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Suppressed black hole production from minimal length

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Abstract

Large extra dimensions lower the Planck scale to values soon accessible. Motivated by string theory, the models of large extra dimensions predict a vast number of new effects in the energy range of the lowered Planck scale, among them the production of TeV-mass black holes. But not only is the Planck scale the energy scale at which effects of modified gravity become important. String theory as well as non-commutative quantum mechanics suggest that the Planck length acts a minimal length in nature, providing a natural ultraviolet cutoff and a limit to the possible resolution of spacetime. The minimal length effects thus become important in the same energy range in which the black holes are expected to form.

In this Letter we examine the influence of the minimal length on the expected production rate of the black holes.

1. Extra dimensions

The study of models with Large eXtra Dimensions (LXDs) has recently received a great deal of attention. These models, which are motivated by string theory [1], provide us with an extension to the Standard Model in which observables can be computed and predictions for tests beyond the Standard Model can be addressed. This in turn might help us to extract knowledge about the underlying theory. The models of LXDs successfully fill the gap between theoretical conclusions and experimental possibilities as the extra hidden dimensions may have radii large enough to make them accessible to experiment. The need to look beyond the Standard Model infected many experimental groups to search for such Standard Model violating processes, for a summary see, e.g., [2].

Arkani-Hamed, Dimopoulos and Dvali [3] proposed a solution to the hierarchy problem by introducing d additional compactified spacelike dimensions in which only the gravitons can propagate. The Standard Model particles are bound to our 4-dimensional submanifold, often called our 3-brane. Due to its higher-dimensional character, the gravitational force at small distances then is strengthened as it goes in the radial distance r with the power $-d-1$ instead of the usual -1. This results in a lowering of the Planck scale to a

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new fundamental scale, $M_{\rm f}$, and gives rise to the exciting possibility of TeV scale GUTs [4]. The radius R of the extra dimensions lies in the range mm to 10^3 fm for d from 2 to 7, or the inverse radius $1/R$ lies in energy range eV to MeV, respectively. Throughout this paper the new fundamental scale is fixed to $M_{\rm f} = 1$ TeV as a representative value.

2. Black holes in extra dimensions

Using the higher-dimensional Schwarzschild metric [5], it can be derived that the horizon radius $R_{\rm H}$ of a black hole is substantially increased in the presence of LXDs, reflecting the fact that gravity at small distances becomes stronger. For a black hole of mass M one finds

$$R_{\rm H}^{d+1} = \frac{1}{\sqrt{\pi}}\frac{8}{d+3}\Gamma\frac{d+3}{2}\frac{1}{M_{\rm f}^{d+1}}\frac{M}{M_{\rm f}}. \tag{1}$$

The horizon radius for a black hole with mass $\approx$ TeV is then $\approx 10^{-3}$ fm, and thus $R_{\rm H} \ll R$ for black holes which can possibly be produced at colliders or in ultra high energetic cosmic rays (UHECRs).

Black holes with masses in the range of the lowered Planck scale should be a subject of quantum gravity. Since there is no theory available yet to perform this analysis, we treat the black holes as semi classical objects.

Consider two partons moving in opposite directions. If the center of mass energy of the partons, $\sqrt{\hat{s}}$, reaches the fundamental scale, $M_{\rm f} \sim 1$ TeV, and if the impact parameter is less than $R_{\rm H}$, a black hole with mass $M \approx \sqrt{\hat{s}}$ can be produced. The total cross section for such a process can be estimated on geometrical grounds [6] and is of order

$$\sigma(M) \approx \pi R_{\rm H}^2 \Theta\left(\sqrt{\hat{s}} - M_{\rm min}\right), \tag{2}$$

where Θ denotes the Heaviside function and it is assumed that black hole formation is only possible above some minimal mass, $M_{\rm min} < \sqrt{\hat{s}}$, which is of order $M_{\rm f}$. The possibility of forming these TeV-scale black holes in the lab, or in UHECRs, respectively, has been examined in a vast number of publications [7–9], for only to mention a few. The status has been nicely summarized in [10].

The expression for the cross section contains only the fundamental Planck scale as a coupling constant. We want to mention that the given classical estimate of the black hole production cross section has been under debate [11], but further investigations by [12,13,34] justify the use of the classical limit. However, the topic is still under discussion [14]. Setting $M_{\rm f} \sim 1$ TeV and $d = 2$ one finds $\sigma \approx 1\ \text{TeV}^{-2} \approx 400$ pb. With this it is further found that these black holes will be produced at LHC in number of $\approx 10^9$ per year [7].

The above cross section can be derived in string theory approximations as well as using the Aichelburg–Sexl metric [13]. In the latter case, the Schwarzschild metric is boosted to form two colliding shockfronts in which trapped surfaces can be calculated, their occurrence depending on the impact parameter. Using this ansatz it is assumed that the shock waves can be boosted to thin fronts, thus neglecting the uncertainty of the quantum particles. This treatment is justified as the particles with energies $\sqrt{\hat{s}} > M_{\rm f}$ have a position uncertainty smaller than their horizon. We will see that this feature is modified under the assumption of a generalized uncertainty arising from the minimal length.

3. Minimal length

Even if a full description of quantum gravity is not yet available, there are some general features that seem to go hand in hand with all promising candidates for such a theory. One of them is the need for a higher-dimensional spacetime, one other the existence of a minimal length scale. As the success of string theory arises from the fact that interactions are spread out on the world-sheet and do no longer take place at one singular point, the finite extension of the string has to become important at small distances or high energies, respectively. Now, that we are discussing the possibility of a lowered fundamental scale, we want to examine the modifications arising from this as they might get observable soon. If we do so, we should clearly take into account the minimal length effects.

In perturbative string theory [15,16], the feature of a fundamental minimal length scale arises from the fact that strings cannot probe distances smaller than the string scale. If the energy of a string reaches this scale $M_s = \sqrt{\alpha'}$, excitations of the string can occur and increase its extension [17]. In particular, an examination of the spacetime picture of high-energy string

scattering shows, that the extension of the string grows proportional to its energy [15] in every order of perturbation theory. Due to this, uncertainty in position measurement can never become arbitrarily small. For a review, see [18,19].

The minimal length scale does not only appear within string theoretical framework but also arises from various approaches, such as non-commutative geometries, quantum loop gravity, non-perturbative implications of T-duality [20] or an very interesting *gedanken experiment* using micro black holes as the limiting Planck scale [21].

Naturally, the minimum length uncertainty is related to a modification of the standard commutation relations between position and momentum [22]. With the Planck scale as high as 10^{16} TeV, applications of this are of high interest mainly for quantum fluctuations in the early universe and for inflation processes and have been examined closely [23].

In [24,25] we used a model for the effects of the minimal length in which the relation between the wave vector k and the momentum p is modified. We assume, no matter how much we increase the momentum p of a particle, we can never decrease its wavelength below some minimal length L_f or, equivalently, we can never increase its wave vector k above $M_\mathrm{f} = 1/L_\mathrm{f}$. Thus, the relation between the momentum p and the wave vector k is no longer linear $p = k$ but a function[1] $k = k(p)$.

For massless particles, $m = 0$, this function $k(p)$ has to fulfill the following properties:

(a) For energies much smaller than the new scale we reproduce the linear relation: for $p \ll M_\mathrm{f}$ we have $p \approx k$.
(b) It is an uneven function (because of parity) and $k \parallel p$.
(c) The function asymptotically approaches the upper bound M_f.

In general, the above properties have to be fulfilled in the limit $m \to 0$. A particle with a rest mass close to the new scale would experience an additional uncertainty even at rest. However, for all particles of the Standard Model it is $m^2/M_\mathrm{f}^2 \ll 1$ and these effects can be neglected.

The quantization in this scenario is straightforward and follows the usual procedure. The commutators between the corresponding operators $\hat{k}$ and $\hat{x}$ remain in the standard form. Using the well known commutation relations

$$[\hat{x}_i, \hat{k}_j] = \mathrm{i}\delta_{ij} \tag{3}$$

and inserting the functional relation between the wave vector and the momentum then yields the modified commutator for the momentum

$$[\hat{x}_i, \hat{p}_j] = +\mathrm{i}\frac{\partial p_i}{\partial k_j}. \tag{4}$$

This results in the generalized uncertainty principle (GUP)

$$\Delta p_i \Delta x_j \geqslant \frac{1}{2}\left|\left\langle \frac{\partial p_i}{\partial k_j}\right\rangle\right|, \tag{5}$$

which reflects the fact that by construction it is not possible anymore to resolve spacetime distances arbitrarily good. Since $k(p)$ gets asymptotically constant, its derivation $\partial k/\partial p$ drops to zero and the uncertainty in Eq. (5) increases for high energies. The behavior of our particles thus agrees with those of the strings found by Gross and Mende as mentioned above.

The arising modifications derived in [24,25] can be summarized in the effective replacement of the usual measure in momentum space by a modified measure which is suppressed at high momentum

$$\frac{\mathrm{d}^3 k}{(2\pi)^3} \to \frac{\mathrm{d}^3 p}{(2\pi)^3}\left|\frac{\partial k_\mu}{\partial p_\nu}\right|, \tag{6}$$

where the absolute value of the partial derivative denotes the Jacobian determinant.

In the following, we will use the specific relation [25] for $p(k)$ by choosing

$$k_\mu(p) = \hat{e}_\mu \int_0^p \mathrm{d}p'\, e^{-\epsilon(p'^2+m^2)}, \tag{7}$$

where $\hat{e}_\mu$ is the unit vector in μ direction, $p^2 = \vec{p}\cdot\vec{p}$, and $\epsilon = L_\mathrm{f}^2\pi/4$. The factor $\pi/4$ is included to assure that for high energies the limiting value is indeed $1/L_\mathrm{f}$. Is easily verified that this expression fulfills the requirements (a)–(c).

[1] Note, that this is similar to introducing an energy dependence of Planck's constant $\hbar$.

The Jacobian determinant of the function $k(p)$ is best computed by adopting spherical coordinates and can be approximated for $p \sim M_{\rm f}$ by

$$\left|\frac{\partial k_\mu}{\partial p_\nu}\right| \approx e^{-\epsilon(p'^2+m^2)}. \tag{8}$$

With this parametrization of the minimal length effects the modifications read

$$\Delta p_i \Delta x_i \geqslant \frac{1}{2} e^{\epsilon(p'^2+m^2)}, \tag{9}$$

$$\frac{\mathrm{d}^3 k}{(2\pi)^3} \to \frac{\mathrm{d}^3 p}{(2\pi)^3} e^{-\epsilon(p'^2+m^2)}. \tag{10}$$

4. Black holes and the minimal length

The properties of Planck size black holes raise a bunch of fundamental questions as they exist in a regime where quantum physics and gravity are of equal importance. Even an examination within a not fully consistent treatment can reveal some of the exciting and new issues on the interplay between quantum physics and gravity. One of the features arising is the evaporation of black holes, which has first been derived in a semi classical treatment by Hawking in 1975 [26] and since that time has been reproduced within various approaches.

In particular, the analysis of the last section raises the question for the final state of the black hole. This topic has been discussed in the literature extensively and is strongly connected to the information loss puzzle. The black hole emits thermal radiation whose sole property is its temperature whatever the initial state of the collapsing matter has been. So if the black hole first captures all information behind its horizon and then completely vanishes into thermally distributed particles the basic principle of unitarity can be violated. This happens when the initial state was a pure quantum state and then evolves into a mixed one [27].

When we try to escape the information loss problem we have two possibilities left: the information is released back by some unknown mechanism or a stable black hole remnant is left which keeps the information. Besides the fact that it is unclear in which way the information should escape the horizon [28] there are several more arguments for the black hole relics [29].

The most obvious one is the uncertainty relation. The Schwarzschild radius of a black hole with Planck mass is of the order of the Planck length. Since the Planck length is the wavelength corresponding to a particle of Planck mass we see that we get in trouble when the mass of the black hole drops below the Planck mass. Then we have a mass inside a volume which is smaller than the uncertainty principle allows [30]. For this reason is was proposed by Zel'dovich that black holes with masses below Planck mass should be associated with stable elementary particles [31]. The question for black holes with regard to the minimal length was also raises by Gross and Mende [15]. They found by an investigation of the spacetime picture for such string scattering that, with an increasing number of the order in perturbation theory, the size of the string decreases relative to the Schwarzschild radius of the collision region. The production of black holes thus does not become impossible but increasingly difficult within the minimal length approach.

5. Black holes and the minimal length in extra dimensions

It has been examined which modifications from the GUP arise for the Hawking spectrum of the black hole and it has been shown by Cavaglià, Das and Maartens [32] that the black hole is hotter and decays faster into a smaller number of high energetic particles, finally leaving a stable relic. These results agree with our analysis of the Hawking spectrum using a geometrical quantization approach [33].

In the following we will examine the production rates for those black holes under the assumption of an minimal length.

For this purpose, consider again two partons with a center of mass energy $\sqrt{\hat{s}}$ approaching head on in a collision. Now, their modified uncertainty principle will smear out their focussing at energies $\sqrt{\hat{s}} > M_{\rm f}$. This will lead to an effective suppression of the black hole formation since the probability of the partons to get trapped inside the horizon is diminished.

Using the GUP formalism, we can derive this modification. The cross section Eq. (2) assumes that the black hole captures the total energy of the collision and thus, the mass of the created black hole is highly peaked around $M = \sqrt{\hat{s}}$. Due to the high rest mass of the black hole, its remaining momentum is negligi-

ble. However, the precise mass of the black hole might be smeared out by a form factor of order one due to energy losses during the formation and modifications of the horizon radius by a non-zero angular momentum [35].

We will neglect this form factor and further assume the distribution

$$\mathrm{d}\sigma = \sigma\big(\sqrt{\hat{s}}\big)\delta\big(M - \sqrt{\hat{s}}\big)\,\mathrm{d}^3p \tag{11}$$

which is easily translated into the minimal length scenario by using Eq. (10)

$$\mathrm{d}\tilde{\sigma} = \sigma\big(\sqrt{\hat{s}}\big)\delta\big(M - \sqrt{\hat{s}}\big)e^{-\epsilon\hat{s}}\,\mathrm{d}^3p. \tag{12}$$

This can also be understood by considering the above mentioned picture of the colliding partons. Caused by the impossibility to focus the particles, we would expect the damping to be approximately $R_{\mathrm{H}}/\Delta x$. With $1/R_{\mathrm{H}} \approx \Delta p$ and Eq. (9) this yields an exponential suppression factor $\exp(-\epsilon M^2)$ for the cross section. Thus, agreeing with the result found earlier.

The only colliders which can reach energies above the TeV-scale and therefore potentially produce the discussed black holes are hadron colliders. To obtain the cross section for proton–proton (pp) collisions the partonic cross section Eq. (12) must be integrated over a folding with the parton distribution functions (PDFs) $f_i(x, Q^2)$. Here, the index i labels the constituent partons of the hadron and $s = \hat{s}/xy$ is the center of mass energy of the pp-collision.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}M} = \sum_{i,j}\int_0^1 \mathrm{d}x\,\frac{2\sqrt{\hat{s}}}{xs}\,f_i(x,\hat{s})\,f_j(y,\hat{s})\sigma(\hat{s})e^{-\epsilon\hat{s}}. \tag{13}$$

By definition, the PDFs parametrize the probability of finding a parton i with momentum fraction x of the hadrons momentum at a given inverse length scale Q associated with the scattering process. Usually, this scale is chosen to be the momentum transfer, that is in the s-channel $Q^2 \sim s$. Here, investigating the production of black holes, the length scale of the scattering process is limited by the Schwarzschild radius and the generalized position uncertainty,[2] we thus have $1/Q \sim R_{\mathrm{H}}$.

Further modifications for the PDFs in the GUP scenario, in addition to the modified scaling in Q, are not to be expected. To see this, one has to keep in mind the way in which the experimental data is extracted and further used for the common PDFs, such as the CTEQ4-Tabulars [37].

The non-perturbative physics of an hadron–hadron scattering process can be characterized by functions of x alone at a fixed small Q_0 at which the minimal length effects are negligible. This measured experimental input at small Q_0 is then extrapolated to high Q using the DGLAP[3] equations [36]. The scale dependence goes with $\ln(Q^2/Q_0^2)$, signaling that Q-independent Bjorken scaling is violated by QCD-effects at high Q^2.

An exact analytical examination of the DGLAP is beyond the scope of this Letter. However, since the minimal length disables a further resolution of the hadron structure with increasing energy, the effects can effectively be captured in the above assumed Q-definition: above the new fundamental scale, the structure of the hadron is cloaked behind the generalized uncertainty (and it is left to the realm of philosophy to decide whether it would exist at all in that case).

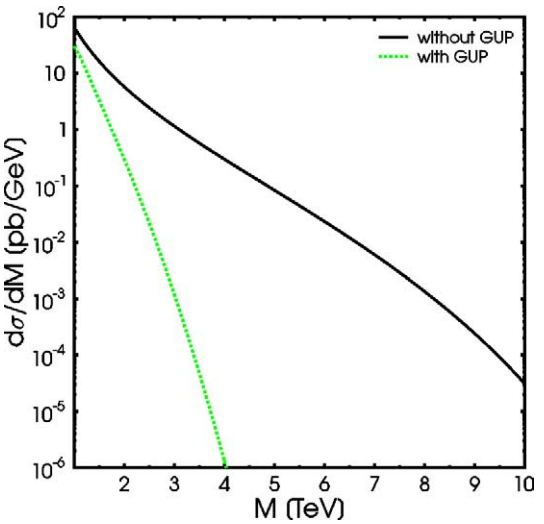

Fig. 1. Differential cross section for black hole production with minimal length, $1/L_{\mathrm{f}} = 1$ TeV, for LHC energy $\sqrt{s} = 14$ TeV. The differential cross section depends on d only by an factor of order 1, here $d = 4$.

[2] It turns out numerically that the results do not depend on this distinction.

[3] Dokshitzer–Gribov–Lipatov–Altarelli–Parisi.

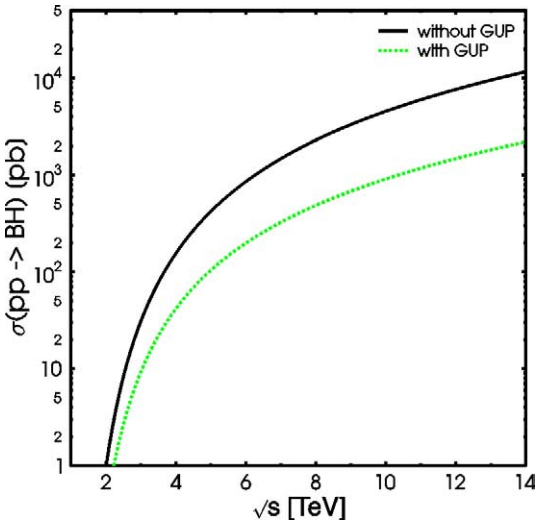

Fig. 2. The total cross section for black hole production with minimal length $1/L_f = 1$ TeV as a function of the center of mass energy $\sqrt{s}$ with $M_{min} = M_f$. The ratio of the total cross section with and without GUP for the expected LHC-energy $\sqrt{s} = 14$ TeV is ≈ 0.19.

The results for the above derived differential cross section and the integrated total cross-section with use of the CTEQ4-Tabulars [37] are shown in Figs. 1 and 2. The calculations for the differential cross section is done for the expected LHC-energies $\sqrt{s} = 14$ TeV.

It can be seen that the effect on the production of black holes is noticeable but does not exceed one order of magnitude and thus stays in the range in which several other uncertainties might come into play (such as M_{min}, form factors, energy losses during collapse, numerical factors from the analysis of trapped surfaces, d, angular momentum, etc.).

6. Conclusion

In this Letter the influence of a minimal length scale on the production of black holes in a model with large extra dimensions was examined. It was found that the finite resolution of spacetime which is caused by the minimal length results in an exponential suppression of the black hole cross-section. Calculation of the total cross-section for LHC-energies in this scenario shows a decrease of the expected number of black holes by a factor ≈ 5.

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References

[1] I. Antoniadis, Phys. Lett. B 246 (1990) 377;
I. Antoniadis, M. Quiros, Phys. Lett. B 392 (1997) 61;
K.R. Dienes, E. Dudas, T. Gherghetta, Nucl. Phys. B 537 (1999) 47.
[2] G. Azuelos, et al., hep-ph/0204031.
[3] N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, Phys. Lett. B 429 (1998) 263;
I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, Phys. Lett. B 436 (1998) 257;
N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, Phys. Rev. D 59 (1999) 086004.
[4] K.R. Dienes, E. Dudas, T. Gherghetta, Phys. Lett. B 436 (1998) 55.
[5] R.C. Myers, M.J. Perry, Ann. Phys. 172 (1986) 304.
[6] T. Banks, W. Fischler, hep-th/9906038.
[7] S. Dimopoulos, G. Landsberg, Phys. Rev. Lett. 87 (2001) 161602.
[8] P.C. Argyres, S. Dimopoulos, J. March-Russell, Phys. Lett. B 441 (1998) 96.
[9] I. Mocioiu, Y. Nara, I. Sarcevic, Phys. Lett. B 557 (2003) 87;
A.V. Kotwal, C. Hays, Phys. Rev. D 66 (2002) 116005;
Y. Uehara, Prog. Theor. Phys. 107 (2002) 621;
R. Emparan, M. Masip, R. Rattazzi, Phys. Rev. D 65 (2002) 064023;
S. Hossenfelder, S. Hofmann, M. Bleicher, H. Stocker, Phys. Rev. D 66 (2002) 101502;
A. Ringwald, H. Tu, Phys. Lett. B 525 (2002) 135;
D. Kazanas, A. Nicolaidis, Gen. Relativ. Gravit. 35 (2003) 1117.
[10] P. Kanti, hep-ph/0402168;
G. Landsberg, hep-ph/0211043;
M. Cavaglia, Int. J. Mod. Phys. A 18 (2003) 1843.
[11] M.B. Voloshin, Phys. Lett. B 518 (2001) 137;
M.B. Voloshin, Phys. Lett. B 524 (2002) 376;
S.B. Giddings, in: N. Graf (Ed.), Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics, Snowmass, CO, 2001, eConf C010630 (2001) P328.
[12] S.N. Solodukhin, Phys. Lett. B 533 (2002) 153;
A. Jevicki, J. Thaler, in: N. Graf (Ed.), Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics, Snowmass, CO, 2001, eConf C010630 (2001) P339.
[13] D.M. Eardley, S.B. Giddings, Phys. Rev. D 66 (2002) 044011.
[14] V.S. Rychkov, hep-ph/0401116.
[15] D.J. Gross, P.F. Mende, Nucl. Phys. B 303 (1988) 407.
[16] D. Amati, M. Ciafaloni, G. Veneziano, Phys. Lett. B 216 (1989) 41.
[17] E. Witten, Phys. Today 50 (5) (1997) 28.

[18] L.J. Garay, Int. J. Mod. Phys. A 10 (1995) 145.
[19] A. Kempf, hep-th/9810215.
[20] A. Smailagic, E. Spallucci, T. Padmanabhan, hep-th/0308122.
[21] F. Scardigli, Phys. Lett. B 452 (1999) 39.
[22] A. Kempf, G. Mangano, R.B. Mann, Phys. Rev. D 52 (1995) 1108;
A. Kempf, G. Mangano, Phys. Rev. D 55 (1997) 7909.
[23] U.H. Danielsson, Phys. Rev. D 66 (2002) 023511;
S. Shankaranarayanan, Class. Quantum Grav. 20 (2003) 75;
L. Mersini, M. Bastero-Gil, P. Kanti, Phys. Rev. D 64 (2001) 043508;
A. Kempf, Phys. Rev. D 63 (2001) 083514;
A. Kempf, J.C. Niemeyer, Phys. Rev. D 64 (2001) 103501;
J. Martin, R.H. Brandenberger, Phys. Rev. D 63 (2001) 123501;
R. Easther, B.R. Greene, W.H. Kinney, G. Shiu, Phys. Rev. D 67 (2003) 063508;
R.H. Brandenberger, J. Martin, Mod. Phys. Lett. A 16 (2001) 999;
S.F. Hassan, M.S. Sloth, Nucl. Phys. B 674 (2003) 434.
[24] S. Hossenfelder, M. Bleicher, S. Hofmann, J. Ruppert, S. Scherer, H. Stöcker, Phys. Lett. B 575 (2003) 85.
[25] S. Hossenfelder, Running coupling with minimal length, in preparation.
[26] S.W. Hawking, Commun. Math. Phys. 43 (1975) 199.
[27] S. Hawking, Commun. Math. Phys. 87 (1982) 395;
J. Preskill, hep-th/9209058;
I.D. Novikov, V.P. Frolov, Black Hole Physics, Kluwer Academic, 1998.
[28] D.N. Page, Phys. Rev. Lett. 44 (1980) 301;
G. 't Hooft, Nucl. Phys. B 256 (1985) 727;
A. Mikovic, Phys. Lett. B 304 (1992) 70;
E. Verlinde, H. Verlinde, Nucl. Phys. B 406 (1993) 43;
L. Susskind, L. Thorlacius, J. Uglum, Phys. Rev. D 48 (1993) 3743;
D.N. Page, Phys. Rev. Lett. 71 (1993) 3743.
[29] Y. Aharonov, A. Casher, S. Nussinov, Phys. Lett. B 191 (1987) 51;
T. Banks, A. Dabholkar, M.R. Douglas, M. O'Loughlin, Phys. Rev. D 45 (1992) 3607;
T. Banks, M. O'Loughlin, Phys. Rev. D 47 (1993) 540;
T. Banks, M. O'Loughlin, A. Strominger, Phys. Rev. D 47 (1993) 4476.
[30] M.A. Markov, in: M.A. Markov, P.C. West (Eds.), Proc. 2nd Seminar in Quantum Gravity, Plenum, New York, 1984.
[31] Y.B. Zel'dovich, in: M.A. Markov, P.C. West (Eds.), Proc. 2nd Seminar in Quantum Gravity, Plenum, New York, 1984.
[32] M. Cavaglia, S. Das, R. Maartens, Class. Quantum Grav. 20 (2003) L205;
M. Cavaglia, S. Das, hep-th/0404050.
[33] S. Hossenfelder, M. Bleicher, S. Hofmann, H. Stocker, A.V. Kotwal, Phys. Lett. B 566 (2003) 233.
[34] T. Banks, W. Fischler, hep-th/9906038.
[35] S.B. Giddings, S. Thomas, Phys. Rev. D 65 (2002) 056010.
[36] G. Altarelli, G. Parisi, Nucl. Phys. B 126 (1977) 298;
Y.L. Dokshitzer, Sov. Phys. JETP 46 (1977) 641;
L.N. Lipatov, Sov. J. Nucl. Phys. 20 (1975) 94;
V.N. Gribov, L.N. Lipatov, Yad. Fiz. 15 (1972) 1218.
[37] H.L. Lai, et al., Phys. Rev. D 55 (1997) 1280.