathematics, vol. 26. The Mathematical Association of America, Providence.

Tournes, D., 2000. Notes et Débats Pour Une Histoire Du Calcule Graphique. Revue d'Histoire des Mathematiques 6, 127–161.

Tournes, D. (ed.), in press. Histoire Du Calcul Graphique. Cassini, Paris.

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From a Geometrical Point of View: A Study of the History and Philosophy of Category Theory

By Jean-Pierre Marquis. Logic, Epistemology, and the Unity of Science. (Springer). 2010. ISBN 978-9048181179. 320 pp. $199.

Felix Klein's 1872 Erlangen Program has always been more famous than familiar. In fact, Klein's greatness lay less in his own mathematics than in his ability to recognize and promote the best new mathematics of his time. It is natural that his program be known for its overall vision more than for the particulars.

The program grew from many sources including Plücker's projective geometry, Lie's symmetry groups, Clebsch on algebraic invariants, and the rise of topology under the name *analysis situs*. Projective geometry and invariant theory faded from prominence over the next decades while topology, Lie groups, and wider ideas of symmetry and transformation exploded, notably encouraged by general relativity and quantum mechanics.

The high opinion of Klein's program today reflects that later trend. And this is entirely fair. Klein never suggested limiting geometry to the methods he knew. He called attention to the rise and the unity of a broad family of great new methods; and he had a synoptic vision of those methods far ahead of anyone else.

Eilenberg and Mac Lane gave a nice historical nod in 1945 in their first paper on general category theory, saying "This may be regarded as a continuation of Klein's *Erlanger Programm*, in the sense that a geometrical space with its group of transformations is generalized to a category with its algebra of mappings" (quoted on p. 9). People have wondered ever since precisely what they meant, and Marquis makes that the start for a wide-ranging reflection on category theory.

Marquis has achieved two things. One is a very interesting history and philosophy of category theory accessible to readers with little knowledge of category theory yet valuable to anyone interested in the subject. The other is to open several lines of inquiry into just how category theory relates to Klein's program. The upshot gives Klein no very specific role in fact. Marquis argues that Eilenberg and Mac Lane "did not extend Klein's program as such, although they clearly made an effort to extend a part of it" (p. 4), and even this claim must be qualified since "the connection with Klein's program probably came as an afterthought" (p. 66). Most precisely he puts his "central claim" this way: "with hindsight one can argue that Klein's program is one very special case of the power, richness, and persuasiveness of categorical methods" (p. 3).